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Three-Axis Attitude Control Using Redundant Reaction Wheels with Continuous Momentum Dumping

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A description of an attitude control system for a three-axis stabilized spacecraft is presented. A globally stabilizing nonlinear feedback control law is derived that enables tracking of an arbitrary time-varying reference attitude. This new control incorporates integral feedback while avoiding any quadratic rate feedback components. A redundant cluster of four or more reaction wheels is used to control the spacecraft attitude, and three magnetic torque rods are used for purposes of continuous autonomous momentum dumping. The momentum dumping strategy can employ general torque rod orientations, and it is developed to take advantage of a redundant set of reaction wheels.

Nomenclature

$\boldsymbol{B}$	=	Earth magnetic field vector
$\hat{\boldsymbol{e}}$	=	principal rotation vector
$[G_s]$	=	matrix of reaction wheel spin axes
$[G_t]$	=	matrix of torque bar alignment vectors
$\hat{\boldsymbol{g}}_{si}$	=	spin axis of ith reaction wheel
$\hat{\boldsymbol{g}}_{ti}$	=	alignment axis of ith torque bar
h_i	=	momentum of reaction wheel about spin axis
$[I]$	=	spacecraft inertia tensor
J_{si}	=	inertia of ith reaction wheel about spin axis
K, $[K_I]$, $[P]$	=	feedback gains
$\boldsymbol{L}$	=	external torques acting on the spacecraft
u_{si}	=	motor torque of ith reaction wheel
μ_i	=	dipole magnitude of ith torque bar
$\boldsymbol{\sigma}$	=	modified Rodrigues parameter set
$\boldsymbol{\tau}_{TB}$	=	torque on spacecraft resulting from magnetic torque bars
Φ	=	principal rotation angle
Ω_i	=	rotational velocity of reaction wheel about spin axis relative to spacecraft
$\boldsymbol{\omega}$	=	spacecraft angular velocity
$\boldsymbol{\omega}_r$	=	reference angular velocity

I. Introduction

THE use of momentum exchange devices, such as reaction wheels, is a common method of spacecraft attitude control. Such devices work through momentum transfer between the spacecraft body and one or more spinning wheels. To detumble a spacecraft, wheel speeds are modified in such a way as to effectively absorb the spacecraft momentum. The same principle may be used by a cluster of reaction wheels for arbitrary three-axis spacecraft pointing. Here, motor torques drive wheel accelerations; these motor torques, in turn, act equally and opposite on the spacecraft frame. By carefully controlling the wheel accelerations, torque is created, which allows for general attitude corrections [1–4]. Due to the interface between the reaction wheel assemblies and the spacecraft, the total momentum of the system is constant. The limitation of reaction wheel actuation is the speed to which a flywheel can be accelerated before reaching the physical wheel speed limit. This saturation can lead to stability or performance concerns. Thus, it is of interest to consider reaction wheel control strategies that seek to reduce necessary control efforts in order to lessen the likelihood of reaching saturation. In prior work, attitude control strategies were developed that were quadratic in terms of angular velocity [3]. If the spacecraft angular velocities were large, this quadratic term required significant control effort, as it was proportional to the angular velocity squared.

A practical consideration with the use of reaction wheels is the need for momentum dumping [5,6]. Because the system of spacecraft and reaction wheels conserves momentum, as the spacecraft loses momentum, the wheel speeds must increase. Additionally, any perturbing torques acting on the spacecraft must be absorbed by the wheels if precise pointing is to be maintained. Due to the inherent size and speed limitations of physical systems, wheel speeds may become saturated after a period of time, preventing further attitude control of the spacecraft. If the momentum storage capacity of the reaction wheel cluster is not large enough to absorb all of the spacecraft momentum, the wheel speeds reach their maximum values and further wheel acceleration is not possible. As a method of torquing, the spacecraft besides the reaction wheels is needed in order to despin the wheels while still meeting the attitude control requirements. One method for accomplishing this task is the use of thrusters [7]. By angling the thrusters offcenter from the spacecraft center of mass, torques are created that can be used to lower wheel speeds.

Another option for momentum dumping is the use of magnetic torque rods [5,8,9]. Here, coils of wire are wrapped around a ferrous core, such as iron. Applying a current to the wire produces a magnetic dipole that, in turn, interacts with the Earth's magnetic field to produce a torque on the spacecraft. This torque is then used to despin the wheels. A challenge with using magnetic torque rods lies in their inability to produce an arbitrary three-dimensional torque. In fact, a torque can only be produced perpendicular to the Earth's magnetic field, providing only two degrees of freedom available for dumping the wheel momentum [10,11].

Reference [12] considers attitude control with a redundant cluster of four reaction wheels. An attitude control solution that minimizes the norm of the reaction wheel motor torques is provided using Euler angles, the direction cosine matrix, the Euler axis of rotation, and quaternions. To handle momentum buildup in the wheels, the commanded motor torques are augmented to drive the total momentum of all wheels to zero. However, with redundant wheels, the total momentum of the wheel cluster can be zero, even for very large wheel speeds, due to the four degrees of freedom spanning three-dimensional space. Thus, there does not appear to be a guarantee that wheel speeds will actually return to zero due to momentum dumping. This method also does not provide a means to bias the wheels to nonzero wheel speeds, though this has been considered in other work [5].

