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An impulsive feedback control is developed to establish specific relative orbits for spacecraft formation flying. The relative orbit tracking errors are expressed in terms of mean orbit elements. The feedback control, based on Gauss' variational equations of motion, allows specific orbit elements or orbit element sets to be controlled with minimal impact on the remaining osculating orbit elements. This is advantageous when J_2 invariant orbits are to be controlled, where only the argument of perigee and mean anomaly will drift apart at equal and opposite rates. The advantage of this impulsive feedback control, compared to optimal control solutions, is that it can operate with little computational effort and in a near-optimal manner, while commanding only a marginal penalty in fuel cost. While applied to the spacecraft formation flying problem, this control could also be used to perform general orbit corrections. Formulas are developed providing accurate estimates of the sensitivities of the mean semi-major axis and mean eccentricity with respect to the osculating inclination angle. With these sensitivities, the tracking error in semi-major axis, eccentricity and inclination angle can be cancelled within one orbit.

Introduction

Spacecraft formation flying is an interesting and challenging topic as is seen in references 1–7. In a gravity dominated environment, where control is a relatively small perturbation on the overall motion, it is important to seek relative motions which are natural to the prevailing dynamics and require little control effort to maintain. If all spacecraft involved are of equal type and build (i.e. have the same ballistic coefficient), then the differential J_2 perturbation is the dominant perturbative effect experienced by the various spacecraft. For this case the differential drag effect is negligible on the relative motion over the time period of several orbits. The Earth oblateness perturbation causes secular drifts in the ascending node Ω, the argument of perigee ω and the mean anomaly M, along with short and long period oscillations in all six orbit elements. As shown in reference 5, it is beneficial to describe the relative orbits of a spacecraft formation in terms of mean orbit element differences, as compared to using cartesian position and velocity vectors. By using mean orbit elements, the long term behavior of the spacecraft formation is immediately evident and short term deviations are not considered. In trying to achieve bounded relative motion, controlling the short term oscillation from the desired relative trajectories would be an unnecessary fuel fuel expense.

For general formation flying it is not possible to set up the orbit element differences between two neighboring orbits such that the three relative secular mean orbit element drifts are zero. However, it is possible to find two constraints which enforce equal ascending node and mean latitude angle rates,[5] where the mean latitude angle is defined as the sum of the argument of perigee and the mean anomaly. With these constraints it is possible to establish specific orbit element differences which render the resulting relative spacecraft orbit J_2 invariant. Any control maintaining these types of orbits will naturally go to zero as the desired relative orbit is asymptotically approached.

One drawback to the above orbit design methodology is that while it does achieve equal nodal and mean latitude drift rates between various spacecraft, it is still possible for the individual argument of perigees and mean anomalies to drift apart. Two neighboring orbits will not drift apart in the classical sense, but as the lines of perigee drift apart from their initial values, the relative orbit will either expand or contract. To counter this effect, it will be necessary to periodically compensate for both the argument of perigee and mean anomaly drift. The following impulsive control strategy was born out of the quest to find a method to correct the argument of perigee and mean anomaly while minimally impacting the remaining orbit elements. While the presented method is attractive to compensate specific sets orbit elements, it is also possible to use this method to correct for arbitrary relative orbit errors in a near-optimal manner.

To maintain desired orbit element differences, a multitude of control strategies may be employed. This paper studies a sequential impulsive algorithm which depends on mean orbit element errors to establish a specific relative orbit. Using Gauss' variational equations of motion, a firing sequence is established which allows only certain orbit element errors to be corrected during an orbit with little or no effect on the remaining orbit element differences. However, Gauss' variational equations of motion are derived for osculating orbit elements. Since specific mean orbit element differences are desired, modifications are introduced to account for the small differences between mean and osculating elements. In particular, first order relationships between osculating orbit inclination changes and the thus induced changes in mean semi-major axis and eccentricity are introduced. These relationships allow for a more efficient impulsive thrusting scheme to establish the desired mean orbit element differences faster. While this impulsive feedback control is demonstrated and applied to the spacecraft formation flying problem, it can also be applied to the general orbit correction problem.

