JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 40, No. 5, May 2017

Engineering Notes

Consider-Filter-Based On-Orbit Coarse Sun Sensor Calibration Sensitivity

Stephen A. O'Keefe* and Hanspeter Schaub†
University of Colorado, Boulder, Colorado, 80309

DOI: 10.2514/1.G001692

I. Introduction

IN RECENT years, there has been a significant increase in interest in smaller satellites as a lower-cost alternative to traditional satellites. The size and cost limitations of these satellites are a driving factor for making the most of inexpensive components and sensors. Cosine-type coarse sun sensors (CSSs) that output a scalar voltage relative to the angle between the input irradiance and the sensor normal are an example of such a small, inexpensive sensor. CSSs are often used in concert with other sensors [1,2] to perform coarse attitude determination, to point a spacecraft's solar arrays at the sun, or for use during safe-mode operations.

Commercial sensors that have been calibrated are quite expensive, whereas custom photodiodes can be manufactured for significantly less cost. Highly accurate calibration can be done on the ground for in-house sensors; however, this requires significant financial costs and testing facilities. A coarse ground calibration, with a detailed calibration done on orbit, requires less manpower and costs less and therefore presents an attractive alternative for small satellite budgets. Methods for performing initial sun heading estimation using poorly calibrated sensors, in an underdetermined configuration, have been presented [3]. Performing calibration on orbit improves the performance of safe-mode operations and reduces the covariance of the CSS parameters, allowing for autonomous fault detection to be performed.

Ortega et al. [4] and Wu and Steyn [5] present calibrations of two-axis digital sun sensors specific to individual models. However, literature on calibration methods for scalar coarse CSS calibration is limited. Springmann and Cutler [6] present a CSS calibration filter capable of calculating the CSS scale factor and misalignment. Both a quaternion-based extended Kalman filter (EKF) approach and an unscented Kalman filter approach are presented, and the filter performance is shown for flight data. Analyzed here are the sensitivities of a modified Rodrigues parameter (MRP)-based EKF approach to the level of albedo model accuracy and attitude measurement accuracy. The MRP-based approach provides an alternative to a quaternion approach while still characterizing the information necessary to perform meaningful calibration and can be easily added onto an existing MRP-based attitude estimation framework, such as that presented in [7].

Numerical simulations are used to demonstrate the performance of the filter in which significant noise and biases are added to exercise the estimator in a realistic scenario. Reference [3] shows that simultaneous sun-direction estimation and pointing can be performed when scale factor uncertainties are normally distributed with a standard deviation of 2%. Here, those scale factors are distributed by 30%, an order of magnitude larger. For comparison, photodiode calibration is typically on the order of 5% for visible light.

II. Coarse Sun Sensors

The unit direction vector for a CSS is spherically expressed in the spacecraft body frame as

$$\mathcal{B}_{n} = [\cos(\phi)\ \cos(\theta) \quad \cos(\phi)\ \sin(\theta) \quad \sin(\phi)\,]^T$$

where θ is the azimuth angle, measured positive from the body $+x$ axis around the $+z$ axis, and ϕ is the elevation angle, measured positive toward the body $+z$ axis from the x-y plane, of the CSS direction vector. Assuming Lambert's cosine law [8], including the effects of Earth's albedo, and accounting for field-of-view limitations of actual hardware, the output voltage of an individual CSS is modeled as

$$V = C\cdot(V_d + V_\alpha + \nu_V)$$
$$V_d = \begin{cases} n^T\frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} & \text{if } \left(n^T\frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} \geq \cos\psi\right) \wedge (B \notin \mathcal{S}) \\ 0 & \text{if } \left(n^T\frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} < \cos\psi\right) \vee (B \in \mathcal{S}) \end{cases}$$
$$V_\alpha = \begin{cases} -\frac{1}{\pi}\iint_A \frac{\alpha}{\|\boldsymbol{r}_{AB}\|^2}\left(n_A^T\frac{\boldsymbol{s}_\oplus}{\|\boldsymbol{s}_\oplus\|}\right)\left(n_A^T\frac{\boldsymbol{r}_{AB}}{\|\boldsymbol{r}_{AB}\|}\right)\left(n^T\frac{\boldsymbol{r}_{AB}}{\|\boldsymbol{r}_{AB}\|}\right)\mathrm{d}A & \text{if } B \notin S \\ 0 & \text{if } B \in S \end{cases} \tag{1}$$