In this paper, an autonomous control is presented for a three-axis stabilized spacecraft. This pointing control is achieved using a redundant combination of four reaction wheels for attitude control and three magnetic torque rods for momentum dumping. However, the methodology developed is general enough to be applicable for an arbitrary number of redundant reaction wheels and magnetic torque

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bars. The control algorithm is parameterized using modified Rodrigues parameters (MRPs) as the attitude description [13–16], and a nonlinear feedback control law capable of tracking a time-varying arbitrary attitude is developed. Although MRPs are chosen as the attitude measure, the developments are easily modified to incorporate alternative attitude descriptions such as Euler angles and quaternions. A new feedback control is developed for a spacecraft with a redundant set of reaction wheels that still guarantees global asymptotic stability but includes an integral error measure while avoiding quadratic rate feedback terms. The integral feedback provides robustness to unmodeled disturbance torques. This combination of integral feedback while avoiding quadratic angular velocity feedback components is a new development that helps avoid control saturation issues when handling initial detumbling after being released from the launch vehicle.

The redundancy of having four reaction wheels presents a challenge for momentum dumping. For any desired torque, there are an infinite number of wheel accelerations that may be used to achieve it. If a magnetic torque is created for momentum dumping purposes, a solution for the motor torques must be chosen to offset it such that the wheels actually spin down. There is no guarantee that any arbitrary solution will not further increase wheel speeds, leading to wheel saturation. To that end, a momentum dumping strategy is investigated for the case of a redundant reaction wheel cluster. The solution should be very general, in that the reaction wheels and torque rod actuation axes can have general body-fixed orientations. This more general solution should provide effective autonomous momentum dumping and integrate well with the attitude control law. If the reaction wheel cluster is redundant, the momentum dumping strategy is desired to exploit the reaction wheel null space. The end result should be a single strategy for general configurations.

II. Background

In this paper, a rigid-body spacecraft outfitted with a redundant set of reaction wheels ($n > 3$) and a set of magnetic torque bars is considered. The developments in this paper are formulated in a general way that can account for different numbers of these actuation devices. The primary function of the reaction wheels is to provide three-axis pointing. Due to the process of momentum exchange between the wheels and spacecraft, a method for dumping momentum is needed [5]. To that end, a set of magnetic torque bars is used for the purpose of momentum dumping. For a single torque bar, the resulting torque is [17]

$$\boldsymbol{\tau}_{TBi} = \mu_i \hat{\boldsymbol{g}}_{ti} \times \boldsymbol{B}$$

where μ_i is the strength of the magnetic dipole of the torque bar, $\hat{\boldsymbol{g}}_{ti}$ is the alignment axis of the torque bar, and $\boldsymbol{B}$ is the Earth's magnetic field. It is important to note that a magnetic torque bar is only capable of producing torque in the plane perpendicular to the magnetic field. For the full set of N torque bars, the resulting torque on the spacecraft is expressed as

$$\boldsymbol{\tau}_{TB} = -[\tilde{\boldsymbol{B}}][G_t]\boldsymbol{\mu} \tag{1}$$

where $[\tilde{\boldsymbol{B}}]$ denotes the skew-symmetric matrix, $[G_t] = [\hat{\boldsymbol{g}}_{t1} \quad \hat{\boldsymbol{g}}_{t2} \quad \dots \quad \hat{\boldsymbol{g}}_{tN}]$, and $\boldsymbol{\mu} = [\mu_1 \quad \mu_2 \quad \dots \quad \mu_N]^T$.

The spin axis of each reaction wheel is denoted as $\hat{\boldsymbol{g}}_{si}$. The momentum exchange between the spacecraft and wheels is accomplished through careful application of motor torques u_{si}. These torques, in turn, act to change the wheel speeds, which are defined relative to the spacecraft and referred to as Ω_i. Thus, the momentum of a single wheel about its spin axis is

$$h_i = J_{si}(\Omega_i + \hat{\boldsymbol{g}}_{si}^T \boldsymbol{\omega}) \tag{2}$$

where J_{si} is the inertia of the wheel about its spin axis, and $\boldsymbol{\omega}$ is the spacecraft angular velocity. In accordance with Euler's equation, the wheel speeds evolve as a result of applied motor torques according to

$$u_{si} = J_{si}(\dot{\Omega} + \hat{\boldsymbol{g}}_{si}^T \dot{\boldsymbol{\omega}}) \tag{3}$$

In the current study, the modified Rodrigues parameter set is used as the attitude measure [18–20]. Although MRPs are chosen for the analysis in this paper, the control developments could also be similarly derived using other attitude parameters, such as Euler angles or quaternions. The kinematic differential equation for the MRP set is [14]

$$\dot{\boldsymbol{\sigma}} = \frac{1}{4}[(1 - \boldsymbol{\sigma}^T\boldsymbol{\sigma})\mathbb{I} + 2[\tilde{\boldsymbol{\sigma}}] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T]\delta\boldsymbol{\omega} \tag{4}$$

where $\mathbb{I}$ is the 3×3 identity matrix, and $\delta\boldsymbol{\omega} = \boldsymbol{\omega} - \boldsymbol{\omega}_r$ is the angular velocity of the spacecraft body frame relative to a reference frame. The spacecraft angular velocity is denoted as $\boldsymbol{\omega}$, and the reference frame angular velocity is denoted as $\boldsymbol{\omega}_r$. Being a minimal parameter set for attitude description, modified Rodrigues parameters do have singularities. However, the nonuniqueness of the MRP set allows for avoidance of the singularities by switching back and forth between the original and shadow sets [3].