Problem Formulation

Gauss' variational equations are convenient to determine the effect of a control vector $\boldsymbol{u} = (u_r, u_\theta, u_h)^T$ on the osculating orbit elements, where the u_r vector component is the thrust along the orbit radial direction, u_h is in the orbit normal thrust and the thrust u_θ is perpendicular to the previous two directions. Let a be the semi-major axis, e be the eccentricity and i be the orbit inclination angle, then

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Gauss' variational equations of motion are given by[8]

$$\frac{da}{dt} = \frac{2a^2}{h}\left(e\sin f u_r + \frac{p}{r}u_\theta\right) \tag{1a}$$

$$\frac{de}{dt} = \frac{1}{h}\left[p\sin f u_r + ((p+r)\cos f + re)\,u_\theta\right] \tag{1b}$$

$$\frac{di}{dt} = \frac{r\cos\theta}{h}u_h \tag{1c}$$

$$\frac{d\Omega}{dt} = \frac{r\sin\theta}{h\sin i}u_h \tag{1d}$$

$$\frac{d\omega}{dt} = \frac{1}{he}\left[-p\cos f u_r + (p+r)\sin f u_\theta\right] - \frac{r\sin\theta\cos i}{h\sin i}u_h \tag{1e}$$

$$\frac{dM}{dt} = n + \frac{\eta}{he}\left[(p\cos f - 2re)u_r - (p+r)\sin f u_\theta\right] \tag{1f}$$

where p is the semi-latus rectum, h is the orbit angular momentum, r is the scalar orbit radius and θ is the true latitude angle defined as $\theta = \omega + f$. The parameter $\eta = \sqrt{1-e^2}$ is another convenient eccentricity measure. These equations are written in matrix form as

$$\dot{\boldsymbol{e}}_{osc} = [B(\boldsymbol{e}_{osc})]\boldsymbol{u} \tag{2}$$

where $\boldsymbol{e}_{osc}$ is the osculating orbit element vector. Note that Eqs. (1a)-(1f) do not necessarily show what influence the control vector will have on the mean orbit elements. Let the mean orbit element vector $\boldsymbol{e}''$ be written as a function of $\boldsymbol{e}_{osc}$ as

$$\boldsymbol{e}'' = \boldsymbol{f}(\boldsymbol{e}_{osc}) \tag{3}$$

where the function $\boldsymbol{f}()$ could be formed from Brouwer's analytical artificial satellite theory.[9] Using the chain rule of differentiation, the mean orbit element rate equation is written as

$$\dot{\boldsymbol{e}}'' = \left[\frac{\partial \boldsymbol{e}''}{\partial \boldsymbol{e}_{osc}}\right]^T \frac{d\boldsymbol{e}_{osc}}{dt} \tag{4}$$

However, using a first order truncation of Brouwer's transformation between osculating and mean orbit elements, it is evident that the sensitivity matrix $[\partial\boldsymbol{e}''/\partial\boldsymbol{e}_{osc}]$ is essentially the identity matrix with the off diagonal terms being of order J_2 or smaller.[6] Therefore, a reasonable first cut control strategy is to approximate Eq. (4) as

$$\dot{\boldsymbol{e}}'' \approx \dot{\boldsymbol{e}}_{osc} \approx [B(\boldsymbol{e})]\boldsymbol{u} \tag{5}$$

To refine this control approach, solving for the Jacobian matrix of the mean orbit elements would be very valuable. Since Brouwer's analytical transformation between osculating and mean orbit elements is very complex, solving for the complete 6×6 sensitivity matrix is a challenging task. For the presented impulsive control strategy, two important partial derivatives are the sensitivities of the mean semi-major axis and the mean eccentricity with respect to the osculating orbit inclination angle. Appropriate analytical expressions are derived for these partial derivatives and incorporated into the control scheme to yield an improved convergence rate to the desired mean orbit elements.

Control Strategy

Studying the $d\Omega/dt$ and di/dt expressions in Eq. (1), it is evident that the individual ascending node or inclination angles are adjusted best when the spacecraft passes through either the polar or the equatorial regions respectively. However, if both an inclination angle and nodal correction are to be performed, it is more fuel efficient to perform both corrections with one impulse only. Both elements are adjusted with an orbit normal impulsive Δv_h as shown in Eq. (1). The corresponding inclination angle and ascending node corrections are given by

$$\Delta i = \frac{r\cos\theta}{h}\Delta v_h \tag{6}$$

$$\Delta\Omega = \frac{r\sin\theta}{h\sin i}\Delta v_h \tag{7}$$

Dividing Eq. (7) by (6), the critical true latitude angle θ_c at which to perform this orbit normal thrusting maneuver is

$$\theta_c = \arctan\frac{\Delta\Omega\ \sin i}{\Delta i} \tag{8}$$

Squaring and summing Eqs. (6) and (7), the required Δv_h to perform the desired inclination correction Δi and ascending node correction $\Delta\Omega$ is

$$\Delta v_h = \frac{h}{r}\sqrt{\Delta i^2 + \Delta\Omega^2\sin i^2} \tag{9}$$

Note that applying this Δv_h only affects the orbit elements i, Ω and ω. This cross-coupling between the (i,Ω) correction and ω is the only coupling between osculating orbit element set corrections in this firing scheme. Note that while there always exists two possible critical true latitude angles θ_c from Eq. (8), only the solution corresponding to a positive Δv_h is used in this control method. Thus (i,Ω) are only corrected at one point in the orbit.

