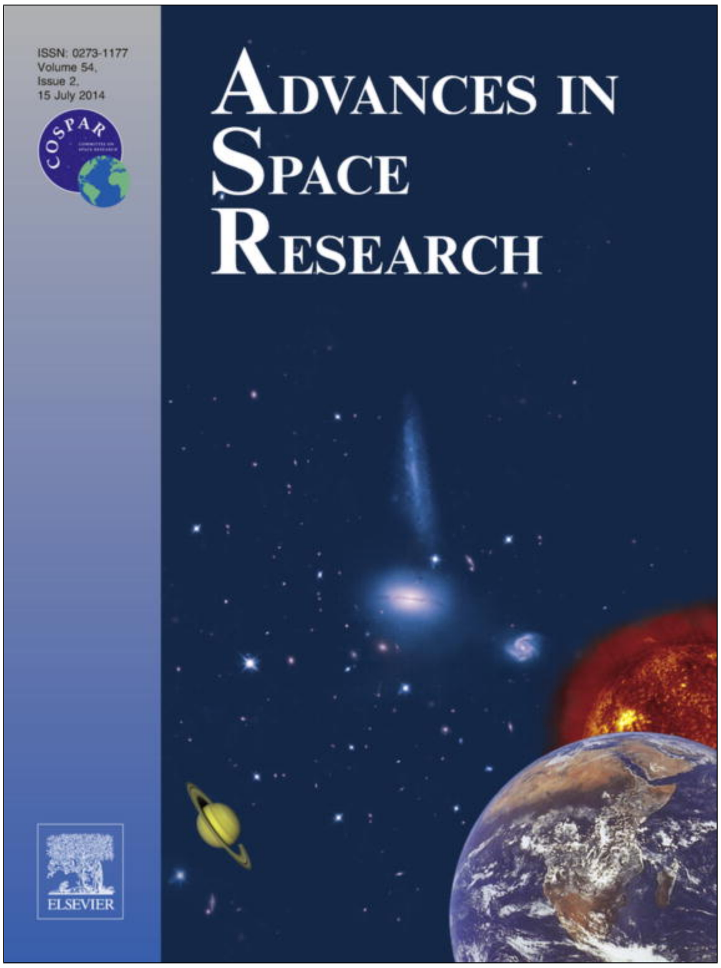
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Effective Coulomb force modeling for spacecraft in Earth orbit plasmas

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Abstract

Coulomb formation flight is a concept that utilizes electrostatic forces to control the separations of close proximity spacecraft. The Coulomb force between charged bodies is a product of their size, separation, potential and interaction with the local plasma environment. A fast and accurate analytic method of capturing the interaction of a charged body in a plasma is shown. The Debye–Hückel analytic model of the electrostatic field about a charged sphere in a plasma is expanded to analytically compute the forces. This model is fitted to numerical simulations with representative geosynchronous and low Earth orbit (GEO and LEO) plasma environments using an effective Debye length. This effective Debye length, which more accurately captures the charge partial shielding, can be up to 7 times larger at GEO, and as great as 100 times larger at LEO. The force between a sphere and point charge is accurately captured with the effective Debye length, as opposed to the electron Debye length solutions that have errors exceeding 50%. One notable finding is that the effective Debye lengths in LEO plasmas about a charged body are increased from centimeters to meters. This is a promising outcome, as the reduced shielding at increased potentials provides sufficient force levels for operating the electrostatically inflated membrane structures concept at these dense plasma altitudes.

Keywords: Coulomb formation flight; Electrostatic force modeling; Plasma effective Debye length

1. Introduction

John Cover proposed the use of Coulomb forces in space in the 1960s as a means to inflate large-scale parabolic antennas (Cover et al., 1966). Cover's design involved the use of a charging source to inflate, electrically conductive surfaces with a mutually repulsive or attractive Coulomb force. By holding a charge, the reflector maintains its position relative to a radio frequency feed. It was proposed that a 30–40 foot diameter reflector at Geosynchronous Earth Orbit (GEO) requires potentials on the order of one to several tens of kilovolts and watt to kilowatt levels of power depending on the environment (Cover et al., 1966).

More recently, the use of electrostatics in space is a progressive research area that encompasses many conceptual applications. In 2001, King and Parker proposed the use of Coulomb forces to control the relative dynamics of a free-flying formation of spacecraft (King et al., 2002). Their study concluded that it is feasible to operate a 20–30 m size synthetic aperture for interferometry from GEO and the concept warrants further analysis. Building on this theoretical work, static equilibrium configurations are examined (Berryman and Schaub, 2007; Vasavada and Schaub, 2007; Hogan and Schaub, 2010), as well as the development of algorithms to control these naturally unstable formations (Natarajan and Schaub, 2006; Wang and Schaub, 2011; Izzo and Pettazzi, 2006; Pettazzi et al., 2008). An illustration of a simple two spacecraft formation using Coulomb forces for separation distance control is shown in Fig. 1.

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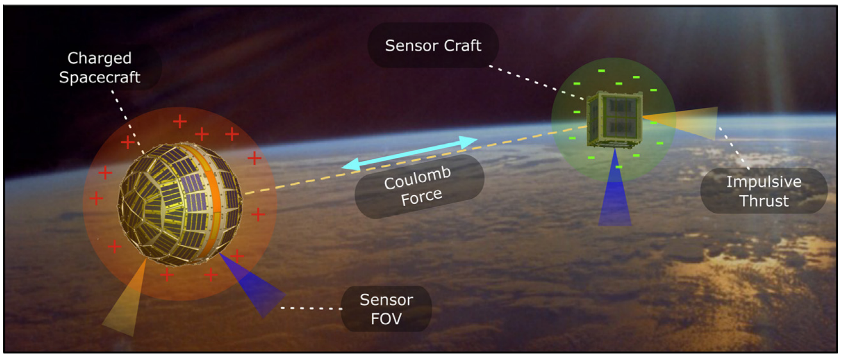


Fig. 1. Two-craft Coulomb formation flight concept; active charge emission is used to charge the craft to kilovolt-level potentials and control the separation distance with electrostatic forces.

The use of Coulomb forces in "tractor" applications to manipulate the orbit of an object is also investigated. Schaub and Moorer present a concept that uses electrostatic forces to tug a GEO debris body (Schaub and Moorer, 2010). In this scenario, the tug craft uses conventional thrusters to re-orbit the formation that is under electrostatic attraction without needing inter-spacecraft contact. In another application, Murdoch et al. propose the "electrostatic tractor" to deflect Near Earth Objects (NEO) (Murdoch et al., 2008).

