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ADAPTIVE CONTROL OF NONLINEAR ATTITUDE MOTIONS REALIZING LINEAR CLOSED LOOP DYNAMICS

Hanspeter Schaub*, Maruthi R. Akella† and John L. Junkins‡

An adaptive attitude control law is presented to realize linear closed loop dynamics in the attitude error vector. The Modified Rodrigues Parameters (MRPs) are used as the kinematic variables since they are nonsingular for all possible rotations. The desired linear closed loop dynamics can be of either PD or PID form. Only a crude estimate of the moment of inertia matrix is assumed to be known. An open loop nonlinear control law is presented which yields linear closed loop dynamics in terms of the MRPs. An adaptive control law is then developed which enforces these desired linear closed loop dynamics in the presence of large inertia and external disturbance model errors. Since the unforced closed loop dynamics are nominally linear, standard linear control methodologies, such as pole placement, can be employed to satisfy design requirements such as control bandwidth. The adaptive control law is shown to track the desired linear performance asymptotically without requiring apriori knowledge of either the inertia matrix or external disturbance.

Introduction

While the traditional approach to attitude control is based on linear control theory, recent efforts by several authors indicate a shift towards nonlinear control methods. For example, Wie et. al. in References 1 and 2 develop the rotational equations of motion using the redundant set of Euler parameters. In contrast, Dwyer outlines in References 3 and 4 an approach based on a minimal set of three Euler parameters wherein a nonlinear transformation maps the complete equations of motion into a locally valid linear model which may encounter singular attitudes. The work of Slotine and Li based on Euler angles also has the same limitation.[5]

To overcome the problem of singular orientations while using a minimal set of three rigid body attitude coordinates, more recently the Modified Rodrigues parameters (MRPs) have been proposed as an attractive set of coordinates for attitude motions which are nonsingular for all possible $\pm 360°$ rotations.[6–9] Any orientation can be described through two numerically distinct sets of MRPs which abide by the same differential kinematic equation. By switching between the original and alternate set (also referred to as the shadow set) of MRPs it is possible to achieve a globally nonsingular attitude parameterization.[6,8] Another very attractive property of MRPs is in tracking rotations where we are virtually assured of always remaining within the linear range with these coordinates.[10,11]

Given all these advantages, there have been several recent attitude control applications employing MRPs as the rotational kinematic variables.[12–15] In all these efforts and other approaches by Meyer[16,17] and Slotine and Li[5] the control law is based on a stability analysis driven by an associated Lyapunov function. While such attitude feedback control laws can be found by first defining a candidate Lyapunov function and then extracting the corresponding stabilizing nonlinear control, certain very important concepts from linear control theory, such as closed-loop damping and bandwidth, are not very well defined, since the corresponding closed loop dynamics are generally nonlinear. To achieve a desired closed loop behavior, the closed loop dynamics are linearized about a reference motion in order to use linear control theory techniques to pick the feedback gains. Depending on the nonlinearity of the exact closed loop equations of motion, the desired closed loop performance will only be achieved in a local neighborhood and not globally.

Instead of first finding a feedback control law and then analyzing the closed loop dynamics stability, it is possible to start out instead with a desired (or prescribed) set of stable closed-loop dynamics and then extract the corresponding nonlinear control law using a variation of the "inverse dynamics" approach common in robotics path planning problems. For example, the closed loop dynamics could be a stable linear differential equation. This technique is very general and can be applied to a multitude of systems. However, depending on the nonlinearity of the dynamical system, the such extracted nonlinear control laws can be potentially very complex. Paielli and Bach[18] present such an attitude control law derived in terms of the Euler parameter components which is remarkably simple. Compared to standard Lyapunov function derived attitude control laws, their control law expression is only slightly more complex. Further, Paielli and Bach illustrate that this type of control law is rather robust for attitude control problems. However, this control law feeds back the Gibbs vector[7] as an attitude measure which is singular at ± 180 degree rotations about any axis. To alleviate this problem, we develop a control law based on the MRP vector which achieves the desired set of stable closed-loop trajectories without encountering singular orientations. This paper also addresses the issue of uncertainty in the moment of inertia matrix. While the open loop attitude control law is robust with respect to inertia uncertainties, the closed loop dynamics will no longer exhibit the desired performance if the incorrect inertia matrix is used in the feedback control law. The inertia matrix is assumed to be essentially unknown in this development, yet the feedback control law should still produce the desired closed loop dynamics. To accomplish this task, time-varying update laws for the feedback gain matrices are developed that *learn* the dynamics of the system adaptively. While classical adaptive control theory due to Narendra (Ref. 19) and Sastry (Ref. 20) has also been employed in attitude control problems previously,[1,13,21,22] the present approach is unique in that sense that it explicitly enables linear closed-loop dynamics to be chosen and motivated by useful physical concepts such as damping ratio and loop bandwidth.