Substituting the Δv_h in Eq. (7) into Eq. (1e), the $\Delta\Omega$ correction results in the following $\Delta\omega$ change:

$$\Delta\omega(\Delta v_h) = -\cos i\Delta\Omega \tag{10}$$

This secondary effect will be taken into account when specifying the impulse required to correct the argument of perigee.

The argument of perigee and the mean anomaly are also corrected together together as an orbit element pair, but with two impulsive maneuvers over one orbit. Each impulsive thrust is in the orbit radial direction only and is applied at both the orbit perigee and apogee. Let Δv_{r_p} be the radial impulse applied at perigee and Δv_{r_a} be the impulse at apogee. Computed over one orbit, and taking into account that an ascending node correction $\Delta\Omega$ could be occurring (which causes an additional change in ω), the Δv_{r_p} and Δv_{r_a} impulses cause the following osculating orbit element changes.

$$\Delta\omega = \frac{1}{he}\left(-p(\Delta v_{r_p} - \Delta v_{r_a}) - \Delta\Omega\cos i\right) \tag{11}$$

$$\Delta M = \frac{\eta}{he}\left((p - 2r_p e)\Delta v_{r_p} - (p + 2r_a e)\Delta v_{r_a}\right) \tag{12}$$

To solve these two equations for the radial Δv's, the following identities are useful

$$p - 2r_p e = p\frac{1-e}{1+e} \tag{13a}$$

$$p - 2r_a e = p\frac{1+e}{1-e} \tag{13b}$$

along with $h/p = na/\eta$. Substituting these expressions into Eqs. (11) and (12) we find

$$\Delta v_{r_p} - \Delta v_{r_a} = -(\Delta\omega + \Delta\Omega\cos i)\frac{nae}{\eta} \tag{14}$$

$$(1-e)^2\Delta v_{r_p} - (1+e)^2\Delta v_{r_a} = nae\Delta M \tag{15}$$

Solving these two equations for the required radial impulses to achieve a desired $\Delta\omega$ and ΔM we find

$$\Delta v_{r_p} = -\frac{na}{4}\left(\frac{(1+e)^2}{\eta}(\Delta\omega + \Delta\Omega\cos i) + \Delta M\right) \tag{16}$$

$$\Delta v_{r_a} = \frac{na}{4}\left(\frac{(1-e)^2}{\eta}(\Delta\omega + \Delta\Omega\cos i) + \Delta M\right) \tag{17}$$

Note that if a $\Delta\Omega$ correction is performed during this orbit, then its effect is immediately taken into account in the above two equations.

The argument of perigee and mean anomaly corrections, provided by Eqs. (16) and (17), are convenient to compensate for the natural secular drift in these orbit elements that will occur with the J_2 invariant orbit presented in reference 5. Only ω and M of the six orbit elements will not have an equal relative drift rate, but rather their sum will. This relative drift difference is not very large, but depending on the tolerances of the relative orbit it will have to be compensated for periodically. Further, the smaller the eccentricity of the orbit, the less effect the relative drift of ω and M will have on the orbit geometry. However, Eqs. (16) and (17) provide an impulsive control method which is able to directly readjust the argument of perigee and mean anomaly while minimally affecting the other osculating orbit elements.

The remaining two orbit elements to be corrected are the semi-major axis a and the eccentricity e. As is the case with the argument of perigee and mean anomaly corrections, the semi-major axis and eccentricity are adjusted together through two impulsive maneuvers over one orbit. However, these impulsive thrusts are fired in the tangential u_θ direction. One impulsive correction Δv_{θ_p} is fired at perigee and the other impulse Δv_{θ_a} is fired at apogee. With this firing sequence a and e are adjusted efficiently and without disturbing the other osculating orbit elements. From Eq. (1), the a and e corrections over one orbit are

$$\Delta a = \frac{2a^2}{h}\left(\frac{p}{r_p}\Delta v_{\theta_p} + \frac{p}{r_a}\Delta v_{\theta_a}\right) \tag{18}$$

$$\Delta e = \frac{1}{h}\Big((p + r_p + r_p e)\Delta v_{\theta_p} + (-p - r_a + r_a e)\Delta v_{\theta_a}\Big) \tag{19}$$

Note that in deriving Eqs. (18) and (19) it is assumed that the orbit corrections Δa and Δe are relatively small. Otherwise a and e could not be held constant during the two maneuvers. To solve these two equations for the tangential Δv's, the following identities are used.