The techniques of Coulomb formation control led to the development of the Tethered Coulomb Structure (TCS) concept (Seubert and Schaub, 2008; Seubert and Schaub, 2010). Combining the features of large space structures and free-flying formations, a TCS uses Coulomb forces to repel a formation of spacecraft nodes that are connected with fine, low-mass tethers, creating large quasi-rigid and lightweight space structures. Most recently there is the proposed Electrostatic Inflated Membrane Structures (EIMS) concept for inflation and stiffening of gossamer structures (Stiles et al., 2011). Utilizing connected, lightweight conductive membranes, Coulomb forces inflate the structure for applications such as drag de-orbiting or radiation shielding.

A common requirement of these theoretical studies and applications is a model of the Coulomb force. The interaction between charged bodies and their electric fields (E-fields) and the resulting Coulomb force can be complex even for simple spherical shapes. Modeling this Coulomb force is further complicated with the interactions of the variable plasma environment, or the inclusion of multiple charged bodies, non-spherical objects and attitude dependencies. To capture all design intricacies and correctly model the electrostatic interactions in a space plasma environment, a finite-element numerical solver is typically used. While these solvers offer an accurate solution that includes plasma dependencies, they can require significant processing power and time. An additional limitation with numeric solvers is that they may only offer a solution for a fixed geometry and particular formation distribution. Consequently, for simplicity in theoretical developments, analytic models are often desired and used. A review of these analytic Coulomb force models is presented in this paper. Ultimately, an adapted analytic model that is both easily computable and captures approximate plasma interactions is proposed here. It specifically focuses on the use of close proximity charged craft in Earth orbit plasmas.

The Coulomb force between two point charges in a vacuum, based on Laplace potential fields, is often used and has validity in certain applications where the plasma charge shielding properties are negligible, such as nominal geosynchronous space weather conditions (Pettazzi et al., 2008; Wang and Schaub, 2006). However, for finite charged bodies in a plasma the electrostatic force is partially screened by the free flying particles. In this scenario the vacuum model overpredicts the electrostatic force magnitude. A common practice to account for the partial charge shielding in a plasma is to use the conservative Debye–Hückel force model. This analytic representation has been used with point charges by King et al. (2002), Izzo and Pettazzi (2006), Lappas et al. (2007) and Schaub et al. (2006). It is demonstrated here, that this analytic approximation gives a conservative lower bound of the Coulomb force between points in a plasma. This analytic Debye–Hückel representation is the basis of the force model in a plasma investigated in this paper.

Another parameter to consider in electrostatic force modeling is the variation in capacitance due to having finite bodies in close proximity and the local plasma interactions. An increased capacitance consequently increases the charge carrying capacity and force for a fixed potential. For studies of Lorentz-augmented orbits, Peck includes the increased capacitance that the plasma has on an isolated charged craft (Peck, 2005; Streetman and Peck, 2007). It is calculated by Peck that a 1.5 m charged craft in a LEO plasma can enhance its capacitance by a factor as great as 151 (Peck, 2005). This fundamental interaction of the plasma and a charged craft is further explored and applied to the multi-craft scenario in this paper.

The true Coulomb force between charges in a plasma is bounded by the vacuum and Debye–Hückel models. An alternative method to more accurately capture the force between charged spheres is suggested by Murdoch et al. in Murdoch et al. (2008). Murdoch proposes the use of an effective Debye length to use in a Debye–Hückel force model. This effective Debye length, which is longer and consequently reduces the extent of force shielding, is computed from numerically fitted solutions. The Murdoch application is for deflection of NEO asteroids, hundreds of meters in size and a representative plasma of deep space. In this paper, an effective Debye length approach is pursued to study the plasma-shielded electrostatic forces for smaller, meter-level bodies operating in Earth orbit plasmas. Of importance here is the computation of effective Debye lengths in Earth orbit plasmas and the resulting charge production for meter-size craft and inflatable membrane structures.

The goal of this paper is to model the Coulomb force between spacecraft operating in Earth orbit plasmas. This analytic approximation includes the variation in system capacitance due to both the close proximity spheres and the plasma interaction. An independent, plasma numerical solver is used to verify the analytic force between a sphere and point charge. The analytic model quantifies the force capabilities for the Coulomb formation flight (CFF) concept as well as extending to Coulomb applications such as the TCS and EIMS. This is intended for spacecraft operating in close separations (dozens of meters) as well as membrane and actuation devices operating at the cm-level. Numerical solutions are used to compute effective Debye lengths for meter-size craft operating at tens of kilovolts in both LEO and GEO plasma conditions. The effect of these effective Debye lengths on the total charge of the craft and the resulting Coulomb force magnitude is explored.

2. Review of Earth orbit plasmas and Debye lengths

Coulomb spacecraft applications require the use of a charge control devices to maintain a desired potential. This is achieved with an ion or electron emitter. Spacecraft will also naturally charge due to the interaction with the local plasma and solar environment. Orbital missions such as Spacecraft Charging AT High Altitudes (SCATHA) and the Applications Technology Spacecraft (ATS-5 and ATS-6) were designed and launched specifically to characterize and quantify the extent of natural spacecraft charging (Mullen et al., 1986; Whipple and Olsen, 1980; McPherson et al., 1975). On-orbit studies such as these have established that a GEO spacecraft can naturally charge to kilovolt-level potentials (Mullen et al., 1997; Fennell et al., 2001), similar to what is envisioned by a CFF mission.

In order to quantify Coulomb force magnitudes and the extent of plasma partial shielding, it is necessary to have representative models of the plasma. The parameters of the Earth plasma (densities and temperatures) and the corresponding Debye lengths (λ_D) are used to represent the orbit environment. Although offering simple insight, it is difficult to model plasma environments with only nominal density and velocity values (Maxwellian distributions) as conditions can vary rapidly and with large fluctuations

Table 1
Representative GEO and LEO single Maxwellian plasma parameters and Debye lengths.

Conditions	Temperature [eV]	Density [m^{-3}]	λ_D [m]	$e_c V = \kappa T$ Potential [V]
LEO Nominal	0.2	0.1	0.01	0.2
GEO Quiet	3	1×10^{-5}	4	3
GEO Nominal	900	1.25×10^{-6}	200	900

(DeForest, 1977). The local plasma conditions depend on the local time as well as solar interactions with the geomagnetic activity (Hastings and Garrett, 1996). For the purposes of providing an analytic force model, the Maxwellian distribution is appropriate and used in this paper.

