Quantum-Resistance Meets White-Box Cryptography: How to Implement Hash-Based Signatures against White-Box Attackers?

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Abstract. White-box cryptography challenges the assumption that the endpoints are trusted and aims at providing protection against an adversary more powerful than the one in the traditional black-box cryptographic model. Motivating by the fact that most existing white-box implementations focus on symmetric encryption, we present implementations for hash-based signatures so that the security against white-box attackers (who has read-only access to data with a size bounded by a space-hardness parameter M) depends on the availability of a white-box secure cipher (in addition to a general one-way function). We also introduce parameters and key-generation complexity results for white-box secure instantiation of stateless hash-based signature scheme SPHINCS+, one of the NIST selection for quantum-resistant digital signature algorithms, and its older version SPHINCS. We also present a hash tree based solution for one-time passwords secure in a white-box attacker context. We implement the proposed solutions and share our performance results.

Keywords: white-box cryptography · digital signature · white-box signature · quantum-safe signature · hash chain · one-time password · hash tree · SPHINCS+.

1 Introduction

The standard cryptographic model (black-box model) assumes the end points are trusted hence the secret keys in cryptographic implementations cannot be observed while they are in use. The first work that challenged this assumption is by Chow et al. [1] in 2002. The authors proposed an implementation of AES

algorithm to prevent secret key extraction even when an attacker has a full access to the execution environment. Although their specific implementation was later broken, their core idea of building up a key-dependent lookup table(s) from which the encryption (and decryption) could be performed without a need for the cryptographic key remains highly relevant. Later, some dedicated white-box ciphers have been designed with the same philosophy, which are not broken till now e.g., SPACE [2] and SPNbox [13].

White-box implementations have the objective of preventing extraction of cryptographic keys useful on a different platform. Instead of using keys directly, an attacker may attempt to isolate the complete implementation code from the environment, carry it to his own device and directly use it like a larger key. These so-called code lifting attacks are assumed to be mitigated by the help of the notion so called *space-hardness* (security against code lifting is quantified by the amount of code that needs to be extracted from the implementation). We also note that with additional software protection techniques such as device binding and code obfuscation, the use of software in other hardware may not be possible.

Up to now, white-box cryptography is mostly studied in symmetric encryption context. Proposals for white-box implementation of digital signature algorithms are rare and not sufficiently analyzed from security point of view. In our work, we present simple and elegant designs for white-box implementation of hash-based signatures and cryptographic primitives desirable in authentication protocols. Although known for a long time, hash-based signatures have received a new surge of interest due to their ability to remain post-quantum safe [3]. We contribute to the literature by presenting parameters for white-box secure instantiation of hash-based digital signatures including SPHINCS+ algorithm, which will become part of NIST's post-quantum cryptographic standard [22] so that the security against white-box attacker depends not more than the availability of a white-box secure pseudo-random function implemented as a cipher (in addition to a general one-way function). We also show a hash tree based alternative to the hash chain primitive (useful for entity authentication) to remain secure against white-box attackers on an untrusted environment.

The rest of our paper is organized as follows: Section 2 overviews Lamport's one-time signature scheme and section 3 presents its white-box secure implementation. Extensions to sign multiple messages is mostly based on Merkle tree. Section 4 generalizes our secure white-box implementation approach for signing multiple messages using Merkle tree. Section 5 presents an overview for the viability of new design ideas in hash based signature research in a white-box security context. Section 6 introduces WB-SPHINCS+ and WB-SPHINCS as stateless constructions with suggested parameters and performance figures for a white-box setting. Section 7 and section 8 proposes a white-box alternative to hash chains and time-based OTPs, respectively. Section 9 is for the analysis of security and space-hardness of the proposed schemes. Section 10 provides implementation details. Section 11 summarizes the earlier work. Finally, section 12 concludes our paper.

2 Lamport's Signature Scheme

Lamport's construction of one-time signature (OTS) is the first scheme which relies solely on one-way (hash) functions for its security [4]. Although the efficiency of this scheme has been improved in subsequent studies, for pedagogical reasons, we prefer to use it to explain our core idea for making hash-based signatures strong against white-box attackers.

As always, there are three algorithms defining Lamport's one-time signature scheme:

Let f be a one-way hash function with an output length of N.

Key Generation:
Input: *Parameters* L, N
L: *the length of random numbers*
$2N$: *total number of random numbers*
Output:
For one-time private key, generate:
$2N$ *L-bit random numbers* $r_1, r_2, \ldots, r_{2N}$
As one-time public key, compute:
$p_k = f(r_k)$ *for* $1 \leq k \leq 2N$ *(Distribute the public key securely as usual).*
As another more useful notation, random numbers (pre-images) and hash values (hash-images) could be indexed as follows, respectively:
$r_{i,j} (1 \leq i \leq N$ *and* $1 \leq j \leq 2)$ *and* $p_{i,j} (1 \leq i \leq N$ *and* $1 \leq j \leq 2)$

Signing:
Input: M
M: *message to be signed*
$h = f(M)$ *(h has a length of N)*
Output:
for $1 \leq s \leq N$ */* index value for bits of h */*
if $h_s = 0$ *reveal* $r_{s,1}$
else reveal $r_{s,2}$
as part of the signature

Verifying:
Input: *Parameters* $M', r'_{s,j}, p_{i,j}, h'$
M': *message received*
$r'_{s,j}$: *signature received* $(1 \leq s \leq N)$
$p_{i,j}$: *public key* $(1 \leq i \leq N$ *and* $1 \leq j \leq 2)$
$h' = f(M')$
Output:
Accept *if for each* $1 \leq s \leq N$
if $h'_s = 0 \quad h(r'_{s,1}) = p_{s,1}$
else $\quad h(r'_{s,2}) = p_{s,2}$
Reject *otherwise*

3 White-Box Implementation of Lamport's Scheme

Instead of storing all the random numbers constituting the one-time private key, one can use a cryptographically secure pseudo random function (PRF) to generate all the random numbers using a single secret (private) key. Rather than using a general-purpose PRF (practically implemented using standard block ciphers such as AES), we now introduce an implementation of Lamport's scheme secure in a white-box model [2] assuming that there is a white-box attack-resistant (secure) block cipher which also behaves as a PRF.