The rotational motion of a spacecraft equipped with n reaction wheels and N magnetic torque bars is described by [3]

$$[I]\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times ([I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) - [G_s]\boldsymbol{u}_s - [\tilde{\boldsymbol{B}}][G_t]\boldsymbol{\mu} + \boldsymbol{L} \tag{5}$$

where $[I]$ is the inertia tensor of the spacecraft, and

$$[G_s] = [\hat{\boldsymbol{g}}_{s1} \quad \hat{\boldsymbol{g}}_{s2} \quad \dots \quad \hat{\boldsymbol{g}}_{sn}] \tag{6a}$$

$$\boldsymbol{h}_s = [h_1 \quad h_2 \quad \dots \quad h_n]^T \tag{6b}$$

$$\boldsymbol{u}_s = [u_{s1} \quad u_{s2} \quad \dots \quad u_{sn}]^T \tag{6c}$$

whereas $\boldsymbol{L}$ is the external torque acting on the spacecraft. Note that the term $[I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s$ is the total angular momentum of the spacecraft and reaction wheels.

III. Attitude Control

The tracking problem of an arbitrary, possibly time-varying reference attitude is considered. Of interest is developing a nonlinear three-axis attitude control with a redundant set of reaction wheels. The control should avoid quadratic rate feedback terms to avoid saturation during a detumbling maneuver while providing a robust solution to small unmodeled torques. This baseline control will then be enhanced in the following section by superimposing a continuous momentum management system.

Let the reference frame be denoted as $\mathcal{R}$. The goal of the attitude tracking control law is to reorient the spacecraft body frame $\mathcal{B}$ such that it matches $\mathcal{R}$. The attitude error between $\mathcal{B}$ and $\mathcal{R}$ is described using the MRP description $\boldsymbol{\sigma}$. It then follows that, by driving $\boldsymbol{\sigma} \to 0$, attitude tracking is achieved. Furthermore, if the reference attitude is time varying, then the spacecraft angular velocity $\boldsymbol{\omega}$ must track that of the reference frame $\boldsymbol{\omega}_r$.

Consider the candidate Lyapunov function

$$V(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}, z) = \frac{1}{2}\delta\boldsymbol{\omega}^T[I]\delta\boldsymbol{\omega} + 2K \ln(1 + \boldsymbol{\sigma}^T\boldsymbol{\sigma}) + \frac{1}{2}z^T[K_I]z \tag{7}$$

where $\delta\boldsymbol{\omega} = \boldsymbol{\omega} - \boldsymbol{\omega}_r$, K is a scalar gain, $[K_I]$ is a gain matrix, and z is the integral term [3,21]:

$$z = \int_0^t (K\boldsymbol{\sigma} + [I]\delta\dot{\boldsymbol{\omega}})\,\mathrm{d}t \tag{8}$$

This integral term is added to provide robustness in the presence of unmodeled external torques. The Lyapunov function is positive definite about $\boldsymbol{\sigma} = 0$, $\boldsymbol{\omega} = \boldsymbol{\omega}_r$, $z = 0$. Here, the attitude error is formulated in terms of MRPs, but alternate attitude parameter sets could be similarly used. Lyapunov error functions incorporating other attitude parameter sets may be found in [3]. Computing the derivative of the Lyapunov function yields

$$\dot{V}(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}, z) = (\delta\boldsymbol{\omega} + [K_I]z)^T([I]\dot{\boldsymbol{\omega}} - [I](\dot{\boldsymbol{\omega}}_r - \boldsymbol{\omega}\times\boldsymbol{\omega}_r) + K\boldsymbol{\sigma}) \tag{9}$$

Note that, to arrive at this result, the derivative of the right-hand side of Eq. (7) is taken with respect to the spacecraft body frame. Evaluating the body-frame derivative of $\delta\boldsymbol{\omega}$ results in the introduction of the $\boldsymbol{\omega}\times\boldsymbol{\omega}_r$ term. For more detail on this development, the reader is referred to [3]. Plugging in Eq. (5), the Lyapunov rate becomes

$$\dot{V}(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}, z) = (\delta\boldsymbol{\omega} + [K_I]z)^T(-\boldsymbol{\omega}\times\left([I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s\right) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} - [I](\dot{\boldsymbol{\omega}}_r - \boldsymbol{\omega}\times\boldsymbol{\omega}_r) + K\boldsymbol{\sigma}) \tag{10}$$

Note that the magnetic torque has been omitted in this development. It is considered in the following section regarding momentum dumping. Its omission does not impact the stability guarantees derived here. As discussed later on, the added magnetic torques are compensated exactly by the reaction wheels, and the net effect on the spacecraft dynamics is zero. Thus, the magnetic torques affect neither the Lyapunov function nor the Lyapunov rate, and the stability guarantees remain valid.