Table 1 lists the plasma temperature and densities used in this study. The plasma thermal energy is shown in units of eV in this table, but for calculations throughout the paper it is converted to Kelvin.[1] The GEO values are derived from on-orbit measurements of the SCATHA, ATS-5 and ATS-6, interpreted by Garrett and DeForest (1979), Garrett and Whittlesey (2012), Lennartsson and Reasoner (1978), Purvis et al. (1984), Tribble (2003) and Pisacane (2008). The LEO data is obtained from King et al. (2002). The plasma representations are also supported by data obtained from the Magnetospheric Plasma Analyzer instruments onboard the Los Alamos National Laboratory spacecraft, that continue to operate at a range of longitudinal locations around the GEO belt.

Two representative GEO plasma conditions are used for this analysis (quiet and nominal). The quiet is an extreme bound at GEO that represents the "worst-case" conditions and highest shielding of Coulomb force. Nominal plasma conditions are a closer representation of the typical operating conditions at GEO.

Also presented in Table 1 is the spacecraft potential required to match the plasma thermal energy, i.e. $e_c V = \kappa T$. This is a quantitative measure of the energies and is important in the analytic developments used later in this paper. For all plasma conditions except the rare disturbed environment the potentials required to match the plasma thermal energy are well below the kilovolt-levels that are required for the Coulomb formation flight concept.

The Debye length is a quantitative measure of the temperature and density of the local plasma (Pisacane, 2008). It is a dimensional scale that parameterizes the shielding of the electric fields (E-fields) of a charged body in a plasma. The sphere of influence of a charged body is defined by the Debye sphere that has a radius of one Debye length. The plasma screens the potential field of the charged body so that outside the Debye sphere the charge is effectively shielded (Gurnett and Bhattacharjee, 2005).

At GEO the plasma has Debye lengths ranging from 4 to 1000 m with a nominal value of approximately 200 m

[1] Temperature conversion: T_e[eV]= $(\kappa/e_c) \times T_e$[K], where κ [JK^{-1}] is the Boltzmann constant and e_c [C] is an elementary charge.

(Denton et al., 2005). Debye lengths of this scale allow the use of Coulomb repulsion when operating with spacecraft separations of dozens of meters. The LEO Debye lengths are typically at the cm-level and the interplanetary medium is typically at the 10 meter-level (Murdoch et al., 2008; Garrett and Whittlesey, 2012; Pisacane, 2008).

The 'electron' Deybe length, used throughout this paper for unperturbed plasmas, is computed using

$$\lambda_D = \sqrt{\frac{\epsilon_0 \kappa T_e}{n_e e_c^2}}, \tag{1}$$

where ϵ_0 is the permittivity of vacuum. Although the plasma is a neutral mix of electrons and ions the Debye length is computed using solely the electrons, neglecting the influence of the more massive ions. Neglecting the ions is relevant when the timescales of the process are short relative to the mobility of the ions (Hutchinson, 2002).

For a body charged to kilovolt-level potentials, the local plasma within the Debye sphere is perturbed. To approximately incorporate the effects of charged bodies on their local plasma, an effective Debye length ($\bar{\lambda}_D$) has been proposed (Murdoch et al., 2008). This effective Debye length is linearly proportional to the electron Debye length using a scaling parameter α and the relationship

$$\bar{\lambda}_D = \alpha \lambda_D. \tag{2}$$

The effective Debye length is computed with numerical simulations and is a function of parameters such as craft potential and size as well as the unperturbed plasma conditions. The benefit of using the effective Debye length is that it allows efficient analytic force computations while more accurately representing the local plasma environment about a charged body.

In Murdoch et al. (2008), Murdoch et al. compute effective Debye lengths for the electrostatic tug application of altering the orbit of a NEO asteroid. Their study indicates, with particular examples, that the Coulomb application works best for 100 m size NEO, charges of 20 kV and mission durations up to 20 years. In this NEO application the interplanetary Debye length is 7.4 m, however with potentials up to 20 kV, Murdoch calculates that the effective Debye lengths can be as great as 349 m (Murdoch et al., 2008). This is a scale increase of approximately 50, and can result in significantly less plasma partial shielding of the Coulomb forces. The effective Debye length study by Murdoch et al. (2008) is used as a basis here to analyze the force production in a plasma for small spacecraft, operating in close proximities in Earth orbit plasmas. The size of the charged object impacts the Debye Length calculation. Thus, the study on shielding about asteroids is not directly applicable to man-made space objects.

3. Overview of Coulomb force modeling in a vacuum

An overview of Coulomb force models used for the CFF, TCS or EIMS applications is given. This commences with the force between point charges in a vacuum and is expanded finite spherical bodies and charges in plasma. The electrostatic force between two infinitesimally small point charges q_A and q_B is computed with the well known Coulomb's law

$$\boldsymbol{F} = k_c \frac{|q_A q_B|}{d^2} \hat{d}, \tag{3}$$

where $k_c = 1/(4\pi\epsilon_0) \approx 8.99 \times 10^9$ Nm^2 C^{-2} is the Coulomb's constant, and d is the radial distance between the point charges. The force on each point charge is of equal magnitude and directly opposite to one another.

3.1. Force between sphere and point charge

Consider now that charge A is a finite sphere of radius R_A. In a vacuum, without neighboring charged objects the potential on the surface of this isolated sphere is represented by its capacitance equation

$$V_A = k_c \frac{q_A}{R_A}. \tag{4}$$

At a radial distance from the center of this sphere ($r \geqslant R_A$), the potential field strength that radiates isotropically from this isolated charge is computed with

$$\Phi(r) = k_c \frac{q_A}{r} = \frac{V_A R_A}{r}. \tag{5}$$

The E-field strength of this charge is then

$$E(r) = -\nabla_r \Phi(r) = \frac{\Phi(r)}{r} = k_c \frac{q_A}{r^2} = \frac{V_A R_A}{r^2}. \tag{6}$$

If an infinitesimally small point charge, q_B is placed in this E-field at a distance d, the Coulomb force magnitude felt by both the point charge and the sphere is

$$F = E(r = d) \cdot q_B = k_c \frac{q_A q_B}{d^2} = \frac{V_A R_A}{d^2} q_B. \tag{7}$$

The infinitesimal charge q_B has no effect on the overall charge on the sphere q_A, except that a force is exerted.

For the Coulomb formation flight concept development it is assumed that the potential of the bodies, not the charge, is directly controlled to a desired level. It is envisioned that the craft will have a conductive outer material with an equipotential surface charge density. From an application standpoint it is necessary to control the potential as it is more readily measurable than the entire charge of the body. The force produced between two finite bodies is a result of the total charge of the bodies. Therefore it is advantageous to model this charge to force relationship.