The paper first develops all the theory necessary to develop the inverse dynamics approach to obtaining stable closed-loop rigid body dynamics. In particular, the MRPs are chosen as the attitude parameters. An adaptive control law is presented which includes an integral feedback term in the desired closed loop dynamics and achieves asymptotic stability even in the presence of unmodeled external disturbances. These results are illustrated through various numerical simulations.

Linear Closed-Loop Dynamics

The modified Rodrigues Parameter vector $\boldsymbol{\sigma}$ is adopted as a rigid body attitude measure relative to the target attitude. Note that the vector $\boldsymbol{\sigma}$ contains information about both

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the principal rotation axis $\hat{\boldsymbol{e}}$ and principal rotation angle Φ, since they are related through

$$\boldsymbol{\sigma} = \hat{\boldsymbol{e}} \tan \frac{\Phi}{4} \tag{1}$$

Therefore, if $\boldsymbol{\sigma} \to 0$, then the orientation has returned back to the origin. As a complete revolution is performed (i.e. $\Phi \to$360 degrees), this particular MRP set goes singular. As is shown in References 6 and 8, it is possible to map the original MRP vector $\boldsymbol{\sigma}$ to its corresponding shadow counterpart $\boldsymbol{\sigma}^S$ through

$$\boldsymbol{\sigma}^S = -\frac{1}{\sigma^2}\boldsymbol{\sigma} \tag{2}$$

where the notation $\sigma^2 = \boldsymbol{\sigma}^T\boldsymbol{\sigma}$ is used. By choosing to switch the MRPs whenever $\sigma^2 > 1$, the MRP vector remains bounded within a unit sphere. Switching when the $\sigma^2 = 1$ surface is penetrated also results in the corresponding MRPs always indicating the shortest rotational distance back to the origin.[6, 11]

Let's assume that we desire the closed loop dynamics to have the following prescribed *linear* form

$$\ddot{\boldsymbol{\sigma}} + P\dot{\boldsymbol{\sigma}} + K\boldsymbol{\sigma} = 0 \tag{3}$$

where P and K are the positive scalar velocity and position feedback gains. Observe that both P and K could be chosen to be symmetric, positive definite matrices. However, doing so greatly complicates the resulting algebra. Note that this differential equation only contains kinematic quantities and no system properties such as inertia terms are present. Linear control theory states that, for any initial $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}}$ vectors, the resulting motion is asymptotically stable. If desired, one could also easily add an integral feedback term to the desired closed loop equations and still retain asymptotic stability.

$$\ddot{\boldsymbol{\sigma}} + P\dot{\boldsymbol{\sigma}} + K\boldsymbol{\sigma} + K_i\int_0^t \boldsymbol{\sigma} dt = 0 \tag{4}$$

Note that instead of the MRP vector $\boldsymbol{\sigma}$, any attitude or position vector could have been used. In particular, in Reference 18, Paielli and Bach chose to express their linear closed loop equations in terms of the vector components of the Euler parameters.

Let the vector $\boldsymbol{u}$ be an external control torque vector which is applied to a rigid body with the inertia matrix $[I]$. The vector $\boldsymbol{F}_e$ is the unmodeled torque vector due to such influences as atmospheric or solar drag or bearing friction. The vector $\boldsymbol{\omega}$ is the body angular velocity vector. Euler's rotational equations of motion state that

$$[I]\dot{\boldsymbol{\omega}} + [\tilde{\boldsymbol{\omega}}][I]\boldsymbol{\omega} = \boldsymbol{u} + \boldsymbol{F}_e \tag{5}$$

where the tilde matrix $[\tilde{\boldsymbol{\omega}}]$ is the vector cross product operator defined as

$$[\tilde{\boldsymbol{\omega}}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{6}$$