$$p + r_p + r_p e = 2p \tag{20}$$

$$-p - r_a + r_a e = -2p \tag{21}$$

Eqs. (18) and (19) are now rewritten as

$$(1+e)\Delta v_{\theta_p} + (1-e)\Delta v_{\theta_a} = \frac{h^2}{2a^2}\Delta a \tag{22}$$

$$\Delta v_{\theta_p} - \Delta v_{\theta_a} = \frac{h}{2p}\Delta e \tag{23}$$

Using $h/a = n a \eta$, with $\eta = \sqrt{1-e^2}$, the required tangential impulses are found to be

$$\Delta v_{\theta_p} = \frac{n a \eta}{4}\left(\frac{\Delta a}{a} + \frac{\Delta e}{1+e}\right) \tag{24}$$

$$\Delta v_{\theta_a} = \frac{n a \eta}{4}\left(\frac{\Delta a}{a} - \frac{\Delta e}{1-e}\right) \tag{25}$$

Note that in both the (ω, M) and (a, e) corrections, the sequence of impulsive maneuvers over an orbit is irrelevant. The first maneuver may occur at either perigee or apogee.

To implement these impulsive Δv's, the mean orbit element errors are established at some arbitrary point in the orbit, and are then held constant during the orbit while appropriate Δv's are applied as discussed earlier. This impulsive firing scheme assumes that all the mean orbit element errors will remain constant over an orbit. If the a, e and i elements do not satisfy the J_2 invariant conditions setup in Reference,[5] then Ω, ω and M will experience some J_2 induced secular relative drift. However, this drift is relatively small over an orbit and can be ignored. The impulsive feedback control will correct, or at least substantionally reduce, any remaining mean orbit element errors during the following orbit. The exception is if the deputy semi-major axis is substantionally different from that of the chief. In this case the different orbit periods will cause the mean anomaly to exhibit substantial relative drift over one orbit. In this case it cannot be assumed that ΔM is constant over an orbit. Thus, the (ω, M) corrections do not begin until the second orbit. Doing this allows the a, e and i variables to be corrected during the first obit, which will set the orbit periods equal between deputy and chief satellite. During further orbits, any remaining relative mean anomaly errors will remain constant over an orbit. If the (ω, M) corrections are applied during the first orbit with a large semi-major axis error present, then the impulsive feedback control law still corrects the relative orbit. However, the fuel cost typically increases since incorrect (ω, M) corrections are performed during the first orbit.

Since it is advantageous to describe the relative orbit in terms of orbit element differences of the deputy satellite relative to the chief satellite, this impulsive firing sequence is a convenient method to correct orbit errors from the desired orbit element differences. If only one or two elements are to be adjusted, then this control solution is essentially optimal. If several orbit elements are to be corrected, then preliminary studies have shown this method to still yield a near-optimal solution with a fuel cost increase of only a few percent over the multi-impulse optimal solution. The advantage of this method is that through its simplicity and low computational overhead, it lends itself well to be implemented in an autonomous manner. Little ground support would be required for a cluster of spacecraft to maintain their formation as long as they are able to sense their inertial orbits themselves. This could be achieved through GPS measurements or direct line of sight measurements between the various satellites. Feeding back mean orbit element errors has the benefit that any short period oscillations are ignored.

Further, it is convenient to be able to adjust only certain orbit elements, leaving the remaining elements virtually untouched. For relative orbits designed using the approach outlined in reference 5, the resulting relative orbit will be J_2 invariant in an angular sense. This means that the neighboring orbits will have equal nodal and mean latitude drift rates. However, the argument of perigee and mean anomaly will still drift apart at equal and opposite rates. The consequence of this drift is that the relative orbit will go through cycles of symmetrically growing and shrinking as the chief satellite completes one orbit. This effect is more noticeable for satellite clusters with larger eccentricities. For a cluster with nominally zero eccentricity, having the argument of perigee and mean anomaly grow apart at equal and opposite rates has no affect on the overall relative orbit geometry. Further, this impulsive firing scheme could also be used as the initial conditions for an optimizer solving for the true minimum fuel orbit correction. Often indirect optimizing methods are sensitive to initial conditions, and the presented impulsive feedback law could provide reasonable initial guess as to the structure of the optimal control solution.

Selected Mean Orbit Element Sensitivities

An important issue not considered so far is that mean orbit elements are to be controlled with the spacecraft formation flying, not osculating orbit elements. As a first approximation, it is feasible to assume that the $\partial \boldsymbol{e}/\partial \boldsymbol{e}_{osc}$ matrix, given in Eq. (4), is a 6×6 identity matrix. However, for the tight tolerances required with formation flying, the effects of $\partial \boldsymbol{e}/\partial \boldsymbol{e}_{osc}$ must also be considered. For example, whereas the inclination angle and ascending node correction should not affect any other osculating elements, besides the argument of perigee, it does affect particular mean orbit elements.