3.2. Force between finite spheres

Consider two charged bodies with finite dimensions in close proximity. The overlapping potential fields will raise or lower the effective potential of each body and consequently the Coulomb force between them. This can be significant at kilovolt level potentials when the center-to-center separation is low relative to the sphere radii (separations less than approximately 10 sphere radii, $r < 10R$). The net potential of the spheres is computed by combining Eqs. (4) and (5) to produce the set of equations in matrix form Stevenson and Schaub (2013) and Jasper and Schaub (2011)

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = k_c \begin{bmatrix} 1/R_A & 1/d \\ 1/d & 1/R_B \end{bmatrix} \begin{bmatrix} q_A \\ q_B \end{bmatrix}, \tag{8}$$

where d is the center to center separation of the bodies. Given that the potentials V_A and V_B of the bodies are controlled, then this equation is inverted to yield the resulting net charges on each body

$$\begin{bmatrix} q_A \\ q_B \end{bmatrix} = \underbrace{\frac{d}{k_c(d^2 - R_A R_B)} \begin{bmatrix} dR_A & -R_A R_B \\ -R_A R_B & dR_B \end{bmatrix}}_{C_V} \begin{bmatrix} V_A \\ V_B \end{bmatrix}. \tag{9}$$

Here C_V is the matrix of mutual capacitance for the charged system in a vacuum. This set of equations is expandable to N number of charged bodies of both positive and negative potentials. The charge solution of these equations is then used in Eq. (3) to compute the Coulomb force between the spheres with surface potentials V_A and V_B.

4. Coulomb force modeling in a plasma

The resulting force between two charges in a plasma is affected by the free flying charged particles. The objective here is to use the vacuum force developments to explore representative analytic plasma E-force models.

4.1. Electric fields from a sphere

Plasma shields a charged body causing its potential field to drop off more rapidly than the vacuum expression of Eq. (5). The properties of a plasma surrounding a charged body are governed by the Poisson–Vlasov coupled equations. These second order partial differential equations cannot be solved analytically for the potential field about even a simple point charge in a plasma. Numerical solutions can be employed with techniques such as the turning point method (Parrot et al., 1982). However, if thermodynamic equilibrium is reached and the body has a low potential compared to the local plasma thermal energy

$$e_c V \ll \kappa T_e,$$

then a first order solution to the Taylor series expansion can be used to obtain the Debye–Hückel approximation of the craft potential field (Gurnett and Bhattacharjee, 2005; Whipple, 1981)

$$\Phi(r) = \frac{V_A R_A}{r} e^{-(r-R_A)/\lambda_D}. \tag{10}$$

The advantage of using this Debye–Hückel potential field is that it provides a simplified analytic solution without the need for numerically solving the full Poisson–Vlasov equations. The consequence of neglecting higher order terms in the Poisson's partial differential equations is that the plasma shielding of the electrostatic fields is not as steep. Thus, this is a conservative estimate on the potential function that might actually exist about the charged body in a plasma (Murdoch et al., 2008).

Fig. 2 demonstrates graphically the differences in the potential field from the surface of an isolated 1 m sphere charged to a potential of 50 kV between the vacuum and Debye–Hückel models. The vacuum potential field bounds the upper limit of the potential curve, while the Debye–Hückel lower limit is computed for a worst-case GEO (quiet plasma) Debye length $\lambda_D = 4$ m. The true potential field decay will lie in the shaded region between these curves. As the Debye length increases the shaded area is reduced as the lines converge.

Taking the gradient of the potential function of Eq. (10) yields the spherically symmetric E-field for $r \geqslant R_A$

$$E(r) = -\nabla_r \Phi(r) = \frac{V_A R_A}{r^2} e^{-(r-R_A)/\lambda_D} \left(1 + \frac{r}{\lambda_D}\right). \tag{11}$$

The E-field of a charged body in a plasma is also bound by the limits of this Debye–Hückel and Laplace fields, which are also shown in Fig. 2. Due to the gradient of the potential function being larger at very close separations, the E-field for the Debye–Hückel model is actually larger than the Laplace, consequently the force in this region can also be larger. For the CFF concept this is of importance for deployment or docking conditions.

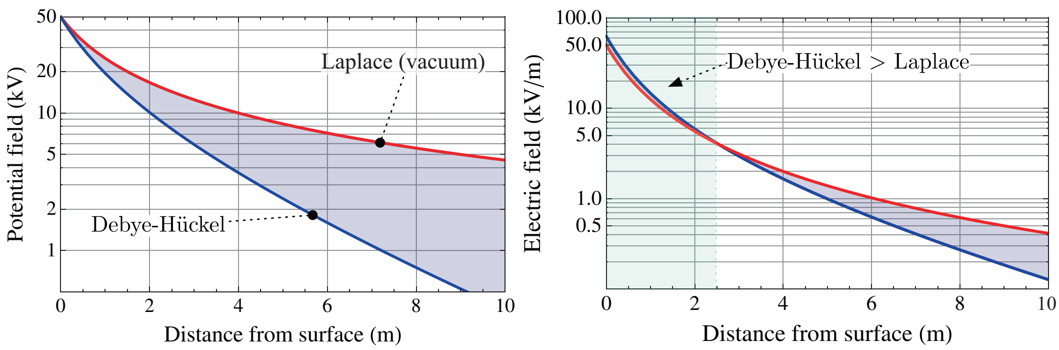


Fig. 2. Potential and electric fields from an isolated, 1 m diameter sphere, charged to 50 kV (quiet GEO plasma, $\lambda_D = 4$ m, used for the Debye–Hückel model).

Further, this plasma enhanced capacitance may be advantageous for close proximity Coulomb concepts such as the membrane structure developments. For fundamental CFF studies it is suitable to use the analytic Debye–Hückel potential model in Eq. (10) as it provides a conservative lower limit of the resulting force production in a plasma.

