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Portfolio perspectives

Diversification still works: Exploring the equity-bond relationship with K-means clustering

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- Some investors have become concerned about the effect of a low-interest-rate environment on the role of bonds in a multi-asset portfolio.
- By applying a machine learning technique called "K-means clustering" to periods when rates were low, we show that government bonds have historically acted as intended in an equity-bond portfolio, performing positively when equities have fallen.
- Although in some periods both equities and bonds fell, this can be considered part of market volatility and distinctly different from the typical outcomes that can be regarded as recurrent market states.

Introduction

The main concept underpinning the equity-bond portfolio is the idea of diversification: While equities typically offer higher returns over time, bonds' investment properties make them an excellent tool for reducing risk. However, in recent times, investors have become more concerned about the effect of a low-interest-rate environment on the role of bonds in a portfolio. Since increases in interest rates result in bond price declines and rates are at historic lows, the worry is that bond investments can only fall in value. If that is the case, then why hold fixed income in the first place?

Although this is a reasonable question, it overlooks one of the key properties of bonds: They act as a hedge against equity market drops. These scenarios can include a "flight to safety," where investors react by allocating more toward safer investments such as government bonds, leading to a rise in their price. Moreover, equity market crashes themselves are often due to negative economic shocks (such as COVID-19). Central banks typically respond by cutting rates and conducting quantitative easing, which usually results in positive fixed income performance. For these reasons, investment principles would suggest that bonds still have a role to play in a portfolio.

We have demonstrated the equity downside risk-hedging properties of bonds—particularly government bonds—in a previous study.[1] This research note expands on the topic and provides an additional perspective by applying an unsupervised machine learning technique called "K-means clustering" to historical returns during periods of low government bond yields. This allows us to identify the market states that govern the equity-bond relationship during these periods and suggests a powerful and enduring role for bonds as a hedge against equity risk even when yields are low.

1 See Renzi-Ricci and Baynes (2021).

Equity-bond relationship: The past two decades

Compared to corporate bonds, government bonds tend to have a lower correlation to equities. Therefore, in this analysis we focus on the relationship between equity and government bonds. In particular, we look at the monthly performance of equities and government bonds between October 2000 and March 2021 for U.S. and global indexes.

We also specify that the government bond yield for these returns must be below a certain threshold, in order to isolate the relationships that have held historically when interest rates are low. As we will see, in some cases over the period both equities and bonds have positive returns (the pairwise returns are in an "up-up" state), and in others, equities have positive returns but bonds have negative ("up-down"), or pairs of returns are "down-up" or "down-down." From an investor's perspective, an ideal world would be one in which returns are only in the up-up state. However, this is rarely the case, and what we are really concerned with here is how bonds perform when equities have negative performance.

One approach to assess this could be to simply count the observations falling into each of the four different quadrants to determine how frequently each market state happens. For example, in our data set, when U.S. equities have negative performance, U.S. government bonds have positive performance roughly 63% of the time. However, this tells us nothing about magnitude; perhaps bonds have small positive performance most of the time but on some occasions (the other 37% of the time) both equities and bonds drop significantly. If this were the case, it would be unreasonable to say that bonds are good hedges against poor equity performance; the last thing investors want is for both of their asset classes to lose money in times of turmoil.

An alternative approach might be to perform a linear regression and determine a "line of best fit," giving us a sense of bonds' average response to a given change in equities. However, as we will show later, a linear regression would be unable to capture even significant dispersion or noise. Moreover, a regression approach would provide no sense as to whether certain scenarios (such as when both bonds and equities have negative returns) can be considered as actual market states likely to recur in the future or are simply the result of market volatility and noise.

We require a methodology that addresses both of these issues. K-means clustering allows us to identify clusters of pairwise returns, indicating which scenarios can be considered market states (and which cannot) and how bonds perform when equities fall. This approach lets us shed more light on the equity-bond relationship in a way that other techniques cannot and can be used as an additional tool for investors.