Performing a numerical study, it is evident that adjusting the osculating orbit inclination correction does have a noticeable effect on the mean semi-major axis and eccentricity. However, adjusting the osculating semi-major axis and eccentricity, as shown in Eqs. (24) and (25), has a negligible effect on the remaining mean elements.

This section develops algebraic formulas for the sensitivities of the mean semi-major axis and the eccentricity with respect to the osculating inclination angle. With these formulae, it is possible to predict the effect that the inclination angle correction will have on these selected mean orbit elements, and incorporate this information when computing the required a and e corrections. During the first orbit, these formulas will result in near-perfect cancellations of tracking errors in a, e and i.

Using a first order truncation of the Brouwer artificial satellite theory,[9] the mean semi-major axis a'' is given by

$$a'' = a - a\frac{J_2}{2}\frac{r_e^2}{a^2}\Big[(3\cos i^2 - 1)\left(\left(\frac{a}{r'}\right)^3 - \frac{1}{\eta^3}\right) + 3(1-\cos^2 i)\left(\frac{a}{r'}\right)^3\cos(2\omega' + 2f')\Big] \quad (26)$$

where r_e is the Earth radius, f is the true anomaly and r is the current orbit radius. Using Brouwer's notation, double primed variables are the mean orbit elements, single-primed variables have the long-period terms removed and un-primed variables are the osculating parameters. Note that Eq. (26) involves both un-primed and single-primed variables. This makes the precise development of $\partial a''/\partial i$ very challenging. Thus it is assumed that the difference between using un-primed and primed ω and f is minimal. The mean semi-major axis is now expressed as

$$a'' = a - a\frac{J_2}{2}\frac{r_e^2}{a^2}\Big[(3\cos i^2 - 1)\left(\left(\frac{a}{r}\right)^3 - \frac{1}{\eta^3}\right) + 3(1-\cos^2 i)\left(\frac{a}{r}\right)^3\cos(2\omega + 2f)\Big] \quad (27)$$

Taking the partial derivative of Eq. (27) we find

$$\frac{\partial a''}{\partial i} = -\frac{3}{2}J_2\sin(2i)\frac{r_e^2}{a}\left(\frac{1}{\eta^3} + \left(\frac{a}{r}\right)^3(\cos(2\omega + 2f) - 1)\right) \quad (28)$$

Thus, for a given orbit inclination angle correction Δi, the corresponding change in mean semi-major axis $\Delta a''$ is given by

$$\Delta a'' = \frac{\partial a''}{\partial i}\Delta i \quad (29)$$

If only an inclination correction is performed, then the impulse is applied at either $\theta = \omega + f = 0$ or 180 degrees. Eq. (29) is then reduced to the simpler form:

$$\Delta a'' = -J_2\frac{3}{2}\frac{r_e^2}{a}\frac{\sin(2i)}{\eta^3}\Delta i \quad (30)$$

From Brouwer's artificial satellite theory, a first order approximation of the mapping between the mean and osculating eccentricity is given by

$$e'' = e - \delta_1 e - \frac{J_2}{4}\frac{\eta^2}{e}\frac{r_e^2}{a^2}\Big[(3\cos^2 i - 1)\left(\left(\frac{a}{r'}\right)^3 - \frac{1}{\eta^3}\right) + 3(1-\cos^2 i)\left(\left(\frac{a}{r'}\right)^3 - \frac{1}{\eta^4}\right) - \frac{e}{\eta^4}(1-\cos^2 i)\big(3\cos(2\omega' + f') + \cos(2\omega' + 3f')\big)\Big] \quad (31)$$

with the variable $\delta_1 e$ given by

$$\delta_1 e = \frac{J_2}{16}\frac{r_e^2}{a^2}\frac{e}{\eta^2}\left(1 - 11\cos^2 i - 40\cos^4 i(1 - 5\cos^2)^{-1}\right)\cos(2\omega) \quad (32)$$

Making the same assumptions as were done when developing the partial derivative of a'', the partial derivative of e'' with respect to the osculating inclination angle i is:

$$\frac{\partial e''}{\partial i} = -\frac{J_2}{4}\frac{r_e^2}{a^2}\frac{\sin(2i)}{\eta^2}\Big[\frac{e}{4}\left(11 + \frac{80\cos^2 i}{1 - 5\cos^2 i} + \frac{200\cos^4 i}{(1-5\cos^2 i)^2}\right)\cos(2\omega) + \frac{3}{e\eta^4}\left(\left(\left(\frac{a}{r}\right)^3 - \frac{1}{\eta^3}\right) + \left(\left(\frac{a}{r}\right)^3 - \frac{1}{\eta^4}\right)\cos(2\omega + 2f)\right) - 3\cos(2\omega + f) - \cos(2\omega + 3f)\Big] \quad (33)$$