where C is a calibration factor; ν_V is a zero-mean Gaussian random variable included to compensate for sensor noise and model errors; $\boldsymbol{s}$ is the sun-direction vector in the body frame; ψ is the half angle of the CSS field of view; $\boldsymbol{A}$ is the surface of the Earth visible to the spacecraft that is also illuminated by the sun; $\boldsymbol{n}_A$ is the unit normal of a differential area of $\boldsymbol{A}$; $\boldsymbol{s}_\oplus$ is the direction vector from the Earth to the sun; $\boldsymbol{r}_{AB}$ is a vector from dA to the body of the spacecraft; α is the albedo, or reflectivity coefficient, of dA; B is the spacecraft's position in orbit; and S is the region of the spacecraft's orbit in the shadow of the Earth.

The value of the Earth's albedo varies with position, due to seasonal, ground cover, and cloud cover changes, and accounts for a large part of Eq. (1). The mean value of the Earth's albedo varies between 0 and 0.9 depending on the relative positions of the Earth, sun, and spacecraft. The standard deviation of the Earth's albedo varies between 0 and 0.3 with lower variations over the poles and Greenland and higher variations over land where seasonal variations and cloud cover can significantly change the albedo in relatively short time scales. As a reference, a spacecraft in a 400 km circular, polar orbit will receive irradiance due to albedo equal to 0 to 50% of that received from direct sunlight.

For this study, daily measurements from 2000 to 2005, corresponding to a 1° × 1.25° latitude–longitude grid, are used to calculate mean and standard deviation values for the Earth's albedo. The data used in this study to model the Earth's albedo constant were acquired as part of the NASA's Earth–Sun System Division and archived and distributed by the Goddard Earth Sciences Data and Information Services Center Distributed Active Archive Center. These values are used to generate statistically accurate randomly selected values for the Earth's albedo coefficient used in the numerical simulations.

III. Coarse Sun Sensor Calibration

For the calibration of the CSS, a consider Kalman filter approach is used. The measurement equations and system dynamics considered for calibration contain both noise and systematic biases, most notably due to Earth's albedo. Systematic bias errors are typically treated in four ways:

Received 10 September 2015; revision received 16 February 2016; accepted for publication 17 February 2016; published online 17 August 2016.

*Graduate Research Assistant, Aerospace Engineering Sciences Department; saokeefe@gmail.com. Member AIAA.

†Alfred T. and Betty E. Look Professor of Engineering, Aerospace Engineering Sciences Department; hanspeter.schaub@colorado.edu. Associate Member AIAA.

neglected, compensated for with process noise, estimated, or considered [9,10]. Neglecting the impact of the biases is a reasonable solution when the parameters have low impact on the dynamics. Compensation via process noise usually involves Monte Carlo analysis to numerically bound the uncertainty that is unaccounted for mathematically and can be time consuming and tedious. Expanding the state of the system to estimate such biases is an excellent way to account for their effects, if doing so does not overly increase the computational burden and the system is sufficiently observable. Finally, the effect of biases can be "considered"; the biases are not estimated directly, but their uncertainty is included in the calculation of the system covariance. Consider analysis provides a middle ground between ignoring and estimating the biases when the biases themselves have low observability. In addition, a consider Kalman filter approach is easily incorporated into existing attitude filters such as the MRP based EKF presented in [7].

Initially presented by Schmidt [11], more recent derivations of the consider Kalman filter have been published by Jazwinski [12], Tapley et al. [9], Woodbury and Junkins [10], and Zanetti and Bishop [13]. A continuous-discrete extended consider Kalman filter is used here, adapted from the work of Zanetti and Bishop with modifications for continuous-time propagation and nonconstant biases.

A. Full CSS Calibration Filter

The full CSS calibration filter assumes the spacecraft has CSSs, inertial attitude, and angular rate measurements available, as well as an orbit solution and an estimate of the reference Earth–sun vector, which can be calculated from the current date. The state vector and process noise vectors are set to

$$\begin{aligned} \boldsymbol{x}(t) &= [\,\boldsymbol{\sigma}^T(t) \quad \mathcal{G}\boldsymbol{\omega}_\beta^T(t) \quad \boldsymbol{C}^T(t) \quad \boldsymbol{\theta}^T \quad \boldsymbol{\phi}^T\,]^T, \\ \boldsymbol{\eta}(t) &= [\,\mathcal{G}\boldsymbol{\eta}_\omega^T(t) \quad \mathcal{G}\boldsymbol{\eta}_{\omega_\beta}^T(t) \quad \boldsymbol{\eta}_C^T(t)\,]^T \end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}(t)$ is the MRP attitude description of the spacecraft, $\boldsymbol{\theta}$ is a vector of CSS azimuth angles, and $\boldsymbol{\phi}$ is a vector of CSS elevation angles.