It is desired to find a nonlinear control law $\boldsymbol{u}$ that will render the closed loop dynamics to be of the stable form in Eq. (3) or (4), assuming the system inertia matrix is perfectly known. To achieve this, we treat the body angular acceleration vector $\dot{\boldsymbol{\omega}}$ as the control variable in the following development. Once the necessary vector $\dot{\boldsymbol{\omega}}$ is found, then the physical control torque is found through Eq. (5). To extract $\dot{\boldsymbol{\omega}}$ from either Eq. (3) or (4), all velocities and accelerations in these closed loop equations must be expressed in terms of the body angular velocity vector. Assume the target attitude is stationary, the MRP kinematic differential equations are[6–9]

$$\dot{\boldsymbol{\sigma}} = \frac{1}{4}[B(\boldsymbol{\sigma})]\boldsymbol{\omega} \tag{7}$$

where the matrix $[B] = [B(\boldsymbol{\sigma})]$ is conveniently expressed as[6, 7]

$$[B] = \left[\left(1 - \boldsymbol{\sigma}^T\boldsymbol{\sigma}\right) I_{3\times3} + 2\left[\tilde{\boldsymbol{\sigma}}\right] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T\right] \tag{8}$$

with the skew-symmetric matrix operator being defined in Eq. (6). Differentiating the MRP kinematic differential equation in Eq. (7) we find

$$\ddot{\boldsymbol{\sigma}} = \frac{1}{4}[B]\dot{\boldsymbol{\omega}} + \frac{1}{4}[\dot{B}]\boldsymbol{\omega} \tag{9}$$

Substituting Eqs. (7) and (9) into the desired linear closed loop dynamics in Eq. (3), the following constraint condition is found.

$$\begin{aligned} \ddot{\boldsymbol{\sigma}} + P\dot{\boldsymbol{\sigma}} + K\boldsymbol{\sigma} = 0 = \\ \frac{1}{4}[B]\left[\dot{\boldsymbol{\omega}} + P\boldsymbol{\omega} + [B]^{-1}\left([\dot{B}]\boldsymbol{\omega} + 4K\boldsymbol{\sigma}\right)\right] \end{aligned} \tag{10}$$

The following algebra is greatly simplified by making use of the explicit expression of the matrix inverse of $[B]$ given by[11]

$$[B]^{-1} = \frac{1}{(1+\sigma^2)^2}[B]^T \tag{11}$$

This expression is readily verified by using it to confirm that $[B]^{-1}[B] = I_{3\times3}$. Since for $|\boldsymbol{\sigma}| \leq 1$ the matrix $[B]$ is always invertible, from Eq. (10), the following expression must be true.

$$\dot{\boldsymbol{\omega}} + P\boldsymbol{\omega} + [B]^{-1}\left([\dot{B}]\boldsymbol{\omega} + 4K\boldsymbol{\sigma}\right) = 0 \tag{12}$$

Eq. (12) yields the necessary $\dot{\boldsymbol{\omega}}$ term to calculate the actual torque vector $\boldsymbol{u}$ in Eq. (5). The vector $\dot{\boldsymbol{\omega}}$ is written as

$$\dot{\boldsymbol{\omega}} = -P\boldsymbol{\omega} - [B]^{-1}\left([\dot{B}]\boldsymbol{\omega} + 4K\boldsymbol{\sigma}\right) = \boldsymbol{\phi} \tag{13}$$

where the expression of the right hand side of Eq. (13) is set equal to the new state vector $\boldsymbol{\phi}$. Using the vector product definition of the $[B]$ matrix in Eq. (8), the product $[\dot{B}]\boldsymbol{\omega}$ is expressed as

$$\begin{aligned} [\dot{B}]\omega \quad = \quad & \boldsymbol{\sigma}^T\boldsymbol{\omega}\left(1-\sigma^2\right)\boldsymbol{\omega} - \left(1+\sigma^2\right)\frac{\omega^2}{2}\boldsymbol{\sigma} \\ & -2\boldsymbol{\sigma}^T\boldsymbol{\omega}[\tilde{\boldsymbol{\omega}}]\boldsymbol{\sigma} + 2\left(\boldsymbol{\sigma}^T\boldsymbol{\omega}\right)^2\boldsymbol{\sigma} \end{aligned} \tag{14}$$

where the shorthand notation $\omega^2 = \boldsymbol{\omega}^T\boldsymbol{\omega}$ is used. The expression in Eq. (14) is obtained after considerable algebraic manipulations using the identities $[\tilde{\boldsymbol{a}}]\boldsymbol{a} = 0$ and

$$[\tilde{\boldsymbol{a}}][\tilde{\boldsymbol{a}}] = \boldsymbol{a}\boldsymbol{a}^T - \boldsymbol{a}^T\boldsymbol{a}I_{3\times3}, \quad \text{any } \boldsymbol{a} \in \mathcal{R}^3 \tag{15}$$