Let f be a one-way hash function with an output length of N.

Let E_K be a white-box secure block cipher (e.g., SPNbox [13]). E_K is represented as one big key-dependent lookup table denoted as $WBT\text{-}E_K$. We assume key K is securely erased after $WBT\text{-}E_K$ is ready.

Key Generation:

Input: *Parameters L, N, IP*

L: *the length of random numbers (as well as block length of E_K)*

$2N$: *total number of random numbers*

IP: *(randomly generated and stored) initial plaintext for $WBT\text{-}E_K$*

Output:

For one-time private key, generate $2N$ L-bit pseudo-random numbers:

for $1 \leq k \leq 2N$ $\quad r_k = WBT\text{-}E_K\ (IP + k)$

As the one-time public key, compute:

$p_k = f(r_k)$ for $1 \leq k \leq 2N$ (distribute the public key securely as usual).

For a more useful notation, random numbers (pre-images) and hash values (hash-images) could be indexed as follows, respectively:

$r_{i,j} (1 \leq i \leq N$ and $1 \leq j \leq 2)$ and $p_{i,j} (1 \leq i \leq N$ and $1 \leq j \leq 2)$

After the public key is generated, r_k values are securely erased.

Signing:

Input: M

M: *message to be signed*

$h = f(M)$ (h has a length of N)

Output:

for $1 \leq s \leq N$ / index value for bits of h */*

if $h_s = 0$ compute and reveal $r_{s,1} = WBT\text{-}E_k(IP + 2s - 1)$

else compute and reveal $r_{s,2} = WBT\text{-}E_k(IP + 2s)$

as part of the signature.

Verifying:

Same as the original case (nothing is changed).

4 Signing Multiple Messages using Merkle Tree

Lamport's OTS scheme is only useful for signing a single message per single public key hence its utility is quite limited. The problem of extending Lamport's OTS for multiple messages has already been extensively studied in the literature. Most of the proposed schemes are variations of the early work by Merkle [5].

In Merkle's original scheme, the tree is built in top-down for certification of additional OTS public keys i.e., starting with the root node, every node has three public keys, one for the message itself, one for the left child node, and one for the right child node. With this scheme, an infinite number of messages could be signed using a single root one-time public key. Another implementation choice for Merkle's scheme is adopting a bottom-up approach rather than top-to-bottom one to sign multiple but finite pre-determined number of messages. Here, first, the leaf N nodes (one-time private keys and corresponding public keys) are prepared. Then, using hash values of public keys, a binary tree is built. The final public key is the root of the node. See Fig. 1 for an example of Merkle tree with 8 leaf nodes.

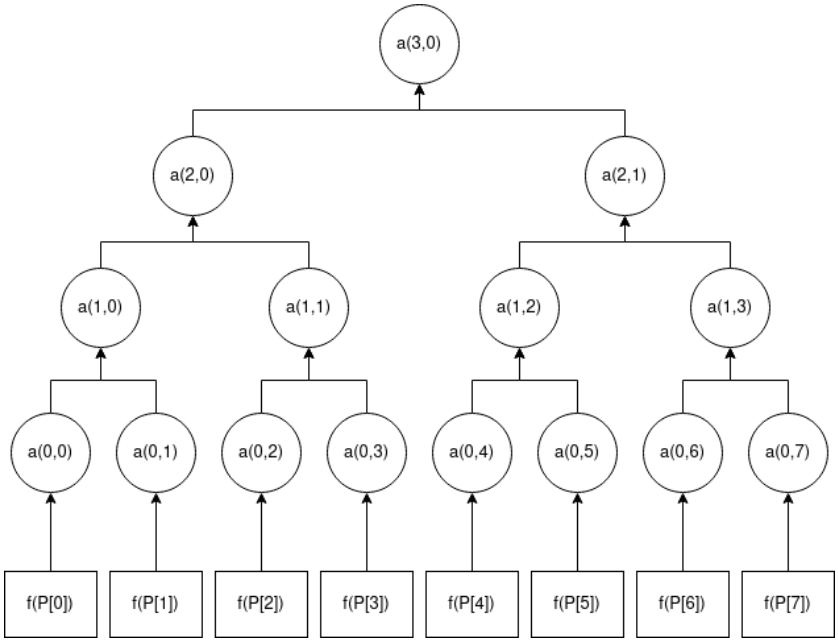

Fig. 1. Merkle tree to sign 8 messages.

Below, we first describe an insecure implementation of Merkle's scheme and then show how to make it secure in a white-box model.

