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Spacecraft Formation Flying Control using Mean Orbit Elements

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Abstract

Two nonlinear feedback control laws are presented to reestablish a desired J_2 invariant relative orbit. Since it is convenient to describe the relative orbit of a deputy satellite with respect to a chief satellite in terms of mean orbit element differences, and because the conditions for a relative orbit being J_2 invariant are expressed in terms of mean orbit elements, the first control law feeds back errors in terms of mean orbit elements. Dealing with mean orbit elements has the advantage that short period oscillations are not perceived as tracking errors; rather, only the long term tracking errors are compensated for. The second control law feeds back traditional cartesian position and velocity tracking errors. For both of the control laws, the desired orbit is computed using mean orbit elements. A numerical study compares and contrasts the two feedback laws.

Introduction

In recent years the challenging concept of spacecraft formation flying has been studied by various authors.[1–6] These spacecraft may be in a simple along-track string formation or a more dynamic formation where several deputy spacecraft orbit relative to a chief reference spacecraft. With these formations, the purpose is to increase the baseline of scientific instruments placed on the spacecraft. These instruments could form a radio-telescope or surface-mapping radar array.

One method to find natural relative orbits about a reference spacecraft is to use the Clohessy-Wiltshire (CW) equations.[7] Here a circular reference orbit and spherical Earth model is assumed and the equations of motion of the orbiting spacecraft are linearized relative to the rotating frame of the reference spacecraft. These equations of motion are

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sometimes also referred to as the Hill-Clohessy-Wiltshire equations.[8] Reference 3 successfully demonstrates that these linear equations of motion can be used to establish a large family of relative orbits which require a small amount of fuel to maintain.

However, the drawback of this method is that the resulting "natural" orbits ignore the nonlinear effects in that the method doesn't take into account the effects of higher order gravitational perturbations. Further, the results are generally limited to the special case of circular orbits. Reference 9 presents another method to analytically seek natural relative orbits for the case where all satellites within the formation are of equal geometry and mass (i.e. equal ballistic coefficient). In this situation differential drag has a negligible effect on the *relative* motion compared to the perturbative effect of J_2. Brouwer's analytical solution to the two-body problem with zonal harmonics was used to find relative spacecraft orbits that are invariant as seen in the Local-Vertical-Local-Horizon (LVLH) reference frame. Matching conditions on the orbit elements of the two spacecraft were developed that guarantee that the resulting relative orbit is invariant to the gravitational J_2 effect up to first order. These matching conditions are applicable for circular and elliptical reference orbits at all orbit inclinations except for the polar orbit case.

Reference 9 suggests that the relative orbit geometry should be setup with *mean orbit elements*, with differences in the three momenta orbit elements being subject to two constraints. In the absence of J_i perturbations, the six orbit elements remain constant. Due to the influence of J_2 some orbit elements will oscillate about a nominal value, while others will exhibit secular drifts. The orbit averaged values of the orbit elements are called the *mean elements*, while their instantaneous, time-varying counterparts are referred to as the *osculating elements*. By specifying the relative orbit geometry in mean element space and then translating the resulting initial conditions into corresponding osculating elements, the true relative spacecraft motion does not deviate from the prescribed relative orbit geometry.

This paper studies methods to reestablish these J_2 invariant relative orbits by feedback of mean orbit element errors or by nonlinear feedback of the cartesian position and velocity error vectors. By feeding back errors in mean orbit elements, advantage is taken of celestial mechanics insight to avoid trying to correct orbit elements at ill-suited times. For example, studying Gauss' variational equations of the classical orbit elements, it is clear that the inclination angle is easiest to change near the equator and worst near the polar regions.

Review of J_2 Invariant Orbits

In Reference 9, Schaub and Alfriend presented two necessary conditions for two neighboring orbits to be J_2 invariant. Being invariant to the perturbation of the J_2 gravitational harmonic is understood to mean that both orbits will exhibit the same secular angular drift rates. In particular, the conditions guarantee that the ascending node rates $\dot{\Omega}$ and mean latitude rates $\dot{\theta}_M = \dot{M} + \dot{\omega}$ are equal, where M is the mean anomaly and ω is the argument of perigee. Let the orbit elements with the subscript 1 indicate quantities of the chief's orbit. The differences in the momenta elements of the J_2 invariant relative orbit of the deputy must be:

$$\delta\eta = -\frac{\eta_1}{4}\tan i_1 \delta i \tag{1}$$

$$\delta a = 2Da_1\delta\eta \tag{2}$$

with

$$D = \frac{J_2}{4a_1^2\eta_1^5}(4 + 3\eta_1)(1 + 5\cos^2 i_1) \tag{3}$$

where a is the semi-major axis, i is the inclination angle and the eccentricity measure η is defined as

$$\eta = \sqrt{1 - e^2} \tag{4}$$

with e being the eccentricity. After choosing a particular mean element difference δa, δe or δi, the remaining two momenta element differences are set by the two constraints in Eqs. (1) and (2). Since only the momenta elements a, e and i affect the J_2 induced secular drift, the mean angles M, ω and Ω can be chosen at will.

Particular relative orbits are setup by first choosing desired orbit element differences in *mean orbit element space*. Here the short and long term oscillations caused by J_2 do not appear, only the secular drift of M, ω and Ω is present. To find the corresponding inertial position and velocity vectors, the mean orbit elements are first translated to corresponding osculating orbit elements using Brouwer's artificial satellite theory in Reference 10. Note that only a first order truncation is used of Brouwer's theory in this study and the transformation back and forth between mean and osculating elements thus is imprecise. Transformation errors in the position vectors following a forward and backward transformation to mean orbit elements can range in the dozens of meters. This will have an effect on how the relative position or orbit element errors are computed.

Note that it is the mean orbit element differences which define the geometry of the relative orbit of the deputy satellite to the chief satellite. Due to imprecise positioning and various perturbative influences, these specific mean orbit element differences will not be maintained and will have to be reestablished periodically.

Mean Orbit Element Feedback Law

Since the relative orbit is being described in terms of relative differences in mean orbit elements when establishing J_2 invariant relative orbits, we examine a feedback law in terms of mean orbit elements instead of the more traditional approach of feeding back position and velocity vector errors. Doing so will allow us to control and correct specific orbit element errors. Not all orbit position errors are created equal. An error in the ascending node should be controlled at a different time in the orbit than an error in the inclination angle.