Using this $[\dot{B}]\boldsymbol{\omega}$ expression and Eq. (11) together, we obtain the following

$$\begin{aligned} [B]^{-1}\left([\dot{B}]\boldsymbol{\omega} + 4K\boldsymbol{\sigma}\right) = \\ \left[\boldsymbol{\omega}\boldsymbol{\omega}^T + \left(\frac{4K}{1+\sigma^2} - \frac{\omega^2}{2}\right)I_{3\times3}\right]\boldsymbol{\sigma} \end{aligned} \tag{16}$$

Making use of this result in Eq. (13), the vector $\boldsymbol{\phi}$ is finally given by the elegantly simple expression

$$\boldsymbol{\phi} = -P\boldsymbol{\omega} - \left[\boldsymbol{\omega}\boldsymbol{\omega}^T + \left(\frac{4K}{1+\sigma^2} - \frac{\omega^2}{2}\right)I_{3\times3}\right]\boldsymbol{\sigma} \tag{17}$$

Therefore the desired Linear Closed Loop Dynamics (LCLD) in Eq. (3) can be rewritten as

$$\ddot{\boldsymbol{\sigma}} + P\dot{\boldsymbol{\sigma}} + K\boldsymbol{\sigma} = \frac{1}{4}[B]\left(\dot{\boldsymbol{\omega}} - \boldsymbol{\phi}\right) = 0 \tag{18}$$

Substituting $\dot{\boldsymbol{\omega}} = \boldsymbol{\phi}$ into Euler's rotational equations of motion in Eq. (5) yields the required nonlinear feedback control law vector $\boldsymbol{u}$.

$$\boldsymbol{u} = [\tilde{\boldsymbol{\omega}}][I]\boldsymbol{\omega} + [I]\boldsymbol{\phi} - \boldsymbol{F}_e \tag{19}$$

We remark that these developments are parallel to those in Reference 18 where the three vector components of the Euler parameters are used instead of the MRP vector used in this paper. However, it can easily be seen that the singularity at ± 180 degrees is removed by using the MRP vector. It will also be recognized that this control law contains the inertia matrix $[I]$ linearly. When the inertia matrix is unknown, we cannot directly implement Eq. (19). In the following section, we develop an adaptive controller for such situations.

An attractive component of this methodology when dealing with *known* system parameters is that the structure of the closed loop equations can easily be modified using standard linear control theory techniques by appropriate choice of the constants P and K. If it is necessary that the feedback control reject external disturbances, an integral measure of the attitude error is added to the closed loop equations as shown in Eq. (4). Following similar steps as were done previously in this section, the linearizing body angular acceleration vector $\dot{\boldsymbol{\omega}} = \boldsymbol{\phi}$ for closed loop dynamics with an attitude integral measure are written as

$$\begin{aligned}\boldsymbol{\phi} &= -P\boldsymbol{\omega} - \left[\boldsymbol{\omega}\boldsymbol{\omega}^T + \left(\frac{4K}{1+\sigma^2} - \frac{\omega^2}{2}\right) I_{3\times 3}\right]\boldsymbol{\sigma} \\ &\quad -4K_i\ [B]^{-1}\int_0^t \boldsymbol{\sigma}dt\end{aligned} \tag{20}$$

For this choice of $\boldsymbol{\phi}$, the corresponding physical control vector $\boldsymbol{u}$ is of the same form as shown in Eq. (19).

Adaptive Control Formulation

While the vector $\boldsymbol{\phi}$ is a kinematic quantity depending only on the state vectors $\boldsymbol{\sigma}$ and $\boldsymbol{\omega}$, to compute the proper linearizing control vector $\boldsymbol{u}$, the system inertia matrix $[I]$ and the external torque vector $\boldsymbol{F}_e$ must be known precisely. In the following development it is assumed that only very crude estimates of the inertia matrix and external torque vector are known. In this case the vector $\boldsymbol{\phi}$ is no longer equal to $\dot{\boldsymbol{\omega}}$ and the actual closed loop dynamics will not be linear.