To ensure Lyapunov stability, the motor torques $\boldsymbol{u}_s$ are employed to cause the Lyapunov rate to take the negative semidefinite form

$$\dot{V}(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}, z) = -(\delta\boldsymbol{\omega} + [K_I]z)^T[P](\delta\boldsymbol{\omega} + [K_I]z) \tag{11}$$

where $[P]$ is a positive definite gain matrix. In prior work [3], this is accomplished by directly compensating the natural dynamics present in Eq. (10). Such an approach yields a control law of

$$[G_s]\boldsymbol{u}_s = -[I](\dot{\boldsymbol{\omega}}_r - \boldsymbol{\omega}\times\boldsymbol{\omega}_r) + K\boldsymbol{\sigma} + [P]\delta\boldsymbol{\omega} + [P][K_I]\boldsymbol{z} - \boldsymbol{\omega}\times([I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) + \boldsymbol{L} \tag{12}$$

Although this solution achieves the desired negative semidefinite form and can be shown to be asymptotically stabilizing, it suffers from the presence of the quadratic angular velocity term $-\boldsymbol{\omega}\times[I]\boldsymbol{\omega}$. If the angular velocity is high, this quadratic term can become large enough to lead to control saturation, invalidating the analytic stability guarantees. Note that this issue is most likely to occur following kickoff from the launch vehicle, with spacecraft tumble rates of a few degrees per second.

In the current paper, we present a novel control formulation that eliminates the need for this quadratic term while still providing asymptotic stability. Consider the control law

$$[G_s]\boldsymbol{u}_s = -[I](\dot{\boldsymbol{\omega}}_r - \boldsymbol{\omega}\times\boldsymbol{\omega}_r) + K\boldsymbol{\sigma} + [P]\delta\boldsymbol{\omega} + [P][K_I]z - ([\widetilde{\boldsymbol{\omega}_r}] - [\widetilde{K_I}]z)([I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) + \boldsymbol{L} \tag{13}$$

where the overtilde is used to denote the skew-symmetric matrix. Substituting this into Eq. (10) yields

$$\dot{V}(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}, z) = (\delta\boldsymbol{\omega} + [K_I]z)^T[(-[\widetilde{\delta\boldsymbol{\omega}}] - [\widetilde{K_I}]z)([I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) - [P]\delta\boldsymbol{\omega} - [P][K_I]z] \tag{14}$$

Noting the identity

$$(\delta\boldsymbol{\omega} + [K_I]z)^T(-[\widetilde{\delta\boldsymbol{\omega}}] - [\widetilde{K_I}]z)([I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) = 0$$

the Lyapunov rate reduces to the negative semidefinite form in Eq. (11). Thus, the system will converge to the set $\delta\boldsymbol{\omega} + [K_I]z = 0$.

To determine asymptotic stability, higher-order derivatives of the Lyapunov function are evaluated on the set $\delta\boldsymbol{\omega} + [K_I]z = 0$ [22]. The second derivative is identically zero ($\ddot{V} = 0$), and the third derivative reduces to

$$\dddot{V}(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}, z) = -2K^2\boldsymbol{\sigma}^T[I]^{-1}[P][I]^{-1}\boldsymbol{\sigma} \tag{15}$$

which is negative definite. Thus, the control law is asymptotically stabilizing, i.e., $\boldsymbol{\sigma}\to 0$. Furthermore, the kinematic coupling between the MRP set and the angular velocity guarantees that, if $\boldsymbol{\sigma}$ converges to 0, then $\delta\boldsymbol{\omega}$ must also converge to zero. Lastly, the integral term z must converge to 0 to satisfy $\delta\boldsymbol{\omega} + [K_I]z = 0$.

The new control law in Eq. (13) does not contain a quadratic function of $\boldsymbol{\omega}$. It does, however, contain a term proportional to $\boldsymbol{\omega}\times\boldsymbol{\omega}_r$. This is unavoidable if time-varying reference tracking is desired. The absence of the quadratic term is most beneficial for initial detumbling maneuvers following release of the spacecraft, where tumbling rates may be high enough to cause control saturation if the quadratic term is included. To compute the necessary motor torques, the $[G_s]$ matrix must be inverted. In a redundant system, this matrix will be of dimension $3\times n$ and a minimum norm inverse may be used.

The integral term z is included to provide robustness in the presence of unmodeled torques. A torque that is not accounted for will cause the Lyapunov rate to lose its negative semidefiniteness. Instead, the Lyapunov rate will be

$$\dot{V}(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}, z) = -(\delta\boldsymbol{\omega} + [K_I]z)^T([P](\delta\boldsymbol{\omega} + [K_I]z) - \Delta\boldsymbol{L}) \tag{16}$$

where $\Delta\boldsymbol{L}$ is the unmodeled torque. Though the stability guarantees no longer hold, $\delta\boldsymbol{\omega}$ and z cannot grow unbounded because the quadratic $(\delta\boldsymbol{\omega} + [K_I]z)^T[P](\delta\boldsymbol{\omega} + [K_I]z)$ term will eventually dominate, making $\dot{V}$ negative. If $\boldsymbol{\sigma}$ did not converge to 0, then the integral term z would grow unbounded. Therefore, $\boldsymbol{\sigma}$ must converge to 0, along with $\delta\boldsymbol{\omega}$. So, asymptotic stability still holds for the attitude, but the integral term will no longer converge to 0. For further discussion on this matter, the reader is referred to [3].

