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Formation Flying Satellites: Control By an Astrodynamicist

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Abstract. Satellites flying in formation is a concept being pursued by the Air Force and NASA. Potential periodic formation orbits have been identified using Hill's (or Clohessy Wiltshire) equations. Unfortunately the gravitational perturbations destroy the periodicity of the orbits and control will be required to maintain the desired orbits. Since fuel will be one of the major factors limiting the system lifetime it is imperative that fuel consumption be minimized. To maximize lifetime we not only need to find those orbits which require minimum fuel we also need for each satellite to have equal fuel consumption and this average amount needs to be minimized. Thus, control of the system has to be addressed, not just control of each satellite. In this paper control of the individual satellites as well as the constellation is addressed from an astrodynamics perspective.

Keywords: Formation Flying, astrodynamics, satellite theory

1. Introduction

Previous studies on the relative motion of spacecraft in Earth orbit have typically used the Clohessy-Wiltshire (CW) equations (Carter, 1998; Miller, 1999; Kapila, 1999; Xing, 1999) to describe the relative equations of motion. With these linearized equations periodic motions in the relative motion reference frame have been identified. These periodic motions include in-plane, out-of-plane, and combinations of these two motion types. When one includes perturbations, some of these periodic orbits are no longer achievable without control to overcome the deviations. A simple example demonstrates this fact. Consider an out-of-plane relative motion caused by a difference in inclination angles. Due to the J_2 perturbation, the inclination difference will cause a differential nodal precession rate between the two satellites resulting in an oscillatory out-of-plane motion with increasing amplitude. However, the linear CW equations do not show this motion; they indicate an out-of-plane oscillatory motion with a constant amplitude. To maintain a relative orbit designed with the CW equations, periodic orbit corrections are necessary to cancel deviations caused by the J_2 perturbations. Further, a reference motion and the accompanying state transition matrix might result in an out-of-plane control that changes inclination because the state transition matrix does not indicate the increasing amplitude caused by the inclination difference. For these reasons it is necessary for the reference motion to include at least the J_2 gravitational perturbation effect. The satellites considered are assumed to be equal in size and shape. Therefore, compared to the J_2 effect, the differential drag and solar radiation effects are of lesser importance in this study and are neglected. In other formation flying scenarios they may be the dominant perturbations.

For two satellites to remain close together their periods, nodal precession rates,and perigee drift rates must be equal. For close satellits the only way for this to happen is for the two satellites to be in the same orbit. therefore, one of the conditions must be relaxed. For small eccentricities the differential perigee drift has the least effect and is the easiest to control, therefore we relax this condtion. The resulting orbits that match period and right ascension rate we call c. First we derive the conditions for these orbits. Then, we consider control of the constellation, not just one satellite relative to the Chief. For some relative motion orbits J_2 Invariant Orbits are not practical and the differential nodal precession rate must be controlled. Since this rate is proportional to the difference in inclination the fuel required to control this rate is likely to be different for each satellite. The system lifetime will be defined by the failure of the first satellite, therefore it is desirable that each satellite have the same fuel consumption. This is the focus of our constellation control.

2. J_2 Invariant Orbits

The orbit geometry is described through the Delaunay orbit elements l (mean anomaly), g (argument of perigee) and h (longitude of the ascending node) with the associated generalized momenta L, G and H defined as

$$L = \sqrt{\mu a}, \qquad G = \sqrt{\mu a(1-e^2)} = L\eta, \qquad H = G\cos i \tag{1}$$

where a is the semi-major axis, e is the eccentricity and i is the inclination angle.

From Brouwer's (Brouwer, 1959) orbit theory the mean anomaly, argument of perigee and right ascension secular rates with $\epsilon = -J_2$ are

$$\dot{l} = \frac{1}{L^3} + \epsilon \frac{3}{4L^7} \left(\frac{L}{G}\right)^3 \left(1 - 3\frac{H^2}{G^2}\right) = \frac{1}{L^3} + \epsilon \frac{3}{4L^7 \eta^3} \left(1 - 3\cos^2 i\right) \quad (2)$$

$$\dot{g} = \epsilon \frac{3}{4L^7} \left(\frac{L}{G}\right)^4 \left(1 - 5\frac{H^2}{G^2}\right) = \epsilon \frac{3}{4L^7 \eta^4} \left(1 - 5\cos^2 i\right) \quad (3)$$

$$\dot{h} = \epsilon \frac{3}{2L^7} \left(\frac{L}{G}\right)^4 \left(\frac{H}{G}\right) = \epsilon \frac{3}{2L^7 \eta^4} \cos i \quad (4)$$

For two satellites to remain close the secular growth of the three angles of the two satellites must be equal. When described by mean elements as above these secular rates are a function of the momenta or semi-major axis, eccentricity and inclination. When described by osculating elements they are a function of all the orbital elements. Therefore, definition of the relative motion orbits is best described by mean elements. The transformation to osculating elements yields the initial conditions.