The following adaptive control law requires that the unknown states appear linearly in the control formulation. Therefore we rewrite Eq. (19) as

$$\boldsymbol{u} = [L^*]\boldsymbol{g} + [M^*]\boldsymbol{\phi} - \boldsymbol{F}_e^* \tag{21}$$

where the matrices $[L^*]$ and $[M^*]$ are defined as

$$[L_1] = \begin{bmatrix} 0 & I_{23} & -I_{23} \\ -I_{13} & 0 & I_{13} \\ I_{12} & -I_{12} & 0 \end{bmatrix} \tag{22}$$

$$[L_2] = \begin{bmatrix} I_{13} & I_{33}-I_{22} & -I_{12} \\ -I_{23} & I_{12} & I_{11}-I_{33} \\ I_{22}-I_{11} & -I_{13} & I_{23} \end{bmatrix} \tag{23}$$

$$[L^*] \equiv \left[L_1 \vdots L_2\right], \quad [M^*] \equiv [I] \tag{24}$$

the vector $\boldsymbol{F}_e^*$ is the *true* external torque vector and the 6×1 vector $\boldsymbol{g}$ is defined as

$$\boldsymbol{g} \equiv \left[\omega_1^2\ \omega_2^2\ \omega_3^2\ \omega_1\omega_2\ \omega_2\omega_3\ \omega_3\omega_1\right]^T \tag{25}$$

The control vector expression in Eq. (21) is rewritten by introducing the 3×10 matrix $[Q^*]$

$$[Q^*] = [L^* \vdots M^* \vdots \boldsymbol{F}_e^*] \tag{26}$$

and the 10×1 state vector $\boldsymbol{x}$

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{\phi} \\ -1 \end{bmatrix} \tag{27}$$

into the compact form

$$\boldsymbol{u} = [Q^*]\boldsymbol{x} \tag{28}$$

Note that Eq. (28) still assumes that all plant parameters are perfectly known. From here on we assume that the inertia matrix and the external torque vector are not known precisely. The actual control vector $\boldsymbol{u}$ which is implemented is then given by

$$\boldsymbol{u} = [Q(t)]\boldsymbol{x} \tag{29}$$

where $[Q(t)] = [L(t) \vdots M(t) \vdots \boldsymbol{F}_e(t)]$ contains the time varying adaptive estimates of the unknown system parameters. The difference between the adaptive estimates and true system parameters is expressed through the matrix $[\tilde{Q}]$ as

$$[\tilde{Q}] \equiv [Q(t)] - [Q^*] \tag{30}$$

Assume that the desired LCLD are to be of the linear PID form given in Eq. (4), then the actual closed loop dynamics, due to the imperfect control vector $\boldsymbol{u}$ in Eq. (29), are found to be

$$\begin{aligned}\ddot{\boldsymbol{\sigma}} + P\dot{\boldsymbol{\sigma}} + K\boldsymbol{\sigma} + K_i\int_0^t \boldsymbol{\sigma}dt &= \frac{1}{4}[B]\left(\dot{\boldsymbol{\omega}} - \boldsymbol{\phi}\right) \\ &= \frac{1}{4}[B][I]^{-1}\left(-[L^*]\boldsymbol{g} + \boldsymbol{u} + \boldsymbol{F}_e^* - [M^*]\boldsymbol{\phi}\right) \\ &= \frac{1}{4}[B][I]^{-1}\left([Q(t)]\boldsymbol{x} - [Q^*]\boldsymbol{x}\right) \\ &= \frac{1}{4}[B][I]^{-1}[\tilde{Q}]\boldsymbol{x}\end{aligned} \tag{31}$$

A key feature of this method is that the desired LCLD *do not depend* on the unknown inertia matrix. This makes it possible to design a desired performance without any knowledge of the actual system parameters.

The goal of the following adaptive control law is to find learning laws for the inertia matrix quantities $[L]$ and $[M]$, and if necessary for the external torque vector $\boldsymbol{F}_e$, such that the actual closed loop dynamics asymptotically approaches the desired linear form. The main advantage of this control law is that standard linear feedback gain techniques can be employed to find appropriate feedback gains P, K and K_i that meet system requirements such as control bandwidth and performance. These quantities are typically difficult to enforce with general nonlinear control laws. With the adaptation superimposed on the linearizing control law, we will be guaranteed that the desired closed loop performance is achieved asymptotically, even in the presence of severe system parameter ignorance.

