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Technical Comments

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Reply by the Authors to Y. Kim

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IN "Comment on '*N*-Impulse Formation Flying Feedback Control Using Nonsingular Element Description,'" the author states that the original nonlinear program (NLP) proposed in [1] has a unique minimum, such that "any initial guess shall yield the unique (thus global) minimum." As Table 3 in [1] illustrates, varying the initial components of the impulse sequence $\Delta \boldsymbol{v}_{\text{seq}}$, or varying the true anomaly increment $f_{\Delta v}$ (i.e., the number of impulses N in the sequence), leads to a variety of *fmincon* solutions with suboptimal costs above the global minimum J^*. Table 3 in [1] (replicated as Table 1 in this reply) demonstrates that the cost of the impulse sequence decreases, in general, with increasing N. As convergence becomes more challenging with larger N, a continuation method is proposed to overcome this potential difficulty in computer implementation [1].

In "Comment on '*N*-Impulse Formation Flying Feedback Control Using Nonsingular Element Description,'" the author formulates the NLP in [1] as the following semidefinite program (SDP) using the auxiliary vector of impulse magnitudes $\hat{\boldsymbol{y}} = [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_N]^T$, where $\hat{y}_j \equiv \|\Delta \boldsymbol{v}_j\|$:

$$\begin{aligned} &\underset{\hat{\boldsymbol{y}}, \Delta \boldsymbol{v}_{\text{seq}}}{\text{minimize}}\ J \equiv \sum_{j=1}^{N} \hat{y}_j \quad \text{subject to } [B(\boldsymbol{e}_j)]\Delta \boldsymbol{v}_{\text{seq}} - \Delta \boldsymbol{e} = 0 \\ &(\Delta v_r^j)^2 + (\Delta v_i^j)^2 + (\Delta v_c^j)^2 \le \hat{y}_j^2 \quad \text{for } j = 1, 2, \ldots, N \end{aligned} \tag{1}$$

To investigate the claim that "any initial guess shall yield the unique (thus global) minimum," the NLP in [1] is reformulated as the SDP in Eq. (1), and the sensitivity study in Table 1 is repeated; results are shown in Table 2. Although fmincon produces similar solutions in the case of the original NLP when the number of impulses per revolution increases (cf. second two columns of Tables 1 and 2), SDP performance is superior to that of the NLP when the initial components in the sequence $\Delta \boldsymbol{v}_{\text{seq}}$ are varied (cf. first two columns of Tables 1 and 2). In general, the SDP formulation produces solutions with costs beneath those generated by the original NLP in Table 1, but Table 2 still shows variability in these fmincon solutions about the global minimum, which could arise as a result of computer implementation (e.g., fmincon converging on locally "flat" regions in parameter space), therefore failing to achieve this global minimum. In particular, the fmincon solution for the case of $\Delta v^j_{r,i,c} = 2$ m/s with $f_{\Delta v} = 5$ deg is more expensive for the SDP than for the original NLP.

Table 1 Solution sensitivity to initial guess and number of impulses (NLP) [1]

Initial $\Delta \boldsymbol{v}_{\text{seq}}$ guess [$f_{\Delta v} = 5$ deg]	J, m/s	$f_{\Delta v}(N)$ [$\Delta v^j_{r,i,c} = 1$ m/s]	J, m/s
$\Delta v^j_{r,i,c} = 0.1$ m/s	17.706	90 deg (4)	19.427
$\Delta v^j_{r,i,c} = 1$ m/s	17.664	45 deg (8)	18.858
$\Delta v^j_{r,i,c} = 2$ m/s	17.682	30 deg (12)	20.009
$\Delta v^j_{r,i,c} = 5$ m/s	17.715	10 deg (36)	17.987
$\Delta v^j_{r,i,c} = 10$ m/s	17.777	5 deg (72)	17.664

Table 2 Solution sensitivity to initial guess and number of impulses (SDP)

Initial $\Delta \boldsymbol{v}_{\text{seq}}$ guess [$f_{\Delta v} = 5$ deg]	J, m/s	$f_{\Delta v}(N)$ [$\Delta v^j_{r,i,c} = 1$ m/s]	J, m/s
$\Delta v^j_{r,i,c} = 0.1$ m/s	17.435	90 deg (4)	19.428
$\Delta v^j_{r,i,c} = 1$ m/s	17.437	45 deg (8)	18.870
$\Delta v^j_{r,i,c} = 2$ m/s	17.872	30 deg (12)	20.009
$\Delta v^j_{r,i,c} = 5$ m/s	17.481	10 deg (36)	17.993
$\Delta v^j_{r,i,c} = 10$ m/s	17.434	5 deg (72)	17.437

Regardless, Table 2 illustrates that restructuring the NLP as an SDP improves the quality of the fmincon solution for this particular test case, inasmuch as a small true anomaly increment is used to best approximate continuous thrusting. Depending on computer implementation and the numerical challenges of the choice of optimizer, convergence could remain problematic for a particular set of orbit element corrections. In these cases, the authors of this reply recommend using a continuation method in tandem with the SDP modification proposed by the author of the comment.

References

[1] Anderson, P. V., and Schaub, H., "*N*-Impulse Formation Flying Feedback Control Using Nonsingular Element Description," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 2, March–April 2014, pp. 540–548.
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