Gravity: Newtonian, Post-Newtonian, Relativistic. Errors, typographical and otherwise

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The following errors were reported by our faithful readers. They have our boundless gratitude. Emanuele Berti reported a large number of typos before the book was published; the list would be much longer without his invaluable help.

Chapter 1

1. Paragraph below Eq. (1.2), page 3. The 2014 CODATA recommended value for the gravitational constant is $G = 6.67408(31) \times 10^{-11}$ $\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$. The value we quote is slightly different, but the most important mistake is that we got the unit wrong. Reported by Emanuele Berti.

2. Line below Eq. (1.226), page 57. The equation is derived with the help of Eq. (1.160a), not Eq. (1.159a). Reported by Emanuele Berti.

3. Exercise 1.3, page 61. Each term on the right-hand side of the equation should come with a factor of G.

4. Exercise 1.6, page 61. The left-hand side of the equation should read $\frac{1}{2}d^2I^{jk}/dt^2$.

5. Exercise 1.14, page 62. The left-hand side of the displayed equation should read $e^{\langle pL\rangle}n^{\langle qL\rangle}$ instead of $e^{\langle qL\rangle}n^{\langle pL\rangle}$.

Chapter 2

1. Equation (2.66), page 78. The equation for R is correct, but the equation for M should read $M = \sqrt{2\pi K^3/G^3}\rho_c$. Reported by Jacob Stanton.

2. Figure 2.4, page 87. The scale of the vertical axis is actually 10^6 m; meters, not kilometers. Reported by Nico Yunes.

3. Equation (2.166), page 99. The left-hand side of the equation should read $p/\pi G\rho^2$. Reported by Junjie Zhao.

4. Figure 2.5, page 103. In the Jacobi sequence defined by the dashed curve, the eccentricity is defined by $e := \sqrt{1-(a_3/a_1)^2}$. Reported by Junjie Zhao.

5. Box 2.3, page 113. In the first displayed equation, the first term in the expression for U should be $+GM/r$ instead of $-GM/r$.

6. Equation (2.250), page 118. In the rightmost expression, the number within the square root should be $4\pi/5$ instead of $4\pi/3$. Equation (2.251) is correct. Reported by Nico Yunes.

7. Equation (2.283), page 126. The equation should read $dS/dt = (2/3)k_2\tau R^5\epsilon_{jkn}e^j\mathcal{E}^k_p\dot{\mathcal{E}}^{pn}$; the factor of e^j is missing on the right-hand side. Reported by Gerui Chen.

Chapter 3

1. Equation (3.17), page 145. The quantity $u := 1/r$ introduced a few lines previously should not be confused with the eccentric anomaly u introduced in Eq. (3.30). Reported by Nico Yunes.

2. Equation (3.85c), page 165. The expression for $\langle\Delta\omega\rangle$ presents us with a paradox, because it doesn't reduce to Eq. (3.83) when $\iota = 0$, that is, when the perturbing body moves in the same plane as the binary system. The coplanar case is treated in Sec. 3.4.1, where we point out that since the line of nodes is not defined in this case, Ω is redundant and can be set equal to zero. With this convention, ω is the angle between the pericenter and the fixed X-direction. Our convention is different in Sec. 3.4.2, since ω now refers to the line of nodes, which is itself moving with respect to the fixed X-direction. And as we point out in Sec. 3.3.2 below Eq. (3.67), the motion of the pericenter relative to the fixed X-direction is properly captured by $d\omega + \cos\iota\, d\Omega$. The quantity, therefore, that should be compared with the $\langle\Delta\omega\rangle$ of Eq. (3.83) is $\langle\Delta\omega\rangle + \cos\iota\langle\Delta\Omega\rangle$ in the limit $\iota \to 0$, with $\langle\Delta\omega\rangle$ now standing for the expression of Eq. (3.85c). An expression for $\langle\Delta\Omega\rangle$ is not provided in Sec. 3.4.2, but a simple computation returns

$$\langle\Delta\Omega\rangle = -\frac{3\pi}{2}\frac{m_3}{m}\left(\frac{a}{R}\right)^3(1-e^2)^{-1/2}\cos\iota\left(1+4e^2-5e^2\cos^2\omega\right).$$

With this we get

$$\langle\Delta\omega\rangle + \cos\iota\langle\Delta\Omega\rangle = \frac{3\pi}{2}\frac{m_3}{m}\left(\frac{a}{R}\right)^3(1-e^2)^{1/2}\left[(4-5\cos^2\omega)\cos^2\iota+5\cos^2\omega-3\right],$$

and we see that this does indeed reduce to the $\langle\Delta\omega\rangle$ of Eq. (3.83) when $\iota = 0$. The paradox was reported by Katerina Chatziioannou and Nico Yunes, who helped us resolve it.

