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Studying earthquake ground motion in Prague from Wiechert seismograph records

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With 12 Figures

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Summary

The Wiechert seismograms obtained at the Prague station were processed quantitatively. The processing was complicated by (i) interruptions of records, mainly in the vicinity of minute marks; (ii) geometric distortion of records due to the recording device, and (iii) the absence of knowledge on some recording device parameters (the seismograph is not available any more). The method for solving these problems was developed and is described in detail. The corrected records were used to compute approximate variations of the ground displacement $d(t)$, the velocity $v(t)$ and the acceleration $a(t)$. The following two factors made the computation difficult, namely a) the instrumental transfer functions for $d(t)$ and $v(t)$ tend to zero for $f \to 0$, and b) these transfer functions correspond only to the pendulum, not to the recording device as a whole. The complication (a) was removed by suitably modifying transfer functions in the low-frequency interval. The complication (b) was evaluated indirectly, its effect seems negligible.

1. Introduction

Many records of strong ground motion obtained by means of older types of seismographs with a mechanical recording device have been processed in qualitative way only. The quantitative processing has not been attempted since the opinion has prevailed that the problems associated with their digitization and with their correction for apparatus effects could not be solved satisfactorily. However, for certain regions of Czechoslovakia for example, high-quality records of strong ground motion are not yet available. Hence, the only accessible sources of information on strong ground motion are represented by macroseismic observations and, occasionally,by the above-mentioned historical records. In connection with extensive building projects (nuclear power plants etc.), the question of quantitative evaluation of the older records has become highly topical. Therefore, we have undertaken an extensive work with the aim to make the most of available historical records.

2. Procedures

In the processing records obtained by means of mechanical seismographs, we have met the following problems: 1. Many records were of rather poor quality which made digitization difficult, 2. practically all the records were in some way interrupted, most

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frequently in the vicinity of minute marks, 3. the records were geometrically distorted which was quite naturally associated with the device applied and, finally, 4. some parameters of the recording device were not known since the seismograph was not available any more.

If the above problems are satisfactorily solved, that means if complete and geometrically undistorted records are obtained, calculation of time dependences of ground motion displacement, velocity and acceleration can be attempted. However, the calculation is complicated by the following two factors: a) The displacement and velocity transfer functions have singularities for the frequency approaching zero, b) the transfer functions correspond only to the pendulum, not to the recording device as a whole.

We succeeded to develop a method enabling a satisfactory solution of most of the above problems. First, the method was applied to a set of records of strong ground motion obtained by the WIECHERT seismograph at the Prague station. In Prague, the macroseismic intensity of these motions amounted to 3 to 5 degrees of the MSK scale. The record processing was rather complicated since neither the seismograph itself nor its technical documentation was available. Recently, KOLÁŘ (1986) has applied this method to a more extensive set of records taken by the MAIŇKA seismograph located at the Hurbanovo station. In this case, the situation was more favourable: the seismograph has been available and, as a rule, the records were of much higher quality.

In the following, a brief description of the procedure used to evaluate a set of Prague records will be given.

The set is represented by 10 records of horizontal ground motion obtained during the period 1935–1967 by a mechanical WIECHERT seismograph of the pendulum mass of 1000 kg and the eigen period of about 10 s. The seismograph was equipped to record the time variation of ground motion displacement. In Fig. 1, a typical example of a WIECHERT record is illustrated.

