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A systematic approach to constructing high-income portfolios

- When it comes to investing for retirement, best practices suggest that investors formulate a spending strategy based on a portfolio's expected total return (dividends and interest plus capital gains) and draw down their assets using a combination of income and capital gains.
- However, because of "mental accounting," investors typically have an affinity for higher-income-producing assets so that, instead of selling appreciated shares, they can use the income generated to fund their living expenses. Unfortunately, most income-focused portfolios are built with an ad hoc approach that ignores portfolio construction best practices.
- Using an enhanced version of the Vanguard Asset Allocation Model (VAAM) and a time-varying strategy, our analysis shows that higher-income ingredients can add value through both their income-producing potential and their ability to maximize the utility of portfolio wealth derived from total returns. However, taxes can significantly alter the results for investors in high marginal tax brackets.

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Introduction

When it comes to retirement, best practices suggest that investors formulate a spending strategy based on a portfolio's expected total return (dividends and interest plus capital gains) and draw down their assets using a combination of income and capital gains. This approach is generally preferred in academic and practitioner research because it tends to lead to more diversified and risk-return-efficient portfolios (Bupp et al., 2021).

However, because of "mental accounting," investors typically have an affinity for high-income-producing assets because they do not require selling appreciated holdings to fund their living expenses, which often provides a greater sense of confidence in not outliving one's assets. Unfortunately, most income-focused portfolios are built using an ad hoc approach that ignores portfolio construction best practices. They typically have higher income but often lower total return, high concentration, and higher volatility.

Blanchett and Watner (2015) compare optimizations based on total return and income return and conclude that investors with high aversion to income fluctuations, limited liquidity needs, and a high tolerance for principal fluctuations can benefit from an income-oriented portfolio. Yet, there has been limited work on how to develop portfolios that incorporate both total and income returns. After all, assets with higher income are not necessarily risk-return-inefficient; they just are often suboptimally allocated to based solely on their ability to produce income.

In this paper, we introduce a new methodology for building high-income portfolios using sophisticated optimization techniques based on each asset's total return, risk premiums, correlations, and volatility while allowing for higher-income ingredients and yield targeting. We compare this with higher-income portfolios developed through a more conventionally ad hoc, income-focused approach and find significant efficiency gains for comparable levels of portfolio income.

Our process optimizes portfolios that maximize an investor's utility of wealth while accounting for their yield preferences. This tends to result in more diversified portfolios that are expected to achieve higher risk-adjusted returns. Lastly, we find that taxes can significantly alter the after-tax total returns of higher-income assets and subsequently the value they add for investors in high marginal tax brackets.

Notes on risk

All investing is subject to risk, including possible loss of principal. Diversification does not ensure a profit or protect against a loss. Investments in bonds are subject to interest rate, credit, and inflation risk. Investments in stocks or bonds issued by non-U.S. companies are subject to risks including country/regional risk and currency risk. These risks are especially high in emerging markets.

IMPORTANT: The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from VCMM are derived from 10,000 simulations for each modeled asset class. Simulations are as of September 30, 2020. Results from the model may vary with each use and over time. For more information, please see Appendix 4.

Challenges of the current market environment

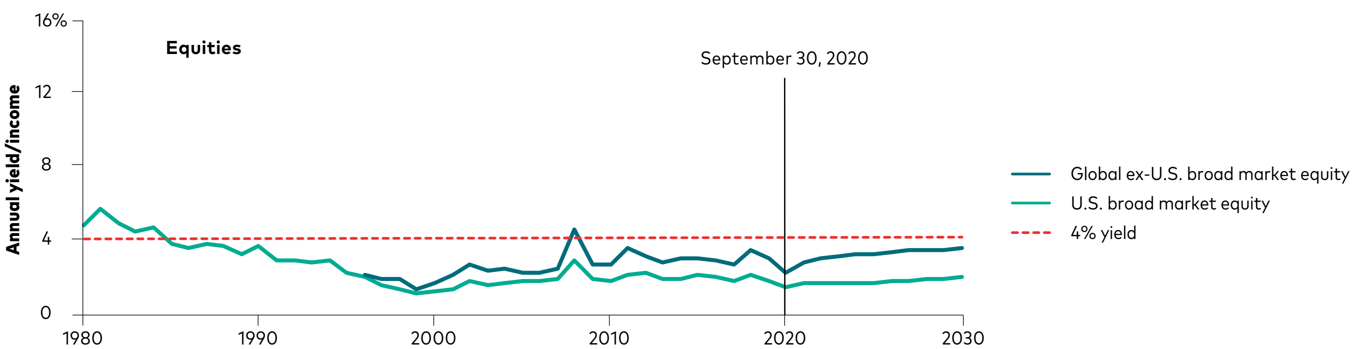
Constructing portfolios that produce high levels of income (defined here as 4% or more) has been challenging for more than a decade. And the low-interest-rate environment that resulted from the global financial crisis is expected to continue for the foreseeable future because of the coronavirus pandemic.

Figure 1 displays the historical annual yields for broad market equity and fixed income asset classes in the U.S. and abroad since 1980. The steady decline is notable, particularly for broad market U.S. bonds, which yielded more than 13% in 1980 and 6% at the turn of the century compared to around 2% today. On a forward-looking basis, the Vanguard Capital Markets Model® (VCMM) (see Davis et al., 2020) projects that annual income for all broad market asset classes will remain below 4% for the next decade.