Gauss' variational equations of motion provide a convenient set of equations relating the effect of a control acceleration vector $\boldsymbol{u}$ to the osculating orbit element time derivatives.[11]

$$\frac{da}{dt} = \frac{2a^2}{h}\left(e\sin f u_r + \frac{p}{r}u_\theta\right) \tag{5a}$$

$$\frac{de}{dt} = \frac{1}{h}\left(p\sin f u_r + \left((p+r)\cos f + re\right)u_\theta\right) \tag{5b}$$

$$\frac{di}{dt} = \frac{r\cos\theta}{h}u_h \tag{5c}$$

$$\frac{d\Omega}{dt} = \frac{r\sin\theta}{h\sin i}u_h \tag{5d}$$

$$\frac{d\omega}{dt} = \frac{1}{he}\left[-p\cos f u_r + (p+r)\sin f u_\theta\right] - \frac{r\sin\theta\cos i}{h\sin i}u_h \tag{5e}$$

$$\frac{dM}{dt} = n + \frac{\eta}{he}\left[(p\cos f - 2re)u_r - (p+r)\sin f u_\theta\right] \tag{5f}$$

where the control acceleration vector $\boldsymbol{u}$ is written in the deputy Local-Vertical-Local-Horizontal (LVLH) frame components as

$$\boldsymbol{u} = \begin{pmatrix} u_r & u_\theta & u_h \end{pmatrix}^T \tag{6}$$

with u_r pointing radially away from Earth, u_h being aligned with the orbit angular momentum vector and u_θ being orthogonal to the previous two directions. The parameter f is the true anomaly, r is the scalar orbit radius, p is the semilatus rectum and the latitude angle is $\theta = \omega + f$. The mean angular velocity n is defined as

$$n = \sqrt{\frac{\mu}{a^3}} \tag{7}$$

Note that these variational equations were derived for Keplerian motion. In matrix form they are expressed as

$$\dot{\boldsymbol{e}}_{osc} = (0,0,0,0,0,n)^T + [B(\boldsymbol{e}_{osc})]\boldsymbol{u} \tag{8}$$

with $\boldsymbol{e}_{osc} = (a, e, i, \Omega, \omega, M)^T$ being the osculating orbit element vector and the 6×3 control influence matrix $[B]$ being defined as

$$[B(\boldsymbol{e})] = \begin{bmatrix} \frac{2a^2 e \sin f}{h} & \frac{2a^2 p}{hr} & 0 \\ \frac{p \sin f}{h} & \frac{(p+r)\cos f + re}{h} & 0 \\ 0 & 0 & \frac{r\cos\theta}{h} \\ 0 & 0 & \frac{r \sin\theta}{h \sin i} \\ -\frac{p\cos f}{he} & \frac{(p+r)\sin f}{he} & -\frac{r\sin\theta\cos i}{h\sin i} \\ \frac{\eta(p\cos f - 2re)}{he} & -\frac{\eta(p+r)\sin f}{he} & 0 \end{bmatrix} \tag{9}$$

Let the vector $\boldsymbol{e} = \begin{pmatrix} a & e & i & \Omega & \omega & M \end{pmatrix}^T$ be the classical mean orbit element vector. Let

$$\boldsymbol{e} = \boldsymbol{\xi}(\boldsymbol{e}_{osc}) \tag{10}$$

be an analytical transformation from the osculating orbit elements $\boldsymbol{e}_{osc}$ to the mean elements $\boldsymbol{e}$. In this study, a first order truncation of Brouwer's analytical satellite solution is used.[10] Incorporating the J_2 influence, we write Gauss' variational equations for the mean motion as

$$\dot{\boldsymbol{e}} = [A(\boldsymbol{e})] + \left[\frac{\partial \boldsymbol{\xi}}{\partial \boldsymbol{e}_{osc}}\right]^T [B(\boldsymbol{e}_{osc})]\boldsymbol{u} \tag{11}$$

with the 6×1 plant matrix $[A(\boldsymbol{e})]$ being defined as

$$[A(\boldsymbol{e})] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 n \cos i \\ \frac{3}{4} J_2 \left(\frac{r_{eq}}{p}\right)^2 n(5\cos^2 i - 1) \\ n + \frac{3}{4} J_2 \left(\frac{r_{eq}}{p}\right)^2 \eta n (3\cos^2 i - 1) \end{bmatrix} \tag{12}$$

Studying Brouwer's transformation between osculating and mean orbit elements it is evident that the matrix $\left[\frac{\partial \boldsymbol{\xi}}{\partial \boldsymbol{e}_{osc}}\right]$ is approximately a 6×6 identity matrix with the off-diagonal terms being of order J_2 or smaller. Therefore it is reasonable to approximate the mean orbit element rate equation as

$$\dot{\boldsymbol{e}} \approx [A(\boldsymbol{e})] + [B(\boldsymbol{e})]\boldsymbol{u} \tag{13}$$

The plant matrix $[A]$ in Eq. (12) rigorously describes the behavior of the mean orbit elements. As is clearly seen here, the J_2 perturbation has no secular effect on the elements a, e and i. The control influence matrix $[B]$, developed in Gauss' variational equations shown in Eq. (5), allows us to compute a change in osculating orbit elements due to a control acceleration vector $\boldsymbol{u}$. It is assumed that these osculating orbit element changes, as indicated in Eq. (13), are directly reflected in corresponding mean orbit element changes. For example, if a thrust is applied to change the osculating inclination angle by one degree, then the corresponding mean inclination angle is also changed by one degree. The errors introduced by this assumption will be of order J_2. Further, since the difference in osculating and mean orbit elements is relatively small for J_2 perturbations, the numerical difference in computing $[B]$ using osculating or mean orbit elements is typically negligible. In our use of Eq. (13) below, we assume that the $[B(\boldsymbol{e})]$ matrix is computed using mean orbit elements.

Let us assume that the relative orbit was set up such that the deputy satellite has a specific mean orbit element difference $\Delta\boldsymbol{e}$ relative to the chief mean orbit elements $\boldsymbol{e}_1$. At any instance, the desired deputy satellite location $\boldsymbol{e}_d$ is expressed in terms of mean orbit elements as

$$\boldsymbol{e}_d = \boldsymbol{e}_1 + \Delta\boldsymbol{e} \tag{14}$$

Note that $\Delta\boldsymbol{e}$ is a fixed mean orbit element difference. Therefore it doesn't matter if the chief orbit was slightly perturbed by other influences such as atmospheric or solar drag. The relative orbit is always defined as a specific difference relative to the current chief mean orbit elements, in order to maintain a specific relative motion.

Given the actual set of mean orbit elements $\boldsymbol{e}_2$ of the deputy satellite, the relative orbit tracking error $\delta\boldsymbol{e}$ is expressed in terms of mean orbit elements as

$$\delta\boldsymbol{e} = \boldsymbol{e}_2 - \boldsymbol{e}_d \tag{15}$$

We define the Lyapunov function V as a positive definite measure of the mean orbit element tracking error $\delta\boldsymbol{e}$.

$$V(\delta\boldsymbol{e}) = \frac{1}{2}\delta\boldsymbol{e}^T\delta\boldsymbol{e} \tag{16}$$

Since the desired relative orbits are J_2 invariant, the derivative of $\boldsymbol{e}_d$ is

$$\dot{\boldsymbol{e}}_d = [A(\boldsymbol{e}_d)] \tag{17}$$

where no control is required to maintain this evolving orbit. Clearly non-J_2 perturbations are being treated as minor disturbances and are not considered in Eq. (17). Taking the derivative of V and substituting Eqs. (13) and (15), we find

$$\dot{V} = \delta\boldsymbol{e}^T\delta\dot{\boldsymbol{e}} = \delta\boldsymbol{e}^T\left([A(\boldsymbol{e}_2)] - [A(\boldsymbol{e}_d)] + [B(\boldsymbol{e})]\boldsymbol{u}\right) \tag{18}$$

Setting $\dot{V}$ equal to the negative definite quantity

$$\dot{V} = -\delta \boldsymbol{e}^T [P] \delta \boldsymbol{e} \tag{19}$$

where $[P]$ is a positive definite feedback gain matrix, we arrive at the following control constraint for Lyapunov stability of the closed-loop departure motion dynamics.

$$[B]\boldsymbol{u} = -([A(\boldsymbol{e}_2)] - [A(\boldsymbol{e}_d)]) - [P]\delta \boldsymbol{e} \tag{20}$$