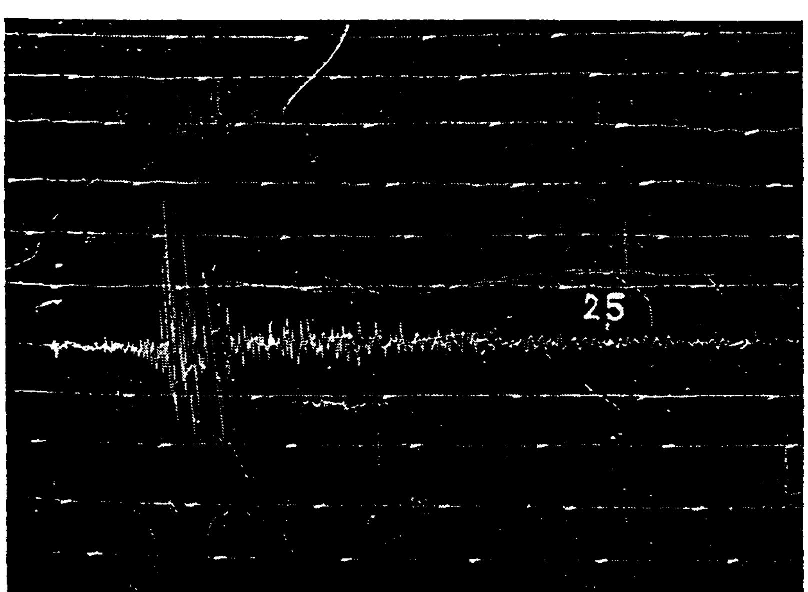

Fig. 1. A typical example of a WIECHERT seismograph record. The EW component of the German earthquake of June 27, 1935. Magnification 1.1

The records were magnified 20 times, manually re-drawn and, finally, digitized. Sufficient experience and skill is required to find the most adequate shape of the curve in the spots, where the original record is of rather poor quality.

As mentioned above, practically all the records were in some way interrupted. They contained gaps which frequently coincided with minute marks and appeared also in the spots of record damage. A number of ways was attempted to complete the missing parts of records. The suitability of these ways was tested as follows: First, the FOURIER spectrum was calculated for a real record without gaps. Second, gaps were artificially created in this record; then these gaps were filled in by the method the suitability of which was tested. The FOURIER spectrum of the record completed as described was calculated and compared with that of the original record. The suitability of the method was then inferred from the difference of both spectra. Surprisingly enough, a very simple method consisting in replacement of missing segments of the record by segments taken from the nearest vicinity of gaps yields the most satisfactory results. Of course, the replacement of missing segments of the record has to provide a continuous curve.

Further, the geometric distortion of the record had to be eliminated. The origin of the geometric distortion is shown in Fig. 2. The sheet of paper for recording the seismogram is placed on a cylinder of the radius r. This cylinder revolves slowly while the pen arm of the length R turns in point 0. The shape and the magnitude of the geometric distortion follow from the drum radius r, pen arm length R as well as from the dimension b and the angle α to be seen from the figure.

The geometric correction represents a transformation of the distorted records digitized in CARTESIAN coordinates $[x, y]$ into an undistorted time series $[t, y]$. Provided the angle α is small, the following correction relation can be derived (see Appendix)

$$t = \frac{60}{d} \cdot \left(x - r \cdot \operatorname{arsin} \frac{2r^2 + y^2 - 2by}{2ar} + r \cdot \operatorname{arsin} \frac{r}{a} \right), \qquad (1)$$

where

$$a = \sqrt{r^2 + R^2 - b^2}\,.$$

In this relation d is a minute mark spacing. The correction can be made easily if the parameters r, R and b are known. If the apparatus is available, the parameters r and R can be simply measured, whereas the parameter b can be measured only immediately

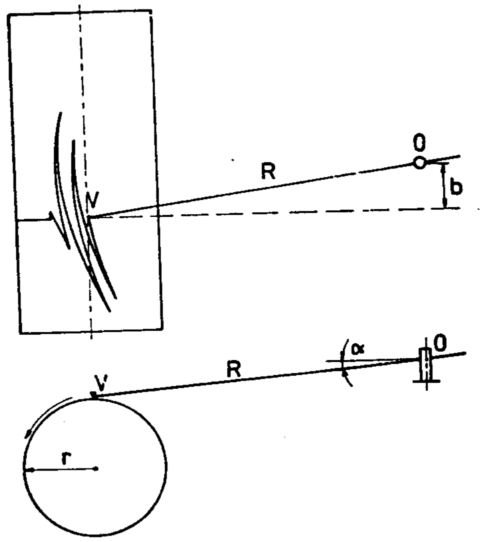

Fig. 2. The origin of geometric distortion

after the event since, as a rule, after the change of paper b changes as well. Thus, the parameter b is usually not known. That is why the slightly modified procedure described by Crouse and Trevor Matuschka (1983) was used in the present work. For each record its uptime and downtime were computed. The uptime and downtime were defined as the total times, the pen was moving in the positive and negative directions, respectively. For the parameter b such a value was sought to which a minimum absolute value of the difference between uptime and downtime would correspond. The procedure was based on a trial-and-error method involving eq. (1). In Fig. 3, values of the uptime are plotted against those of the downtime. The points corresponding to non-corrected records are represented by cross symbols, while those corresponding to corrected records are represented by full-circle symbols. It can be seen that the correction led to a result which is more than satisfactory, especially when the fact is taken into account that neither b nor r and R were known in the case under consideration. (As mentioned above, the seismograph does not exist any more so that only the records mostly of rather poor quality were available.) An example of a record geometrically corrected as described above is shown in Fig. 4.