Let the vector $\boldsymbol{\sigma}_r$ be the solution of the differential equation

$$\ddot{\boldsymbol{\sigma}}_r + P\dot{\boldsymbol{\sigma}}_r + K\boldsymbol{\sigma}_r + K_i\int_0^t \boldsymbol{\sigma}_r dt = 0 \tag{32}$$

where $\boldsymbol{\sigma}_r(t_0) = \boldsymbol{\sigma}(t_0)$ and $\dot{\boldsymbol{\sigma}}_r(t_0) = \dot{\boldsymbol{\sigma}}(t_0)$. Thus the trajectory $\boldsymbol{\sigma}_r(t)$ represents the desired closed loop performance. Any deviations from this performance are assumed to be due system model errors $[\tilde{Q}]$. Let the augmented 9×1 state vector $\boldsymbol{\epsilon}$ express the difference between the actual states and the reference states.

$$\boldsymbol{\epsilon} = \begin{pmatrix} \int_0^t (\boldsymbol{\sigma} - \boldsymbol{\sigma}_r) dt \\ \boldsymbol{\sigma} - \boldsymbol{\sigma}_r \\ \dot{\boldsymbol{\sigma}} - \dot{\boldsymbol{\sigma}}_r \end{pmatrix} \tag{33}$$

Using Eq. (31) and (32), note that $\dot{\boldsymbol{\epsilon}}$ is given by

$$\dot{\boldsymbol{\epsilon}} = \underbrace{\begin{bmatrix} 0 & I_{3\times3} & 0 \\ 0 & 0 & I_{3\times3} \\ -K_i I_{3\times3} & -K I_{3\times3} & -P I_{3\times3} \end{bmatrix}}_{[A]} \boldsymbol{\epsilon} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \boldsymbol{\xi} \end{pmatrix}}_{\boldsymbol{b}} \tag{34}$$

with the vector $\boldsymbol{\xi}$ being defined as

$$\boldsymbol{\xi} = \frac{1}{4}[B][I]^{-1}[\tilde{Q}]\boldsymbol{x} \tag{35}$$

We then define the following positive definite Lyapunov function V around the desired reference performance.

$$V = \boldsymbol{\epsilon}^T [S] \boldsymbol{\epsilon} + \operatorname{tr}\left([\tilde{Q}]^T [\Gamma][\tilde{Q}][\gamma]^{-1} \right) \tag{36}$$

where $[S]$ and $[\Gamma]$ are yet to be determined positive definite gain matrices and $[\gamma]$ is a diagonal matrix containing the various learning rates γ_i. Note that the trace operator in Eq. (36) can be written as

$$\operatorname{tr}\left([\tilde{Q}]^T [\Gamma][\tilde{Q}][\gamma]^{-1} \right) = \sum_{i=1}^{10} \frac{1}{\gamma_i} \begin{pmatrix} \tilde{Q}_{1i} \\ \tilde{Q}_{2i} \\ \tilde{Q}_{3i} \end{pmatrix}^T [\Gamma] \begin{pmatrix} \tilde{Q}_{1i} \\ \tilde{Q}_{2i} \\ \tilde{Q}_{3i} \end{pmatrix} \tag{37}$$

which is clearly a positive definite function in $[\tilde{Q}]$. Taking the derivative of Eq. (36) and using Eq. (34) we find

$$\dot{V} = \boldsymbol{\epsilon}^T \left([S][A] + [A]^T [S] \right) \boldsymbol{\epsilon} + 2\boldsymbol{\epsilon}^T [S] \boldsymbol{b} + 2\operatorname{tr}\left([\tilde{Q}]^T [\Gamma][\dot{\tilde{Q}}][\gamma]^{-1} \right) \tag{38}$$

By partitioning the 9×9 matrix $[S]$ into three 9×3 submatrices $[S_i]$

$$[S] = \left[S_1 \ \vdots \ S_2 \ \vdots \ S_3 \right] \tag{39}$$

the Lyapunov rate $\dot{V}$ is rewritten as

$$\dot{V} = \boldsymbol{\epsilon}^T \left([S][A] + [A]^T [S] \right) \boldsymbol{\epsilon} + 2\boldsymbol{\epsilon}^T [S_3] \boldsymbol{\xi} + 2\operatorname{tr}\left([\tilde{Q}]^T [\Gamma][\dot{\tilde{Q}}][\gamma]^{-1} \right) \tag{40}$$