Key Generation:

Input: *Parameters* L, N, IP, T

L: *the length of random numbers (as well as block length of* E_K*)*

$2N$: *total number of random numbers*

IP: *(randomly generated and stored) initial plaintext for* $WBT\text{-}E_K$ *(representation of* E_K *as one big key-dependent lookup table)*

T: *number of messages to be signed* ($T = 2^n$) (*n is the height of the tree)*

Output:

a. *for* $0 \leq t \leq T-1$

for $1 \leq k \leq 2N$

generate $2N$ *L-bit pseudo-random numbers:*

$r_{k,t} = WBT\text{-}E_K(IP + t \times 2N + k)$

compute $p_{k,t} = f(r_{k,t})$

For a more useful notation, random numbers (pre-images) and hash values (hash-images) could be indexed as follows, respectively:

$r_{i,j,t}(1 \leq i \leq N$ *and* $1 \leq j \leq 2$ *and* $0 \leq t \leq T-1)$

$p_{i,j,t}(1 \leq i \leq N$ *and* $1 \leq j \leq 2$ *and* $0 \leq t \leq T-1)$

b. *Generate hashes of public keys as follows:*

for $0 \leq t \leq T-1$

$P[t] = p_{1,1,t}||p_{1,2,t}||\ldots||p_{N,1,t}||p_{N,2,t}$

$a(0,t) = f(P[t])$

c. *Generate the root public key and distribute it securely (note that only a single hash value constitutes the public key here):*

As an example, consider the case given in Figure 1:

$a(3,0) = f(f(f(a(0,0)||a(0,1))||f(a(0,2)||a(0,3)))||f(f(a(0,4)||a(0,5))||f(a(0,6)||a(0,7))))$

We note that a naive implementation either requires the random numbers to be stored for later use or erase all data (except IP*) for later generation once needed (soon, we will show why both of these are insecure options).*

Signing:

Input: *Parameters* M, t

M: *message to be signed*

$t =$ *index of the leaf node for signing* $(0 \leq t \leq T-1)$

Output:

for $1 \leq s \leq N$ */* index value for bits of h */*

compute (assuming not already stored):

$r_{s,1,t} = WBT\text{-}E_K(IP + t \times 2N + 2s - 1)$

$r_{s,2,t} = WBT\text{-}E_K(IP + t \times 2N + 2s)$

if $h_s = 0$

reveal $r_{s,1,t}$ *and compute and reveal* $f(r_{s,2,t})$

else

reveal $f(r_{s,1,t})$ *and compute and reveal* $r_{s,2,t}$
as part of the signature.

Also additional nodes (hash values) up to the root node should be sent as auxiliary information to make it possible to compute and verify the root public key) (for instance $a(0,1), a(1,1)$ *and* $a(2,1)$ *should be sent for* $t = 0$ *in the example shown in Figure 1).*

Verifying:
Input:
M': *message received*
Signature received: $r'_{s,j,t}$ *or* $f'(r_{s,j,t})(1 \leq s \leq N)(0 \leq t \leq T-1)$ *and auxiliary information received e.g.,* $a'(0,1)$, $a'(1,1)$, $a'(2,1)$
Public key: e.g., $a(3,0)$
$h' = f(M')$
Output:
for each $1 \leq s \leq N$
if $h'_s = 0$
compute $f'(r'_{s,1,t}) = p_{s,1,t}$
else
compute $f'(r'_{s,2,t}) = p_{s,2,t}$

compute $a'(0,t) = f(p_{1,1,t}||p_{1,2,t}|| \ldots ||p_{N,1,t}||p_{N,2,t})$
Accept *if* $a(3,0) = f(f(f(a'(0,t)||a'(0,1))||a'(1,1))||a'(2,1))$ *(for t=0)*
Reject *otherwise*

Now, we show that the above scheme is not secure in a white-box model. The underlying reason is that all the random numbers are computed (or already stored) during signature generation although not revealed as part of the signature. This is required in order to compute the hash values for the random numbers not revealed. Hash values are sent as part of the signature to let the verifier to compute the value of $a'(0,t)$ and verify the signature. However, a white-box attacker having the ability to observe the internal state information could identify and extract all the random numbers and use them to forge a signature for any message he wants. Below, we show a slight change in the implementation to make it secure against white-box attacker.

The change we require is to prepare all the hash values required to build the signature once the message is ready, without a need to generate the random numbers not required as part of the signature itself [5]. For this purpose, after all the components of one-time public key are computed by $p_{k,t} = f(r_{k,t})$ for $1 \leq k \leq 2N$ and $0 \leq t \leq T-1$, the values of $p_{k,t}$ are stored on the signer side. Once a signature is required, signing algorithm is changed slightly as follows:

for $1 \leq s \leq N$ */* index value for bits of h */*

[5] We also note that auxiliary information could also be pre-computed to improve computational efficiency of signature generation.

if $h'_s = 0$
    compute $r_{s,1,t} = WBT\text{-}E_K(IP + t * 2N + 2s - 1)$
    reveal $r_{s,1,t}$ *and* $f(r_{s,2,t})$ /* $f(r_{s,2,t})$ *has been previously stored* */
else
    compute $r_{s,2,t} = WBT\text{-}E_K(IP + t * 2N + 2s)$
    reveal $f(r_{s,1,t})$ *and* $r_{s,2,t}$ /* $f(r_{s,1,t})$ *has been previously stored* */

With this change, a white-box attacker could not observe a random number required to forge a signature for any message different than the message the user has already signed.

To summarize, an implementation of hash-based signatures is not secure in a white-box model if any pre-image(s) not used in the signature itself is generated during signature generation or it is already stored after key generation is completed. While this might be just an implementation choice in some of the schemes (e.g., the above Merkle's scheme) (without any white-box security concern, one might still prefer different variations of the basic scheme for leveraging different storage-computation tradeoffs), in some others, secure implementations are not possible at all (e.g., Winternitz scheme with hash chaining [10]). The next section analyzes prominent design ideas in the rich literature of hash-based signatures in this context.

5 Research Progress in Hash-based Signatures and its White-Box Implications

In this section, we put a lense through which we analyze the research progress in hash-based signatures in a white-box security context. Instead of studying each proposal one by one, our focus will be on underlying design ideas. For the sake of brevity, we do not discuss these ideas in detail, which was already done elsewhere (e.g., [26]), and only give an assessment of their viability in a secure white-box implementation.