4.2. Force between sphere and point charge

An analytic expression of the force between a sphere and point charge using the Debye–Hückel potential is developed. First, it is necessary to compute the charge of the sphere that maintains a desired surface potential V_A. Even for an isolated sphere, the plasma alters the capacitance and the relationship between charge and potential in Eq. (4). Assuming a homogenous surface charge density σ across the sphere (suitable, given an isolated sphere and a well-mixed, neutral plasma), the total charge q residing on the surface is calculated with

$$E(r = R_A) = \frac{\sigma}{\epsilon_o} = \frac{q}{A\epsilon_o}. \quad (12)$$

Using the Coulomb's constant $k_c = 1/(4\pi\epsilon_0)$ and defining $A = 4\pi R^2$ as the spherical surface area, the total charge on sphere A is estimated as (Whipple, 1981)

$$q_A = V_A \frac{R_A}{k_c}\left(1 + \frac{R_A}{\lambda_D}\right). \quad (13)$$

The resulting capacitance of an isolated sphere in a plasma is (Whipple, 1981; Peck, 2005)

$$C_S = \frac{R_A}{k_c}\left(1 + \frac{R_A}{\lambda_D}\right). \quad (14)$$

This indicates that a craft that maintains a fixed potential will hold a charge that depends on the local plasma. If the plasma Debye length is very small (i.e. LEO regime), the space weather can have a significant impact on the sphere's capacitance, and its effective charge. If the plasma has minimal interaction (large Debye lengths, $R_A \ll \lambda_D$) the charge on the isolated sphere reduces to the vacuum formulation of Eq. (4). Placing an infinitesimal point charge, that does not affect the charge of the sphere, in the E-field results in a Coulomb force computed using Eq. (11)

$$F = \frac{V_A R_A q_B}{d^2} e^{-(d-R_A)/\lambda_D}\left(1 + \frac{d}{\lambda_D}\right). \quad (15)$$

4.3. Violating the $e_c V \ll \kappa T$ assumption

The Debye–Hückel potential field and resulting Coulomb force model is an analytic expression that is derived by assuming $e_c V \ll \kappa T$. Table 1 quantifies the spacecraft surface potential required to equal the plasma thermal energy ($e_c V = \kappa T$). If the craft potential is much less than plasma energy ($e_c V \ll \kappa T$), than the Debye–Hückel potential of Eq. (11) is a good approximation. If the craft potential is significantly greater than the plasma ($e_c V \gg \kappa T$) than the plasma-based potential field is closer to the vacuum model of Eq. (6) for small r, such as in the Debye sphere. For the Coulomb formation flight application the potentials and plasma properties have similar magnitudes. Consequently the two approximations available provide bounds on potential decay from a charged body in a plasma. The resulting Coulomb force that is derived from these potential fields is also bound by these analytic representations. One method to analytically compute the force within this range with higher accuracy is with the effective Debye length, as proposed by Murdoch et al. (2008) for the charged asteroid scenario. The suitability of the effective Debye length for partially plasma shielded E-Force evaluations of man-made spacecraft is investigated here.

5. Effective Debye lengths in Earth orbit

One method to more accurately compute the Coulomb force analytically, within the bounds of the vacuum and Debye–Hückel potential equations, is with an effective Debye length. This effective Debye length is larger than the true Debye length and consequently reduces the screening of the potential field. It is then substituted directly into the Debye–Hückel Coulomb force model to more appropriately match the true force.

In this paper, the effective Debye lengths are computed using numerical solutions of the Poisson's equation along with the collision less Vlasov equation for a non-flowing plasma. These Poisson–Vlasov coupled equations are solved for a charged sphere in an isotropic Maxwellian plasma using spherical symmetry and conservation of particle energy and angular momentum. The solver provides a one-dimensional E-field model from the sphere's surface and the corresponding force on a charged particle in the field. An α scalar value described by Eq. (2) is determined by fitting an effective Debye shielded E-field model to the numerical solution across separations up to several Debye lengths from the sphere. The E-field model used is based on Eq. (11), using effective Debye lengths

$$E(r) = -\nabla_r \Phi(r) = \frac{V_A R_A}{r^2} e^{-(r-R_A)/\alpha\lambda_D}\left(1 + \frac{r}{\alpha\lambda_D}\right). \quad (16)$$

The effective Debye length is computed in Earth orbit plasma conditions with spacecraft sizes and potentials specifically tailored for the Coulomb formation flight concept. The nominal GEO plasma conditions are not investigated as the plasma shielding in a Debye length of 200 m is minimal and the Debye–Hückel model closely resembles the vacuum values.

5.1. GEO effective Debye lengths

The E-fields surrounding a charged craft in a quiet GEO plasma ($\lambda_D = 4$ m) are examined. Fig. 3 compares the E-field models and numerical solution for a 1 m craft with two surface potential cases examined: 1 kV potential (left) and 30 kV (right). The analytic E-field models are; vacuum,

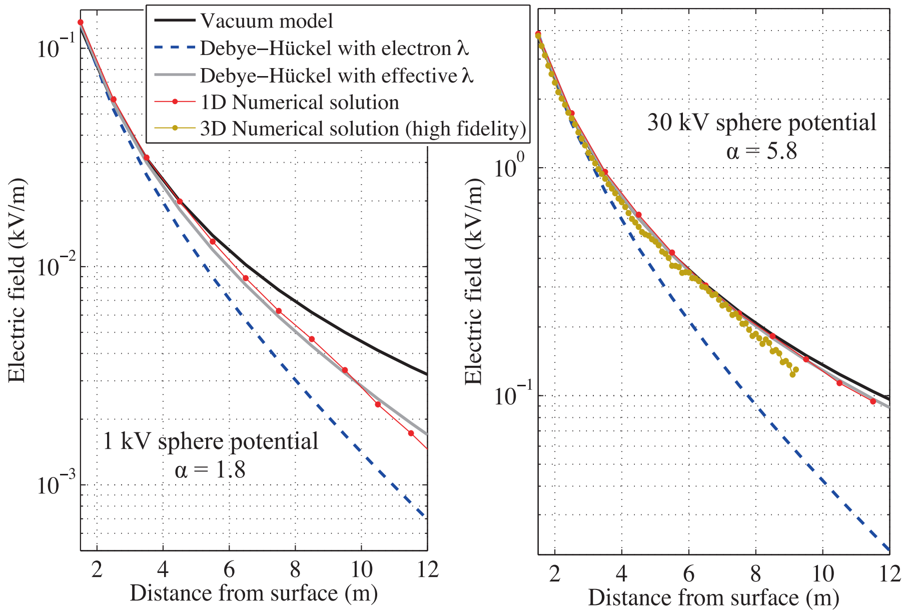


Fig. 3. E-field models with electron and effective Debye lengths compared to numerical solutions for two surface potentials in a GEO quiet plasma ($\lambda_D = 4$ m).

Debye–Hückel with electron Debye length and effective Debye length, and the numerical simulation results. In both surface potential cases, the numerical solution has a stronger E-field than classically predicted, but have an upper bound of the vacuum potential of Eq. (5). As the craft potential increases, the actual E-field values stray further from the Debye–Hückel model and approach the vacuum E-field.