The completed and geometrically corrected records were then used to calculate approximate time variations of the displacement, the velocity and the acceleration.

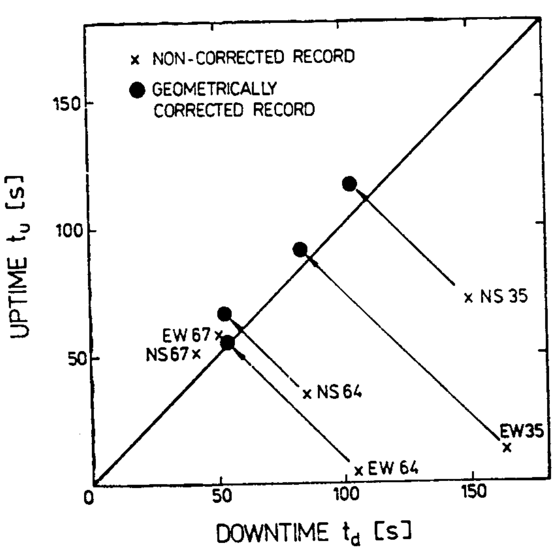

Fig. 3. Uptime versus downtime of Wiechert records before and after the geometrical correction

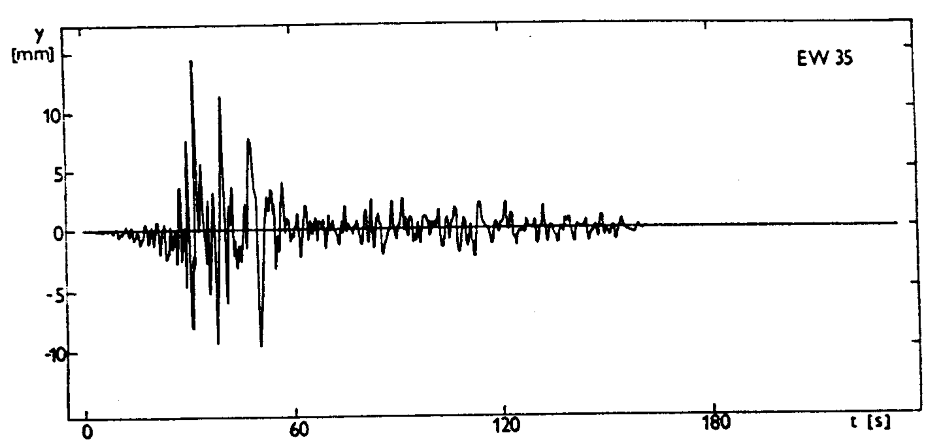

Fig. 4. The completed and geometrically corrected record of the EW component of the German earthquake of June 27, 1935. C. f. Fig. 1

For this calculation the equation of the mechanical seismograph was used as the basic one. This equation to be solved for $\zeta(t)$ is as follows

$$\ddot{y} + 2\varepsilon\dot{y} + (2\pi f_0)^2 y = -V_0\ddot{\zeta}\,, \tag{2}$$

where $y(t)$ relates to the corrected record, $\zeta(t)$ is the real time variation of the displacement of ground motion, V_0 the static magnification, f_0 the eigen frequency of the undamped seismograph and ε is the constant of seismograph damping. Applying to eq. (2) the FOURIER transformation the following relations are obtained for FOURIER patterns of the displacement, the velocity and the acceleration

$$\mathfrak{F}\zeta = \frac{P}{V_0}\cdot\mathfrak{F}y\,, \qquad P = \frac{f_0^2 - f^2}{f^2} + \frac{\varepsilon i}{\pi f}\,, \tag{3}$$

$$\mathfrak{F}\dot{\zeta} = \frac{P_v}{V_0}\cdot\mathfrak{F}y\,, \qquad P_v = -2\varepsilon - \frac{2\pi\cdot i(f^2 - f_0^2)}{f} \tag{4}$$

and

$$\mathfrak{F}\ddot{\zeta} = \frac{P_A}{V_0}\cdot\mathfrak{F}y\,, \qquad P_A = 4\pi^2(f^2 - f_0^2) - 4\pi\varepsilon i f\,, \tag{5}$$

respectively. In eqs. (3), (4) and (5) the FOURIER pattern of the function F is denoted $\mathfrak{F}F$, $F = y, \zeta, \dot{\zeta}, \ddot{\zeta}$.