Since $[A]$ is a stable matrix, Lyapunov's stability theorem for linear systems states that for any symmetric, positive definite matrix $[R]$, we are guaranteed that there exists a corresponding symmetric, positive definite matrix $[S]$ such that[23]

$$[S][A] + [A]^T [S] = -[R] \tag{41}$$

Therefore, we can pick $[R]$ and numerically solve for a corresponding positive definite matrix $[S]$ for a given stable matrix $[A]$. Using Eqs. (35) and (41), the Lyapunov rate $\dot{V}$ is reduced to

$$\dot{V} = -\boldsymbol{\epsilon}^T [R] \boldsymbol{\epsilon} + 2\boldsymbol{\epsilon}^T [S_3] \frac{1}{4}[B][I]^{-1}[\tilde{Q}]\boldsymbol{x} + 2\operatorname{tr}\left([\tilde{Q}]^T [\Gamma][\dot{\tilde{Q}}][\gamma]^{-1} \right) \tag{42}$$

Using several matrix identities listed in Ref. 24, it can be shown that

$$\begin{aligned} \frac{1}{4}\boldsymbol{\epsilon}^T [S_3][B][I]^{-1}[\tilde{Q}]\boldsymbol{x} &= \frac{1}{4}\operatorname{tr}\left(\boldsymbol{x}^T [\tilde{Q}]^T [I]^{-1}[B]^T [S_3]^T \boldsymbol{\epsilon} \right) \\ &= \frac{1}{4}\operatorname{tr}\left([\tilde{Q}]^T [I]^{-1}[B]^T [S_3]^T \boldsymbol{\epsilon}\boldsymbol{x}^T \right) \end{aligned} \tag{43}$$

Using Eq. (43), the Lyapunov rate is expressed as

$$\dot{V} = -\boldsymbol{\epsilon}^T [R] \boldsymbol{\epsilon} + 2\operatorname{tr}\left([\tilde{Q}]^T \left(\frac{1}{4}[I]^{-1}[B]^T [S_3] \boldsymbol{\epsilon}\boldsymbol{x} + [\Gamma][\dot{\tilde{Q}}][\gamma]^{-1} \right) \right) \tag{44}$$

Assuming that the true external torque vector $\boldsymbol{F}_e^*$ is constant, then

$$[\dot{\tilde{Q}}] = [\dot{Q}] - [\dot{Q}^*] = [\dot{Q}] \tag{45}$$

Studying Eq. (44), it is evident that if we set the system parameter learning rate $[\dot{Q}]$ to be

$$[\dot{Q}] = -\frac{1}{4}[\Gamma]^{-1}[I]^{-1}[B]^T [S_3] \boldsymbol{\epsilon}\boldsymbol{x}^T [\gamma] \tag{46}$$

the Lyapunov rate function is guaranteed to be of the negative definite form

$$\dot{V} = -\boldsymbol{\epsilon}^T [R] \boldsymbol{\epsilon} \tag{47}$$

Since $\dot{V}$ in Eq. (47) is *negative definite* in the state vector $\boldsymbol{\epsilon}$, this performance error vector will decay to zero asymptotically. The adaptive system parameter estimate errors $[\tilde{L}]$, $[\tilde{M}]$ and $\tilde{\boldsymbol{F}}_e$ are stable. Since the reference motion $\boldsymbol{\sigma}_r(t)$ is globally, asymptotically stable, having $\boldsymbol{\epsilon} \to 0$ implies that the actual closed loop dynamics are also globally, asymptotically stable. Note however that Eq. (46) cannot be implemented directly, since it explicitly depends on the unknown true inertia matrix $[I]$. This problem is circumvented by setting $[\Gamma] = [I]^{-1}$. Using this specific $[\Gamma]$ matrix in the analysis, the final system parameter learning law is given by the compact expression

$$[\dot{Q}] = -\frac{1}{4}[B]^T [S_3] \boldsymbol{\epsilon}\boldsymbol{x}^T [\gamma] \tag{48}$$