At any instant of time, the current inertial position and velocity vectors can be transformed into corresponding instantaneous orbit elements. In the absence of perturbations, these elements are constants. Adding the J_2 perturbation causes the elements to vary according to three types of motion, namely secular drift, short period motion and long period motion. The long period term is the period of the apsidal rotation. Over a short time this looks like a secular growth of order J_2^2. The short period growth manifests itself as oscillations of the orbit elements, but does not cause the orbits to drift apart. The relative secular growth is the type of growth that needs to be avoided for relative orbits to be J_2 invariant. This growth is best described through *mean orbit elements*. These are orbit averaged elements which do not show any of the short period oscillations. Mean elements can be obtained analytically or numerically. Highly accurate mean elements that must include atmospheric drag, tesseral harmonic and third body effects probably require numerical averaging. In this paper we use an analytical approach to help determine the accuracy that will be required. By studying the relative motion through the use of mean orbit elements, we are able to ignore the orbit period specific oscillations and address the secular drift directly. It is not possible to set the drift of each orbit to zero. However, instead we choose to set the difference in mean orbit element drifts to zero to avoid *relative secular growth*.

Using the fact that $\delta L = O(\epsilon)$ the resulting equations for the angle rate differences are

$$\delta\dot{\theta} = -\frac{3}{{L_0}^4}\delta L - \epsilon\frac{3}{4L_0^7\eta_0^5}\left[3\eta_0(1-3\cos^2 i_0) + 4(1-5\cos^2 i_0)\right]\delta\eta + \epsilon\frac{3}{2L_0^7\eta_0^4}(3\eta_0+5)\cos i_0 \sin i_0 \delta i \quad (5)$$

$$\delta\dot{g} = \epsilon\frac{3}{L_0^7\eta_0^5}[-2(1-5cos^2 i_0)\delta\eta + 5\eta_0 sini_0 cosi_0 \delta i] \quad (6)$$

$$\delta\dot{h} = -\epsilon\frac{3}{2L_0^7\eta_0^5}[4cosi_0)\delta\eta + \eta_0 sini_0\delta i] \quad (7)$$

For the relative motion orbits to not drift apart the relative rates given by eqs. (5)-(7) must be zero. Unfortunately, the only solution is the trivial solution $\delta L = \delta\eta = \delta i = 0$. Therefore, we can select only two conditions and the best decision is to control the perigee drift and let $\delta\dot{\theta} = \delta\dot{h} = 0$. These conditions lead to the orbits called J_2 *Invariant Orbits*. The conditions are

$$\delta\eta = -\frac{\eta_0}{4}tani_0\delta i$$
$$\delta L = -\frac{\epsilon}{4L_0^4\eta_0^5}(4+3\eta_0)(1+5cos^2 i_0)L_0\delta\eta \quad (8)$$

Two problems can arise when imposing these conditions. The first is the required large change in eccentricity (η) in near polar orbits resulting from the $tani_0$ term. The second problem occurs in near circular orbits. Since $\eta^2 = (1-e^2)$, $\delta\eta = -(\eta/e)\delta e$ and large changes in eccentricity are required to counter the differential nodal precession. If the required changes in eccentricity result in unacceptable relative motion orbits it is best to invoke the condition that the projection of the deputy angular velocity vector along the chief orbit normal be zero. That is,

$$\delta\dot{\theta} + \delta\dot{h}cosi_0 = 0 \quad (9)$$

This leads to the condition

$$\delta L = \frac{\epsilon}{4L_0^3\eta_0^5}(3\eta_0+4)[\eta_0 sin2i_0\delta i + (3cos^2 i_0 - 1)\delta\eta] \quad (10)$$

3. Constellation Control

As shown in the previous section the increase in the out of plane displacement caused by J_2^2 cannot be countered in some cases by small changes in the orbital element. From Gauss' variational equations it is easily shown that the Δv to counter this growth is

$$\Delta v = 3\pi J_2(\frac{R_e}{R})^2 v sin^2 i\delta i \quad (11)$$

Thus, the fuel consumption is a function of the differenee in inclination. This means that the satellites in constellations that have out of plane motion will have different fuel maintenance requirements. From a lifetime and design viewoint it is desirable that all satellites consume the same amount of fuel over the system lifetime. To demonstrate the concept for that results in equal fuel consumption for each satellite we will use as an example the relative motion orbit whose projection in the horizontal plane is a circle of radius ρ. Referring to Figure 1, which is a snapshot taken at the Chief's equator crossing, the in track and out of plane displacements can be expressed as