IV. Momentum Management with Redundant Reaction Wheels

As the wheels are spun up to provide attitude control, they will eventually reach their saturation limit with regard to wheel speeds if left unchecked. To eliminate momentum from the spacecraft/reaction wheel system, an external means of torquing is required. The magnetic torque bars are used for this purpose. By creating a magnetic torque on the spacecraft in a controlled manner and compensating for it with the reaction wheels, the wheels can be despun while simultaneously achieving attitude control. In practice, reaction wheels are often biased toward nonzero wheel speeds. These nonzero wheels speeds result in a desired bias angular momentum $\boldsymbol{h}_B$.

In prior work [5,8,9], a cross-product desaturation control law was used. The magnetic dipole produced by the torque bars was proportional to

$$\boldsymbol{\mu}\sim\Delta\boldsymbol{h}\times\boldsymbol{B} \tag{17}$$

where $\Delta\boldsymbol{h} = \boldsymbol{h}_W - \boldsymbol{h}_B$. At any given time, the angular momentum vector of the reaction wheels $\boldsymbol{h}_W$ was given by

$$\boldsymbol{h}_W = [G_s]\boldsymbol{h}_s$$

In the case of three reaction wheels, this strategy is sufficient, as a nonzero $\Delta\boldsymbol{h}$ vector implies the wheel speeds are not biased properly.

In a redundant system, however, such a formulation can be problematic. This is due to the fact that the set of four or more reaction wheels span a three-dimensional space in a manner that allows for nonunique control solutions. For example, for a necessary torque, there are an infinite number of motor torques that may be used to achieve it. Similarly, there are an infinite number of wheel speeds that may result in a given $\boldsymbol{h}_W$. Thus, $\Delta\boldsymbol{h}$ may be 0, or very small, even for large wheel speeds. In such a scenario, the resulting desaturation magnetic torque would be virtually nonexistent, in spite of the fact that the wheels were nowhere near the desired bias.

To prevent these issues, a method for handling desaturation in a redundant system is developed. First, note that the motor torque equation may be approximated by

$$\boldsymbol{u}_s = [J_s]\dot{\boldsymbol{\Omega}} \tag{18}$$

where $[J_s] = \mathrm{diag}([J_{s1}\ J_{s2}\ \dots\ J_{sn}])$ and $\dot{\boldsymbol{\Omega}} = [\dot{\Omega}_1\ \dot{\Omega}_2\ \dots\ \dot{\Omega}_n]^T$. In general, $\dot{\boldsymbol{\omega}}$ will be small and not significantly impact the evolution of the wheel speeds. To impose a despin torque on each wheel, a feedback on wheel speeds is used:

$$\boldsymbol{u}_s^* = -c[J_s](\boldsymbol{\Omega} - \boldsymbol{\Omega}_r) \tag{19}$$

where c is a gain, and $\boldsymbol{\Omega}_r$ are the wheel speed biases. Superimposing these desaturation torques on top of the control solution in Eq. (13) results in a net torque on the spacecraft of

$$\boldsymbol{\tau}_{RW} = -[G_s]\boldsymbol{u}_s^* \tag{20}$$

To counteract this, the magnetic torque bars are used. An attempt is made to perfectly offset the despin torque by controlling the dipoles of the individual magnetic torque bars using

$$-[\tilde{\boldsymbol{B}}][G_t]\boldsymbol{\mu} = [G_s]\boldsymbol{u}_s^* \tag{21}$$

The torque bars are limited, however due to the fact that they can only generate torque perpendicular to the magnetic field vector. Generally, the product $[\tilde{\boldsymbol{B}}][G_t]$ is not full rank, and a direct inverse is not possible. Instead, a singular value decomposition pseudoinverse is performed [23]. The resulting solution for the dipoles is given by

$$\boldsymbol{\mu}^* = -([\tilde{\boldsymbol{B}}][G_t])^{\dagger}[G_s]\boldsymbol{u}_s^* \tag{22}$$

where the superscript † is used to represent the pseudoinverse. This least-squares-like inverse yields dipoles for which the total magnetic torque approximates the desired momentum dumping torque $\boldsymbol{u}_s$ the closest.