It might seem that the problem has been solved since the real time variation of the displacement, the velocity and the acceleration can be obtained by inverse FOURIER transformation. However, this is possible for the acceleration only. In fact, from Fig. 5 it can be seen that the modulus of the function P [eqs. (3)—(5)] in the expression for the displacement (curve D) and for the velocity (curve V) tends to infinity for the frequency approaching zero. It is clear that the inverse FOURIER transformation cannot be made under these conditions. In the present work, this problem was solved by a proper modification of the curve D in the region of the lowest frequencies. Various ways of modification of the function P were tested. The examples of some of them are illustrated in Fig. 6. In this figure, the dashed line represents the original curve, while the curves labelled *1*, *2* and *3* correspond to three ways of modification tested. Way 1 and Way 2 seem rather logical, however, they did not prove acceptably. The reasons will become clear later. On the other hand, Way 3 was found to provide satisfactory results. At inverse FOURIER transformation each of the above modifications of the function P introduces disturbances into the record. This can be seen from Fig. 7. However, the disturbance introduced by Way 3 is quite tolerable since it has a regular harmonic character which is not difficult to remove. The chosen curve labelled *3* is described by the following relation:

$$P' = 4\pi^2(f^2 - f_0^2) - 4\pi\varepsilon i f \quad \text{for} \quad f \in \left\langle 0\,, \frac{1}{2\pi}\right\rangle,$$
$$P' = P \quad \text{for} \quad f > \frac{1}{2\pi}\,. \tag{6}$$

Thus, the function P' was used instead of the function P [in eq. (3)] for the calculation of time variations of the displacement. This modification results in a suppression of

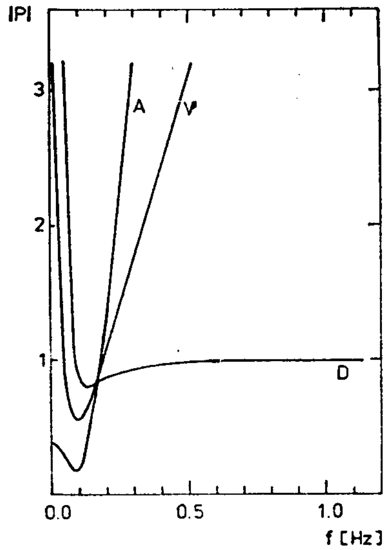

Fig. 5. Modulus of function P [eqs. (3)–(5)] for displacement (curve D), velocity (curve V) and acceleration (curve A)

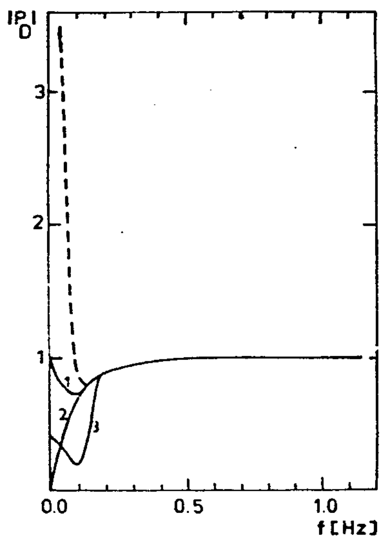

Fig. 6. Examples of tested modifications of the function P. Dashed line represents the original curve. The curve labelled 3 was accepted as the most appropriate one