Note that $[P]$ does not have to be a constant matrix. In fact, later on, we will make use of this fact to encourage certain orbit element corrections to occur during particular phases of the orbit. Using Eq. (9) to study the effectiveness of the control vector to influence a particular orbit element, one choice is to give the feedback gain matrix $[P]$ the following diagonal form

$$P_{11} = P_{a0} + P_{a1} \cos^N \frac{f}{2} \tag{21a}$$

$$P_{22} = P_{e0} + P_{e1} \cos^N f \tag{21b}$$

$$P_{33} = P_{i0} + P_{i1} \cos^N \theta \tag{21c}$$

$$P_{44} = P_{\Omega 0} + P_{\Omega 1} \sin^N \theta \tag{21d}$$

$$P_{55} = P_{\omega 0} + P_{\omega 1} \sin^N f \tag{21e}$$

$$P_{66} = P_{M0} + P_{M1} \sin^N f \tag{21f}$$

with N being an even integer. The various feedback gains are now at a maximum whenever the corresponding orbit elements are the most controllable, and at a minimum or essentially zero when they are the least controllable. The size of N is chosen such that the P_{i1} gain influence drops off and rises sufficiently fast. Clearly there are an infinity of heuristic feedback gain logics that could be used here which belong to the stabilizing family. We could alternatively pose an optimization problem and optimize $[P(t)]$ to extremize some performance measure. For illustration purposes, we simply choose several stable controllers in this text.

One issue of writing the satellite equations of motion in first order form in Eq. (13) becomes quickly apparent. Since the control vector only has three components, and we are attempting to control six orbit elements, we can't directly solve the control constraint equation in Eq. (20) for the control vector $\boldsymbol{u}$. Since the system of equations is over determined, we employ a least-square type inverse to solve for $\boldsymbol{u}$.

$$\boldsymbol{u} = -\left([B]^T[B]\right)^{-1} [B]^T \left(([A(\boldsymbol{e}_2)] - [A(\boldsymbol{e}_d)]) + [P]\delta \boldsymbol{e}\right) \tag{22}$$

Due to the imprecise nature of the least-squares inverse, the resulting control law is no longer guaranteed to satisfy the stability constraint in Eq. (20). However, as numerical simulations show, this control law does successfully cancel mean element tracking errors and reestablish the desired relative orbit.

Other control methods could be employed to control the mean element tracking error defined in Eq. (15). The advantage of this method is the presence of the time varying 6×6 feedback gain matrix $[P]$. In particular, it allows us to selectively cancel particular orbit element errors at any time. A classical example is correcting for ascending node and inclination angle errors. Studying Eq. (5) or (9) it is evident that the feedback gain for $\delta\Omega$ should be large whenever $\theta = \pm 90$ degrees and near-zero whenever $\theta = 0, 180$ degrees. Near the equator it is known that the control effort required to correct for a $\delta\Omega$ would be very large. Therefore nodal corrections are best performed near the polar regions. Analogously, the inclination angle changes are best performed near the equator, with little or no inclination corrections being done near the polar region. Depending on the chief orbit elements, similar statements can be made for the remaining orbit elements. The result is that one can easily design a variable gain control law which will wait for the satellite to be in an advantageous position within the orbit before correcting certain orbit element errors. Note that this approach enables one to simultaneously control the long term secular orbital dynamics (by considering orbit element control and using mean orbit elements) and to effectively time the control corrections during each orbit to "cooperate with the physics" of orbital dynamics.

The feedback law in Eq. (22) contains a term computing the difference in natural mean element rates between the actual mean orbit element vector $\boldsymbol{e}_2$ of the deputy satellite and the desired mean orbit element vector $\boldsymbol{e}_d$. If the difference in actual and desired mean orbit elements of the deputy is small, as is typically the case with spacecraft formation flying, then it can be shown that this difference is very small and has a negligible influence on the control law. Linearizing this difference about the desired mean orbit element vector $\boldsymbol{e}_d$, we find

$$[A(\boldsymbol{e}_2)] - [A(\boldsymbol{e}_d)] \simeq \left[\frac{\partial A}{\partial \boldsymbol{e}}\right]\bigg|_{\boldsymbol{e}_d} \delta\boldsymbol{e} = [A^*(\boldsymbol{e}_d)]\delta\boldsymbol{e} \tag{23}$$

Using Eq. (23), we are able to write the linearized mean element error dynamics as

$$\delta\dot{\boldsymbol{e}} \simeq [A^*(\boldsymbol{e}_d)]\delta\boldsymbol{e} + [B(\boldsymbol{e})]\boldsymbol{u} \tag{24}$$

Note that the plant matrix is time dependent due to $\boldsymbol{e}_d$, and the control influence matrix is state dependent. Because $[A]$ only depends on the mean a, e and i parameters, the 6×6 matrix $[A^*]$ has block structure:

$$[A^*(\boldsymbol{e}_d)] = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ A^*_{21} & 0_{3\times3} \end{bmatrix} \tag{25}$$

Substituting Eq. (23) back into the control law in Eq. (22) we approximate $\boldsymbol{u}$ as

$$\boldsymbol{u} \simeq -\left([B]^T[B]\right)^{-1}[B]^T\left([A^*(\boldsymbol{e}_d)] + [P]\right)\delta\boldsymbol{e} \tag{26}$$

Taking the partial derivatives of Eq. (12) with respect to $\boldsymbol{e}$, the submatrix $[A^*_{21}]$ is found to be

$$[A^*_{21}] = \begin{bmatrix} \frac{21\epsilon}{4a}\cos i & -6\epsilon\frac{e}{\eta^2}\cos i & \frac{3}{2}\epsilon\sin i \\ -\frac{21\epsilon}{16a}(5\cos^2 i - 1) & 3\epsilon\frac{e}{\eta^2}(5\cos^2 i - 1) & -\frac{15}{4}\epsilon\sin(2i) \\ -\frac{3}{2a}\left[n + \frac{7}{8}\epsilon(3\cos^2 i - 1)\right] & 3\epsilon\frac{e}{\eta^2}(3\cos^2 i - 1) & -\frac{9}{4}\epsilon\sin(2i) \end{bmatrix} \tag{27}$$

with the small parameter ϵ being defined as

$$\epsilon = J_2\left(\frac{r_{eq}}{p}\right)^2 n \tag{28}$$

An approximate analysis of the $[A_{21}^*]$ matrix entry magnitudes in terms of metric units yields the following. Because both J_2 and n are of order 10^{-3}, and r_{eq}/p is of order 1, the parameter ϵ is of order 10^{-6}. Most entries of $[A_{21}^*]$ contain ϵ multiplied by either e, a small quantity of order 10^{-2} or smaller, or divided by a, a large quantity of order 10^3. These entries are then at least of order 10^{-8} or smaller. The largest entries contain only ϵ or n/a. Either one is of order 10^{-6}. Therefore, studying Eq. (26) shows that unless the feedback gain matrix $[P]$ is of order 10^{-5} or less, the $[A^*]$ matrix has a negligible influence on the control performance. In fact, if the feedback gain matrix $[P]$ is at least two or more magnitudes larger than the $[A^*]$ matrix, the $([A(\boldsymbol{e}_2)] - [A(\boldsymbol{e}_d)])$ term can be dropped from the control law without any apparent performance loss. As will be shown in the numerical simulations, the feedback gain on the mean orbit elements are typically much larger than this threshold.