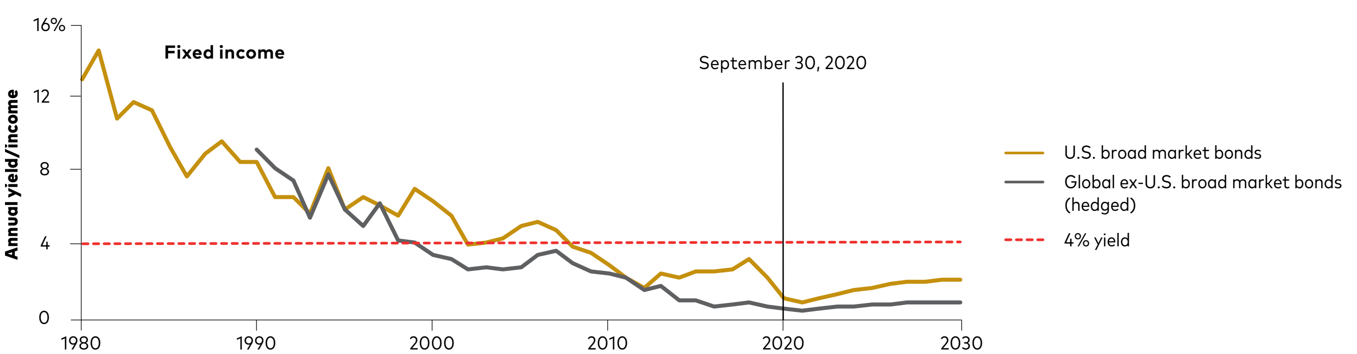
FIGURE 1.

Yields for broad market asset classes have steadily declined for more than a decade

a. Broad market equity



b. Broad market bonds



Notes: Historical yields are represented as follows: U.S. broad market equities by the MSCI USA Index, global ex-U.S. broad market equities by the MSCI ACWI ex USA Index, U.S. broad market bonds by the Bloomberg U.S. Aggregate Bond Index, and global ex-U.S. broad market bonds by the Bloomberg Global Aggregate ex USD Index. Forward-looking income data are represented as the median annual income expectation from the VCMM model. Data are as of September 30, 2020.

Source: Vanguard calculations, using data from Bloomberg and Macrobond.

Past performance is no guarantee of future returns. The performance of an index is not an exact representation of any particular investment, as you cannot invest directly in an index.

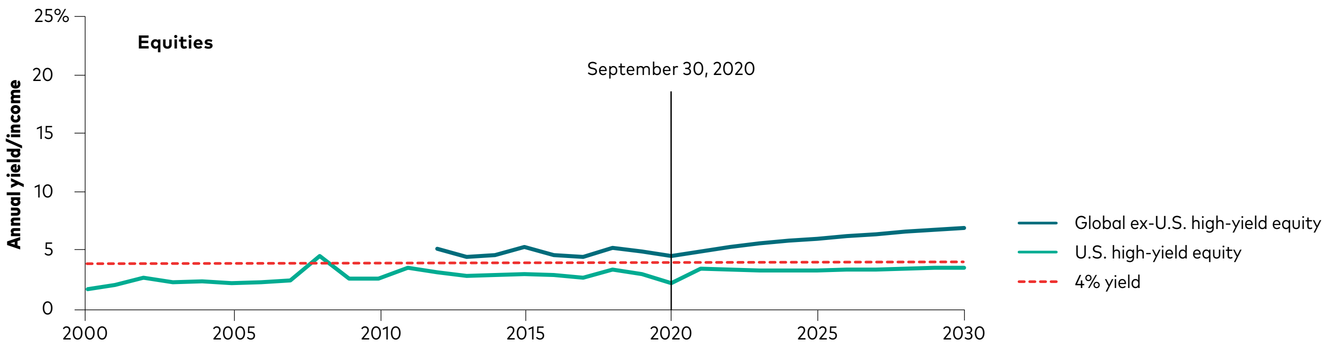
Figure 2 displays similar historical and expected income data for some of the common asset classes investors have turned to over the past decade in pursuit of additional income. Investment-grade credit (both U.S. and global ex-U.S.), high-dividend-paying equities (both U.S. and global ex-U.S.), U.S. high-yield corporate bonds, and emerging-market bonds have all received particular attention.

Currently, only U.S. high-yield corporate bonds and high-dividend-paying equities outside the U.S. are yielding greater than 4%, with emerging-market bonds expected to follow. But U.S. investment-grade credit and U.S. high-dividend-paying equities are yielding considerably more than their broad market counterparts. Because all of these asset classes are to some degree inherently riskier in terms of drawdown risk and volatility, making informed trade-offs in pursuit of higher income is crucial.

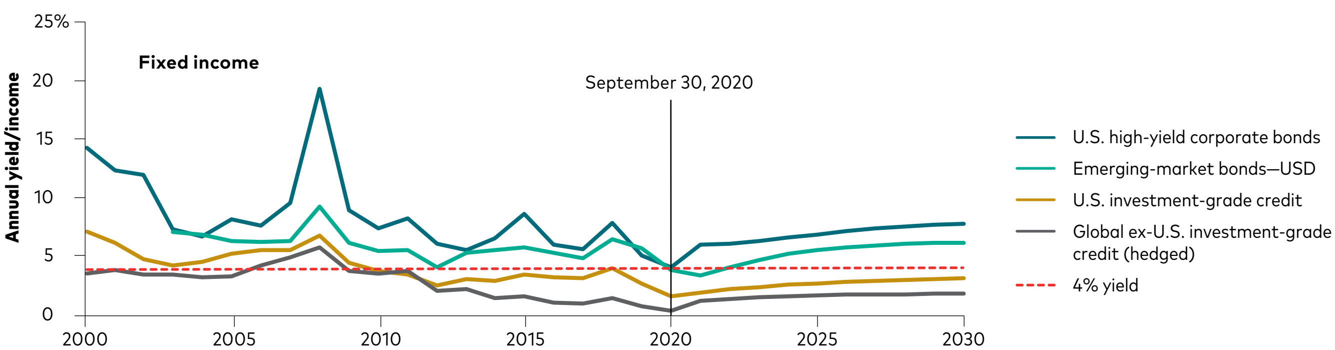
FIGURE 2.