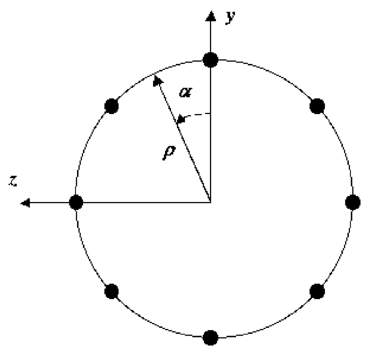

Figure 1. Horizontal Plane Snapshot When the Chief is at the Equator

$$\begin{aligned} x &= 0.5\rho sin(\psi_0 + \alpha) & \dot{x} &= 0.5\rho n_0 cos(\psi_0 + \alpha) \\ y &= \rho cos(\psi_0 + \alpha) & \dot{y} &= -\rho n_0 sin(\psi_0 + \alpha) \\ z &= \rho sin(\psi_0 + \alpha) & \dot{z} &= \rho n_0 cos(\psi_0 + \alpha) \end{aligned} \quad (12)$$

where ψ_0 is the chief argument of latitude and $\psi_0 = n_0 t$, the subscript 0 refers to the chief satellite and $t = 0$ occurs at the equator. Also

$$n_0 = \dot{\theta}_0 + cos i_0 \dot{\Omega}_0 \quad (13)$$

In the presence of J_2 and with the constraint in eq. (9) the modified CW equations (Carter, 1998)are

$$
\begin{aligned}
\ddot{x} - 2n_0\dot{y} - 3n - 0^2 x &= u_x \\
\ddot{y} + 2n_0\dot{x} &= u_y \\
\ddot{z} + n_0^2 z &= u_z + 2An_0 cos\alpha sin\psi_0
\end{aligned} \tag{14}
$$

where

$$
A = 1.5J_2 n_0 (R_e/a_0)^2 \rho sin^2 i_0 \tag{15}
$$

The forcing frequency alongthe z-axis is very nearly equal to the natural frequency causing resonance. We detune the z-oscillator by introducing a small constant value for $\dot{\alpha}$ such that every satellite spends an equal amount of time with the same values of δi and $\delta\Omega$. In this way, the fuel consumption is balanced among all the satellites. The next question is what is the value of $\dot{\alpha}$ that minimizes the fuel consumption. Assuming $\dot{|}alpha$ is very small the controls necessary to rotate the formation and perfectly cancel the J_2 disturbance are

$$
u_x = 2n_0\dot{\alpha}x_r, \quad u_y = -n_0\dot{\alpha}y_r, \quad u_z = -2n_0(\dot{\alpha}z_r + 2Acos\alpha sin\psi) \tag{16}
$$

where x_r, y_r and z_r are given by eqs. (12). The average quadratic cost per satellite, considering an infinite number of satellites, over one orbit of the Chief can be represented by

$$
J = \frac{n_0}{4\pi^2}\int_0^{2\pi/n_0}\int_0^{2\pi}(u_x^2 + u_y^2 + u_z^2)d\alpha dt \tag{17}
$$

Substituting for u_x, u_y and u_z and evaluating the integral gives

$$
J = (3\dot{\alpha}^2\rho^2 + A^2 + 2A\rho\dot{\alpha})n_0^2 \tag{18}
$$

Minimization of the above expression with respect to $\dot{\alpha}$ gives the optimal value of $\dot{\alpha}$

$$
\dot{\alpha}_{opt} = -\frac{1}{2}J_2 n_0 \left(\frac{R_e}{a_0}\right)^2 sin^2 i_0 \tag{19}
$$

Since A is positive $\dot{\alpha}_{opt}$ is negative. If $\dot{\alpha}$ is zero, the average cost per satellite is $J = A^2 n_0^2$. Thus, this detuning strategy has resulted not only in an averaging of the fuel consumption over all the satellites, but also in a 33 percent reduction in the cost function and a substantial reduction in fuel consumption. For Chief orbit parameters of $a_0 = 7100$ km and $i_0 = 70$ deg and a relative motion orbit with a radius of 0.5 km the period for the satellites to rotate around the circle is 179 days.

4. Conclusions

Two new concepts for minimizing fuel consumption for formation flying satellites have been presented. First, the changes in the orbital parameters for negating the out of plane drift and in track drift due to the J_2 perturbation were derived. Then, a strategy for equalizing the fuel consumption over all the satellites in the constellation was derived. This concept also resulted in a substantial reduction in the total fuel consumption for the constellation.

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