The resulting magnetic dipoles interact with the Earth's magnetic field to produce a torque on the spacecraft equivalent to

$$\boldsymbol{\tau}_{MTB} = -[\tilde{\boldsymbol{B}}][G_t]\boldsymbol{\mu}^* \tag{23}$$

Ideally, $\boldsymbol{\tau}_{RW} + \boldsymbol{\tau}_{MTB} = 0$. However, this will rarely, if ever, be the case due to the inability of the torque bars to generate torque in the direction of the magnetic field. If left unchecked, the resulting imbalance between the desaturation torque $\boldsymbol{\tau}_{RW}$ and the torque bar compensation torque $\boldsymbol{\tau}_{MTB}$ will result in a nonzero perturbation on the spacecraft that will drive it away from the desired attitude. A final addition is made to the motor torques to account for this difference, and it is computed as

$$\Delta\boldsymbol{u} = [G_s]^{+}(\boldsymbol{\tau}_{MTB} - [G_s]\boldsymbol{u}_s^*) \tag{24}$$

The superscript + is used to denote a minimum norm inverse. With this correction, the resulting torque acting on the spacecraft due to the desaturation process is zero, i.e.,

$$-[G_s](\boldsymbol{u}_s^* + \Delta\boldsymbol{u}) - [\tilde{\boldsymbol{B}}][G_t]\boldsymbol{\mu}^* = 0 \tag{25}$$

The wheel desaturation algorithm is superimposed upon the attitude control solution and does not impact the stability guarantees derived in the previous section due to the net-zero torque it produces. Assuming the control law in Eq. (13) results in a motor torque solution of $\boldsymbol{u}_r$, the commanded motor torques at any particular time are given by

$$\boldsymbol{u}_s = \boldsymbol{u}_r + \boldsymbol{u}_s^* + \Delta\boldsymbol{u} \tag{26}$$

with the necessary dipoles computed in Eq. (22). This desaturation strategy may be applied continuously. Furthermore, in the case of $\boldsymbol{h}_W = \boldsymbol{h}_B$, even with large wheel speeds, the desaturation strategy will act to bring the wheels to the desired biases.

V. Numerical Simulation

To demonstrate functionality of the control law and momentum dumping strategy, numerical simulation is used. A scenario is considered where the spacecraft is to track the rotating Hill frame [3,24]. Let the instantaneous orbital position of the spacecraft be denoted as $\boldsymbol{r}$ and its velocity by $\boldsymbol{v}$. The Hill frame is then defined by the unit vectors

$$\hat{\boldsymbol{o}}_r = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}, \qquad \hat{\boldsymbol{o}}_\theta = \hat{\boldsymbol{o}}_h \times \hat{\boldsymbol{o}}_r, \qquad \hat{\boldsymbol{o}}_h = \frac{\boldsymbol{r}\times\boldsymbol{v}}{|\boldsymbol{r}\times\boldsymbol{v}|} \tag{27}$$

In this scenario, the goal is to reorient the spacecraft such that $\hat{\boldsymbol{b}}_1 \to \hat{\boldsymbol{o}}_\theta$, $\hat{\boldsymbol{b}}_2 \to \hat{\boldsymbol{o}}_h$, and $\hat{\boldsymbol{b}}_3 \to \hat{\boldsymbol{o}}_r$. The orbital elements used to simulate the orbital motion are given in Table 1. Here, a circular orbit is used, which corresponds to reference angular velocity of

$$\boldsymbol{\omega}_r = n\hat{\boldsymbol{o}}_h \tag{28}$$

where n is the orbital mean motion. The Hill frame rotates about the $\hat{\boldsymbol{o}}_h$ axis at a rate equal to n. To propagate the orbit, two-body dynamics are used.

To simulate magnetic torque bar behavior, a magnetic field model is needed. For this study, the tilted-centered dipole magnetic field model is used, with magnetic field components defined by [17]

$$\begin{bmatrix} B_{\text{North}} \\ B_{\text{East}} \\ B_{\text{Down}} \end{bmatrix} = -\left(\frac{6378\ \text{km}}{r}\right)^3 \times \begin{bmatrix} -\cos\phi & \sin\phi\cos\lambda & \sin\phi\sin\lambda \\ 0 & \sin\lambda & -\cos\lambda \\ -2\sin\phi & -2\cos\phi\cos\lambda & -2\cos\phi\sin\lambda \end{bmatrix} \begin{bmatrix} 29,900 \\ 1900 \\ -5530 \end{bmatrix} nT \tag{29}$$

where λ is the spacecraft latitude, and ϕ is the longitude. Note that, in the Earth-centered/Earth fixed (ECEF) frame, the magnetic field is constant. In the Earth-centered inertial (ECI) frame, the magnetic field rotates along with the Earth. The spacecraft orbit is propagated in the ECI frame, and the rotation of the Earth must be modeled. For the simulation, the ECEF and ECI frames are assumed to be aligned initially, and the Earth rotates about the z axis at a rate of 7.292×10^{-5} rad/s.

The spacecraft is assumed to have three magnetic torque bars and four reaction wheels. In the spacecraft body frame, the alignment axes for these devices are

Table 1 Orbital parameters used in numerical simulation

Parameter	Value
a	6778.14 km
e	0 deg
i	45 deg
Ω	60 deg
ω	0 deg
ν_0 (true anomaly)	0 deg

Table 2 Control gains used for numerical simulation

Parameter	Value
$[K]$	$0.037[\mathbb{I}_{3\times3}]$ N · m
$[P]$	$0.45[\mathbb{I}_{3\times3}]$ N · m
$[K_I]$	$0.001[\mathbb{I}_{3\times3}]$ N^{-1} s^{-2}
c	0.005 s^{-1}

$$[G_t]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},$$

$$[G_s]=\begin{bmatrix}0&0&\cos(45\text{ deg})&-\cos(45\text{ deg})\\\cos(45\text{ deg})&\sin(45\text{ deg})&-\sin(45\text{ deg})&-\sin(45\text{ deg})\\\sin(45\text{ deg})&-\cos(45\text{ deg})&0&0\end{bmatrix}$$