K-means clustering: U.S. returns

K-means clustering is one of the most widely used types of unsupervised machine learning. K represents a given number of clusters decided by the user. Observations are categorized such that those that fall into one cluster are the most similar to each other and as dissimilar as possible from the observations in other clusters.[2]

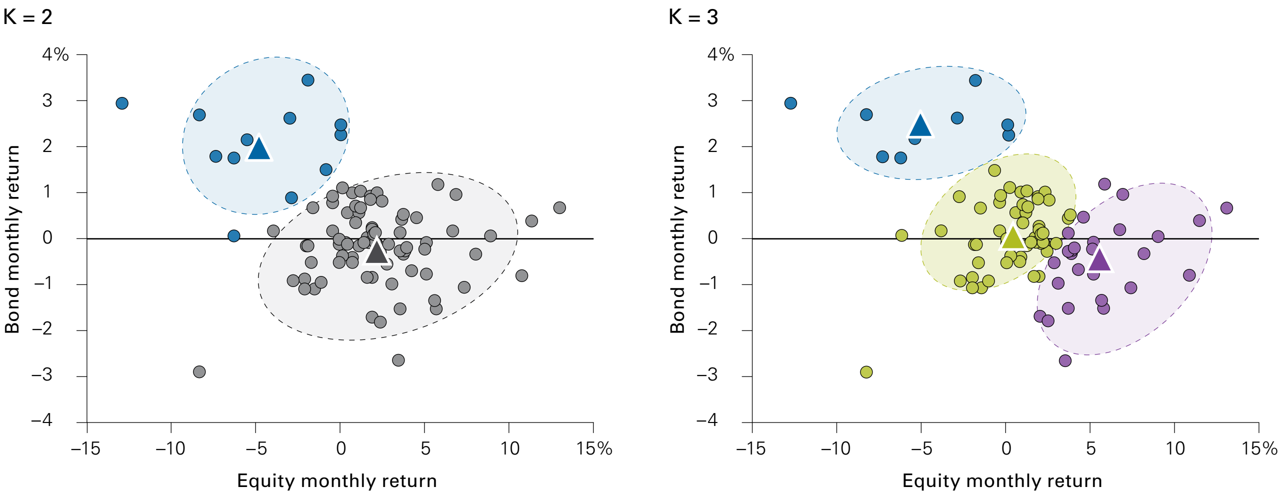
In order to hone in on a low-rate environment, we only consider returns for periods when the government bond yield is below a certain threshold.[3] Our yield threshold for the U.S. is 2.5% (the U.S. 10-year Treasury yield at the beginning of the month must be below 2.5% in order for the return pair to feature in our analysis). This threshold was chosen as being a good balance between ensuring the analysis applies to a low-rate environment and including a reasonably large number of return pairs. **Figure 1** shows the results of the K-means clustering algorithm run on these U.S. returns when K = 2 and 3.[4]

2 Goodfellow et al. (2016) define unsupervised learning as any technique that determines information from a "distribution that (does) not require human labor to annotate examples." By applying unsupervised learning to our return pairs, we do not make any claims as to what type of returns they are. We simply run them through an algorithm that tells us the type based on their similarities.

3 We also conducted this analysis for the entire period (without the threshold) and obtained very similar results.

4 The algorithm is not actually applied to the returns themselves but to their standardized z-scores. We then map the clustered z-scores back to their return equivalents.

Figure 1. K-means clustering of U.S. government bond returns and U.S. equity returns, October 2000 to March 2021



Notes: Equity returns refer to the MSCI USA Total Return Index and bond returns refer to the Bloomberg Barclays U.S. Treasury Total Return Index. The yield threshold of 2.5% was based on the U.S. 10-Year Treasury Bond. All figures are in USD. Clusters' centroids are shown as triangles. Colors are used to identify each cluster.
Source: Bloomberg L.P., using monthly data from October 2000 to March 2021.

Each of the clusters is given a color. We observe that when K = 2, some returns are gray (when equities have almost always had positive performance and bonds are slightly negatively tilted) and some are blue (when bonds have almost always had positive performance and equities are negative). This suggests a negative relationship between U.S. equity and U.S. Treasury returns. When equities go up, the bond return is not easily predictable, but when equities go down, bonds almost always go up. We do not see a cluster in the down-down quadrant, implying that the few pairs of returns that fall in this region are somewhat anomalous and not statistically relevant.