Rate gyroscope measurements are assumed to follow Farrenkopf's approximation [14]

$$\mathcal{G}\tilde{\boldsymbol{\omega}}(t) = [BG](\mathcal{G}\boldsymbol{\omega}(t) + \mathcal{G}\boldsymbol{\omega}_\beta(t) + \mathcal{G}\boldsymbol{\eta}_\omega(t))$$

$$\dot{\boldsymbol{\omega}}_\beta(t) = \boldsymbol{\eta}_{\omega_\beta}(t)$$

where a left superscript $\mathcal{G}$ indicates a quantity expressed in the rate gyro frame; $[BG]$ is the direction cosine matrix describing the rotation from the rate gyro frame to the spacecraft body frame; $\tilde{\boldsymbol{\omega}}(t)$ is the sensed angular velocity; $\boldsymbol{\omega}(t)$ is the true angular velocity; $\boldsymbol{\omega}_\beta(t)$ is the measurement bias drift, modeled as a rate random walk process; and $\boldsymbol{\eta}_\omega(t)$ and $\boldsymbol{\eta}_{\omega_\beta}(t)$ are the zero-mean Gaussian rate and angular acceleration white-noise processes, respectively.

Rewriting the CSS measurement model given by Eq. (1) in terms of Eq. (2) gives

$$\begin{aligned} V &= C(V_d + V_\alpha + \nu_V) \\ V_d &= \begin{cases} \mathcal{B}[\cos\phi\cos\theta \quad \cos\phi\sin\theta \quad \sin\phi][BN]\frac{\mathcal{N}\boldsymbol{s}_\oplus - \mathcal{N}\boldsymbol{r}_B}{\|\mathcal{N}\boldsymbol{s}_\oplus - \mathcal{N}\boldsymbol{r}_B\|} & \text{if } \left(n^T \frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} \ge \cos\psi\right) \wedge (B \notin S) \\ 0 & \text{if } \left(n^T \frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} < \cos\psi\right) \vee (B \in S) \end{cases} \\ [\mathrm{BN}] &= [I_{3\times 3}] + \frac{8[\boldsymbol{\sigma}]_\times^2 - 4(1-\sigma^2)[\boldsymbol{\sigma}]\times}{(1+\sigma^2)^2} \end{aligned} \tag{3}$$

where ν_α is zero-mean Gaussian noise representing the uncertainty in the albedo coefficient of dA calculated from the NASA Total Ozone Mapping Specgtrometer (TOMS) data, $\boldsymbol{r}_B$ is the spacecraft's position relative to the Earth, and a left superscript $\mathcal{N}$ indicates a quantity expressed in an inertial frame. The full filter Jacobians are omitted for brevity but can be found in [15].

B. Reduced CSS Calibration Filter

A reduced version of the CSS calibration filter that assumes the received irradiance due to Earth's albedo is treated as an unmodeled measurement bias is also used; thus, an estimate of the spacecraft's orbit is not necessary. This approach is investigated as a method to reduce the total computation time, at the cost of estimation accuracy, by eliminating the costly evaluation of the irradiance contributions caused by the Earth's albedo. The state and process noise vectors and the attitude measurement update are unchanged from the full CSS calibration filter.

Because the input irradiance due to Earth's albedo is treated as a bias, Eq. (3) is modified to

$$\begin{aligned} V &= C(V_d + V_\alpha + \nu_V) \\ V_d &= \begin{cases} \mathcal{B}[\cos\phi\cos\theta \quad \cos\phi\sin\theta \quad \sin\phi][\mathrm{BN}]\frac{\mathcal{N}\boldsymbol{s}_\oplus - \mathcal{N}\boldsymbol{r}_B}{\|\mathcal{N}\boldsymbol{s}_\oplus - \mathcal{N}\boldsymbol{r}_B\|} & \text{if } \left(n^T \frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} \ge \cos\psi\right) \wedge (B \notin S) \\ 0 & \text{if } \left(n^T \frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} < \cos\psi\right) \vee (B \in S) \end{cases} \end{aligned} \tag{4}$$

where, without an orbit solution, the spacecraft position relative to the Earth is treated as a systematic bias. It is expected that this bias will have a minimal impact on the estimate, especially when compared to the effect of Earth's albedo. Once again, the full Jacobians are omitted for brevity but can be found in [15].