Dropping the $([A(\boldsymbol{e}_2)] - [A(\boldsymbol{e}_d)])$ term from the mean element feedback law, we are able to provide a rigorous stability proof for the special case where the feedback gain matrix $[P]$ is simply a positive constant scalar P.

$$\boldsymbol{u} = -P\left([B]^T[B]\right)^{-1}[B]^T\delta\boldsymbol{e} \tag{29}$$

Note that restraining the feedback gain to be a constant scalar would have a negative impact on the control performance since it is no longer possible to use the celestial mechanics insight to guide when certain orbit elements should be corrected. But, this proof does provide some more analytical confidence in the control law and could be of use when only certain orbit elements have to be controlled.[12] We define a modified time dependent Lyapunov function $V(\delta\boldsymbol{e}, t)$ as[12]

$$V(\delta\boldsymbol{e}, t) = \frac{1}{2}(\alpha_1 + \alpha_2 e^{-\alpha_3 t})\delta\boldsymbol{e}^T\delta\boldsymbol{e} \tag{30}$$

This Lyapunov function is positive definite since there exists a time-invariant positive definite $V_0(\delta\boldsymbol{e})$ such that[13]

$$V(\delta\boldsymbol{e}, t) \geq \frac{\alpha_1}{2}\delta\boldsymbol{e}^T\delta\boldsymbol{e} = V_0(\delta\boldsymbol{e}) \tag{31}$$

Further, this V is decrescent since there exists a time-invariant positive definite function $V_1(\delta\boldsymbol{e})$ such that[13]

$$V(\delta\boldsymbol{e}, t) \leq \frac{\alpha_1 + \alpha_2}{2}\delta\boldsymbol{e}^T\delta\boldsymbol{e} = V_1(\delta\boldsymbol{e}) \tag{32}$$

Since $V(\delta\boldsymbol{e}, t) \to \infty$ if $|\delta\boldsymbol{e}| \to \infty$ it is also radially unbounded. Taking the derivative of Eq. (30) and making use of $\delta\dot{\boldsymbol{e}} = [B(\boldsymbol{e})]\boldsymbol{u}$ and Eq. (29), we find

$$\dot{V}(\delta\boldsymbol{e}, t) = -\alpha_2\alpha_3 e^{-\alpha_3 t}\delta\boldsymbol{e}^T\delta\boldsymbol{e} - (\alpha_1 + \alpha_2 e^{-\alpha_3 t})P\delta\boldsymbol{e}^T[B]([B]^T[B])^{-1}[B]^T\delta\boldsymbol{e} \tag{33}$$

This time dependent function is negative definite since there exists a time-invariant negative definite function

$$\dot{V}_0(\delta\boldsymbol{e}) = -\alpha_2\alpha_3\delta\boldsymbol{e}^T\delta\boldsymbol{e} - \alpha_1 P\delta\boldsymbol{e}^T[B]([B]^T[B])^{-1}[B]^T\delta\boldsymbol{e} \tag{34}$$

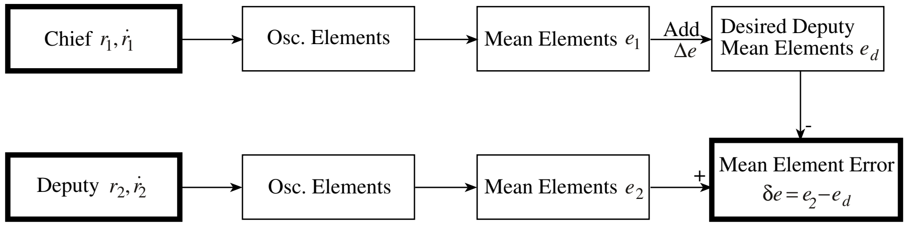


Figure 1: Mean Element Control Illustration

such that[13]

$$\dot{V}(\delta \boldsymbol{e}, t) \leq \dot{V}_0(\delta \boldsymbol{e}) \tag{35}$$

Since $V(\delta \boldsymbol{e}, t)$ is positive definite, decrescent and radially unbounded while $\dot{V}(\delta \boldsymbol{e}, t)$ is negative definite, the simplified control in Eq. (29) provides global uniform asymptotic stability under the assumption that the feedback gain P is large enough such that the term $([A(\boldsymbol{e}_2)] - [A(\boldsymbol{e}_d)])$ can be dropped. Again, it should be noted thought that only having a scalar feedback gain P may provide un-acceptable fuel cost since the feedback control law may try to compensate for orbit element errors when it is very inefficient to do so.

A schematic layout of the mean element control is shown in Figure 1. Inertial position and velocity vectors are assumed to be available for both the chief and deputy satellite. After transforming both sets of vectors into corresponding mean orbit element vectors, the desired deputy mean elements are computed through a specified orbit element difference $\Delta \boldsymbol{e}$ relative to the chief satellite. The tracking error $\delta \boldsymbol{e}$ is then computed as the difference between the desired and actual deputy mean orbit elements. As mentioned earlier, the first order transformation used in this study to transform back and forth between osculating and mean orbit elements is not perfect. Taking a cartesian position and velocity vector, transforming first to mean elements and then back to cartesian coordinates can result in position differences in the dozens of meters. This is not a problem for typical orbit applications. However, for spacecraft formation flying, where the satellite relative orbit is to be controlled very precisely, this transformation error is significant. In the control strategy presented in Figure 1, both sets of mean elements are computed from inertial cartesian coordinates. While there is a minor error associated with this transformation, the error will be roughly the same for both sets of coordinates since the cartesian coordinates are relatively close to begin with. Because a *difference* in mean orbit elements is fed back, these transformation errors are found to approximately cancel each other and do not degrade the controller performance. Of course, this transformation error could be further reduced by expanding the analytic orbit solution to higher order. However, even here it is beneficial to always deal with differences in orbit elements to achieve higher numerical accuracy. The numerical simulations in this study will show what tracking accuracies could be obtained with only a first order truncation.

Nonlinear Feedback Law in Position and Velocity Vectors

Traditional feedback laws depend on cartesian position and velocity error vector measurements. A nonlinear cartesian coordinate feedback law is presented which illustrates the steps necessary to track a prescribed relative orbit expressed in terms of mean orbit element differences. A related nonlinear feedback law is presented in Ref. 14.

The inertial equations of motion of the chief satellite $\boldsymbol{r}_1$ and deputy satellite $\boldsymbol{r}_2$ are

$$\ddot{\boldsymbol{r}}_1 = \boldsymbol{f}(\boldsymbol{r}_1) \tag{36}$$

$$\ddot{\boldsymbol{r}}_2 = \boldsymbol{f}(\boldsymbol{r}_2) + \boldsymbol{u} \tag{37}$$

where the chief satellite is assumed to be in a free, uncontrolled orbit and only the deputy satellite is being controlled to maintain the desired relative orbit. The vector function $\boldsymbol{f}(\boldsymbol{r})$ contains the gravitational acceleration. Expressing the inertial position vector in terms of inertial components $\boldsymbol{r} = (x, y, z)$ and including the J_2 perturbation, this function is defined as

$$\boldsymbol{f}(\boldsymbol{r}) = -\frac{\mu}{r^3}\left[\boldsymbol{r} - J_2\frac{3}{2}\left(\frac{r_{eq}}{r}\right)^2 \begin{pmatrix} 5x\left(\frac{z}{r}\right)^2 - x \\ 5y\left(\frac{z}{r}\right)^2 - y \\ 5z\left(\frac{z}{r}\right)^2 - 3z \end{pmatrix}\right] \tag{38}$$