For K = 3, the story is very similar. The blue down-up state largely remains unchanged, but the gray up-down state splits into two distinct market states, purple and green. This reinforces the hypothesis that bonds provide good diversification for equities as, again, we do not see a cluster toward the bottom left corner mapping a down-down market state.

The triangles depict each cluster's centroid, which is not an actual return pair but rather the notional center of the cluster. For the example of K = 2, the centroids can be thought of as two market state centers, one with equities up and bonds slightly down and one with equities down and bonds up. For K = 3, we still have the centroid in which equities are down and bonds are up, but our equities up state now divides into two—one where bond returns are more centered on 0% and one where they are negative. This further confirms the idea that U.S. Treasuries act as a counterbalance to equity movements.[5]

5 Earlier, we mentioned that an alternative approach might be to run a linear regression to determine bonds' average response to a given change in equities. However, a line of best fit through these data points would be unable to explain a large percentage of the variance (it would have a low R-Squared, equal to 9.9%) because of the large dispersion of the return pairs. Therefore, while it might provide some indication of the typical directionality of the relationship, it would not help much in understanding the market states determined by our clustering analysis.

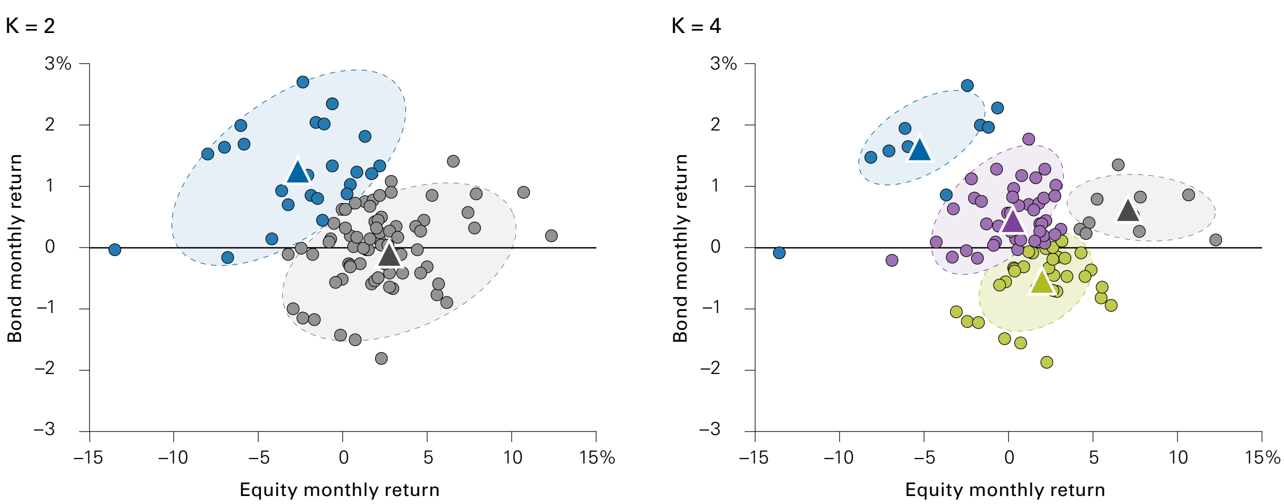
At this stage, two questions occur: Should we use K = 2 or 3, and why should K = 2 or 3? We used two established techniques to determine the appropriate number of clusters (see Appendix). The results of both methods indicate that two clusters is the best choice for the U.S. data.[6] This is important because it suggests that most likely only two main states are consistent with the underlying pattern of equity and bond returns and that these two states (up-down and down-up) correspond to U.S. equities and government bonds balancing one another. Other return pairs are not statistically significant deviations. When we increase the number of clusters to three, the up-down cluster splits into two, and the three centroids are aligned on an imaginary negatively sloped line, confirming the diversification properties of government bonds.