The change in mean eccentricity $\Delta e''$ due to a correction in osculating inclination Δi is then given by

$$\Delta e'' = \frac{\partial e''}{\partial i}\Delta i \quad (34)$$

Again, if only inclination angle corrections are performed individually (i.e. $\theta = 0$ or 180 degrees), Eq. (34) reduces to

$$\Delta e'' = -\frac{J_2}{16}\frac{r_e^2}{a^2}\frac{\sin(2i)}{\eta^2}\Big[\left(11 + 80\cos^2 i(1 - 5\cos^2 i)^{-1} + 200\cos^4 i(1-5\cos^2 i)^{-2}\right)e\cos(2\omega) - 12\frac{1-\eta}{e} \mp 16\cos\omega\Big]\Delta i \quad (35)$$

where the minus sign is used if $\theta = 0$ degrees and the plus sign is used if $\theta = 180$ degrees.

When initializing the mean orbit element tracking errors which are to be corrected during the following orbit, the $\Delta a''$ and $\Delta e''$ are now added to the actual mean orbit element tracking errors to account for the effect of correcting the inclination angle. With this adjustment, numerical simulations illustrate that the mean a, e and i errors can be canceled within one orbit. The remaining mean orbit element sensitivities are to be derived in future work. For the given control problem, obtaining the a'' and e'' sensitivities with respect to i was required to improve the convergence rate.