A 3D electrostatic numerical solver is also used as an independent verification of the potential field model. The numerical computations are performed using the plasma simulation tool VORPAL,[2] developed by Tech-X. VORPAL is a highly flexible and computationally efficient, commercial software package ideally suited to modeling electromagnetic and plasma physics processes. The potential is computed for the 30 kV sphere and shown in Fig. 3. This independent solution confirms the model of a potential field from a charged sphere. Although higher fidelity, the 3D solver does reach its numerical accuracy limits. For this example case, the solution is computed to separations up to 9 m from the sphere surface.

5.2. Trends in GEO effective Debye lengths

For CFF applications it is necessary to quantify the trends in the GEO effective Debye length across a range of Debye lengths, craft diameters and potentials. Multiple numerical simulations are used to fit Eq. (16) to a range of parameters with values shown in Table 2. Fig. 4(a) displays the variation of the Debye length scaling factor, α, for a quiet plasma ($\lambda_D = 4$ m), over a range of craft diameters and potentials. Fig. 4(b) shows the variation of the scaling factor as a function of Debye length and craft potential for a sphere with a 1 m diameter. In contrast to the asteroid charge shielding study which obtained α values as great as $\approx$50 (Murdoch et al., 2008), the meter-sized GEO craft experiences α values ranging up to 7 during space weather conditions with short Debye lengths.

Fig. 4(a) illustrates that larger craft diameters and higher potentials yield larger effective Debye lengths. This relationship to higher craft voltage occurs because as the assumption ($e_c V \ll \kappa T$) is further violated, the shielding reduces. There exist physical limits on craft voltage and size, but this quantifies the increases in Debye lengths, and consequently more effective electrostatic forces, with larger craft and higher potentials.

In the extreme case of a quiet GEO plasma ($\lambda_D = 4$ m) and a craft voltage of 30 kV, the effective Debye length is more than seven times the predicted Debye length. This significantly reduces the plasma partial shielding of the force between two charged spacecraft. As the Debye length increases, the α value approaches unity, and the Debye shielded force approaches the vacuum force. Interpolation of the data shown on these plots can be used to determine the effective Debye lengths for different combinations of craft voltage, Debye lengths, and craft sizes.

Table 2
Ranges of parameters used to quantify trends in the GEO effective Debye lengths.

Parameter	Values
Spacecraft potential (kV)	0.1–30
Spacecraft radius (m)	0.25–1.5
Plasma Debye length (m)	4–90

[2] VORPAL Product Page, Tech-X Corporation, http://vorpal.txcorp.com, 09/09/12

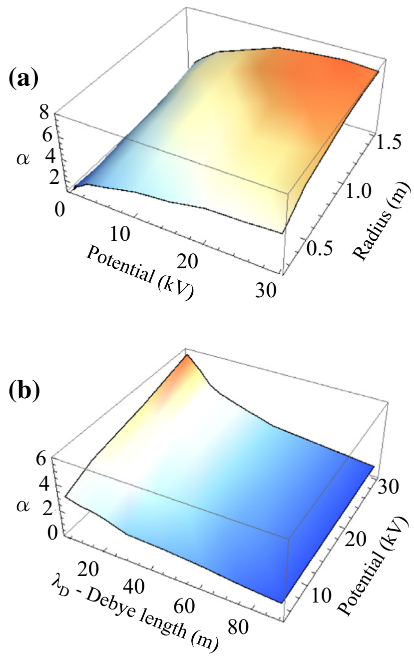


Fig. 4. Trends in effective Debye length scaling factor, α, in a quiet GEO plasma, $\lambda_D = 4$ m, (a) α as a function of craft potential and diameter in a fixed plasma, (b) α as a function of craft potential and electron Debye length for a fixed craft diameter of 1 m.

5.3. Parametric relationship of the effective Debye length

For Coulomb force modeling it is desired to easily compute the effective Debye length for a given set of conditions. For this, a parametric model of the Debye length scaling factor (α) is computed for a range of craft potential and radii. This allows for quick modeling of the E-fields or forces surrounding the craft with improved accuracy from solutions using the electron Debye length.

A nonlinear model is assumed for α using

$$\alpha(V,r) = V \cdot f(V,r) + (1 + e^{V}) \cdot f(V,r). \tag{17}$$

Here f is a first order polynomial that is fit to the three-dimensional surface of α values obtained from a numerical solver. Equal weights are applied for all radii considered. The form of Eq. (17) is found to provide a good least-squares fit with the numerical data, while still yielding an algebraically simple solution to numerically evaluate.

The model fit is computed for the quiet GEO plasma conditions as a function of craft potential (V) in kV and radii (r) in m, using the range of parameters defined in Table 2. The resulting nonlinear fit of the alpha parameter is

$$\begin{aligned}\alpha(V,r) = {} & 1 + 0.4900V - 0.0032V^2 + 0.0089Vr \\ & + \left(1 - e^{-0.5757V}\right)(-0.1045 - 0.2885V \\ & + 2.1719r).\end{aligned} \tag{18}$$

This nonlinear model forces the α parameter to converge to a value of 1 as the craft potential approaches zero. As the potential approaches zero, the $(e_c V \ll \kappa T)$ rule is no longer violated, thus the electron Debye length Debye–Hückel model is satisfactory for predicting E-fields and forces. The form of this nonlinear model is chosen as a summation of a quadratic polynomial, to capture the behaviors at higher voltages (5–30 kV), and an exponential term to capture the low voltage behavior as α converges to one. This generic analytic model allows rapid computation of scaling of Debye lengths to account for the $(e_c V \ll \kappa T)$ violation for dynamic studies of Coulomb spacecraft applications.

To measure the goodness of fit of this and future α models, the coefficient of determination ($\mathcal{R}^2$) is used. The $\mathcal{R}^2$ value of this quiet GEO model (Eq. (18)) is 0.995, very near the maximum value of 1. Therefore, this model is considered a satisfactory prediction of the alpha parameter for effective Debye lengths across the full CFF design space.

To illustrate the results of the parametric relationship of Eq. (18), a surface plot is shown in Fig. 5. This plot displays the original numerically computed alpha parameter of Fig. 4(a), as a colored surface, with the model fit shown as a mesh surface. In this figure, it can be seen that the model closely matches the behavior of the data.