Higher-yielding asset classes offer investors additional income

a. Higher-yielding equity



b. Higher-yielding bonds



Notes: Historical yields are represented as follows: U.S. high-dividend-paying equities by the MSCI USA High Yielding Index, U.S. investment-grade credit by the Bloomberg U.S. Credit Index, global ex-U.S. high-dividend-yielding equities by the MSCI ACWI ex USA High Dividend Yield Index, global ex-U.S. investment-grade credit by the Bloomberg Global ex U.S. Credit Index, U.S. high-yield corporate bonds by the Bloomberg U.S. High Yield Corporate Index, and emerging-market bonds USD by the Bloomberg Emerging Markets Sovereign USD Index. Forward-looking income data are represented as the median annual income expectation from the VCMM model. Data are as of September 30, 2020.

Source: Vanguard calculations, using data from Bloomberg and Macrobond.

Past performance is no guarantee of future returns. The performance of an index is not an exact representation of any particular investment, as you cannot invest directly in an index.

Our portfolio construction methodology

We use the Vanguard Asset Allocation Model (VAAM) (Aliaga-Díaz et al., 2019) as the foundation for developing our high-income portfolios because the goal is for them to be total-return-efficient. VAAM is a utility-based model that assesses risk and return trade-offs to arrive at optimal solutions relative to a level of risk aversion and time horizon. It can assess three dimensions of risk and return sources—passive market betas, investment factors, and active strategies. While it can analyze a range of time horizons, we specifically focus on the next decade because of current market dynamics and later compare this to our long-term market outlook (see **Appendix 1** for more details).

The VAAM optimization process considers approximately 2,000 to 5,000 portfolio combinations.[1] For this analysis, we add an enhancement to sort out those that fall outside of an expected median income band (minimum and maximum threshold for income). As inputs, we analyze the ten-year forward-looking total-return expectations (across a distribution of 10,000 potential paths) for a variety of asset classes from VCMM. Through a sum-of-parts framework, we also break out the corresponding income returns from dividends and interest for each of the 10,000 simulation paths (see **Appendix 2** for more details). Specific to our goal of building total-return-efficient portfolios, we include broad asset classes with low current yields and some common higher-income-producing classes so that the model can identify the optimal combinations.

High-dividend-yielding equities are modeled as active strategies based on Schlanger and Kesidis (2016), which showed that a large portion of their total returns could be explained by investment factors. To generate the income-return distributions for the high-dividend strategies, we used an auto-regressive process to generate income multiples corresponding to each of the broader equity market simulations (see **Appendix 3** for more details). Finally, because higher-income-producing asset classes are also subject to more income taxes, in some parts of our analysis we used estimated after-tax total returns by reducing the distributions by hypothetical amounts for investors in different tax brackets.

The ad hoc process is generally opaque and inefficient

Income-focused portfolios commonly found in the industry are built using an ad hoc, relatively opaque higher-income substitution method. While the specifics may vary from one provider to another, these models typically have a few things in common. First, one or more target equity/bond mixes are selected to gauge overall portfolio risk, regardless of the fixed income credit quality and volatility. Second, within asset classes, sub-asset classes with above-average yields are allocated to with an emphasis on using their prevailing yields to achieve a desired level of portfolio income (typically somewhere between 4% and 6%). Third, it is usually unclear from a methodology standpoint how (if at all) total returns are considered. The main issues with this approach are that it is not fully transparent to the investor and it focuses disproportionately on generating a desired level of income without a holistic consideration of the portfolio, its expected total returns, and risks.

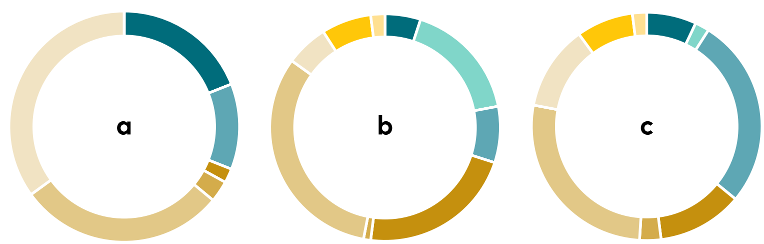
1 The portfolio sample size can be tweaked by the user and can consider as many as 500,000 combinations but typically is within this range.

Figure 3 displays three portfolios and associated metrics derived using our ten-year VCMM capital market expectations (distributions of expected total returns and income returns, correlations, and volatilities) as of September 30, 2020. The higher-income, ad hoc portfolio shown in Figure 3a is representative of a conservative, income-focused portfolio (in this case, with roughly 30% equities and 70% fixed income). It was generated using the ad hoc process described previously and has a juicy expected income return of 5.5% but a total return of more than 100 basis points less (a basis point is one-hundredth of a percentage point). In other words, it is overweighted in assets expected to see capital declines as interest rates normalize over the coming decade, and investors' wealth could decline as a result.