where r is the scalar orbit radius. Let $\boldsymbol{r}_{2_d}$ be the desired inertial position vector of the deputy satellite for a J_2 invariant relative orbit. The position tracking error $\delta\boldsymbol{r}$ is then defined as

$$\delta\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_{2_d} \tag{39}$$

Using this error vector and its derivative, the positive definite Lyapunov function V is defined as

$$V(\delta\boldsymbol{r}, \delta\dot{\boldsymbol{r}}) = \frac{1}{2}\delta\dot{\boldsymbol{r}}^T\delta\dot{\boldsymbol{r}} + \frac{1}{2}\delta\boldsymbol{r}^T[K_1]\delta\boldsymbol{r} \tag{40}$$

where $[K_1]$ is a positive definite 3×3 position feedback gain matrix. Taking the derivative of V we find

$$\dot{V} = \delta\dot{\boldsymbol{r}}^T\left(\ddot{\boldsymbol{r}}_2 - \ddot{\boldsymbol{r}}_{2_d} + [K_1]\delta\boldsymbol{r}\right) \tag{41}$$

Substituting Eq. (37) and making use of the fact that the desired relative orbit is J_2 invariant (i.e. control free), the Lyapunov rate is written as

$$\dot{V} = \delta\dot{\boldsymbol{r}}^T\left(\boldsymbol{f}(\boldsymbol{r}_2) - \boldsymbol{f}(\boldsymbol{r}_{2_d}) + \boldsymbol{u} + [K_1]\delta\boldsymbol{r}\right) \tag{42}$$

Enforcing $\dot{V}$ to be equal to the negative definite quantity

$$\dot{V} = -\delta\dot{\boldsymbol{r}}^T[K_2]\delta\dot{\boldsymbol{r}} \tag{43}$$

where $[K_2]$ is a positive definite 3×3 velocity feedback gain matrix, the asymptotically stabilizing control law $\boldsymbol{u}$ is found to be

$$\boldsymbol{u} = -\left(\boldsymbol{f}(\boldsymbol{r}_2) - \boldsymbol{f}(\boldsymbol{r}_{2_d})\right) - [K_1]\delta\boldsymbol{r} - [K_2]\delta\dot{\boldsymbol{r}} \tag{44}$$

The asymptotic stability property of this control law can be verified by checking the higher order derivatives of V on the set where $\dot{V}$ is zero (i.e. evaluated at $\delta\dot{\boldsymbol{r}} = 0$).[15] The first non-zero higher derivative of V on this set is found to be the third derivative

$$\dddot{V}(\delta\dot{\boldsymbol{r}} = 0) = -\delta\boldsymbol{r}^T[K_1]^T[K_2][K_1]\delta\boldsymbol{r} < 0 \tag{45}$$

which is negative definite in $\delta\boldsymbol{r}$. Thus the order of the first non-zero derivative is odd and the control law is asymptotically stabilizing.

Where the mean orbit element feedback law feeds back a difference in the natural orbit element rates, the cartesian coordinate feedback law in Eq. (44) feedback a difference in gravitational accelerations. Linearizing this difference about the desired motion $\boldsymbol{r}_{2_d}(t)$ we find

$$\boldsymbol{f}(\boldsymbol{r}_2) - \boldsymbol{f}(\boldsymbol{r}_{2_d}) \simeq \left.\left[\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{r}}\right]\right|_{\boldsymbol{r}_{2_d}} \delta\boldsymbol{r} = [F(\boldsymbol{r}_{2_d})]\delta\boldsymbol{r} \tag{46}$$

Using Eq. (46), the closed-loop dynamics are now written in the linear form as

$$\delta\ddot{\boldsymbol{r}} \simeq [F(\boldsymbol{r}_{2_d})]\delta\boldsymbol{r} + \boldsymbol{u} \tag{47}$$

and the control law is linearized as

$$\boldsymbol{u} \simeq -([F(\boldsymbol{r}_{2_d})] + [K_1])\delta\boldsymbol{r} - [K_2]\delta\dot{\boldsymbol{r}} \tag{48}$$

The matrix $[F]$ can be written as $[F] = [F_{Kepler}] + [F_{J_2}]$ where $[F_{Kepler}]$ is the term due to the inverse square gravitational attraction and $[F_{J_2}]$ is the term due to the J_2 perturbation. Doing a similar dimensional study of $[F_{Kepler}]$ and $[F_{J_2}]$, as for $[A^*]$ earlier, the matrix $[F_{Kepler}]$ is found to be of order μ/r and $[F_{J_2}]$ of order $J_2\mu/r^3$. Since both J_2 and $1/r$ are roughly 10^{-3}, this means that $[F_{J_2}]$ is on the order of 10^{-9} smaller than $[F_{Kepler}]$. This means that excluding the J_2 term in the $\boldsymbol{f}(\boldsymbol{r})$ calculation will have a negligible effect on the performance. Therefore the largest component of $[F]$ is of order $\mu/r = 10^1$ in metric units. As the numerical simulations will show, the position feedback gains are typically much smaller than this. For the cartesian feedback law, feeding back the difference in gravitational accelerations has a large influence on the performance. For example, if the gains are very small to allow the maneuver to take several orbit revolutions, then the control effort will still be large due to this gravitational acceleration difference term. This is in contrast to the mean orbit element feedback law where the maneuvers can easily be stretched over several orbit revolutions.

A critical detail in this cartesian coordinate feedback law is how to compute the desired deputy position and velocity vectors, because the relative orbit trajectory is described in terms of mean orbit element differences relative to the chief orbit. Figure 2 illustrates this process. After translating the chief cartesian coordinates into corresponding mean orbit elements, the desired deputy position and velocity vectors are computed by first adding the desired mean orbit element difference vector $\Delta\boldsymbol{e}$ and then transforming these desired elements back to cartesian space. However, if these desired inertial deputy states are differenced with the actual inertial deputy states, serious numerical difficulties may arise. The reason for this is the transformation error that occurs when mapping between osculating and mean orbit elements. The closed loop position errors will stop decaying once the accuracy of this transformation is reached. To avoid this limitation, we don't use the actual states of the deputy when computing the tracking error. Instead, we map these states first to mean orbit elements and then back to cartesian coordinates before differencing them with the desires states. With the difference between the chief and deputy position and velocity vectors being very small, the transformation error due to the forward and backward mapping will be essentially identical and cancel themselves when being differenced.

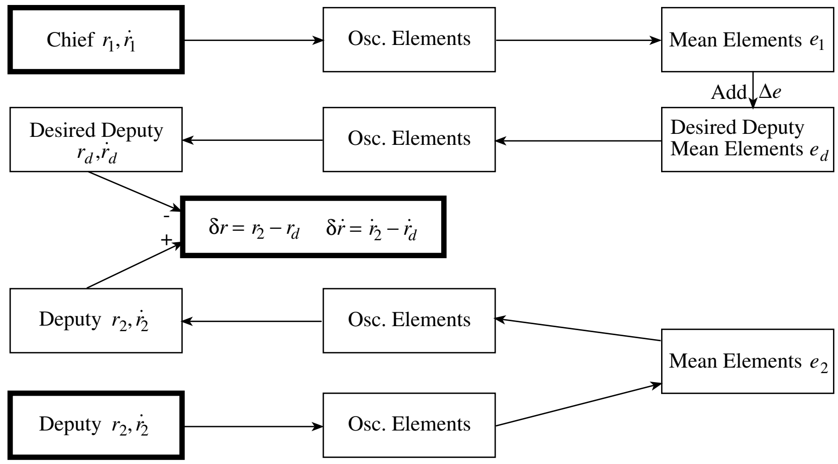


Figure 2: Tracking Error Computation Logic for Cartesian Coordinate Control

This qualitative observation is consistent with our numerical experiments. The result is a nonlinear cartesian coordinate feedback law that is able to establish the J_2 invariant orbit and overcome some of the limitations of having a first-order transformation between the osculating and mean orbit elements.