IV. Numerical Simulations

A. Simulation Description

A spacecraft was modeled in a 400 km altitude circular orbit with an inclination of 90 deg starting on 1 June 2015, 00:00 coordinated universal time. Each simulation was run for one orbit from first illumination by the sun until it returned to the Earth's shadow. A truth trajectory was modeled using the accelerations due to the J_2 through J_6 Earth zonal gravitational perturbations, atmospheric drag, and solar radiation pressure, while the estimation algorithms used a simple two-body orbit propagator updated by position and velocity measurements. Earth and sun positions were simulated using ephemeris from the NASA Navigation and Ancillary Information Facility SPICE toolkit [16]. The spacecraft was assumed to have a mass of 100 kg, a drag area of 0.38 m^2, a ballistic coefficient of 2.1, a cross-sectional area of 1.3 m^2 subject to solar radiation pressure, and an inertia matrix given by $[I] = \mathrm{diag}[\,10.5 \quad 8.0 \quad 6.75\,]\ \mathrm{kg}\cdot\mathrm{m}^2$. The spacecraft was simulated in an uncontrolled tumble, but calibration could be reliably performed with Nadir pointing control active or a similar maneuver in which each sensor was pointed at the sun for some fraction of the orbit.

The spacecraft's initial attitude was uniformly distributed, and its initial angular velocity was varied with a standard deviation equal to

2 °/s. Rate gyroscope measurements were simulated at 10 Hz, and their rate white-noise standard deviation was assumed to be $1 \times 10^{-4}{}^{\circ}/\sqrt{s}$ with a drift stability standard deviation of 1×10^{-6} °/s over 1000 s. Spacecraft attitude vector measurements were simulated at 2 Hz and corrupted by white Gaussian noise with varying standard deviation. Orbit position measurements were simulated at 1 Hz, corrupted by position errors with a standard deviation of 1 km and velocity errors with a standard deviation of $0.1\ \mathrm{K \cdot m \cdot s^{-1}}$.

The alignment azimuth and elevation of each CSS were perturbed by a normally distributed angle with a standard deviation of 1°. All CSS were assumed to have calibration scale factors normally distributed by 30%, 1σ, from a nominal calibration scale factor of $C = 1.0$. CSS measurements were processed at 2 Hz, and white Gaussian noise was added to each sensor with a standard deviation of 0.05. The full $1° \times 1.25°$ latitude–longitude grid of albedo values is used to calculate the reference Earth albedo.

B. Simulation Results

The calibration filter results for inertial attitude measurement uncertainties of 1×10^{-4}, 1×10^{-3}, and 1×10^{-2} rad are shown in Fig. 1. As the accuracy of the inertial attitude measurements is degraded, the attitude uncertainty is increased; however, the CSS calibration statistics show no significant difference and are therefore omitted. Increasing the attitude uncertainty beyond 1×10^{-2} rad results in a large percentage of cases exhibiting filter divergence due to inconsistency in the measurement equations as a result of the discontinuities in Eq. (3).

The reduced calibration filter is compared to the full calibration filter run using $1° \times 1.25°$, $5° \times 5°$, and $10° \times 10°$ resolution albedo data assuming an attitude measurement uncertainty of 1×10^{-4} rad. It is found through Monte Carlo analysis that the measurement noise for the CSS needs to be raised from 0.05, as determined by the CSS noise, to 0.1 for the $1° \times 1.25°$ albedo grid, to 0.175 for the $5° \times 5°$ albedo grid, and to 0.3 for the $10° \times 10°$ albedo grid in order to account for the unmodeled aspects of the partial derivatives. The statistics for a 300 case Monte Carlo are shown in Fig. 2.