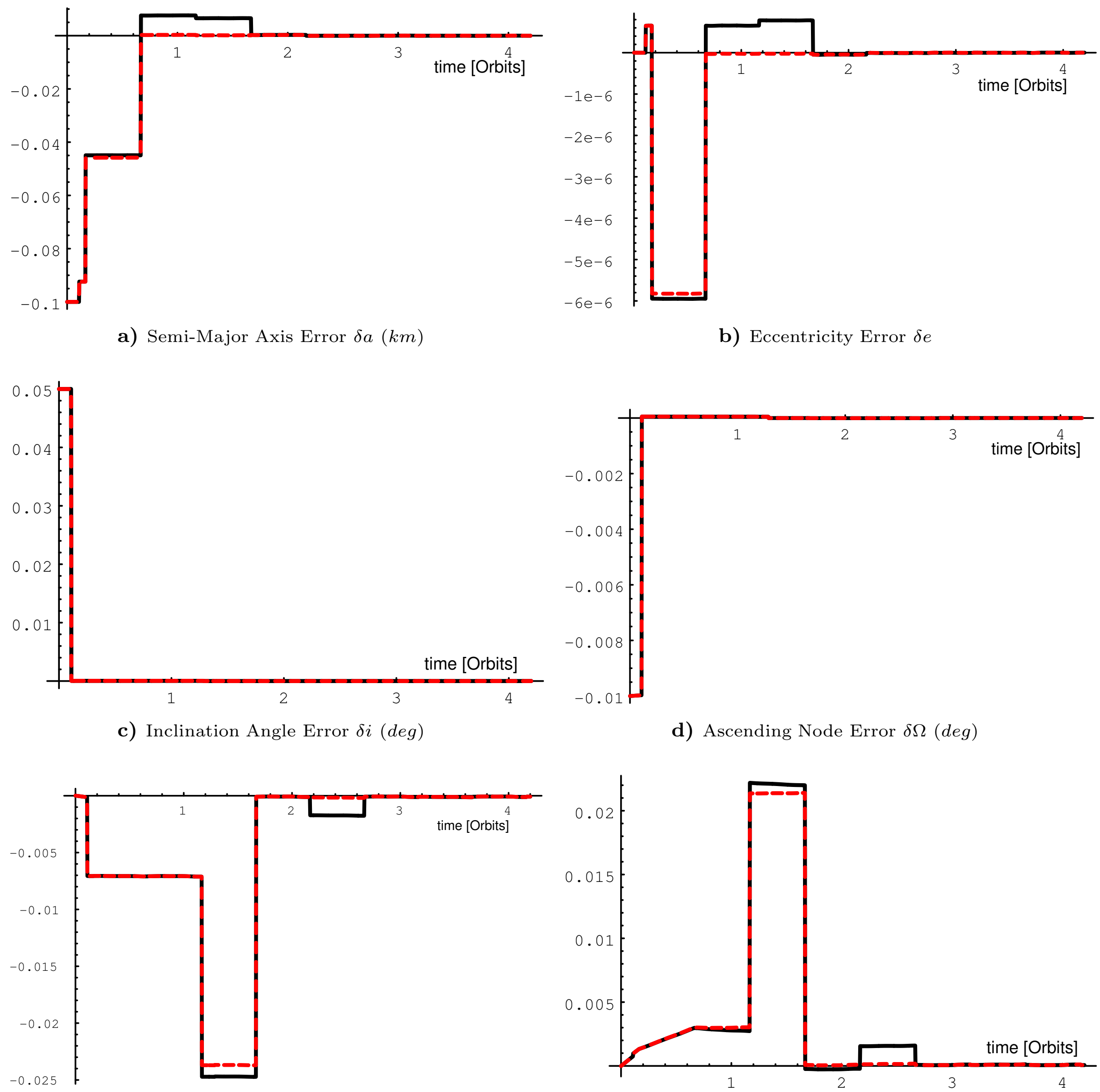
Numerical Simulations

The following numerical simulation establishes a desired J_2 invariant orbit by employing the impulsive control scheme presented in this paper. The chief mean orbit elements and the desired deputy mean orbit element differences are shown in Tables 1 and 2. The relative orbit has a prescribed inclination angle difference of 0.006 degrees, while the semi-major axis and eccentricity are adjusted to compensate for this. The initial mean orbit element errors of the deputy satellite are $\delta a'' = -100$ meters, $\delta i'' = 0.05$ degrees and $\delta\Omega''$ = -0.01 degrees.

Figure 1 illustrates two test runs. The nonlinear equations of motion

$$\ddot{\boldsymbol{r}} + \mu\boldsymbol{r} = \boldsymbol{f}(\boldsymbol{r}, J_2, J_3, J_4, J_5) \quad (36)$$

are integrated for each spacecraft including the gravitational zonal harmonics up to fifth order. This allows for a numerical verification that the predictions based on Gauss' and Brouwer's theories are valid. In Case 1 (shown as a dashed line) the impulsive control scheme is employed without making use of the partial derivatives of a'' and e'' with respect to



a) Semi-Major Axis Error δa (km)

b) Eccentricity Error δe

c) Inclination Angle Error δi (deg)

d) Ascending Node Error $\delta\Omega$ (deg)

e) Argument of Perigee Error $\delta\omega$ (deg)

f) Mean Anomaly Error δM (deg)

Fig. 1 Mean Orbit Element Tracking Errors (Dashed Line is Case 1, Solid Line is Case 2)

Table 1 Chief Mean Orbit Elements of Chief

Mean Chief Orbit Elements	Value	Units
a	7555	km
e	0.05	
i	48	deg
Ω	20.0	deg
ω	10.0	deg
M	120.0	deg

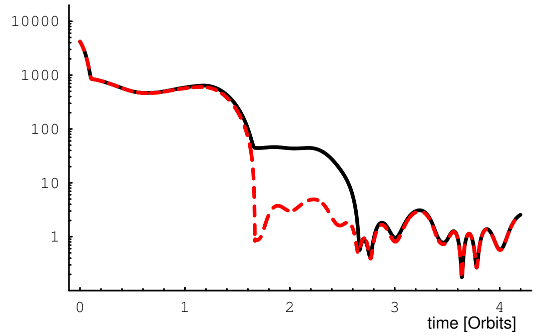
Table 2 Desired Relative Orbit Element Differences

Desired Mean Deputy Orbit Element Differences	Value	Units
Δa	-0.00192995	km
Δe	0.000576727	
Δi	0.006	deg
$\Delta\Omega$	0.0	deg
$\Delta\omega$	0.0	deg
ΔM	0.0	deg

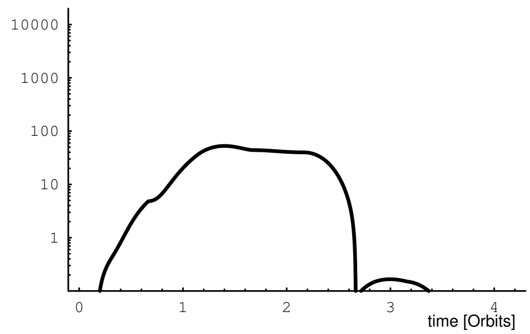
the inclination angle. In Case 2, the same control scheme is used with the addition that if inclination angle corrections are performed, than their effect on a'' and e'' are included. During the first orbit run, the semi-major axis, eccentricity, inclination angle and ascending node are attempted to be corrected. Again, the reason being to first match the orbit periods and then attempt to correct the mean anomaly errors.

Case 1 is able to reduce the initial tracking errors in (a, e, i) substantially during the first orbit. However, it is clear that when the the osculating inclination angle is corrected, the mean semi-major axis and eccentricity are also affected. Studying the osculating orbit elements during the first orbit, no change in the later three orbit elements was observed as i is corrected, as is predicted according to Gauss' variation equations. For the given orbit elements, the effect of changing the inclination by -0.05 degrees is roughly 7.315 meters. In case 2 the sensitivities of the mean a and e with respect to the osculating i are utilized. The result is that after the first orbit the mean elements (a, e, i, Ω) are at the desired values. This verifies that the simplifications performed in deriving the two mean element sensitivities still resulted in a good prediction of these partial derivatives.