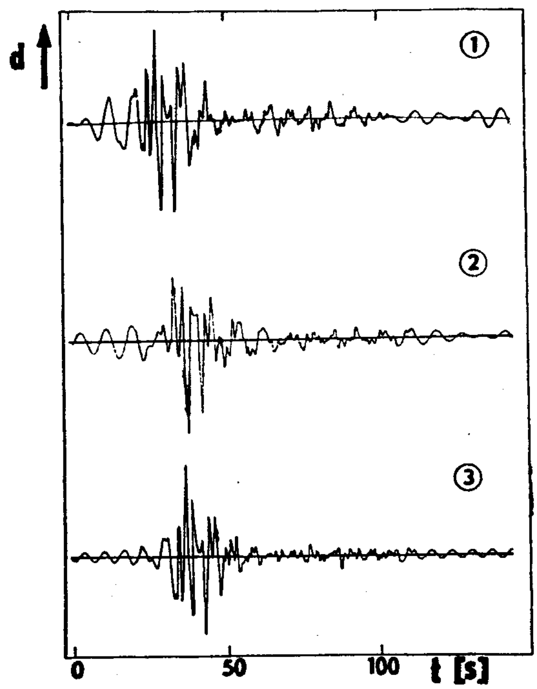

Fig. 7. The disturbances introduced by various modifications of the function P (see Fig. 6)

low-frequency components of the spectrum. However, from the point of view of engineering applications, this fact is immaterial.

The velocity was obtained as the derivative of the time dependence of the displacement. The acceleration was determined without any modification of the transfer function by inverse FOURIER transformation, eq. (5). An example of a time variation of the displacement, the velocity and the acceleration calculated as described above is illustrated in Fig. 8.

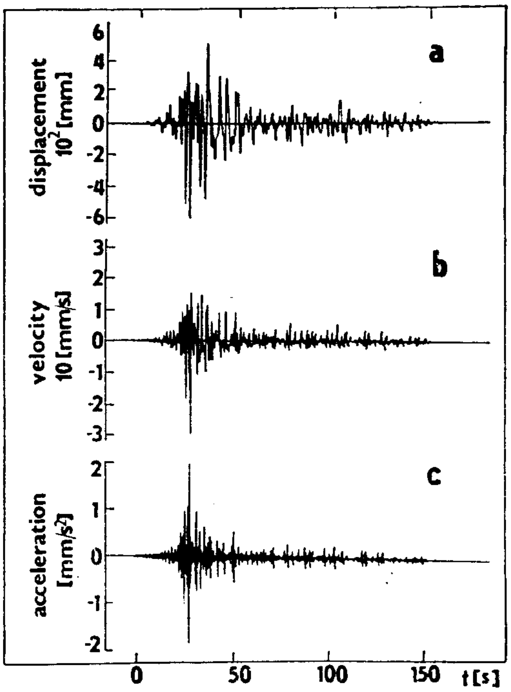

Fig. 8. Calculated time variations of displacement (a), velocity (b) and acceleration (c). The EW component of the German earthquake of June 27, 1935

3. Results and discussion

The spectra of the displacement of all evaluated records are of markedly low frequency. As it can be seen from the example shown in Fig. 9, the main maxima lie at frequencies below 1 Hz. Such a character of the spectra could be expected since we have dealt with records of Alpine earthquakes of epicentral distances over 200 km. On the other hand, a possibility cannot be completely excluded that there is another reason of the low frequency character of displacement spectra. The transfer function used in calculation of the displacement spectra corresponds to the pendulum only. Possible effects of the recording device (particularly of such parts of this device as pen arm and pen pivot) could not be reliably accounted for. Therefore, one cannot exclude the possibility that it was just these parts of the recording device that were responsible for the absence of high frequency motion on the records. However, the spectra obtained in the present work are believed to be essentially correct. This is supported by the fact that many records obtained by means of mechanical seismographs are available that show much higher frequencies.

While the time variation of the displacement can be considered reliable, one has to admit that this need not be the case of time variations of the velocity as well as the acceleration. In fact, because of a high level of digitization noise the analysis had to be restricted to frequencies not exceeding 2 Hz. A typical velocity spectrum is shown in Fig. 10. In Fig. 11, a typical spectrum of the acceleration is illustrated. In both cases, the spectra were cut off at a frequency of 2 Hz. It seems obvious that the