Figure 3b shows the optimal total-return-efficient portfolio for all ten asset classes included in our analysis. It has allocations to most of the higher-income ingredients (showing that they can be total-return-efficient) but is much more diversified than the higher income, ad hoc portfolio. As a result, it has comparable total returns and income returns with lower overall volatility.

FIGURE 3.

Three portfolios and their metrics derived using ten-year VCMM capital market expectations



	Higher-income, ad hoc portfolio	Total-return-efficient portfolio	Higher-income, total-return-efficient portfolio
U.S. broad market equity	0%	0%	0%
U.S. high-yield equity	19%	5%	7%
Global ex-U.S. broad market equity	0%	17%	2%
Global ex-U.S. high-yield equity	12%	8%	27%
U.S. broad market bonds	2%	22%	12%
U.S. investment-grade credit	3%	1%	3%
U.S. high-yield corporate bonds	29%	32%	27%
Emerging-market bonds—USD	35%	6%	12%
Global ex-U.S. broad market bonds (hedged)	0%	7%	8%
Global ex-U.S. investment-grade credit (hedged)	0%	2%	2%
Median income return	5.54%	4.40%	5.01%
Median total return	4.42%	4.35%	4.83%
Median volatility	10.97%	8.92%	10.36%

Note: Portfolio expectations are represented by the median simulation from the VCMM model as of September 30, 2020.
Source: Vanguard.

Figure 3c shows the higher-income, total-return-efficient portfolio with a VAAM-generated asset allocation constrained to optimize only for portfolios expected to generate income of 5% or more to make it more comparable to the ad hoc version. This differs from the total-return-efficient portfolio in allocating a greater share to high-dividend-yielding equities outside the U.S. that have higher expected income and total returns relative to the industry ad hoc portfolio, which allocates a greater share to emerging-market bonds expected to see capital declines.

A close examination of the industry ad hoc portfolio shows what happens when the focus is solely on increasing yield and assets are suboptimally allocated on the basis of their total returns. One way to see this is to plot the expected total return, income return, and volatility of the three portfolios, as shown in **Figure 4**. The higher-income, ad hoc portfolio is expected to realize about 160 basis points (bps) of additional volatility while achieving total returns comparable to the total-return-efficient portfolio (a difference of 7bps).

Another way to quantify efficiency is to compare utility scores via a certainty-free equivalent (CFE), which can be thought of as the additional cost an investor should be willing to pay to invest in one portfolio relative to another. The higher the CFE, the better the expected risk-return trade-off. The CFE of the VAAM optimal allocation is 170 bps greater than that of the higher-income, ad hoc portfolio, whereas the higher-income, total-return-efficient version is only 18 bps lower than the optimal.

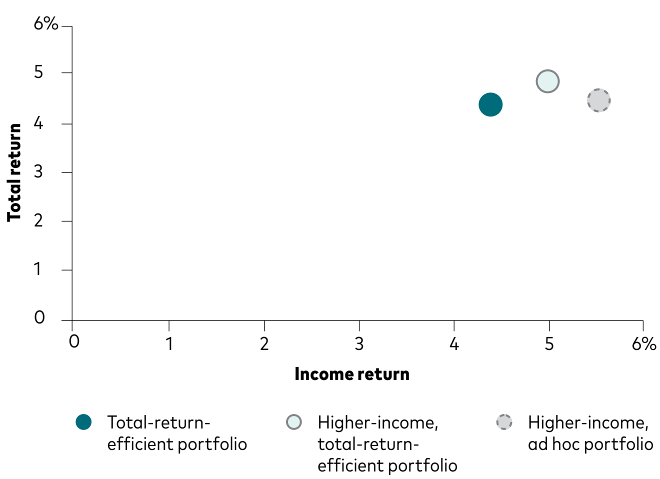
Don't forget to account for taxes

All of the analysis we have done up until this point has been in a tax-free environment, although we know that many investors' assets are subject to taxes assessed differently based on income (dividends and interest) versus capital gains. For investors in lower tax brackets, these differences tend to be small, but they increase dramatically for higher incomes. For example, the top marginal U.S. federal tax rate for a married couple filing jointly is 37% (for combined income above $622,051). When you add state taxes to that,

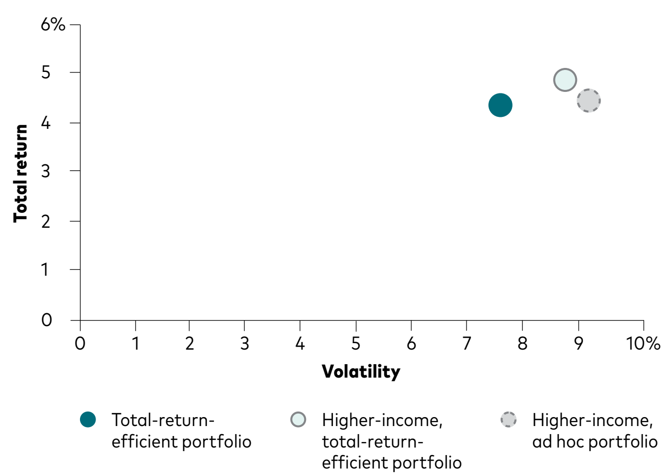
FIGURE 4.