A key advantage of this analysis over a regression-based approach is the identification of market states. In addition to providing information on the (linear) relationship between equities and bonds, K-means enables us to identify any clustering that may be consistent with the presence of a specific market state (such as down-up). The combination of the optimal number of clusters and their locations helps us assess whether there is sufficient evidence for such a state to exist. The absence of clusters can be just as valuable, providing insight into whether observations located in these regions are likely to recur in the future.

K-means clustering: Global returns

Of course, not all multi-asset investors choose to invest exclusively in U.S. funds. **Figure 2** shows the same K-means analysis using K = 2 and 4 for global returns. Here, we set our yield threshold to 1.5% and use a global Treasury index, resulting in a number of return pairs similar to the U.S. example.

Figure 2. K-means clustering of global government bond returns and global equity returns, October 2000 to March 2021



Notes: Equity returns refer to the MSCI All Country World Total Return Index and bond returns refer to the Bloomberg Barclays Global Treasury Total Return Index. The yield threshold of 1.5% was based on the Bloomberg Barclays Global Treasury Total Return Index. All figures are in USD and bond returns are hedged to USD. Clusters' centroids are shown as triangles. Colors are used to identify each cluster.

Source: Bloomberg L.P., using monthly data from October 2000 to March 2021.

6 Some qualitative judgment is still required to choose an appropriate value for the number of clusters, and parsimony is often preferable. In fact, some of the validation methods (such as the Silhouette score) suggest that a large number of clusters is optimal, which can lead to overfitting.

As in Figure 1, two clusters show a clear diversification benefit with similar locations for the two centroids. As a regression analysis would also suggest, the clusters reveal a negative relationship between global equity and government bond returns. Again, there is no mapping to the down-down state, suggesting that the returns that fall in this region can be thought of as anomalies or noise. Therefore, assuming the paradigm of two clusters is appropriate, both U.S. and global returns show the market states of equities up/bonds down and equities down/bonds up.

One difference between the global and U.S. analyses is demonstrated in Figure A-2. Although it suggests that setting K = 2 is still best, with both the maximum Silhouette score and the elbow occurring at this point, in this scenario K = 3 actually has a lower Silhouette score than K = 4, which is why we use K = 4. However, once again no cluster or centroid appears in the bottom left quadrant.

Conclusion

Investors have become concerned about expected returns for fixed income in a low-interest-rate environment. However, our K-means clustering analysis shows that government bonds have historically acted as a counterbalance in an equity-bond portfolio during low-rate periods by hedging against equity market drops for both U.S. and global investors. Although in some months both equities and bonds fall, our analysis suggests that this can be thought of as market noise and therefore as distinctly different from the typical outcomes, which can be regarded as recurrent equities up/bonds down and equities down/bonds up market states.

References

Goodfellow, Ian, Yoshua Bengio, and Aaron Courville, 2016. *Deep Learning*. Cambridge, Mass.: MIT Press.

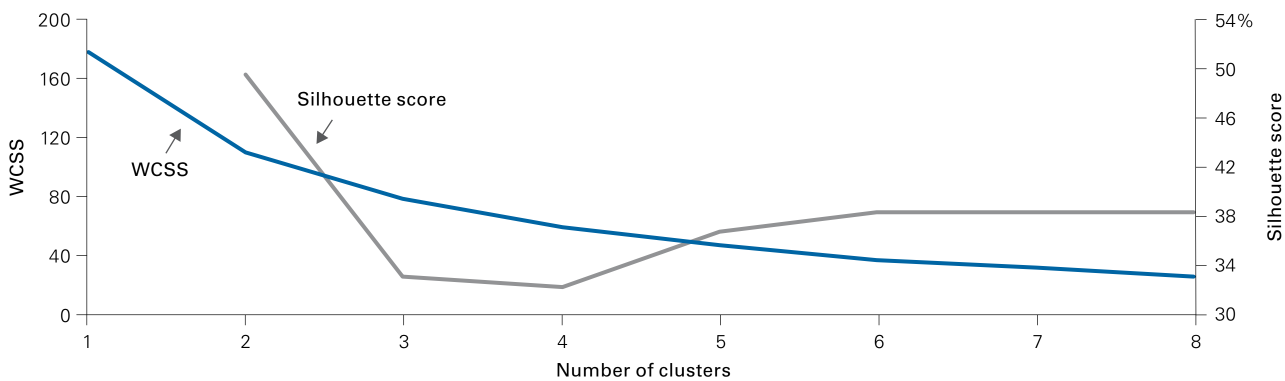
Renzi-Ricci, Giulio, and Lucas Baynes, 2021. *Hedging Equity Downside Risk with Bonds in the Low-Yield Environment*. Valley Forge, Pa.: The Vanguard Group.