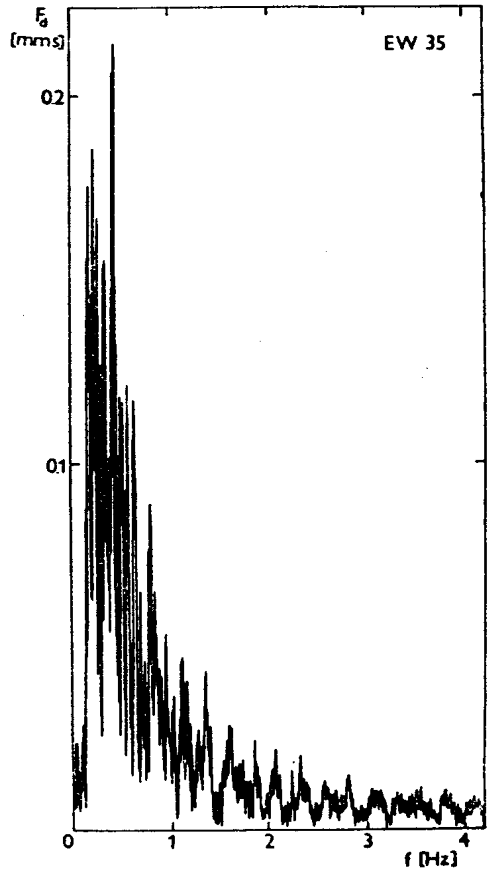

Fig. 9. A typical example of the calculated FOURIER amplitude displacement spectrum

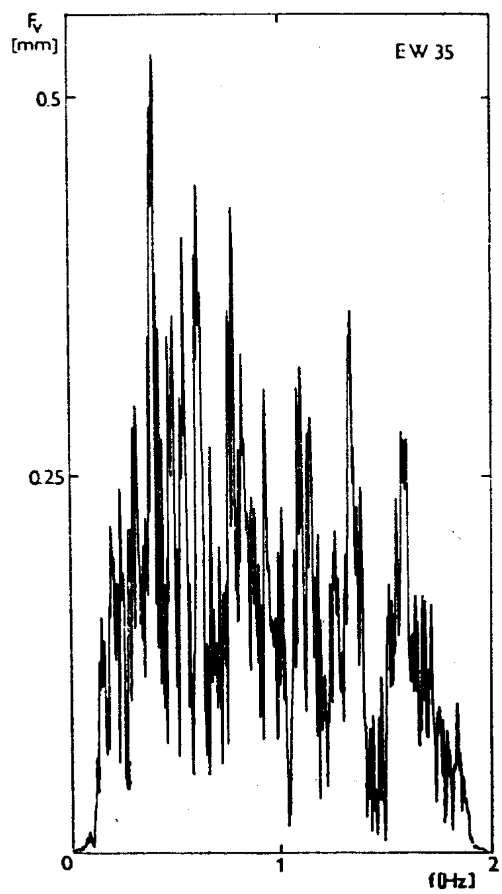

Fig. 10. The same as in Fig. 9 for amplitude velocity spectrum

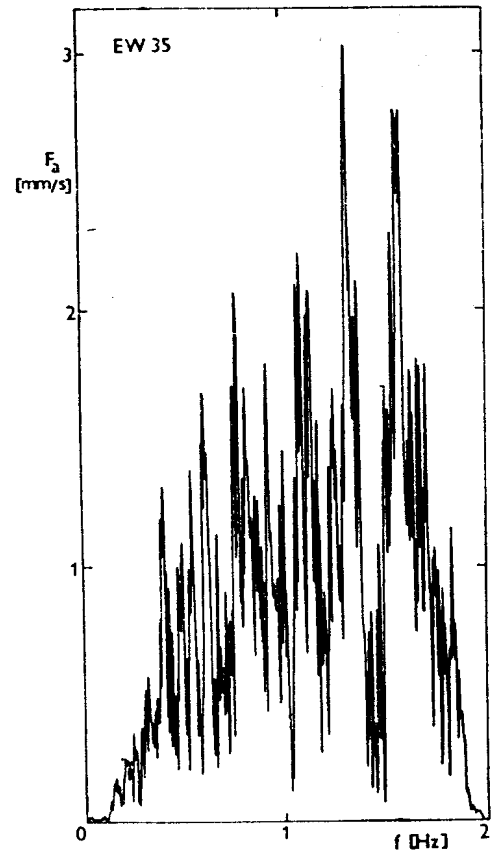

Fig. 11. The same as in Fig. 9 for amplitude acceleration spectrum

existence of the maxima at frequencies higher than 2 Hz cannot be excluded. Especially, this concerns the spectrum of the acceleration. From the point of view of engineering applications, this partly disqualifies the obtained time variations of the velocity and, especially, the acceleration.