3. Box 3.4, page 171. Our discussion of DI Herculis is out of date. A plausible explanation for the discrepancy between the observed and calculated apsidal advance was proposed by S. Albrecht, S. Reffert, I. A. G. Snellen, and J. N. Winn, *Misaligned spin and orbital axes cause the anomalous precession of DI Herculis*, Nature **461**, 373–376 (2009). We thank Scott Hughes for pointing out this reference.

4. Exercise 3.3, page 185. The modified Poisson equation should read $(\nabla^2 - \lambda^{-2})U = -4\pi G\rho$. Reported by Nicholas Loutrel.

5. Exercise 3.4 (c), page 185. The expression for C should be $C = h^2 - J_2R^2[\cdots]$; there is a relative minus sign between the terms.

6. Exercise 3.9 (c), page 187. First, the label should be (c) instead of (b), but you knew that, didn't you? Second, the length of the day should come out to 47 days instead of 48.

Chapter 4

1. Exercise 4.4, page 215. The correct factor on the right-hand side of the displayed equation for V^α_{obs} is c^2, not c. Reported by Eric Zheng Lian.

Chapter 5

1. Last displayed equation in Box 5.7. The equation is actually correct, but the derivation leading to it is wrong. The expression for $\tilde{C}_{\mathrm{T}}^{jk}$ provided in the preceding text should also contain a term $k^j k^k \tilde{D}$, to ensure that the tensor is properly transverse. The tracefree condition on $\tilde{C}^{jk}$ then relates the scalar $\tilde{D}$ to $\tilde{C}$. The displayed equation follows after a rescaling of $\tilde{C}$. All this is a bit cumbersome. It is easier to start with the decomposition

$$\tilde{C}^{jk} = \delta^{jk}\tilde{D} - k^j k^k \tilde{C} + ik^j \tilde{C}_{\mathrm{T}}^k + ik^k \tilde{C}_{\mathrm{T}}^j + \tilde{C}_{\mathrm{TT}}^{jk},$$

in which the last tensor is now both transverse and tracefree. The tracefree condition on $\tilde{C}^{jk}$ then yields $\tilde{D} = \frac{1}{3}k^2\tilde{C}$, and the last displayed equation follows immediately. Reported by Tan Liu.

Chapter 6

1. Equation (6.39a), page 299. A factor of $(-g)$ is missing inside the integral. Reported by Gerui Chen.

2. Equation (6.109), page 324. The factor cr in front of $f^{(n-1)}(\tau)$ should be r/c. Reported by Gerui Chen.

3. Exercise 6.4, page 326. The expression for J^{jk} is incorrect. It should be replaced with

$$J^{jk} = -\frac{c^3}{16\pi G}\oint_\infty r^4 \frac{\partial}{\partial r}\left(\frac{x^j h^{0k} - x^k h^{0j}}{r^2}\right)d\Omega.$$

Chapter 7

1. Equation (7.24c), page 338. The Kronecker delta was typeset incorrectly: it should be δ_{jk} instead of δ_{j_k}. Reported by Gerui Chen.

2. Page 339, at the end of the paragraph before Eq. (7.31), the factor should be $c/2$ instead of $1/2$, so that the expresion reads $(c/2)(\dot{\mathcal{I}}^{jk} - \epsilon^{mjk}J_0^m)$. Reported by Zhuhai Li.

3. Box 7.2, page 340. The equation for $\mathcal{M}^{0jL}$ should read

$$\mathcal{M}^{0jL} = \frac{c}{\ell+1}\left(\dot{\mathcal{I}}^{jL} - \ell\epsilon^{mja_1}\mathcal{J}^{ma_2\cdots a_\ell}\right)(\text{sym } a : L) + \frac{1}{\ell+1}\oint_{\partial\mathcal{M}}\tau^{0m}x^{jL}\,dS_m.$$

Notice the now-absent factor of $1/2$ in the first term.