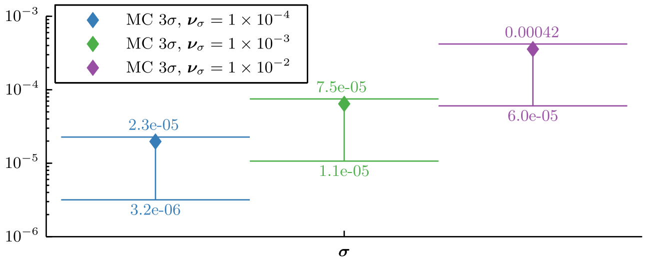

Fig. 1 Monte Carlo generated statistics of attitude uncertainties for spacecraft after one orbit for various levels of inertial attitude measurement uncertainty.

Table 1 Averages and standard deviations of computation times for various filters

ECKF	Albedo data	Propagation update, μs	Measurement update, μs
Full	$1° \times 1.25°$	15.2 ± 2.7	653 ± 457
Full	$5° \times 5°$	14.7 ± 2.0	204 ± 38
Full	$10° \times 10°$	14.8 ± 2.0	167 ± 18
Reduced	— —	16.8 ± 2.0	171 ± 21

As expected, a steady degradation of accuracy is seen as the albedo resolution is decreased, and the estimation of the scale factor benefits the most over such a short time scale. The full filter with $10° \times 10°$ albedo data provides only slightly lower uncertainty in the alignment angles than the reduced filter; however, up to a full order of magnitude lower uncertainty in the calibration coefficient is achieved. While the filter performance is not dependent on the number of CSSs, better estimates result when sensors have access to more direct measurements, a combined function of sensor alignment and satellite orientation.

It is important to consider computation time in addition to estimation accuracy. Statistics on the computation times are shown in Table 1. Because no control effort is applied, the simulations follow exactly the same trajectory and experience identical simulated sensor measurements. All code is written in C and compiled and run on a Windows i7 2.5GHz computer. While not flight hardware, the relative computation times provide insight into the expected trends. As expected, the full calibration filter takes significantly more time than the reduced filter, but reducing the density of the albedo grid greatly increases the computation speed.

V. Conclusions

A modified-Rodrigues-parameter-based coarse sun sensor (CSS) calibration filter, based on an extended consider Kalman filter (ECKF), was analyzed for estimating the calibration coefficient and alignment misalignment angles of CSS onboard a spacecraft in low Earth orbit. The sensitivity to albedo knowledge and attitude measurements and the computation time of the filter are computed using realistic numerical simulations. While the method does require inertial attitude measurements, such as from a star tracker or magnetometer, calibration can be performed using poor inertial attitude measurements, on the order of 1° error standard deviation, with no significant increase in calibration error. At best, CSS calibration scale factors can be estimated to less than 1%, and alignment angles can be estimated to approximately 1 deg. However, computation time can be reduced by a factor of 6, and calibration coefficient accuracies of 2% and alignment accuracies of

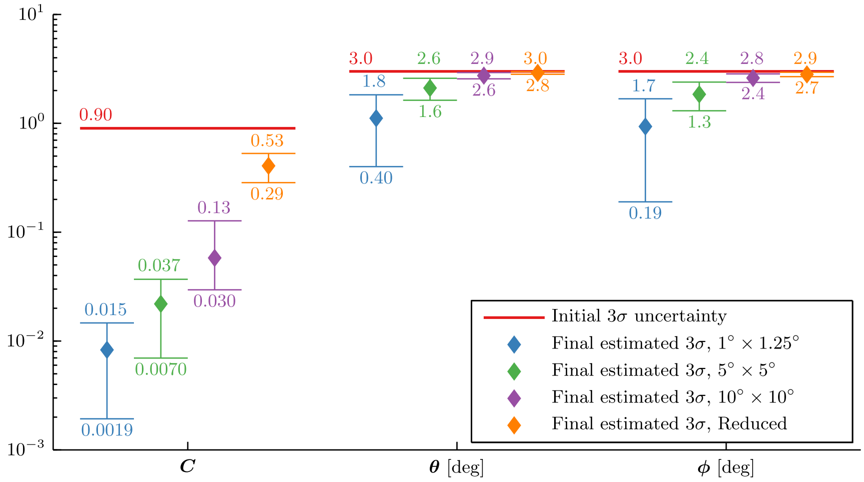

Fig. 2 Monte Carlo generated statistics of CSS calibration parameter uncertainties for spacecraft using a dual CSS configuration after one orbit for various levels of albedo data resolution.

approximately 2° can still be achieved. Such a calibration filter could be used onboard a small satellite in order to reduce necessary ground support, increase autonomy, and improve the functionality of safe-mode operations.