A total-return approach is important

a. Total return relative to income return



b. Risk-return for three portfolios



Note: Portfolio expectations are represented by the median simulation from the VCMM model as of September 30, 2020.

Source: Vanguard.

which can be as much as 13.3% for a California resident, some investors have a marginal tax rate as high as 50.3%. This is considerably above the 20% maximum federal tax investors pay on capital gains (for income over $441,451) and the 15% rate for lower income levels.[2]

Accounting for taxes is important, because ultimately they are a cost and should therefore be subtracted from an investment's total return. To test the tax sensitivity of our VAAM-optimized portfolios, we focus on income taxes because they must be paid every year, whereas capital gains taxes are only incurred when assets are sold.

Figure 5 shows the results of two additional scenarios for our baseline VAAM-optimized portfolio from Figure 3b, for investors with marginal tax rates of 25% and 50%. As one might expect, the higher-income ingredients (such as high-yield corporates, emerging-market bonds and high-dividend-paying equities) become progressively less optimal as the tax rate rises.

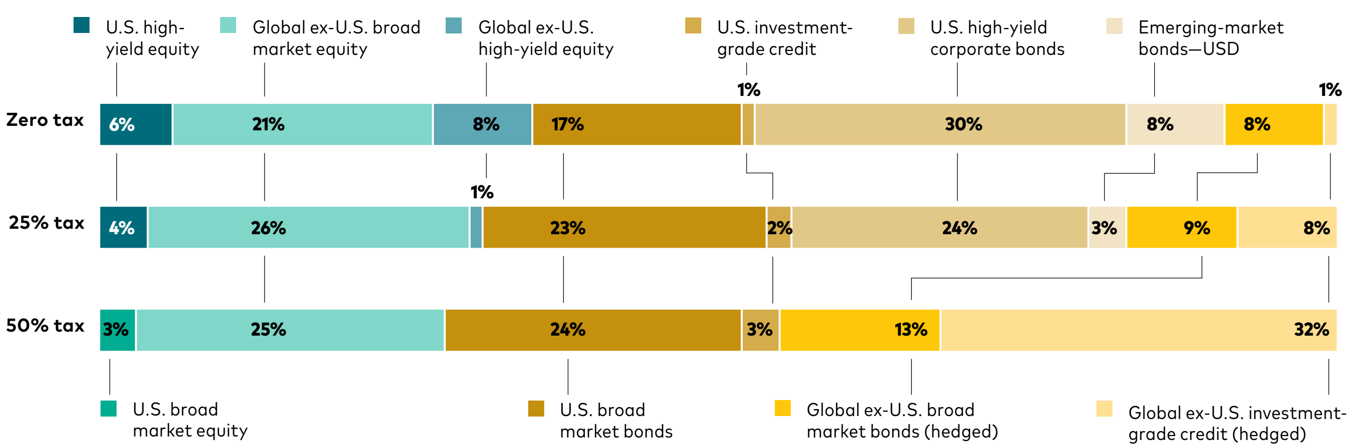
This suggests that allocations to higher-income securities on the basis of their total returns can add value to a portfolio as long as taxes are not a primary concern—for example, for low to moderate incomes or in a tax-deferred account.

A time-varying approach

All of the analysis shown thus far has been based on VCMM capital market expectations as of September 30, 2020. This gives an idea of how the optimization applies to today's environment, but yields are always changing (as shown in Figures 1 and 2), along with valuations and expected total returns that could all affect results (Wallick et al., 2020). **Figure 6a** shows the optimal VAAM-derived allocations for the ten-asset-class portfolio from Figure 3b compared with a similar portfolio based on VCMM long-term equilibrium, or "steady-state," capital market expectations.

FIGURE 5.

The impact of taxes can significantly alter the total-return efficiency of higher-income ingredients



Note: Portfolio optimizations are derived from the VAAM using data as of September 30, 2020.
Source: Vanguard.

2 Some U.S. states also tax capital gains.

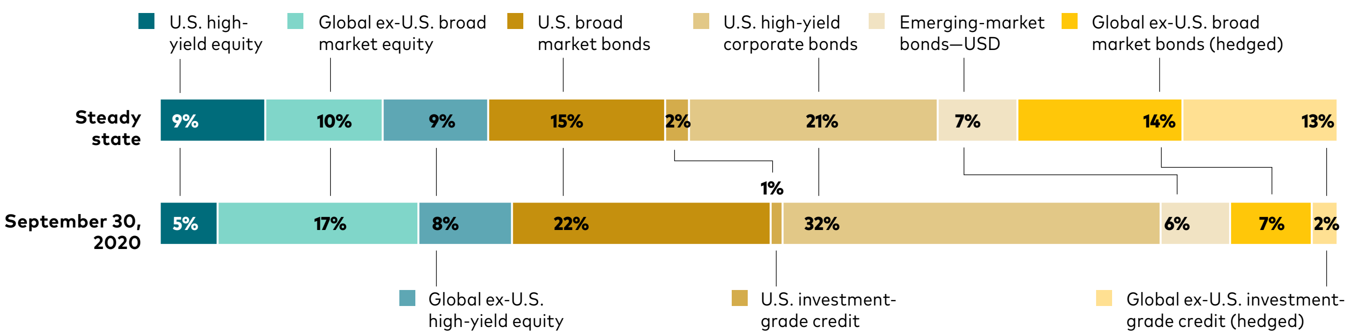
The large allocation differences relative to steady state are notable and symptomatic of today's environment. Similarly, the projected income return distribution presented in **Figure 6b** shows that despite higher allocations to higher-yielding assets in the portfolio with data as of September, 30, 2020, the income returns over the next ten years are likely to be markedly lower than steady state.