4. Equation (7.52c), page 346 and Eq. (7.53), page 347. The scaling of $(16\pi G/c^4)(-g)t_{\mathrm{H}}^{jk}$ is c^{-8} instead of c^{-6}. This can be seen by inserting $h^{00}\propto c^{-2}$, $h^{0j}\propto c^{-3}$, and $h^{jk}\propto c^{-4}$ in the expression for $(-g)t_{\mathrm{H}}^{jk}$, and noticing that the ∂_0 operator comes with an additional factor of c^{-1}. The complete expression for $(-g)t_{\mathrm{H}}^{jk}$ contains more terms than what is shown in Eq. (7.53); the additional terms, however, are not required in any subsequent calculation. The error term in Eq. (7.53) should be of order c^{-10} instead of c^{-8}. Reported by Alain Dirkes and Vitash Kotteeswaran.

5. Equation (7.107b), page 362. The third term in $h_{\mathscr{N}}^{0j}$ should read

$$+\frac{2G}{3c^2}\partial_{kn}\left[\frac{\dot{\mathcal{I}}^{jkn}(\tau) - 2\epsilon^{mjk}\mathcal{J}^{mn}(\tau)}{r}\right].$$

Notice the wrong sign and the factor of 2. This error propagates to the next item.

6. Equation (7.109), page 363. The symbol on the left-hand side should be h^{00}, not $h_{\mathcal{W}}^{00}$. The same remark applies to the line above the equation. Reported by Vitash Kotteeswaran.

7. Box 7.7, page 364. In the second equation, the first occurrence of h^{0j}, the third term within the square brackets should read

$$+ \frac{1}{6} \partial_{kn} \left(\frac{\dot{\mathcal{I}}^{jkn} - 2\epsilon^{mjk} \mathcal{J}^{mn}}{r} \right).$$

And in the second-to-last equation, the second occurrence of h^{0j}, the second term within the square brackets should read

$$+ \frac{1}{6c^2} \left(\dddot{\mathcal{I}}^{jkn} - 2\epsilon^{mjk} \ddot{\mathcal{J}}^{mn} \right) n_k n_n.$$

Here the sign was correct, but we still had a wrong numerical factor and an incorrect factor of c. Reported by Béatrice Bonga.

Chapter 8

1. Equation (1), Box 8.1, page 377. The term $v^2 \boldsymbol{\nabla} g_s$ within brackets should read $\frac{1}{2} v^2 \boldsymbol{\nabla} g_s$.

2. Equation (8.70), page 392. The linear term in the expansion of $\bar{U}_{\text{ext}}$ should read $+\bar{x}^j [\partial_j U_{\text{ext}}(\bar{t}, r^j) - a_j]$. Reported by Gerui Chen.

3. Exercise 8.2, page 411. The term $v^2 \boldsymbol{\nabla} g_s$ within brackets should read $\frac{1}{2} v^2 \boldsymbol{\nabla} g_s$.

4. Exercise 8.3, page 411. The expression for $\bar{g}_{00}$ should read

$$\bar{g}_{00} = g_{00} - \frac{2\lambda}{c^4} \left(\bar{U}^2 + \bar{\Phi}_2 + \bar{\Phi}_W \right).$$

5. Exercise 8.8, page 413. The bracketed term on the second line should read $4U^{k]} + \frac{1}{2} \partial_t^{k]} X$.

Chapter 9

1. Equation (9.54), page 425. The second term in the sum misses a factor of G; it should read $\frac{1}{2} G I_A^{jk} \partial_{jk} s_A^{-1}$. Reported by Gerui Chen.

Chapter 10

1. On the second line of page 492, the averaged rate of advance of the line of nodes of the lunar orbit should be 1.91 arcseconds per century, not 19.1.

Chapter 11

1. Equation (11.21), page 544. In these equations, and in the line below Eq. (11.22), the gauge vector should be denoted ζ^α instead of ξ^α. Reported by Gerui Chen.

2. Equation (11.98), page 562. The numerical factor should be 6.8×10^{-26}. Reported by Rui Xu.

3. Equation (11.137), page 571. The domain of integration should be $\partial \mathcal{M}_y$ instead of $\mathcal{M}_y$. Reported by Alain Dirkes.

4. Equation (11.167), page 577. The error term should read $O(r^{-4})$. Reported by Gerui Chen.

5. Equation (11.168), page 577. The error term should read $O(r^{-5})$. Reported by Gerui Chen.

6. Equation (11.192), page 582. The equation should read

$$F_A^{mnab} = \frac{1}{4\pi} \int_{\mathscr{M}} \frac{y^m y^n y^a y^b}{y^6} \, d^3y + \frac{r_A^a}{4\pi} \int_{\mathscr{M}} \frac{y^m y^n y^b}{y^6} \, d^3y$$
$$+ \frac{r_A^b}{4\pi} \int_{\mathscr{M}} \frac{y^m y^n y^a}{y^6} \, d^3y + + \frac{r_A^a r_A^b}{4\pi} \int_{\mathscr{M}} \frac{y^m y^n}{y^6} \, d^3y.$$

Reported by Gerui Chen.