To conclude, it should be mentioned that the method developed in the present work was recently applied by KOLÁŘ (1986) to analyse a set of records (of higher quality as compared to those analysed in the present work) originating from the MAINKA seismograph at the Hurbanovo station. As to the velocity spectra, this author obtained rather reliable results up to a frequency of 4 Hz.

Appendix

Consider CARTESIAN system (x', y, z) shown in Fig. A1. The coordinates of the point of rotation 0 are $(a, b, 0)$. If the registration drum does not rotate, the pen draws a curve which represents an intersection of the cylindrical surface

$$x'^2 + z^2 = r^2 \tag{A1}$$

and spherical surface

$$(x' - a)^2 + (y - b)^2 + z^2 = R^2 \,. \tag{A2}$$

From eqs. (A1) and (A2) the relation

$$x' = \frac{r^2 + a^2 + (y - b)^2 - R^2}{2a} \tag{A3}$$

follows. The coordinate x' can also be expressed by means of angle φ_y shown in Fig. A1 from which it can be seen that

$$x' = r \cdot \sin \varphi_y \,. \tag{A4}$$

From eqs. (A3) and (A4) one obtains

$$\varphi_y \doteq \arcsin \frac{r^2 + a^2 + (y - b)^2 - R^2}{2a \cdot r}. \tag{A5}$$

The angle φ_0 (Fig. A1) follows from eq. (A5) setting zero for y:

$$\varphi_0 = \arcsin \frac{r^2 + a^2 + b^2 - R^2}{2a \cdot r}. \tag{A6}$$

The difference $\varphi_y - \varphi_0$ represents the geometric distortion which arises from the motion of the pen from the initial position P_1 $(y = 0)$ to a position P_2 $(y \neq 0)$. On the record, this is represented by the increment of coordinate x by $\xi(y) = r \cdot (\varphi_y - \varphi_0)$. Hence, if (x_i, y_i) represent points of the digitized record and d is the minute marks spacing, the correction formula is

$$t_i = \frac{60}{d} \cdot \left(x_i - r \cdot \arcsin \frac{r^2 + a^2 - R^2 + (y_i - b)^2}{2a \cdot r} + \right.$$
$$\left. + r \cdot \arcsin \frac{r^2 + a^2 - R^2 + b^2}{2a \cdot r} \right). \tag{A7}$$

When the angle α is small, a simpler formula

$$t_i = \frac{60}{d} \cdot \left(x_i - r \cdot \arcsin \frac{2r^2 + y^2 - 2y \cdot b}{2a \cdot r} + r \cdot \arcsin \frac{r}{a} \right),$$
$$a = \sqrt{r^2 + R^2 - b^2} \tag{A8}$$

can be used instead of eq. (A7). This was done in Section 2, eq. (1).

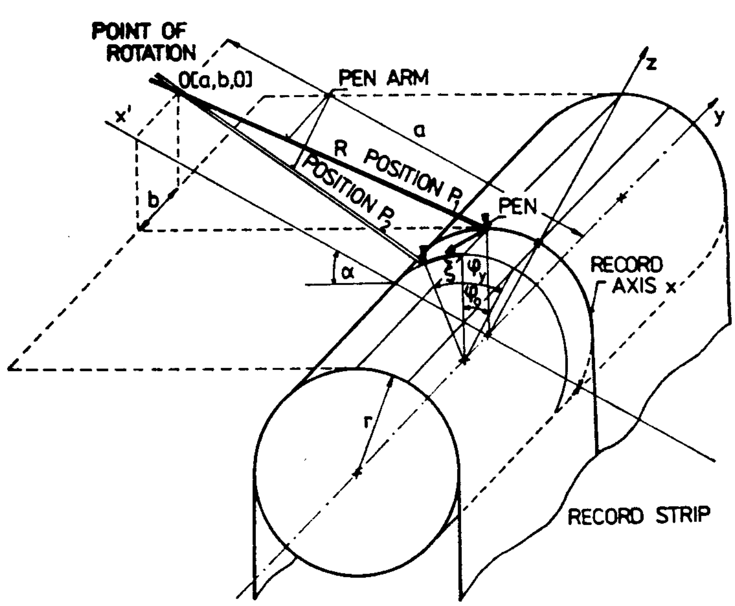

Fig. A1. The geometry of pen offset for the WIECHERT seismograph

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