Numerical Simulations

The following numerical simulations establish a desired J_2 invariant orbit by feeding back either errors in mean orbit elements or by feeding back cartesian position and velocity error vectors. The numerical simulation includes the J_2 through J_5 zonal harmonics. The chief mean orbit elements and the desired deputy mean orbit element differences are shown in Table 1. The relative orbit has a prescribed inclination angle difference of 0.006 degrees, while the semi-major axis and eccentricity are adjusted to compensate for this. The initial mean orbit element errors of the deputy satellite are $\delta a = -100$ meters, $\delta i = 0.05$ degrees and $\delta\Omega$ = -0.01 degrees.

Table 1: Mean Orbit Elements of Chief and Deputy Satellite

Mean Chief Orbit Elements	Value	Units	Desired Mean Deputy Orbit Element Differences	Value	Units
a	7555	km	Δa	-0.00192995	km
e	0.05		Δe	0.000576727	
i	48	deg	Δi	0.006	deg
Ω	0.0	deg	$\Delta\Omega$	0.0	deg
ω	10.0	deg	$\Delta\omega$	0.0	deg
M	120.0	deg	ΔM	0.0	deg

The feedback gain matrices for the cartesian coordinate feedback laws are set to be scalar feedback gains of $K_1 = 0.0000011$ and $K_2 = 0.001$ times a 3×3 identity matrix. The feedback gain matrix $[P]$ for the mean orbit element feedback law is set to be of the form shown in

Eq. (21) with the trigonometry power N is set to be 12. The particular gains are shown in Table 2. These parameters where obtained after running numerous numerical simulation for cases where only one orbit element at a time had an initial tracking error. Streamlining this process would be highly desirable. However, whereas there is a lot of theory available on how to produce position and velocity feedback gains, how to obtain orbit element gains for good performance remains an open and critical issue to be investigated in future work.

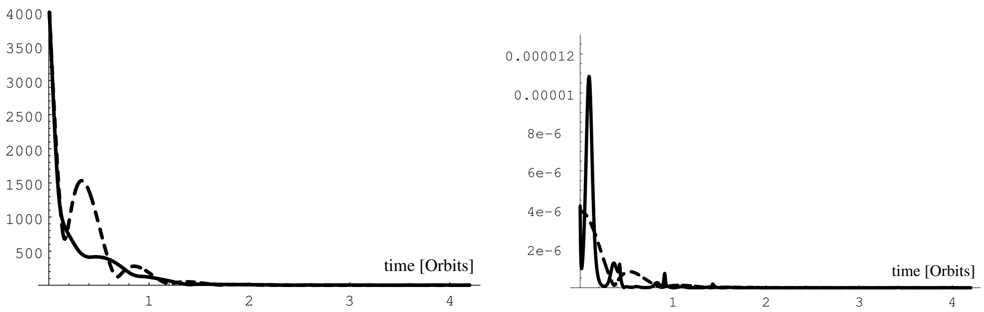
Table 2: Feedback Gain Parameters for $[P]$ Gain Matrix

Parameter	Value	Parameter	Value
P_{a0}	0.024	P_{a1}	0.024
P_{e0}	.020	P_{e1}	0.020
P_{i0}	0.00004	P_{i1}	0.005
$P_{\Omega 0}$	0.00004	$P_{\Omega 1}$	0.005
$P_{\omega 0}$	0.0002	$P_{\omega 1}$	0.040
P_{M0}	0.000001	P_{M1}	0.010

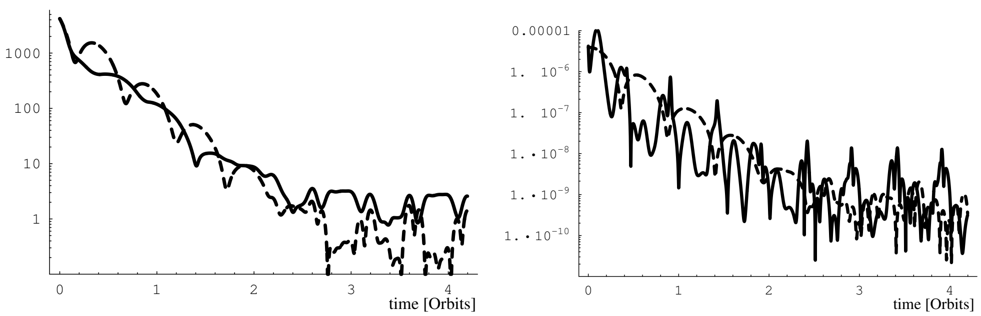
The performance of the mean orbit element feedback law with the gains given in Table 2 is shown in Figure 3. The scalar position tracking error $|\delta \boldsymbol{r}|$ is shown in Figure 3(a) on a linear scale. Within only 2 orbits the tracking error decays to a very small value. The associated control vector magnitude is shown in Figure 3(b). Plotting the data on a linear scale, it is evident that the control is administered in a pulse-like manner during certain phases of an orbit. The first big peak corresponds to when the deputy satellite travels near the equator and the feedback control compensates for the large inclination angle change. Since the inclination angle error is five times that of the ascending node error, the latitude angle at which to perform corrections for these orbit element errors is near the equator. The second peak in the control effort is when the satellite passes the $f=$ -90 degree point. These peaks at true anomalies of ± 90 degrees are caused by the calculation of the inverse of $[B]^T[B]$. The singular values of this 3×3 matrix never go to zero. However, at $f = \pm 90$ degrees the intermediate singular value decreases in size to a magnitude similar to the smallest singular value. Therefore, the mean element control magnitude plots always exhibit sharp peaks whenever $f = \pm 90$ degrees. The third peak in Figure 3(b) is again to compensate for the remaining inclination angle difference. Of the three initial mean orbit element tracking errors that were present at the beginning of the simulation, compensating for the inclination angle error is by far the most costly effort. The total $\Delta \boldsymbol{v}$ fuel cost for this maneuver is 7.482 m/s.

Recall that the the control vector $\boldsymbol{u}$ for this feedback law is solved for by performing a least-squares inverse of the control influence matrix $[B]$. Even though the Lyapunov function wasn't able to prove stability rigorously, no initial conditions were found that caused the system to become unstable. For both small and large mean orbit element tracking errors, the control was always able to reestablish the desired relative orbit.