Each reaction wheel is assumed to have the same spin-axis inertia value of $J_{si}=0.002$ kg · m^2. The torque bars are limited to a maximum dipole of 20 A · m^2. A diagonal inertia tensor is assumed with the values

$$[I]=\begin{bmatrix}10.5&0&0\\0&8&0\\0&0&6.75\end{bmatrix}\text{ kg}\cdot\text{m}^2$$

Lastly, the control gains implemented for the numerical simulation are summarized in Table 2.

The initial conditions used for the simulation are $\boldsymbol{\omega}_0=[6.0\quad 6.0\quad 6.0]^T$ deg/s, $\boldsymbol{\Omega}_i=1000$ revolutions per minute, and $\boldsymbol{\sigma}_0=[0.5\quad -0.5\quad 0.7]^T$. Here, $\boldsymbol{\sigma}_0$ represents the initial attitude of the spacecraft relative to the ECI frame. For the attitude error, $\boldsymbol{\sigma}_{BR}$ is used, which represents the rotation between the reference Hill frame and the spacecraft body frame. To simulate unmodeled torques on the spacecraft, a residual dipole of 1 A · m^2 is applied to all three body axes of the spacecraft.

For the purposes of comparison, two simulations are performed. One is run with the cross-product momentum dumping strategy presented in [5], and the second is run with the momentum dumping strategy developed in this paper. For all three cases, the control law in Eq. (13) is used and the closed-loop dynamics of the system are the same. The resulting attitude and angular velocity errors are shown in Fig. 1. Though the momentum dumping is handled differently for each of the simulations, there is no net effect on the spacecraft attitude errors. Thus, the attitude tracking history in Fig. 1 applies to both cases. Roughly 25–30 min are required for convergence onto the desired attitude.

The momentum dumping strategy in [5] is implemented as follows. Here, a total dipole of

$$\boldsymbol{M}=-\frac{k_M}{B^2}\boldsymbol{H}_D\times\boldsymbol{B} \tag{30}$$

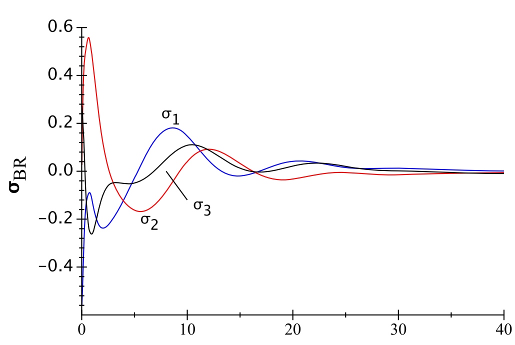
is applied to the spacecraft using the torque bars. Note that k_M is a feedback gain and

$$\boldsymbol{H}_D=[G_s]\boldsymbol{h}_s+[I]\boldsymbol{\omega}-\boldsymbol{H}_B \tag{31}$$

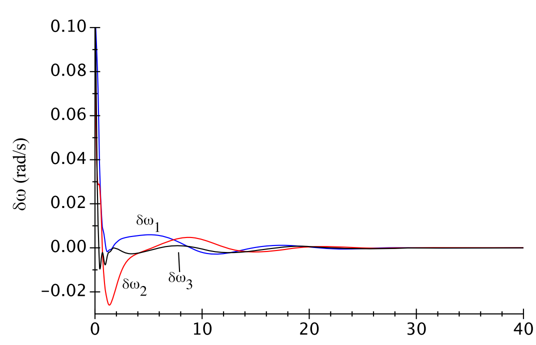
where $\boldsymbol{H}_B$ is the desired bias momentum of the system. In the current study, a value of $k_M=0.003$ 1/s is used. Effectively, $\boldsymbol{H}_D$ is a measure of the difference between the actual system momentum and the desired bias momentum. To compensate the application of $\boldsymbol{M}$, a correction to reaction wheel motor torques is performed to offset the resulting $\boldsymbol{M}\times\boldsymbol{B}$ torque on the spacecraft. Ideally, this correction will drive the wheel speeds toward their desired values. For the case of four reaction wheels, however, there are an infinite number of wheel speeds that will yield a desired $\boldsymbol{H}_B$. Thus, there is no guarantee that the wheel speeds will converge to the desired values.

To illustrate the differences between the momentum dumping algorithms, the wheel speeds for both methods are shown in Fig. 2 for a duration of 125 min. All four reaction wheels are commanded to a desired bias of $\Omega_r=250$ rpm. The differences between the methods are evident. The novel formulation shows all wheel speeds approaching the desired biases. The small oscillations are due to the compensation of the unmodeled torques acting on the spacecraft. The behavior is not the same, however, for the case of the cross-product formulation. In this case, the wheel speeds oscillate about 0 rpm. This is not a failure of the momentum dumping strategy; indeed, the system momentum is converging to the desired bias. Rather, it is a failure in the ability of this formulation to handle the case of redundant reaction wheels. There is no guarantee the wheel speeds will approach their desired values. Although final wheel speeds of 0 rpm may be completely acceptable, this result is dependent on the initial conditions and the desired attitude history. With different initial conditions, or a different reference attitude, the wheel speeds could converge to different final values closer to the saturation limit, limiting future attitude maneuvers. This result shows that the newly developed momentum dumping strategy is an improvement over prior work for the case of redundant reaction wheels.