Therefore, recalibration when there are material changes to capital market expectations and yields is important to a higher-income investment strategy. While this could be done using the ad hoc process based on prevailing yields, the strategy we have outlined provides a more rigorous and holistic approach.

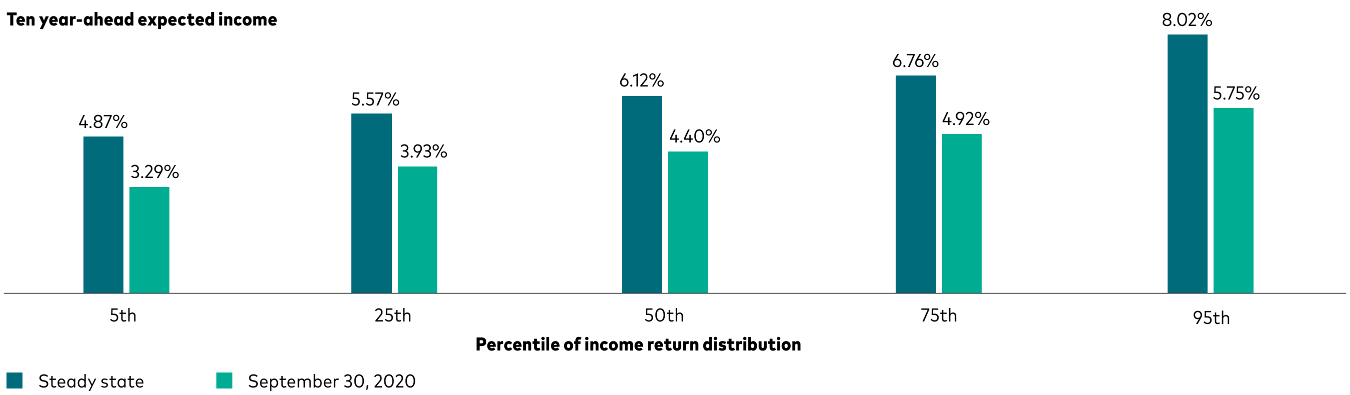
FIGURE 6.

A time-varying approach is important

a. Total-return-optimized portfolios by period



b. Projected income return distribution for optimal-total-return portfolios by period



Note: Portfolio optimizations and expected income returns were derived from the VAAM with data as of September 30, 2020.
Source: Vanguard.

Conclusion

In this paper, we introduced a new methodology for building high-income portfolios using sophisticated optimization techniques based on each asset's total return, risk premiums, correlations, and volatility while allowing for higher-income ingredients and yield targeting. We contrasted this method with similar portfolios developed through a more conventionally qualitative, ad hoc approach and found significant expected efficiency gains for comparable levels of income.

Our process optimizes for portfolios that maximize the investor's utility of wealth based on total returns while accounting for yield preferences. This tends to result in more diversified portfolios that are expected to achieve higher risk-adjusted returns. Lastly, we found that taxes can significantly alter the after-tax total returns of higher-income assets for investors in high marginal tax brackets and subsequently their optimization in taxable accounts.

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Appendix 1. The Vanguard Asset Allocation Model (VAAM)

The Vanguard Asset Allocation Model (VAAM) aims to determine an investor's optimal asset allocation based on two key inputs: that investor's attitude toward the trade-off between risk and return and the output of Vanguard's VCMM model. The former is estimated using a power utility function that aims to capture the investor's subjective perception of the relationship between risk and return through the risk aversion coefficient:

Formula 1.

$$U(W) = \begin{cases} \dfrac{W^{1-\gamma}}{1-\gamma}, & \gamma > 1 \\ \ln(W), & \gamma = 1 \end{cases}$$

where γ is the relative risk aversion (RRA) coefficient and W is the level of terminal wealth relative to starting wealth. W will compound in each time period by the total multiasset portfolio return R_t:

Formula 2.

$$R_t = \sum_{i=1}^{N} x_i r_{i,t} = \sum_{i=1}^{N} x_i^p r_{i,t}^p + \sum_{i=1}^{N}\sum_{f=1}^{F} x_i^f r_{i,t}^f + \sum_{i=1}^{F} x_i^\alpha r_{i,t}^\alpha$$

$$\begin{cases} r_{i,t}^p = r_{i,t}^M \\ r_{i,t}^f = r_{i,t}^M + \delta_{i,t}^f \\ r_{i,t}^\alpha = \alpha_i + \beta_i r_{i,t}^M + \sum_{f=1}^{F} L_i^f \delta_{i,t}^f + \varepsilon_{i,t} \quad \varepsilon_{i,t} \sim t(v)\sqrt{\sigma_{\alpha i}^2} \end{cases}$$

where x_i and r_i are the portfolio weights and relative total returns for each asset class i and superscripts p, f, and α refer to passive, factors, and active, respectively.

The market benchmark return represented by $r_{i,t}^{M}$, $\delta_{i,t}^{f}$ is the excess (to the market benchmark) factor return for factor f and β_i and L_i corresponding to the market beta and factor loading for each asset class. α_i is the factor-adjusted excess active return.