To better visualize the long-term convergence of the tracking errors and control effort, the same plots are shown again in Figures 3(c) and 3(d) on a logarithmic scale. Enforcing a specific *constant* difference in argument of perigee and mean anomaly is not a J_2 invariant condition. In fact, the J_2 condition only enforces equal latitude rates, not equal argument of perigee or mean anomaly rates. Under the influence of J_2 they will drift apart slowly at equal



(a) Tracking Error Magnitude $|\delta \boldsymbol{r}|$ (m) (b) Control Magnitude (km/s^2)

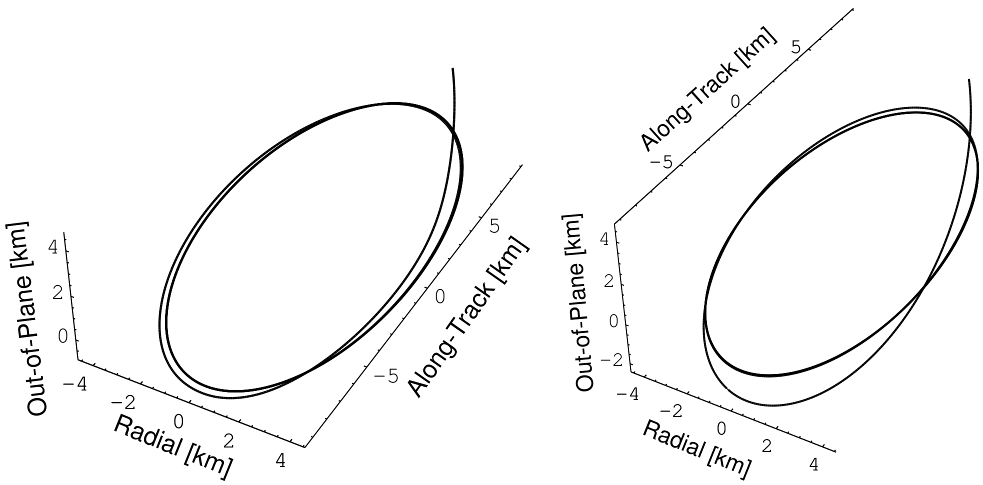


(c) Tracking Error Magnitude $|\delta \boldsymbol{r}|$ (m) (d) Control Magnitude (km/s^2)

Figure 3: Feedback Control Law Performance Comparison

and opposite rates. Having a feedback law attempting to establish the constant differences causes some of the final tracking errors. Another source of the final tracking errors are the transformation errors when performing a first order mapping between osculating and mean orbit elements. However, by dealing with differences in orbit elements, the control law is able to converge down to just 2.5 meters from an initial tracking error of over 4000 meters. Similarly, the control effort converges to very small values, but not precisely to zero. The peaks seen here are also due the the numerical inverse effect of $[B]^T[B]$ at $f = \pm 90$ degrees. Had only osculating orbit element tracking errors been fed back, then the performance would have been noticeably worse. In particular, the final tracking errors would be two to three orders of magnitude larger. Dealing with mean orbit element we avoid chasing the short period oscillations and only deal with secular tracking errors.

Figure 3 shows the performance of the cartesian coordinate feedback law. The position tracking errors take slightly longer than two orbits to decay. The control magnitude shown in Figure 3(b) shows a more continuous control effort compared to the more pulse like control effort demanded by the mean element feedback law. The reason for this is that it continuously compensates for any tracking errors, whereas the mean orbit element feedback law, with the particular time-varying gains chosen, does most of its controlling during particular phases of the orbit. The latter lends itself more naturally for multi-orbit revolution corrections. If the feedback gains are lowered for the cartesian coordinate feedback law to allow it to use more orbits to perform the correction, then the total control cost starts to



(a) Mean Orbit Element Feedback Law

(b) Cartesian Coordinates Feedback Law

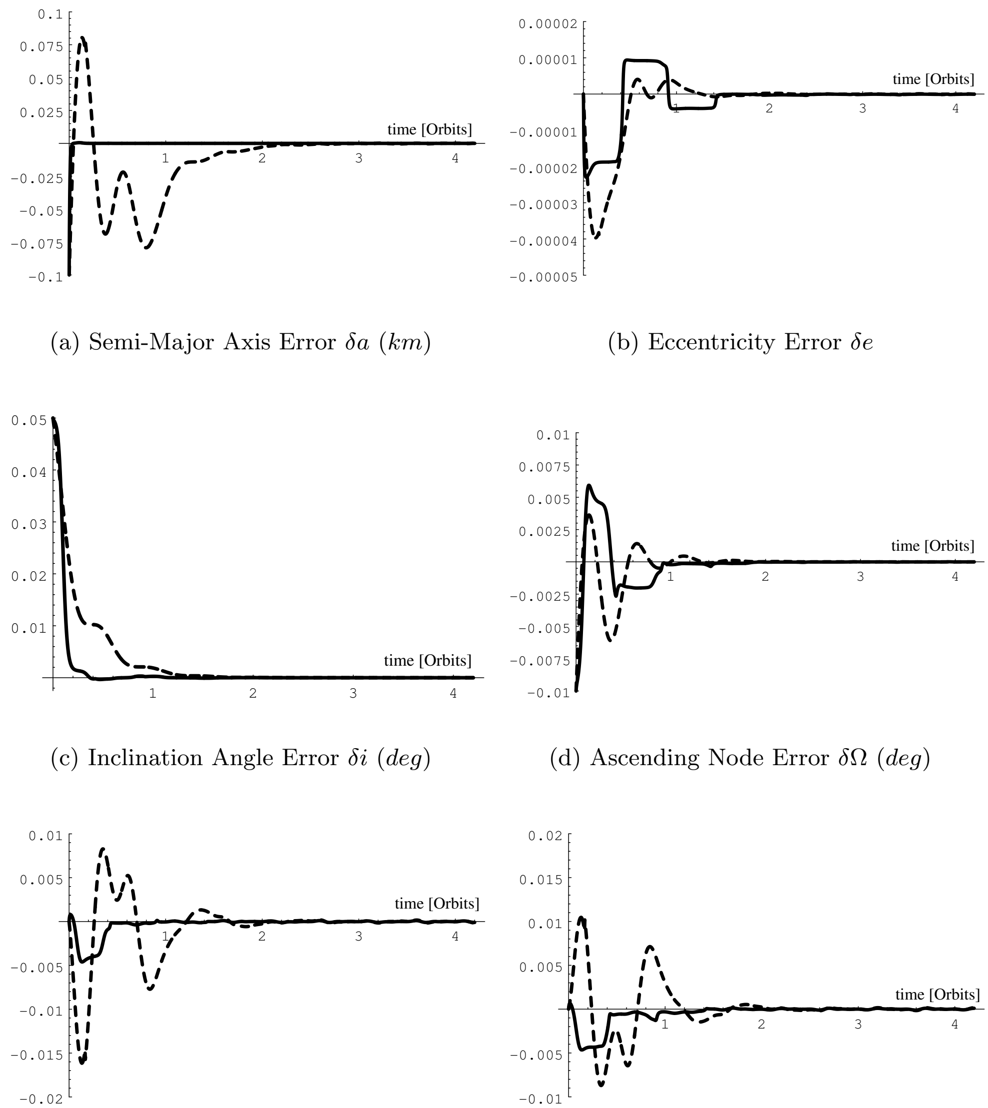
Figure 4: Relative Orbit Trajectories in Rotating Chief LVLH Frame

grow rapidly. The reason for this is due to the relative size of the gravitational acceleration difference matrix $[F(\boldsymbol{r}_{2_d})]$. Even with small gains, it still commands a large cumulative control effort which decreases the effectiveness of multi-orbit corrections with this type of control law. With the mean orbit element feedback law however, because the element rate difference matrix $[A^*(\boldsymbol{e}_d)]$ is relatively small, only a negligible control effort is required if the feedback gains are very small. Therefore the orbit corrections are easier to spread over several orbits with the mean orbit element feedback law.