5.1 WOTS and its derivatives

One problem in Lamport's one-time signature construction described in section 2 is the large size of signatures. For a message of size N, it requires a signature composed of $2N$ random numbers. Later work (e.g., [15]) achieved a reduction of almost half in size by optimizing the mapping between the message to be signed and the random numbers to be revealed. The Winternitz One-Time Signatures (WOTS) could lead to an even shorter signatures by providing a trade-off between the signature time and the signature size [10]. The underlying idea is to apply a hash function to the random numbers not only once but iteratively w times, thus forming hash chains of length w (each chain is used to sign not only one bit but multiple bits, for instance a byte). A computed checksum is added to the message for security reasons. The message is signed together with the checksum. The checksum is added so that it becomes voided if a forged signature is

generated by computing the later elements of hash chains once the signature composed of selected elements of chains is revealed. On the other hand, in a white-box context, the attacker has an additional capability since he could go to either direction in hash chains (by reading the intermediate elements during hash computations). As a result, he could generate a forged signature without voiding the checksum (e.g., an increment in one byte of the message could be canceled out by a decrement in another byte).

In our work, our focus is on white-box secure constructions. As discussed in short, WOTS and its derivatives are not useful in a white-box setting, therefore we do not further discuss them here.

5.2 Few-time Signatures: HORS and others

So far, we have seen some constructions useful to sign only one message per one public-key. We remind that Merkle tree allows signing more than one message by increasing the number of public keys. How could we sign multiple messages securely with a truly single public key? One such early construction allowing to generate not one but several signatures using a single public key is named as HORS (Hash to Obtain Random Subset) [27].

Here, we provide only an informal overview of HORS construction and first revisit Lamport's scheme to remember why it is not secure to sign a second message with the same public key. Signing the first message means we reveal a random subset of N elements out of $2N$ random numbers. Similarly, signing a second message means we reveal another random subset of N elements out of $2N$ random numbers. We expect half of the N elements revealed for the second message is among those not revealed for the first one. As a result, $3N/2$ out of $2N$ random numbers are available to forge a third message (which has a non-negligible probability), therefore it is not secure to sign a second message.

In the HORS construction, the aforementioned security problem is solved by having t random numbers ($t > 2N$) and reveal k of these ($k < N$) as part of a signature. By this way, unlike Lamport's scheme, we could have probabilistic guarantees with a notion called *subset resilience* so that forging a signature for a new message is not feasible even though the same public key is used for more than one signature. Although security degrades with each additional signature, it could still be used to sign more than once. The exact number of signatures depends on selected parameters.

There are many extensions of HORS scheme including HORST (HORS with Trees) [23] and FORS (Forest Of Random Subsets) [28]. The underlying idea in HORST is to add a Merkle tree to HORS to reduce the public key size. In FORS, the key pair does not consist of a single monolithic tree but of k trees of height $log\, t$. HORS and its extensions could be used without any security concern in a white-box setting because a white-box attacker cannot have access to any additional secret values if they are preferred. We will discuss further details with suitable parameters while we introduce white-box versions of SPHINCS and SPHINCS+.

5.3 Hyper-tree / Tree Chaining

As mentioned in section 4, a secure implementation of Merkle tree is possible in a white-box setting with a bottom-up approach where all elements of public keys are made available during the key generation time and only the random numbers constituting the signatures are computed once the message becomes ready and nothing more.

On the other hand, implementing Merkle tree with a top-down approach (also known as certification tree) where the generation of one-time public keys could be deferred to the signature generation time is not secure in a white-box context. This is because of attacker's ability during key generation to obtain random number(s) useful to forge signatures.

Hyper-tree or Tree chaining is a way of combining two approaches (bottom-up and top-down) sketched above by using many layers of one-time keys. Keys on higher layers are used to sign the root values of lower layers prepared at any time. At the final layer, the keys are used to sign the messages. Although these variants bring significant advantages and flexibility over the basic Merkle tree construction, since they cannot be used securely in a white-box setting, we do not explore them here any further.

Table 1 summarizes our discussion on the suitability of hash based signature variants for a white-box secure implementation.

Table 1. Suitability of hash based signature variants for a white-box (WB) secure implementation.

Scheme	Idea	Purpose	Secure in WB setting?
Lamport [4]	Hash-based signature	The first scheme	√
Optimal OTS [15]	Encode message to random numbers optimally	Efficiency improvements	√
Merkle tree [5]	Combine one-time key pairs into a single tree structure	Signing more than once	√
Winternitz [10]	Use hash chains	Sign many bits at once	No
HORS(T) and FORS [27]	Subset resilience	Few-time signatures	√
Hyper-tree [23]	Many layers of trees	Efficiency improvements	No

6 Stateless Constructions

Stateful hash-based signatures such as those we present in section 4 require that OTS at each state must be used at most once. So, our signing algorithm starts from an initial state and increments it at each signature. This hinders multi-threading, load-balancing, and backup of the keys and brings additional complexities. Stateless hash-based signatures do not need to store any state while signing. However, stateless signatures are not as good as stateful signatures in performance. As we will see, especially in the white-box setting, due to WB cipher calls for key generation, stateless white-box signatures require significantly longer key generation times.

In this paper, we consider SPHINCS and SPHINCS+ for stateless white-box signatures. SPHINCS+ is chosen as a post-quantum signature standard by NIST [22]. SPHINCS, the ancestor of SPHINCS+, was published in 2015 [23]. In section 6.1 and 6.2, we investigate suitable parameters of SPHINCS+ and SPHINCS in the white-box model, respectively.