From the second orbit run on all six orbit elements are corrected per orbit. After the second orbit, the orbit element tracking errors for case 2 are essentially zero. Case 1 requires an extra orbit iteration to cancel out the remaining small tracking errors.



a) Scalar Cartesian Tracking Error (m)



b) Tracking Error Difference Between Case 1 and Case 2 (m)

Fig. 2 Relative Orbit Tracking Errors in Cartesian Coordinates

Figure 2(a) shows the scalar, radial tracking error of the relative orbit for both case 1 and 2, with case 1 being again the dashed line. Both cases converge to the same level of tracking accuracy of about 1 meter. This is the same level of accuracy as was achieved with the feedback control laws in reference 6. The reason these control laws don't reduce the tracking error to zero is due to using a first order truncation of Brouwer's artificial satellite theory when translating between osculating and mean orbit elements. Figure 2(b) shows the difference in tracking errors between case 1 and 2. Note that the difference during the second orbit is too small to be seen in Figure 2(a). Note that the tracking error of case 2 has reached its lower limit before the end of the second orbit. The tracking error of case 1 only reaches its lower limit only before the third orbit is completed.

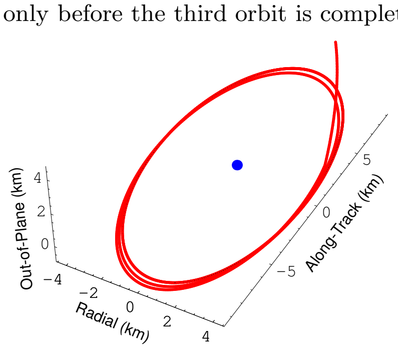


Fig. 3 Relative Orbit as seen in Rotating LVLH Reference Frame (km)

The relative orbit trajectory, as seen in the rotating LVLH frame, is shown for either case as a red line in Figure 3. The differences in the trajectories of either case 1 or 2 are too small to be visible at the scale shown. The reference relative orbit is shown as the black curve, with the location of the chief satellite shown as a blue point. The point where the first inclination angle and ascending node correction occurs is plainly visible here as a an abrupt change in the relative orbit plane. After three orbits, the actual relative orbit does approach the desired trajectory. Note that the impulsive feedback control law does a very good job in converging to the desired relative orbit. Even within one orbit period the actual relative orbit is very close to the desired relative orbit

The total Δv consumed with case 1 is 6.4550 m/s. The Δv for case 2 is reduced slightly to 6.4550 m/s. Even though typically case 2 provided a better fuel economy than case 1, it was possible to setup the initial orbit element tracking errors such that case 1 had a slightly lower fuel consumption. As a comparison, either case has a lower fuel consumption than what was found for the feedback laws in reference 6. There the mean element feedback control law required 7.584 m/s for the same initial errors, while the cartesian coordinate feedback law required 7.428 m/s. The Δv for a two-impulse optimal orbit correction required 6.24 m/s. This means that the impulsive feedback control law commanded only a 3 percent Δv penalty compared to the fuel optimal solution.

The presented impulsive control law is not necessarily intended to replace precomputed, fuel-optimized maneuvers. If the time and computational effort is available, fuel optimal maneuvers should be employed. What the impulsive feedback control does provide is a simple logic with which to do orbit corrections. Since these corrections are near optimal, it is feasible that a spacecraft would be able to perform relative motion station keeping without the extensive ground support required for doing optimal trajectories.

Conclusion

An impulsive feedback control strategy in terms of mean orbit element differences is presented to maintain a cluster of formation flying spacecraft. The control compares the orbit element differences to predefined values and adjusts various orbit elements at particular regions of the orbit. The elements (a, e) and (ω, M) and (i, Ω) are adjusted as pairs. The orbit element corrections are designed such that they only marginally influence the remaining osculating orbit elements. Since mean orbit elements are to be tracked, the sensitivities of the mean semi-major axis and eccentricity with respect to the osculating inclination angle are presented. With these formulae, it is possible to essentially cancel all mean (a, e, i, Ω) errors within the first orbit, while the (ω, M) are corrected during the second orbit. The impulsive feedback control law requires little computational effort compared to fuel-optimal solutions, yet achieves the orbit correction in a near fuel-optimal manner. Due to its simplicity, this control technique lends itself for autonomous relative orbit station keeping without extensive ground support, as long as the individual satellites are able to determine their orbits themselves.

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