Appendix: How to determine the right number of clusters

We use two approaches to determine the optimal number of clusters in our analysis. The first is the elbow chart, in which we plot the within-cluster sum of squared distances (WCSS)—the objective that the clustering algorithm minimizes. The WCSS is then computed as the sum of squared distances between each observation and its associated centroid. Ideally, we want the WCSS to be as low as possible; however, by increasing the number of clusters it will almost always decrease. Therefore, there is a trade-off between explaining more of the variation and preventing overfitting. An elbow is sought in the curve because additional clusters beyond this point produce a relatively small reduction in WCSS. The optimal number of clusters is at the point where the kink occurs.

The second technique is the Silhouette score, which is calculated using the mean intra-cluster distance and the mean nearest-cluster distance for each sample. When the Silhouette score is high, the observation is well-matched to its own cluster and poorly matched to neighboring clusters; therefore, a higher score is better. **Figure A-1** shows the elbow and Silhouette charts for our analysis of U.S. equity and U.S. Treasury returns, and **Figure A-2** refers to global equity and government bond returns.

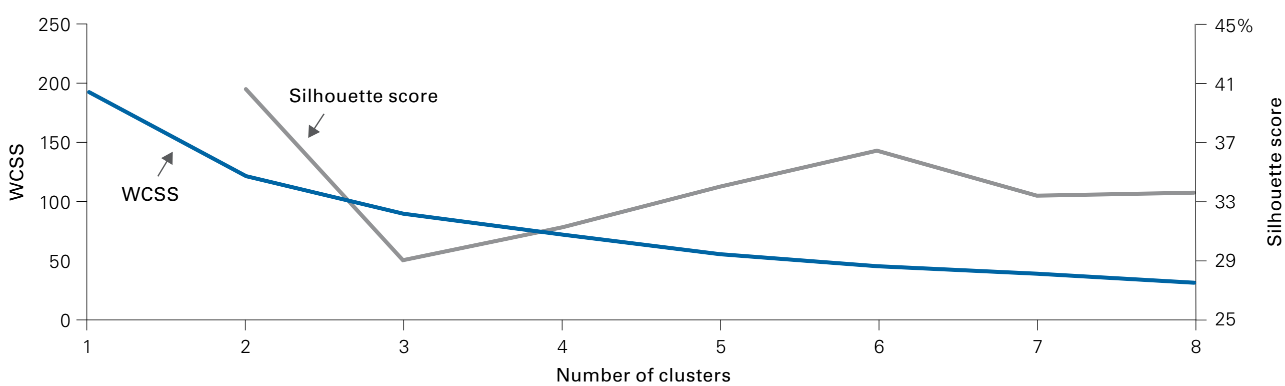
Figure A-1. Testing the value of K for K-means clustering of U.S. government bond returns and U.S. equity returns, October 2000 to March 2021



Notes: Equity returns refer to the MSCI USA Total Return Index and bond returns refer to the Bloomberg Barclays U.S. Treasury Total Return Index. The yield threshold of 2.5% was based on the U.S. 10-Year Treasury Bond. All figures are in USD.

Source: Bloomberg L.P., using monthly data from October 2000 to March 2021.

Figure A-2. Testing the value of K for K-means clustering of global government bond returns and global equity returns, October 2000 to March 2021



Notes: Equity returns refer to the MSCI All Country World Total Return Index and bond returns refer to the Bloomberg Barclays Global Treasury Total Return Index. The yield threshold of 1.5% was based on the Bloomberg Barclays Global Treasury Total Return Index. All figures are in USD and bond returns are hedged to USD.

Source: Bloomberg L.P., using monthly data from October 2000 to March 2021.

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