The multiple layers of trees require key generation calls for bottom layers. As we have already discussed, this is not secure in a white-box setting, therefore we need to work only with one layer (i.e., we set $d = 1$ for both SPHINCS and SPHINCS+). For a similar reason, signing with WOTS+ with $w > 1$ is not an option for us. As a result, we have few time signatures and a hash tree of their public keys. So our schemes are composed of the following building blocks:

- few time signatures (HORST for SPHINCS and FORS for SPHINCS+)
- hash tree of public keys of few time signatures
- WB ciphers with masked inputs

Table 2. SPHINCS+ parameters for white-box implementations.

Number of Maximum Signatures	Security Level	Parameters h	t	k	Signature Size	Key Generation Complexity (Number of WB-cipher calls)
2^{10}	128	0	11	96	18448	196608
	192	0	11	143	41208	292864
	256	0	11	191	73376	391168
2^{20}	128	8	12	197	41120	206569472
	192	8	13	145	48936	304087040
	256	8	13	194	87200	406847488

6.1 White-box SPHINCS+

As mentioned earlier, we have $d = 1$ and no WOTS+ in the scheme. Our scheme consists of 2^h few time signatures with FORS having parameters k and t. We get

help from the parameter search code of SPHINCS+ [24]. For PRF, we use WB-cipher which is much slower than hash functions. Hence, our complexity function counts only WB-cipher calls. In the key generation phase of the scheme, we have $2^h \cdot 2^t \cdot k$ WB-cipher calls. The signing algorithm has k WB-cipher calls. The verification algorithm has no WB-cipher calls. So signing and verification operations are fast as in the stateful settings. However, performance of key generation must be considered for practical applications. Consequently, we limit the maximum number of signatures as 2^{10} and 2^{20} because for larger values the schemes could be considered impractical. Table 2 shows suitable parameters for different security levels.

6.2 White-box SPHINCS

Similar to the WB-SPHINCS+, for white-box applications we set $d = 1$ and we use only few time signatures removing all WOTS options. We search parameters for 128, 192 and 256-bit security limited to 2^{10} and 2^{20} signatures. Table 3 shows suitable parameters.

Table 3. SPHINCS parameters for white-box implementations.

Number of maximum signatures	Security Level	Parameters			Signature Size(Byte)	Key Generation Complexity (Number of WB-cipher calls)
		h	t	k		
2^{10}	128	0	18	64	17792	262144
	192	0	19	64	28128	524288
	256	0	19	128	76416	524288
2^{20}	128	9	19	67	19808	268435456
	192	11	18	68	28680	536870912
	256	8	21	134	88640	536870912

6.3 Comparison of WB-SPHINCS+ and WB-SPHINCS

We opt to use WB-ciphers with 128-bit security level, therefore we compare the performance of white-box stateless hash-based signature schemes for 128-bit security using our implementation of WB cipher which will be discussed in section 10. For maximum 2^{10} number of signatures, WB-SPHINCS and WB-SPHINCS+ has key generation time around 0.14 and 0.1 seconds, respectively. These schemes have signature sizes of around 18KB.

For maximum 2^{20} number of signatures, WB-SPHINCS and WB-SPHINCS+ has key generation time around 146 and 112 seconds, respectively. These schemes have signature sizes of 20KB and 41 KB, respectively. All these schemes take less than a microsecond to sign and verify.

7 A White-Box Alternative to Hash Chains

A hash chain (proposed also by Lamport [6]) is a useful cryptographic primitive where a single shared (public) value is sufficient to verify securely the authenticity of a finite (but potentially large) number of different values. Besides a number of other applications, elements of hash chains could simply be used as one-time passwords (OTPs) for user authentication.

For a better grasp of the advantage of using a hash chain, let us first consider the case where a number of independent one-time passwords are generated on the user (client) side and the hash value of each is sent to the server as part of the initialization. Each OTP is sent one by one during normal operation to be verified by the server using hash values. The disadvantage here is that the storage requirement both on the server and client side increases linearly with the number of OTPs. (Besides, the risk is evident on the client side, any (white-box) attacker having access to the untrusted client machine could easily intercept all the OTPs to be used for later impersonation.)

Could we eliminate some of these problems with hash chains? A hash chain of length m is simply obtained by iteratively applying a one-way (hash) function to a randomly generated seed value for m times:

$$f^m(s) = \underbrace{(f \circ f \circ \cdots \circ f)}_{m \text{ times}}(s)$$

The final value $f^m(s)$ is sent to the server for initialization(registration). The first OTP used for authentication is the element just before the final value: $f^{m-1}(s)$. In this reverse order, in total $m-1$ OTPs could be generated and used while requiring only a single value on the server side for verification. On the client side, there are two options for the storage:

- The client could choose to generate and store all the elements in the hash chain. Later, once one of them is to be used, there will not be any need to do computation.
- The client chooses to store only the seed value and generates the required OTP by iteratively doing the required number of hash computations [6].

It is evident that the first option is not secure against a white-box attacker. It is also easy to see that the second option is similarly insecure simply because on the untrusted machine either the seed value itself (while one of the elements of the hash chain is generated) or other elements prior to a particular element could be intercepted by the white-box attacker for later use. We require a solution where OTPs could be generated independently so that white-box attacker could not gain any advantage even when he could fully observe the internal state of the client-side software. Below, we illustrate a solution for achieving this. In fact,

[6] An amortization technique could also be used to reduce memory-times-computation complexity [11].

the white-box implementation of Merkle's tree discussed in the previous section could be tailored to serve for our purposes so that each leaf node corresponds to the hash of a single random number rather than $2N$ random numbers.

Below, we only show the initialization phase (generation and verification of OTPs are skipped for the sake of brevity).