References

[1] Appel, P., "Attitude Estimation from Magnetometer and Earth-Albedo-Corrected Coarse Sun Sensor Measurements," *Acta Astronautica*, Vol. 56, Nos. 1–2, Jan. 2005, pp. 2–5. doi:10.1016/j.actaastro.2004.09.001.

[2] Jung, H., and Psiaki, M. L., "Tests of Magnetometer/Sun-Sensor Orbit Determination Using Flight Data," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 3, 2002, pp. 582–590. doi:10.2514/2.4920

[3] O'Keefe, S. A., and Schaub, H., "Sun-Direction Estimation Using a Partially Underdetermined Set of Coarse Sun Sensors," *Journal of the Astronautical Sciences*, Vol. 61, No. 1, Sept. 2015, pp. 85–106. doi:10.1007/s40295-015-0058-9

[4] Ortega, P., López-Rodríguez, G., Ricart, J., Domínguez, M., Castañer, L. M., Quero, J. M., Tarrida, C. L., García, J., Reina, M., Gras, A., and Angulo, M., "A Miniaturized Two Axis Sun Sensor for Attitude Control of Nano-Satellites," *IEEE Sensors Journal*, Vol. 10, No. 10, 2010, pp. 1623–1632. doi:10.1109/JSEN.2010.2047104

[5] Wu, S.-F., and Steyn, W. H., "Modelling and In-Orbit Calibration Practice of a Miniature 2-Axis Analogue Sun Sensor," *Aerospace Science and Technology*, Vol. 6, No. 6, Oct. 2002, pp. 423–433. doi:10.1016/S1270-9638(02)01187-2

[6] Springmann, J. C., and Cutler, J. W., "On-Orbit Calibration of Photodiodes for Attitude Determination," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 6, Nov. 2014, pp. 1808–1823. doi:10.2514/1.G000175

[7] Karlgaard, C. D., and Schaub, H., "Nonsingular Attitude Filtering Using Modified Rodrigues Parameters," *Journal of the Astronautical Sciences*, Vol. 57, No. 4, 2010, pp. 777–791. doi:10.1007/BF03321529

[8] Lerner, G. M., "Spacecraft Attitude Determination and Control," *Chapter Sun Sensors*, Reidel, Dordrecht, The Netherlands, 1978, pp. 155–166.

[9] Tapley, B., Schutz, B., and Born, G., *Statistical Orbit Determination*, Elsevier Academic Press, Burlington, MA, 2004, pp. 387–430. doi:10.1016/B978-1-4831-6716-9.50011-4

[10] Woodbury, D. P., and Junkins, J. L., "On the Consider Kalman Filter," *AIAA Guidance, Navigation, and Control Conference*, AIAA Paper 2010-7752, Aug. 2010. doi:10.2514/6.2010-7752

[11] Schmidt, S. F., "Application of State-Space Methods to Navigation Problems," *Advances in Control Systems*, Vol. 3, 1966, pp. 293–340.

[12] Jazwinski, A. H., *Stochastic Processes and Filtering Theory*, Academic Press, New York, 1970, Chap. 6.

[13] Zanetti, R., and Bishop, R. H., "Kalman Filters with Uncompensated Biases," *Journal of Guidance, Control, and Dynamics*, Vol. 35, No. 1, Jan. 2012, pp. 327–335. doi:10.2514/1.55120

[14] Farrenkopf, R. L., "Analytic Steady-State Accuracy Solutions for Two Common Spacecraft Attitude Estimators," *Journal of Guidance, Control, and Dynamics*, Vol. 1, No. 4, 1978, pp. 282–284. doi:10.2514/3.55779

[15] O'Keefe, S. A., and Schaub, H., "On-Orbit Coarse Sun Sensor Calibration Sensitivity to Sensor and Model Error," *AAS/AIAA Space Flight Mechanics Meeting*, AAS Paper 15-392, Williamsburg, VA, Jan. 2015.

[16] Acton, C., "Ancillary Data Services of NASA's Navigation and Ancillary Information Facility," *Planetary and Space Science*, Vol. 44, No. 1, 1996, pp. 65–70. doi:10.1016/0032-0633(95)00107-7