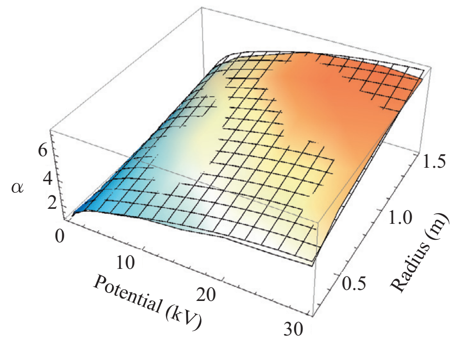


Fig. 5. Alpha parameter values and surface plot of regression (Eq. (18)) for quiet GEO plasma conditions, $\lambda_D = 4$; colored surface represents data and mesh represents model fit.

5.4. LEO effective Debye lengths

In the same procedure as described for GEO plasma conditions, effective Debye lengths at LEO are investigated. As seen in Table 1, the nominal LEO Debye length is on the order of centimeters. The resulting force shielding significantly limits the feasibility of electrostatic actuation for Coulomb formation flying or tethered Coulomb structures. Other Coulomb applications with smaller separation distance, such as electrostatically inflated membrane structures, may be feasible in LEO. These structures use electrostatic forces between layers of conducting membranes as the source of inflation pressure. The distances over which the electrostatic force acts is envisioned to be on the order of a few centimeters. Table 1 also shows that a craft potential of only 0.2 V is sufficient to equal the plasma energy $(e_c V = \kappa T)$, therefore any Coulomb application with kilovolts of potentials will clearly violate the Debye–Hückel model. Presented here is a study of the effective Debye lengths of the dense plasma of LEO to more accurately capture the true potential field of a charged body. This is needed for analyzing the capabilities of electrostatic inflation and other small separation distance Coulomb applications at LEO.

Fig. 6 illustrates the E-field from a craft of 0.25 m radius, charged to 5 kV in a LEO plasma environment ($\lambda_D = 0.011$ m). It demonstrates the variation in the analytic models as well as the solution of the numerical computation, which is taken as reference. The electron Debye length E-field model is not a good approximation of the numeric solution. With the electron Debye length and Debye–Hückel model, the dense plasma quickly drives the E-field to zero. The vacuum model is an improved approximation but in this LEO plasma, underpredicts the true E-field. To more accurately capture the plasma shielding the Debye–Hückel analytic model using an effective Debye length (Eq. (16)) is fit to the numeric data.

The resulting scaling factor for this illustrated case is approximately, $\alpha = 24$, giving an effective Debye length of 0.264 m. It is found that the scaling factors at LEO are significantly larger than those at GEO. LEO effective Debye lengths are on the order of several decimeters up to a meter, as opposed to the centimeter-level electron Debye lengths. This significantly reduces the partial shielding, improving the Coulomb force magnitudes and making them viable for applications such as inflation of membranes at centimeter-level separations.

Fitting the effective Debye length model to the numerical solutions is appropriate for potentials in the range of 5–30 kV. This is an anticipated range of potentials employed for Coulomb applications. At lower potentials, the Debye–Hückel model does not reasonably fit the numerical solutions solely by using an effective Debye length. Alternative models would need to be considered for modeling potentials below 5 kV.

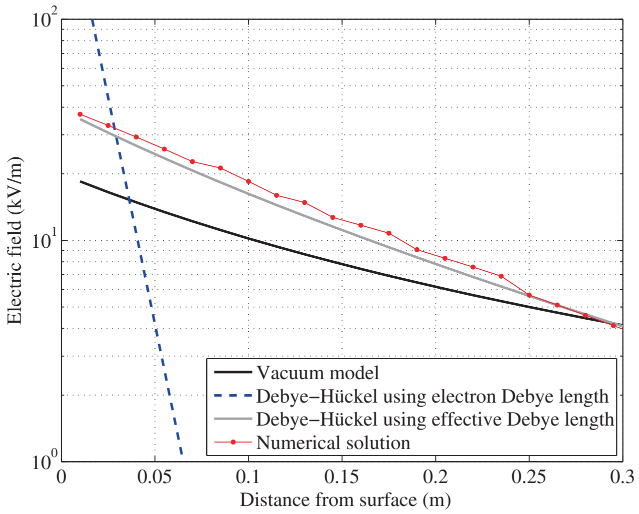


Fig. 6. Comparison of E-field models to numerical simulations for LEO plasma ($\lambda_D = 0.011$ m, $V = 5$ kV, $r = 0.25$ m).

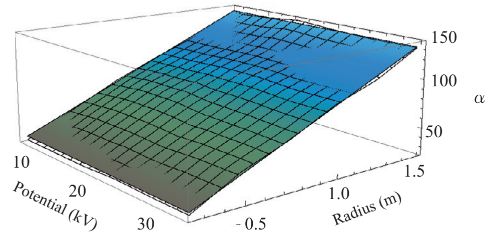


Fig. 7. Alpha parameter values and surface plot of regression (Eq. (19)) for LEO plasma conditions, $\lambda_D = 0.011$; colored surface represents data and mesh represents model fit. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this article.)

Just like the GEO case, it is desired to quantify the α value trends and then fit a parametric model as a function of craft potential and radii. A parametric model of the Debye length scaling factor (α) is shown in Fig. 7 for craft radii of 0.25–1.5 m and potentials of 5–30 kV. Interestingly, the α value is near constant across all craft potentials, which is an alternate trend from the GEO case. This indicates that the effective Debye lengths converge to a limit as $e_c V \gg \kappa T$. Also seen in Fig. 7 is the linear relationship between effective Debye length and craft radius.

A first order polynomial of the form $\alpha(V, r) = f(V, r)$ is fit to the three dimensional surface of α values, giving the expression

$$\alpha = 7.028 - 0.031V + 84.628r. \tag{19}$$

This model allows rapid computation of the Debye length scaling factor for the range of craft potentials and radii used in this LEO plasma study. The scaling factor model is also shown in Fig. 7 as a meshed surface and fits the data with a coefficient of determination value, $\mathcal{R}^2 = 0.994$.

A charged body will have a complex interaction with the dense plasma at LEO altitudes. It is important to note that other plasma mechanisms, such as wake effects, photoelectron and secondary electron emission, and magnetic field interaction are not being considered in this analysis. Two key findings are drawn from this LEO study. Firstly, it gives an indication of the limits of using the electron Debye length value with the Debye–Hückel model and secondly the significantly increased effective Debye lengths offer a promising outlook on certain Coulomb applications in this dense plasma.

6. Spacecraft charges and forces with effective Debye lengths

The developed models of the quiet GEO and LEO effective Debye lengths are now used in spacecraft charge and force computations. The numerical solver for the electric field about a sphere in a plasma is used to compare the proposed analytic models using the effective Debye lengths.