To illustrate the effect of the new control law on control requirements, a simulation using the old quadratic controller in Eq. (12) is run. All simulation parameters are maintained as previously stated, and the novel momentum dumping strategy is used. The resulting attitude error is shown in Fig. 3. The magnitude of the commanded wheel motor torques is also shown, and it is compared with the motor torque commands from the simulation using the novel control law. A few conclusions can be drawn. Using the quadratic control law results in a faster settling time. This faster settling time comes at the cost of higher peak motor torques. Including the gyroscopic $-\boldsymbol{\omega}\times[I]\boldsymbol{\omega}$ term in the controller helps to damp out the angular velocity faster, but it results in larger peak motor torques due to the large angular velocity of the spacecraft. This illustrates that, if motor torque saturation is a concern due to the need to damp out high angular velocities, the novel control law presented here can help reduce the required motor

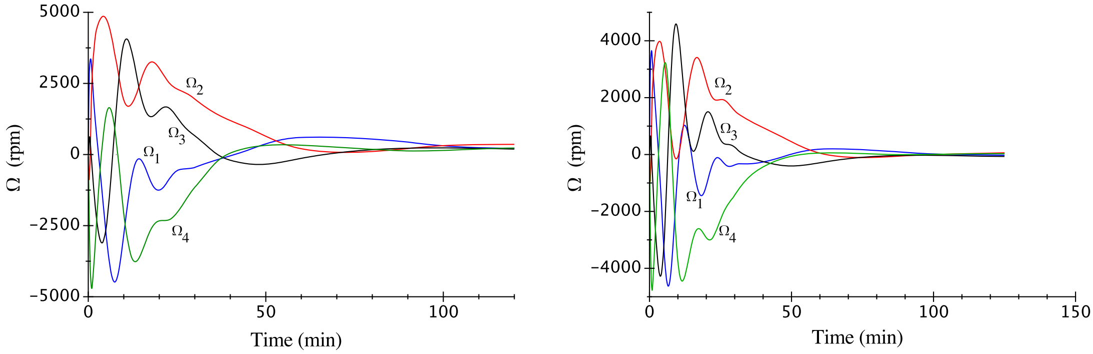


a) Attitude error



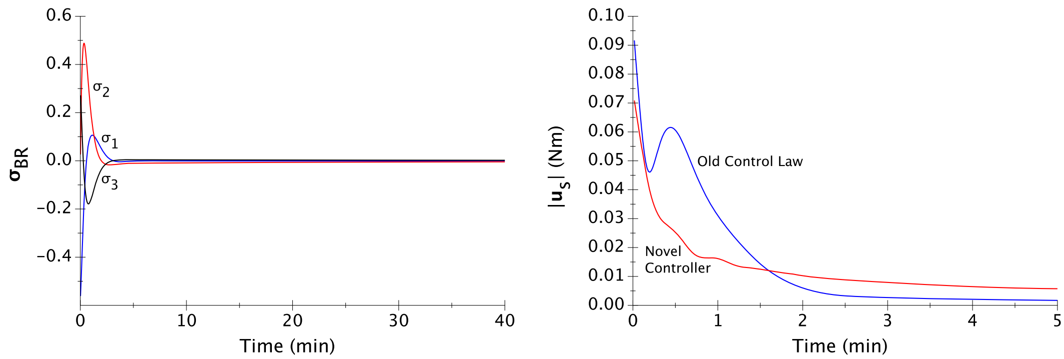
b) Angular velocity error

Fig. 1 Representations of a) relative attitude and b) angular velocity of body frame relative to Hill frame during first 40 min of pointing maneuver.



a) General momentum dumping **b) Cross-product momentum dumping**

Fig. 2 Reaction wheel speed performance illustration for two momentum management scenarios using desired bias wheel speeds of 250 rpm.



a) Attitude convergence **b) Motot torques**

Fig. 3 Representations of a) attitude convergence for old control law in Eq. (12) and b) comparison of motor torque magnitude between old controller and the novel control law.

torques. However, this does result in somewhat reduced performance, as it takes longer to achieve convergence onto the desired attitude. The differences in the necessary motor torques only increase as the initial spacecraft angular velocity increases.

VI. Conclusions

In this paper, a spacecraft attitude control law is developed for a redundant cluster of reaction wheels. The control law, which includes integral feedback to account for unmodeled torques, is an improvement over similar control strategies previously developed, in that it does not contain a term that is quadratic in angular velocity. The issue of momentum dumping is also addressed in the current study. Prior work implements momentum management strategies for nonredundant reaction wheel clusters that have limitations in their application to a spacecraft equipped with four or more reaction wheels. A new method of momentum dumping is developed, which does not experience the shortcomings inherent in prior work.

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