7. Equation (11.203), page 584. In the last line, the first term should read $\frac{1}{6} r_{AB}^{-1} n_{AB}^{\langle mn \rangle} r_B^a r_B^b$. Reported by Gerui Chen.

8. Equation (11.224), page 589. The left-hand side should read $Q^{jkab}[F, \mathscr{M}]$. Reported by Gerui Chen.

9. Line before Eq. (11.241), page 593. The definition of the angular-momentum tensor is given with a wrong sign. It should read $\mathcal{J}^{jk} = \sum_A M_A (r_A^j v_A^k - v_A^j r_A^k) + O(c^{-2})$. Reported by Gerui Chen.

10. Equation (11.246c), page 595. The right-hand side misses a term. The equation should read

$$\partial^j h^p{}_p = \frac{4G}{c^2} \left[-\frac{1}{2c^2} \left(\frac{\dddot{I}}{R^2} + \frac{\ddddot{I}}{cR} \right) N^j + \cdots \right].$$

The missing term correctly appears in Eq. (11.247) and all relevant equations below. Reported by Gerui Chen.

11. Equation (11.250), page 596. In the last term of the third line, the factor of r^{-1} should be replaced by R^{-1}. Reported by Gerui Chen.

Chapter 12

1. Equation (12.46), page 647. The factor of c^3 on the right-hand side should be c^2. Reported by Nico Yunes.

2. Equation (12.56), page 640. In the first line of the equation, the first term on the right-hand side should contain a factor of c^{-1}. We can perhaps be forgiven for this one, because this term vanishes anyway. Reported by Emanuele Berti.

3. Equation (12.116), page 662. The last member of the set should be $\partial_p h^{00} \partial_0 h^{jk}$. Reported by Gerui Chen.

4. Page 686. We misspelled the name of Steven Orszag. It should also be corrected in the references and the index.

Chapter 13

1. Equation (13.38a), page 719. On the right-hand side of the equation, the final term should read $-\alpha_3 n_{AB}^{\langle jl \rangle} (w + v_B)^k \hat{S}_B^{lk}$ instead of $-\alpha_3 n_{AB}^{\langle jl \rangle} (w + v_B)^p \hat{S}_B^{lk}$; the index on $(w + v_B)$ should be k instead of p.

2. Equation (13.63), page 726. The Δ that appears in this equation should be defined by $\Delta := (M_2 - M_1)/(M_1 + M_2)$ instead of the relation provided below the equation. This new definition does not agree with our previous usage. The redefinition is localized to Sec. 13.3.3 only, and it impacts Eqs. (13.63), (13.66), (13.71), and (13.72).

3. Equation (13.72), page 728. Our expression for $\delta r(t)$ is not complete. It omits a number of terms that are either constant or proportional to $\cos(\omega_\oplus t \pm n\Phi)$, where $n = \{0, 1, 2, 3\}$. Collectively, these terms represent an eccentricity perturbation superposed to our initially circular ($A = B = 0$) orbit.

4. Equation (13.103), page 740. The power on the prefactor (ϕ/ϕ_0) should be 1 not 2. Reported by Christopher Devitt.

5. Equations (13.132)–(13.134), page 745. The pseudotensors $t_\phi^{\alpha\beta}$, $t_{\mathrm{LL}}^{\alpha\beta}$, and $t_{\mathrm{H}}^{\alpha\beta}$ should all be adorned with tildes: $\tilde{t}_\phi^{\alpha\beta}$, $\tilde{t}_{\mathrm{LL}}^{\alpha\beta}$, and $\tilde{t}_{\mathrm{H}}^{\alpha\beta}$. Reported by Nico Yunes.

6. Equations (13.156) and (13.157), page 751. In Eq. (13.156), the term proportional to c^{-1} should come with a minus sign. In Eq. (13.157), the right-hand sides should include a factor of c^{-2}; for example, $\mathcal{I}_\mathrm{s}(\tau) := c^{-2} \int_{\mathscr{M}} \tau_\mathrm{s}(\tau, \boldsymbol{x}) \, d^3x$. Reported by Mathias Pavely Nødtvedt.