Initialization Phase:
Input: *Parameters* L, IP, T
L: *the length of random numbers (as well as block length of* E_K*)*
IP: *(randomly generated and stored) initial plaintext for* $WBT\text{-}E_K$
T: *number of OTPs* ($T = 2^n$) *(*n *is the height of the tree)*
Output:
a. *for* $0 \leq t \leq T-1$
$r_t = WBT\text{-}E_K(IP + t)$ /*generate L-bit pseudo-random numbers*/
$p_t = f(r_t)$ /* compute hash of the random numbers */

The values of p_t *also correspond to the leaf nodes of the tree (no need to compute the hash of* p_t *values) i.e.,* $p_t = a(0, t)$

b. *Generate the root node and distribute it securely.*

Table 4. Comparison of OTP schemes (for use of T times).

	Proposed Scheme	w/ Hash Chains	Independent OTPs
White-box resistant	$\surd$	No	No
Client-side computation per OTP	$O(1)$ $WBT\text{-}E_K$ $+O(log\,T)$ hash	None (if elements are stored)	None
Storage on client	$O(T)$	$O(T)$ (if elements are stored)	$O(T)$
Storage on server	$O(1)$	$O(1)$	$O(T)$
Communication cost per authentication	$O(log\,T)$	$O(1)$	$O(1)$
Initialization cost	$O(T)$ $WBT\text{-}E_K$ $+O(T)$ hash	$O(T)$ hash	$O(T)$ hash

To summarize, a Merkle's tree in which the leaf nodes are the hash values of a single random number could be a viable alternative to hash chains if white-box attackers are also of concern. In other words, no alternative provides this

feature. Table 4 compares our proposed scheme with hash chain based OTPs and independent OTPs.

8 White-Box Resistant Time-based OTPs

The idea of using a hash chain for one-time passwords was developed and implemented under the name of S/KEY [7]. As noted in [8], S/KEY has a number of undesirable properties. In particular, the scheme is vulnerable to an attack where the client reveals OTP(s) to attackers for future abuses by various means such as social engineering or by impersonating the server. The need and difficulty for synchronization of the chain between server and client is another concern (*i.e.*, determining which element is the next one).

A widely used solution for OTPs is implemented in the TOTP standard [9]. Here, the server and the client shares a secret key. Using the current time (usually increments in time steps of 30 seconds) as an implicit challenge of the server, this standard actually implements a simple challenge-response protocol. The client computes the MAC of the challenge and transmits the output (actually part of it) as the OTP response. The same computation could be done on the server side if a loose time synchronization is present. On the down side, TOTP depends on a secret stored both on the client and server, hence it is open to attacks on both sides.

To solve this problem for the server side, Kogan et al. proposed T/Key, a time-based OTP scheme [8]. The key idea in T/Key is to map each element of a hash chain to a specific time period so that OTPs are now time dependent [7]. However although no secret is stored on the server side, T/Key is vulnerable to a white-box attacker having access to the client side implementation. Below, we will show that the scheme we proposed in previous section could easily be made time-dependent just like TOTP and T/Key.

In fact, making our proposed scheme time-dependent requires no more than a mapping of each OTP (leaf node of the tree) to a pre-determined specific time period. For instance we could prefer a simple mapping starting from the leftmost and ending at the rightmost leaf node. Suppose we choose such a mapping for a Merkle tree exemplified with eight leaf nodes in Fig. 1. Suppose also the tree is already built and the root node is shared with the server. Then, the time-dependent OTP is generated as follows:

Time-dependent OTP Generation:
Input: *Parameters* I, t_{init}, t
I: *time step (e.g., 30 seconds)*
t_{init}: *setup time t (measured in I) (it has to be shared with the server side during registration)*
t: *current time (measured in I)*

[7] Additionally, in order to make the chain birthday-attack resistant, to generate each of its element, independent hash functions could be obtained from a single hash function using the idea of so-called *domain separation*.

Output:

$r_t = WBT\text{-}E_K(IP + t - t_{init})$ /* generate time-based OTP */

(In addition, nodes (hash values) up to the root node should be computed and sent as auxiliary information to make it possible to compute and verify the root public key) (for instance $a(0,1), a(1,1)$ and $a(2,1)$ should be sent for $t = t_{init}$ in the example shown in Figure 1).

On the down side, as compared to T/Key, one major efficiency drawback of the proposed scheme is that for a binary tree with 1×10^6 leaf nodes (valid approximately for one year), initialization (set up) requires 1×10^6 white-box encryption operations besides the hash operations. This drawback might be addressed by using a lightweight white-box encryption primitive (as further discussed in section 10).

Another concern is the increase in OTP sizes. On the other hand, we argue that additional communication cost for OTP transmissions in our proposed scheme is less of a concern especially in a use case where OTPs are sent to the online server without manual entry (with only with a simple confirmation tap in a mobile authenticator application). Note that the use of a traditional digital signature scheme in this use case might not be preferable due to a white-box attacker threat.

If manual entry of OTPs is performed using QR-codes (the phone displays a QR code containing OTP and the user scans it using his laptop camera) as proposed by T/Key inventors [8], our proposed scheme still seems a viable approach. It requires an OTP length of less than 1 KB for 128-bit security and 32 years of authentication period, which does not exceed the maximum capacity of QR codes [12] (also see Table 9 in section 10).

9 Security Analysis and Space-Hardness of the Schemes

In this section, we first provide a brief informal security analysis of the proposed schemes and then analyze their space-hardness properties.

9.1 Sketch of Security Analysis

A specific attacker goal against a white-box implementation of a hash-based digital signature is to obtain the private key (or part of it) to generate a signature for a message not intended to be signed by the legitimate user. This corresponds to any r_k values not revealed as part of the signature. The attacker has two options for achieving this:

- He could try to invert at least one of the hash images (the hash values in the public key).
- He could try to generate at least one of the unrevealed pseudo-random numbers (possibly using the stored IP value).