The problem consists of finding optimal weights for each passive asset class, factor, active manager, or strategy in the portfolio. Weights are selected to maximize the expected utility of final wealth, expressed as:

Formula 3.

$$\max_{w} \mathbb{E}[U(W_T)] \rightarrow \max_{w} \left\{ \mathbb{E}\left[\frac{W_p^{1-\gamma_p}}{1-\gamma_p}\right] + \mathbb{E}\left[\frac{W_f^{1-\gamma_f}}{1-\gamma_f}\right] + \mathbb{E}\left[\frac{W_a^{1-\gamma_a}}{1-\gamma_a}\right] \right.$$
$$s.t. \{w_i \in \mathbb{R} \mid 0 \leq w_i \leq 1\} \wedge \sum_i w_i = 1$$
$$\sum_i C \cdot w_i \leq b$$

where Wp, Wf, and Wa are the wealth at maturity coming from systematic, factor, and factor-adjusted alpha exposures, γp, γf, and γa are the systematic, factor, and alpha risk aversions, and C and b refer to the set of linear inequality constraints.

Appendix 2. Constraining portfolio income in VAAM

For the income-oriented investor, we use a custom configuration of VAAM that also considers Vanguard's VCMM-generated income-return forecasts. Our VAAM configuration not only accounts for the investor's preferences toward risk and return but also constrains the optimal portfolio to have a median annualized income return within an input target range. This means that VAAM will prescreen the set of portfolios that meets the expected income return constraint and then evaluate it against the investor's utility function and Vanguard's VCMM forecasts to determine the optimal asset allocation.

Using the set of portfolio weights selected by VAAM, we can express the portfolio income return in a given year as:

Formula 4.

$$IR_t = \sum_{i=1}^{N} x_i Ir_{i,t}$$

where x_i and Ir_i are the portfolio weights and income returns for each asset class i. We then take the geometric mean of IR_t across the full time horizon to calculate an annualized portfolio income return:

Formula 5.

$$IR_t = \left(\prod_{t=1}^{T} (1 + IR_t) \right)^{1/T}$$

Because of the simulation-based nature of Vanguard's VCMM model, for a set of portfolio weights X and corresponding annualized income return IR we apply VAAM's income constraint to the median forecast IR.

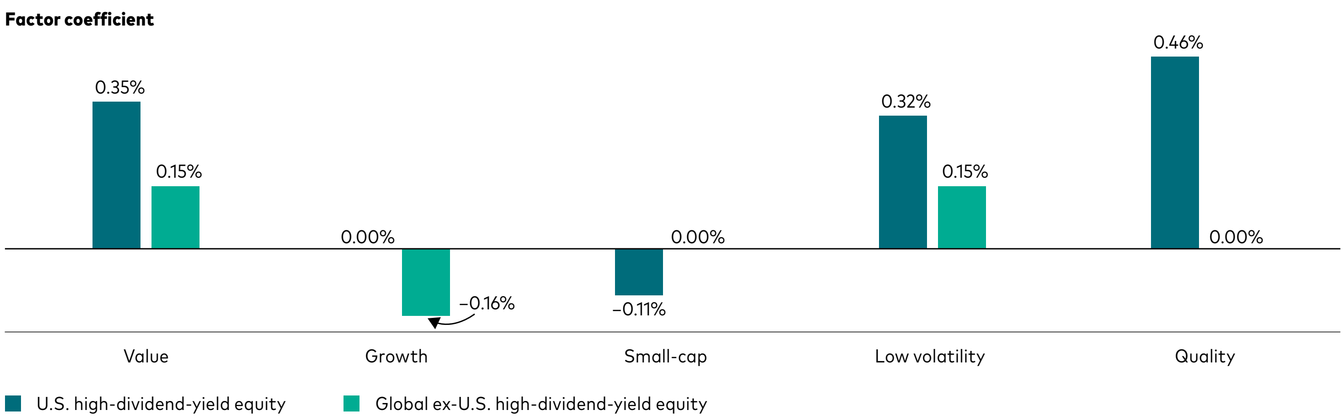
Appendix 3. Constructing a total and income return series for high-dividend-yielding equities

High-dividend-yielding equities can be thought of as rules-based active strategies. Their total returns will be time-varying relative to broader market equities largely based on exposure to common investment factors (Schlanger and Kesidis, 2016).

Figure A-3.1 shows the investment factors for U.S. and overseas high-dividend-yielding equities based on the historical returns of the U.S. and global ex-U.S. components of the FTSE All-World High Dividend Yield Index (we found an R-squared measure of 95% for U.S. high-dividend equities and 98% for global ex-U.S. equities). Using these factor loadings and an alpha and tracking error assumption, we generated forward-looking total returns based on the active and factor modeling process detailed previously in Appendix 1.

FIGURE A-3.1

Factor exposure of high-dividend-yielding equities



Note: U.S. high-dividend equities are represented by the FTSE High Dividend Index, global ex-U.S. high-dividend equity by the FTSE All-World ex US High Dividend Index, U.S. factor indexes by Vanguard's proprietary indexes derived from the Russell 1000 Index Series, and global factors by MSCI World factor indexes.

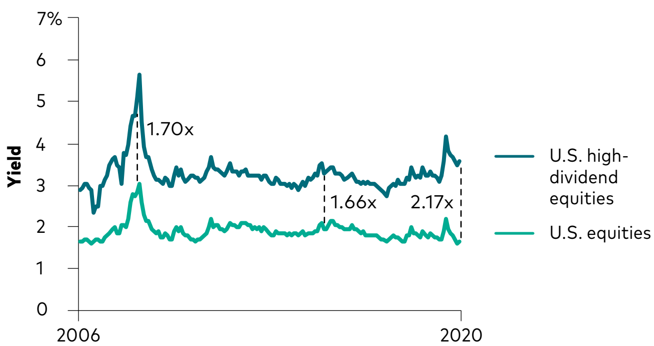
Source: Vanguard calculations, using data from Vanguard, Russell, and MSCI.