The total control cost for the cartesian feedback law is $\Delta\boldsymbol{v} = 7.428$ m/s, slightly less than the element feedback law. The $\Delta\boldsymbol{v}$ cost for both types of control laws is rather close most of the time. Depending on the initial conditions and feedback gains, either may have a slightly smaller or larger $\Delta\boldsymbol{v}$ cost. As a comparison, the $\Delta\boldsymbol{v}$ cost for a two impulse orbit correction for the same initial conditions is as low as 6.24 m/s, depending on how long the transit time is. The feedback control cost is therefore about 20% higher than the two-impulse fuel cost for the given initial conditions.

Whereas the control effort of the mean orbit element feedback law has periodic peaks, the cartesian coordinate feedback law effort is smooth as expected. The position tracking errors converge to roughly 1-2 meters in size, slightly less than with the the element feedback law. The control effort during this end game is of the same order of magnitude, but without the periodic peaks at $f = \pm 90$ degrees. For the cartesian coordinate feedback law, dropping the J_2 term from the $\boldsymbol{f}(\boldsymbol{r})$vector computation in Eq. (44) has no visible effect on the performance or convergence.

The relative orbit trajectories for both control laws are shown in the chief satellite LVLH frame in Figure 4. Both controls do a good job in reestablishing the desired J_2 invariant relative orbit. The mean element feedback does a better job in keeping the relative orbit close to the desired relative orbit. The cartesian coordinate feedback law causes the relative orbit to dip below the desired relative orbit before converging. Staying close to desired



(a) Semi-Major Axis Error δa (km)

(b) Eccentricity Error δe

(c) Inclination Angle Error δi (deg)

(d) Ascending Node Error $\delta\Omega$ (deg)

(e) Argument of Perigee Error $\delta\omega$ (deg)

(f) Mean Anomaly Error δM (deg)

Figure 5: Mean Orbit Element Tracking Errors for (solid) Mean Orbit Element Feedback Law and (dashed) Cartesian Feedback Law

orbit is beneficial when collision avoidance with other deputy satellites is of consideration. However, the flight paths resulting from either control law do not always differ by this amount. Often they are very similar in nature.

Where performances of the two control laws differ substantially is in maintaining the desired mean orbit element of the deputy satellite relative to the chief satellite. Figure 5 shows the various mean orbit element tracking errors for both the mean orbit element feedback law (solid line) and the cartesian coordinate feedback law (dashed line). Recall that the initial mean orbit element tracking errors were a semi-major axis error of 0.1 km, an inclination angle error of 0.05 degrees and an ascending node error of -0.01 degrees.

The element control law is able to correct the mean semi-major axis error rather quickly within a fraction of an orbit. As is expected from Gauss' variational equations, doing so causes an error in eccentricity, argument of perigee and mean anomaly. This is a general trend with the element feedback law. Correcting for one particular orbit element error always causes subsequent errors in other orbit elements. Thus, even if only one element were off initially, using the element feedback law would correct for this error, but in the process the remaining orbit element would experience transient errors. The effect of the time varying feedback gain matrix $[P]$ is clearly seen in the inclination angle corrections. They occur whenever the deputy satellite crosses the favorable latitude angles. While e, ω and M do experience transient tracking errors due to compensating for the other element errors, these tracking errors remain rather small. This is an advantage of the mean element tracking law. While it is not able to hold other orbit elements fixed while correcting the initial mean element tracking errors, it typically does keep them rather close to the desired values. Often this translates into the relative trajectory remaining closer to the desired trajectory during the transient phase.

Most of the mean element tracking errors for the cartesian coordinate feedback law are quite different from the mean element tracking errors of the element feedback law. In particular, the semi-major axis in not maintained at the desired value at all during the transient period. This explains the different transient relative orbits that were observed between the two feedback laws. As with the element feedback law, temporary tracking errors are introduced to the eccentricity, argument of perigee and mean anomaly. Further, these transient errors are much larger than with the element feedback law. Note the following interesting detail. While $\delta\omega$ and δM may grow rather large during the maneuver, their *sum* is maintained close to the desired sum of $\omega_d + M_d$. This behavior is observed for all initial conditions studied. The mean inclination error time history is similar in profile compared to the $\delta i(t)$ of the element feedback law. Major inclination angle correction occur during the equatorial orbit regions.

One reason why the cartesian coordinate feedback law performs so well is that the full nonlinear equations of motion are utilized. In future research it would be interesting to compare this performance to that of traditional linear control laws such as are used in the classical rendez-vous problem.

Conclusions

Two nonlinear feedback laws are presented to establish and recursively reestablish a desired J_2 invariant relative orbit. While these control laws are applied to the J_2 perturbed spacecraft control problem, they are general enough to be used in general orbit control.

The first control law, motivated by orbital mechanics insight, feeds back tracking errors in mean orbit elements. The advantage here is that the relative orbit errors provide more geometric information as to what relative orbit perturbations this error will cause than do the classical position and velocity error expressions. Thus, it is possible to construct the feedback gain matrix to have the control attempt certain orbit element corrections when it more efficient or practical. While the general mean orbit element feedback control law does not have a rigorous stability proof at this point, numerical studies indicate that it is highly stable and is able to correct for both small and large initial mean orbit element tracking errors. A rigorous stability proof is provided for a simplified version of the feedback control law where the feedback gain is a positive scalar. A benefit of this feedback law is that the orbit elements which do not have tracking errors are kept relatively close to the desired values during the maneuver. This makes it simpler to predict what the transient orbit will look like and consider collision avoidance. Further, it is relatively simple to extend the maneuver over several orbits without increasing the fuel cost by reducing the magnitudes of the feedback gains. The reason for this is that in dealing with errors in orbit elements versus errors in position and velocity vectors, we are dealing perturbations over very slowly varying quantities. Being able to easily extent the maneuvers over multiple orbits implies that the thrust requirements can be reduced to fit within practical constraints.

The second control law feeds back traditional cartesian position and velocity errors. Prescribing the desired relative orbit in terms of differences in mean orbit elements poses certain numerical challenges. In particular, a method is shown which partially compensates for the transformation errors between osculating and mean elements. These transformation errors can further be reduced by using a higher order truncation of Brouwer's artificial satellite theory. The fuel cost for this feedback law is similar to that of the mean orbit element feedback law and maneuver times are comparable. However, the thrust of the mean element feedback law is typically applied is a pulse like manner, whereas the cartesian coordinate feedback demands a continuous thrust. This means that the element feedback based control could be realized with a hybrid system consisting of conventional fixed-thrust thrusters and variable-thrust pulsed-plasma thrusters. This cartesian coordinate feedback law does not lend itself well to be performed over multiple orbits without increasing the fuel cost substantially. Thus it is more difficult with this method to extend the maneuver time to bring the thrust magnitudes within practical constraints.

Open questions remain how to find proper orbit element feedback gains and how to construct the matrix $[P(\boldsymbol{e})]$ such that the dominant orbit mechanics are better exploited. These critical questions have a significant influence on the performance and feasibility of the mean orbit element feedback law. With the tight mission requirements of the currently proposed spacecraft formation flying missions, it is critical to exploit the dominant dynamics that are present within the control design. The potential payoffs are better insight into the nature of the relative orbit errors, better control over the transient orbits and the ability to extend the corrective maneuvers over an arbitrary number of revolutions.

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