The first option is not possible due to the one-way property of the hash function used. Similarly, the second option is out of reach if a secure white-box block cipher is available which prevents to extract the key K from the lookup table. (the concern of code lifting will be addressed by the space-hardness analysis of the proposed schemes, given below.)

What if the attacker accesses the implementation environment before or while the key-dependent look-up table is built? Since the encryption key K is available in memory at that time, access to this key brings the ability to generate the whole one-time private key itself. We remind that this is also a legitimate concern for the symmetric encryption case but with a subtle difference. For encryption, the cryptographic key (therefore the key-dependent lookup table) is prepared to encrypt potentially infinite amount of plaintext messages. Hence the time window of vulnerability against a white-box attacker is short and acceptable (other precautions such as building the tables while the untrusted device is offline could be considered). On the other hand, if the lookup table is only used for signing a single message and building a second table is required thereafter, the risk against white-box attacker is significantly increased. In previous sections, we have already shown that a single look-up table could be used to sign multiple (potentially infinite) messages but there are some caveats that implementations should take into account.

9.2 Space-Hardness

For symmetric ciphers, the notion of space-hardness is introduced and a white-box model is defined in [2]:

Definition 1. *A cipher is said to be (M, Z)-space hard if it is infeasible for an adversary to encrypt (decrypt) a randomly chosen plaintext with probability more that 2^{-Z} given code (table) size less than M.*

Similarly, we define the space-hardness for signatures as follows:

Definition 2. *A signature scheme is said to be (M, Z)-space hard if it infeasible for an adversary to forge a (signed) message with probability more that 2^{-Z} given code (table) size less than M.*

According to the alternative and stronger White-Box Attack Context definition by Chow et al. [1], it is assumed that "internal details of cryptographic algorithms are both completely visible and alterable at will". This model is not applicable for the signature schemes as the attacker can change the message to be signed. That means all signature schemes are forgeable accordingly. As a result, we think Definition 2 makes more sense for signature schemes. Here, adversary has *read-only access* where the size of data accessed is bounded by M.

Claim 1 *Let S be the signing scheme described above with $T = 2^n$ number of messages to be signed, N is the message length and one-way hash function f*

with an output length of N. If the S uses (M, Z)-space hard cipher WBT-E_K, then the scheme (M, W)-space hard where

$$W = Z - \log_2(N) - n - 1$$

provided that N is not too small.

In the signature scheme, to forge a message, an adversary needs to find at least one of the preimages $f(r_{s,i,t})$ where $1 \leq s \leq N$, $1 \leq i \leq 2$ and $0 \leq t \leq T-1$. So there is $2N \cdot 2^n$ preimages which are encrypted by WBT-E_k. By the assumption each value can be obtained by the adversary with probability less than 2^{-Z}. Then the probability that at least one of them can be obtained by the adversary is

$$2N \cdot 2^n \cdot 2^{-Z} = 2^{-W}.$$

Here, an adversary can also try to find the an inverse of the hash function f by directly taking random hashes or using some weakness in hash function. Because of this, we assume the scheme uses a secure hash function with enough output length. For example, if SHA256 is used, then the adversary needs to take about $\frac{2^{256}}{2N \cdot 2^n}$ to find at least one of preimages $f(r_{s,i,t})$. For the white-box ciphers such as SPACE or SPNbox, the space hardness level is generally taken as $Z = 64$ or $Z = 128$. So, the security bottleneck of this scheme is explosion of the white box cipher.

For our white-box resistant OTP scheme, a similar calculation can be done.

Claim 2 *A white-box resistant OTP scheme using $T = 2^n$ number of passwords is $(M, Z-n)$-space hard if an (M, Z)-space hard cipher WBT-E_k is used, provided that the scheme is using secure hash function with enough output length.*

We know that SPNbox is $(M, -10 \cdot t \cdot \log_2(M/I))$-space hard for any given M where I is the size of the input-output table of the inner cipher and $t = 16, 8, 5, 4$ for SPNbox-8, -16, -24, -32 respectively [13]. Using the statements above we can calculate space-hardness of the signature and OTP schemes. We assume the signature scheme with $T = 2^{20}$ number of messages to be signed and OTP with $T = 2^{20}$ number of passwords. For $Z = 64$ and $Z = 128$, (M, Z)-space hardness of the schemes with SPNbox instantiations calculated in Table 2.

10 Implementation

We implement SPNbox algorithm [13] and the schemes introduced in section 4 and section 8. We also provide performance figures of white-box implementations of SPHINCS+ and SPHINCS schemes with our SPNbox implementation.

SPNbox algorithm is considered secure when evaluated in terms of various adversary models such as cache timing, key extraction and space-hardness. We have implemented the SPNbox algorithm for different configurations, the results are shown in Table 3 and Table 4 below (The results of SPNbox-32 are omitted due to high memory requirements).

Table 5. (M, Z)-space hardness of the proposed schemes with SPNbox instantiations.

$WBT\text{-}E_K$	Table Size	Signature Space-hardness		OTP Space-hardness	
	U	$Z = 64$	$Z = 128$	$Z = 64$	$Z = 128$
SPNbox-8	256 B	$U/2^{0.58}$	$U/2^{0.98}$	$U/2^{0.53}$	$U/2^{0.93}$
SPNbox-16	132 KB	$U/2^{1.16}$	$U/2^{1.96}$	$U/2^{1.05}$	$U/2^{1.85}$
SPNbox-24	50.3 MB	$U/2^{1.86}$	$U/2^{3.14}$	$U/2^{1.68}$	$U/2^{2.96}$
SPNbox-32	17.2 GB	$U/2^{2.33}$	$U/2^{3.93}$	$U/2^{2.10}$	$U/2^{3.70}$

While implementing the SPNbox algorithm, we aimed to increase the performance by creating ready-made tables for matrix multiplications in the nonlinear and linear layers. We have 4,5,8 and 16 independent table lookups in SPNbox-32, 24, 16 and 8, respectively. As a result, we achieved 15% performance improvement compared to other available implementations [14] (parallel instruction sets were not used).