Since the high-dividend-yielding indexes represent the top 50% of yielding securities, the income will always be higher by a multiple of the broader markets. Using the historical observations of broad market and high-dividend-equity income returns, we construct a time series of the high-yield-income multiplier as shown in **Figure A-3.2**.

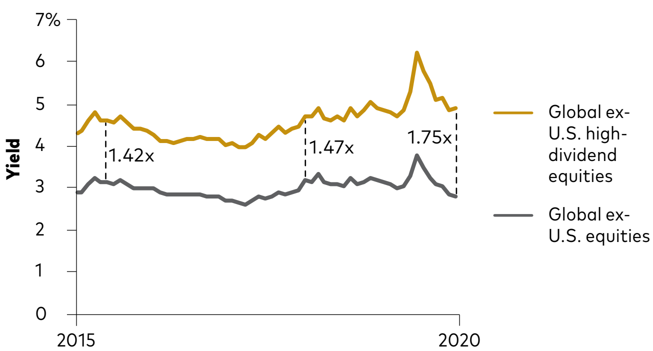
FIGURE A-3.2

High-dividend-yielding equity income returns are determined by an income multiple

a. U.S. high-dividend equities



b. Global ex-U.S. high-dividend equities



Notes: Representative indexes can be found in the footnotes to Figures 1 and 2. Data are through September 30, 2020.

Source: Vanguard calculations, using data from Macrobond.

Past performance is no guarantee of future returns. The performance of an index is not an exact representation of any particular investment, as you cannot invest directly in an index.

Based on this multiplier, we define an AR(1) process and, collecting error terms, generate a matrix of yield multiplier simulations. The elements of this matrix are then scaled by the elements of broad market equity income simulations from Vanguard's VCMM model to produce high-dividend-equity income return simulations for the VAAM income tilt configuration.

Here we define the process for generating simulations of the income multiplier.

Let Y and X represent the historical income returns for high-dividend and broad market equities. From this, we define the time series history of the income multiplier as:

Formula 6.

$$M_t = Y_t / X_t \; \forall \; t$$

We then estimate a demeaned AR(1) process for M:

Formula 7.

$$M_t = \mu + \theta \left(M_{t-1} - \mu \right) + \varepsilon_t$$

where μ is the sample mean of the M series, θ is the parameter to be estimated, and ε_t are the error terms.

We can then define M_{t+1}, the first observation in our yield multiplier forecast series, as:

Formula 8.

$$M_{t+1} = \mu + \theta \,(M_t - \mu)$$

M_{t+1} can be thought of as the expectation of next year's income multiplier. To account for uncertainty, we must simulate some variation around this mean, both cross-sectionally and through time. To do so, we fit a distribution to the error terms collected from the AR(1) model, which should behave as:

Formula 9.

$$\varepsilon_t \sim N\,(0,\sigma^2),\ i.\ i.\ d.$$

Beginning with the cross-sectional variation, we produce N possible scenarios for M_{t+1}:

Formula 10.

$$M_{t+1,j} = M_{t+1} + \varepsilon_j \ \forall \ j \in [1,N]$$

where ε_j is a random sampling of the AR(1) error distribution.

We now have N possibilities for next year's income multiplier. We extend these possibilities across the T-year horizon of the VAAM optimization by combining the parameters of the autoregressive model with the same error sampling technique. Extending the prior notation:

Formula 11.

$$M_{i,j} = \mu + \theta \,(M_{i-1,j} - \mu) + \varepsilon_{i,j} \,\forall\, i \in [2,T] \,\forall\, j \in [1,N]$$

Where $\varepsilon_{i,j}$ is a random sampling of the AR(1) error distribution.

Appendix 4: About the Vanguard Capital Markets Model (VCMM)

IMPORTANT: The projections and other information generated by the Vanguard Capital Markets Model regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. VCMM results will vary with each use and over time.

The VCMM projections are based on a statistical analysis of historical data. Future returns may behave differently from the historical patterns captured in the VCMM. More important, the VCMM may be underestimating extreme negative scenarios unobserved in the historical period on which the model estimation is based.

The Vanguard Capital Markets Model® is a proprietary financial simulation tool developed and maintained by Vanguard's primary investment research and advice teams. The model forecasts distributions of future returns for a wide array of broad asset classes. Those asset classes include U.S. and international equity markets, several maturities of the U.S. Treasury and corporate fixed income markets, international fixed income markets, U.S. money markets, commodities, and certain alternative investment strategies. The theoretical and empirical foundation for the Vanguard Capital Markets Model is that the returns of various asset classes reflect the compensation investors require for bearing different types of systematic risk (beta). At the core of the model are estimates of the dynamic statistical relationship between risk factors and asset returns, obtained from statistical analysis based on available monthly financial and economic data from as early as 1960. Using a system of estimated equations, the model then applies a Monte Carlo simulation method to project the estimated interrelationships among risk factors and asset classes as well as uncertainty and randomness over time. The model generates a large set of simulated outcomes for each asset class over several time horizons. Forecasts are obtained by computing measures of central tendency in these simulations. Results produced by the tool will vary with each use and over time.

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