Both the charge computation of the isolated sphere in a plasma as well as the resulting force between the sphere and a point charge is compared.

Naturally, every numerical solver employs approximations to the full physics that would be encountered in space. Measuring the electrostatic forces in a plasma laboratory or space environment is very challenging, and still the focus of ongoing research. In particular, having a small satellite test mission to experiment in flight with electrostatic actuation, and validate candidate force models, would be very valuable. However, charged actuation and experiments have been performed in atmospheric conditions in Seubert (2011) and Stiles et al. (2011), where the electrostatic force models used in this paper are employed. In Seubert (2011), these models are able to capture the measured electrostatic response of a charged sphere on a 1-D hover track. The atmospheric ionization and interactions are successfully approximated using the Debye-shielded force equations shown in this paper.

6.1. Total charge on an isolated sphere in a plasma

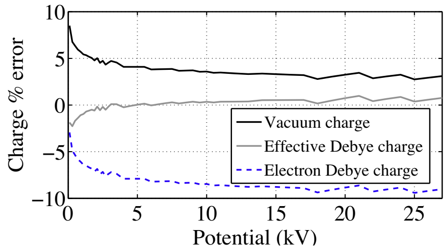
The numerical solver is used to compute the total charge on a sphere, maintaining a fixed potential in a plasma. For the purposes of comparing the analytic models in this study, this charge solution is considered the reference value (q_{ref}). The charge is analytically computed using the vacuum model of Eq. (4) and compared to the Debye–Hückel plasma model of Eq. (13) with both the electron and effective Debye length

$$q_A = V_A \frac{R_A}{k_c}\left(1 + \frac{R_A}{\bar{\lambda}_D}\right). \tag{20}$$

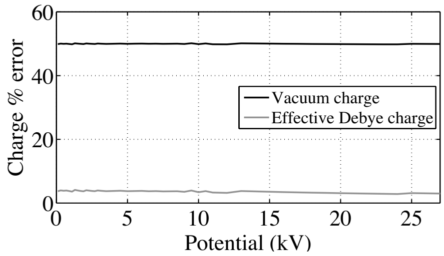
Each model is compared to the numerical reference as a percentage error ($\% = 100 \times (q_{\text{ref}} - q_i)/q_{\text{ref}}$) and shown in Fig. 8 for a 0.5 m radius sphere in each plasma condition. Fig. 8(a) shows the percentage error in the sphere's charge for the quiet GEO plasma. In this situation, the computation of the charge using the effective Debye length is $< 1\%$ across all potentials. The vacuum Debye length under estimates the charge magnitude, while the charge computed with the electron Debye length over estimates the charge of the sphere.

Fig. 8(b) shows the percentage error in the sphere's charge for the LEO plasma. In this case the effective Debye length computation is within 4% of the reference across all potentials. The vacuum computation under estimates the charge magnitude by $\approx 50\%$. The computation using the electron Debye length grossly over estimates the charge by an error of $\approx -2000\%$ and is not shown in this scale.

This comparison shows the importance of using the effective Debye length to compute the self capacitance of a sphere in a plasma. It is necessary to use the effective Debye length in these and similar plasma environments as it leads to accurate computation of the force between sphere and point charge as shown in the next section.



(a) GEO quiet plasma $\lambda_D = 4$ m



(b) LEO plasma $\lambda_D = 0.11$ m

Fig. 8. Computation of charge on an isolated sphere using analytic models compared to the numerical reference as a percentage difference over a range of CFF potentials.

6.2. Force between sphere and point charge

In this section the force between a sphere and a point charge in a quiet GEO plasma is computed with both the numerical solver and analytic models and compared. The force is computed with the electron Debye length shielding using Eq. (15). This equation is then modified for computation using the effective Debye length

$$F = \frac{V_A R_A q_B}{d^2} e^{-(d-R_A)/\bar{\lambda}_D}\left(1 + \frac{d}{\bar{\lambda}_D}\right). \tag{21}$$

To quantify the accuracy of the analytic models, the force is computed between a sphere of 0.5 m radius with a point charge separated from the sphere center by 2, 4, 6, and 8 m. The percentage error of the force computation from the numerical reference is calculated using $\% = 100 \times (F_{\text{ref}} - F_i)/F_{\text{ref}}$ and shown in Fig. 9 for each separation distance. This demonstrates that the force calculated with the effective Debye length model is within 5% of

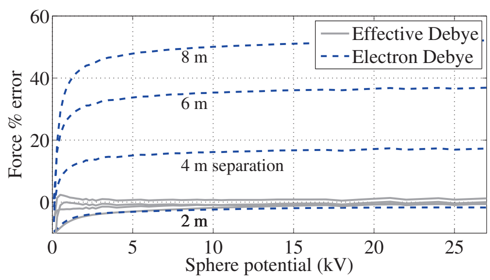


Fig. 9. Comparison of analytic forces with electron and effective Debye lengths as a percentage difference from the reference between a 1 m diameter sphere and point charge over a range of CFF potentials for four different separations in a quiet GEO plasma ($\lambda_D = 4$ m).

the numerical reference for all separations. The force calculated with the electron Debye length in the analytic model is accurate at close separations but at separations of 8 m underestimates the force as much as 50%. This indicates that for the force between a sphere and point charge the analytic model using the parametric α model (effective Debye length) accurately predicts the force magnitude in a GEO quiet plasma. The consequence of this for CFF is that the electron Debye analytic force models used are an underestimate of the force magnitude in a dense plasma that can differ substantially at larger, yet realistic separations.

7. Conclusions

In this paper, the approximate plasma environment effects on Coulomb force calculations for Coulomb spacecraft applications are explored. Computing the Coulomb force between charges in a plasma is a complex task that has to include mutual capacitance and Debye shielding effects. Analytic approximations are developed based on the assumption that the body has a low potential compared to the local plasma thermal energy. This assumption, however, is quickly violated for charged craft in LEO and also in quiet GEO conditions. Numerical simulations allows for a more accurate solution to the forces and a modified 'effective' Debye length can be defined to allow quick use of the analytical equations. These effective Debye lengths are calculated for GEO and show that the effective Debye length can be several times larger than the calculated Debye length for the applications of Coulomb formation flying. In LEO plasma conditions, the 'effective' Debye length can be more than an order of magnitude larger than the electron Debye length. The LEO effective Debye lengths can therefore be up to the meter level, and the resulting Coulomb force improvement from reduced shielding may allow for LEO Coulomb spacecraft applications such as inflation of membranes at cm level separations.

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