Table 6. Software performance of the SPNbox cipher family on the Apple M1(ARM) processor in a white-box setting. Numbers are given in cycles per byte (cpb).

Algorithm	Rounds (outer)	Table Size	Performance [cpb]
SPNbox-24	10	50.3 MB	782
SPNbox-16	10	132 KB	109
SPNbox-8	10	256 B	1187

Table 7. Software performance of the SPNbox cipher family in a black-box setting on the Apple M1(ARM) processor. Numbers are given in cycles per byte (cpb).

Algorithm	Rounds (outer)	Rounds (inner)	Performance [cpb]
SPNbox-24	10	20	8216
SPNbox-16	10	32	11643
SPNbox-8	10	64	12394

While implementing the Merkle tree signature scheme, instead of producing 2N pre-images and hash-images for N-bit hash length, we preferred an optimized message mapping algorithm [15], thereby we achieved almost 50% of improvement in time and memory. Considering the time and memory usage performance, we decided to use the SPNbox-16 configuration in our signature and OTP im-

plementations. Performance results of our signature and OTP implementations are given in Table 5 and Table 6, respectively.

Table 8. Performance results of white-box signature scheme for different number of leaf nodes. Results obtained with the Apple M1(ARM) processor.(L-bit = 128, N-bit = 128, E_K = SPNbox-16)

Number of Leaf Node	Root Hash Generation	Memory	Signing	Verifying	Signature Size (B)
2^{10}	0.129 sec	2 MB	191 μs	153 μs	2272
2^{15}	4.14 sec	67 MB	258 μs	155 μs	2352
2^{20}	129.05 sec	2147 MB	262 μs	155 μs	2432

Table 9. Performance results of white-box one-time password scheme for different number of leaf nodes. Results obtained with the Apple M1(ARM) processor. (L-bit = 128, N-bit = 256, E_K = SPNbox-16)

Number of Leaf Node	Root Hash Generation	Memory	Generation	Verification	Size of OTP
2^{15}	0.041 sec	656 KB	4 μs	6 μs	496 byte
2^{20}	1.308 sec	17 MB	12 μs	9 μs	656 byte
2^{25}	44.11 sec	536 MB	91 μs	11 μs	816 byte

As a future work, we intend to further improve our performance results using AVX (Intel) and NEON (ARMv8) instructions.

Finally, we calculate the performance of white-box implementations of SPHINCS+ and SPHINCS schemes for different number of maximum signatures using our white-box SPNbox cipher implementation and present the results in Table 10.

Table 10. Calculated performance results of white-box implementations of SPHINCS+ and SPHINCS schemes for different number of max signatures.

	Number of max signautes	Key Generation	Memory (signer)	Signing	Verifying	Signature Size
SPHINCS+	2^{10}	25 sec	6 MB	<1 ms	<1 ms	18448
	2^{20}	7 hour	6 GB	<1 ms	<1 ms	41120
SPHINCS	2^{10}	33 sec	8 MB	<1 ms	<1 ms	17792
	2^{20}	9 hour	8 GB	<1 ms	<1 ms	19808

11 Related Work

Joye pointed out that one of the potential applications of white-box cryptography is to transform a MAC into a digital signature [16]. Here, the "MAC verification" algorithm is assumed to have a "certified" white-box implementation. Since the cryptographic key could not be extracted and cannot be used for the generation of a MAC, the implementation is only useful for the verification of (supposedly) a digital signature. However, this implementation choice is restricted in the sense that only those who have obtained a certified white-box implementation could perform the signature verification.

Zhang et. al. presented a white-box implementation of the identity-based signature scheme in the IEEE P1363 standard [17]. Feng et. al. proposed white-box implementation for the classical Shamir's identity-based signature scheme [18]. In recent work, Dottax et. al provided a deeper comprehension of the challenges of white-box ECDSA implementations [19]. Ma introduced a white-box Schnorr signature scheme [20] but provided only a limited security analysis.

Up to our best knowledge, our paper is the first study presenting a general-purpose quantum-safe digital signature algorithm in a white-box security model [8].

12 Concluding Remarks

To motivate our research, we consider a mobile authenticator application supporting multifactor user authentication. In this scenario, digital signatures and hash chains are preferable constructions since no secret information is required to be stored on the verification (server) side. On the other hand, a typical mobile authenticator application is installed on an untrusted client device vulnerable to attacks and therefore these client side attacks should also be considered in a more sophisticated yet realistic threat model. To protect software implementations in such an environment, in this paper, we presented white-box-resistant solutions for hash-based signatures and one-time passwords. We implemented these schemes and showed that these schemes are feasible in practice. Since they can be implemented using only symmetric crypto primitives, the proposed simple and elegant schemes are quantum-resistant and provide an important step in white-box cryptography.

[8] After the first version of our paper appeared in IACR ePrint in 2021, a white-box signature scheme based on multivariate polynomials was published [21]. We note that this new scheme requires 256 GB memory and 62 MB public keys for 80-bit security. We believe our scheme is more practical since it requires 16 B for the public key and the memory requirement is 2, 67, and 2147 megabytes for the schemes that can sign up to $2^{10}, 2^{15}$ and 2^{20} times, respectively.

Acknowledgments

This research (patent pending) is funded by TUBITAK (The Scientific and Technological Research Council of Turkey) under the grant No: 3191520. We thank Assoc. Prof. Dr. Murat CENK for helpful comments and discussion.

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