While the inertia matrix adaptive estimate errors $[\tilde{L}]$ and $[\tilde{M}]$ won't necessary go to zero, the adaptive external disturbance estimate $\tilde{\boldsymbol{F}}_e$ *will go to zero* if the true external disturbance $\boldsymbol{F}_e^*$ is *constant*. Since the overall system is stable, it will come to rest at some steady-state values. At steady-state, the control vector $\boldsymbol{u}_{ss}$ would need to precisely cancel the true, constant external disturbance $\boldsymbol{F}_e^*$.

$$\boldsymbol{u}_{ss} = [L_{ss}]\boldsymbol{g}_{ss} + [M_{ss}]\boldsymbol{\phi}_{ss} - \boldsymbol{F}_{e_{ss}} = -\boldsymbol{F}_e^* \tag{49}$$

This can be rewritten as

$$[L_{ss}]\boldsymbol{g}_{ss} + [M_{ss}]\boldsymbol{\phi}_{ss} = \tilde{\boldsymbol{F}}_{e_{ss}} \tag{50}$$

Both $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}}$ are zero at steady state, which implies that g_{ss} is also zero. Since $\boldsymbol{\sigma}_r$ and its derivatives go to zero, then according to Eq. (32) so does the term $\int_0^t \boldsymbol{\sigma}_r dt$. Since $\boldsymbol{\epsilon} \to 0$, this implies that $\int_0^t \boldsymbol{\sigma} dt \to 0$. Studying Eq. (20) it is then clear that $\boldsymbol{\phi}_{ss}$ is also zero. Therefore, according to Eq. (50), the adaptive external disturbance estimate error is guaranteed to go to zero. It is quite remarkable that the relatively simple adaptive learning law in Eq. (48) results in the desired LCLD and the external disturbance being tracked asymptotically *without any apriori* knowledge of either the system inertia matrix or the disturbances themselves. One reason for this is that the desired LCLD is written as a kinematic expression which does not explicitly depend on any system parameters.

As a practical matter, of course $\boldsymbol{F}_e^*$ need not be constant to obtain good tracking performance. If $\boldsymbol{F}_e^*$ is large and rapidly varying, some tuning may be required to find practical values for $[\gamma]$, P, K and K_i.

Numerical Simulations

A rigid spacecraft with an initial non-zero attitude and body angular velocity vector is to be brought to rest at a zero attitude vector. The desired LCLD are to be of the PID form shown in Eq. (4) in the presence of large ignorance in the inertia matrix and external disturbance model. The simulation parameters are given in Table 1. The initial $[L(t_0)]$ matrix is constructed out of the corresponding $[M(t_0)]$ matrix elements using Eqs. (22) through (24).

Table 1 Numerical Simulation Parameters

Parameter	Value	Units
$\boldsymbol{\sigma}(t_0)$	$[-0.3\ \ -0.4\ 0.2]$	
$\boldsymbol{\omega}(t_0)$	$[0.2\ 0.2\ 0.2]$	rad/s
$[I]$	$\begin{bmatrix} 30 & 10 & 5 \\ 10 & 20 & 3 \\ 5 & 3 & 15 \end{bmatrix}$	kg-m^2
$[M(t_0)]$	$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$	kg-m^2
K_i	0.090	sec^{-3}
K	1.0	sec^{-2}
P	3.0	sec^{-1}
γ_i	100	
$\gamma_{F_e} = \gamma_{10}$	5	
$\boldsymbol{F}_e^*$	$[2\ \ 1\ \ -1]$	N-m
$\boldsymbol{F}_e(t_0)$	$[0\ 0\ 0]$	N-m

The scalar learning rates γ_i are all equal with the exception of $\gamma_{F_e} = \gamma_{10}$, which is set to demand a slower external disturbance learning rate than the other γ_i. The positive definite $[R]$ matrix is chosen to be a block-diagonal matrix of the form

$$[R] = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 100\ I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 100\ I_{3\times3} \end{bmatrix} \tag{51}$$

Solving the algebraic Lyapunov equation in Eq. (41) and extracting the third block column matrix, the matrix $[S_3]$ is found to be

$$[S_3] = \begin{bmatrix} 0.055555\ I_{3\times3} \\ 0.702749\ I_{3\times3} \\ 0.400916\ I_{3\times3} \end{bmatrix} \tag{52}$$

The resulting simulation is illustrated in Figure 1. The MRP attitude vector components σ_i are shown in Figure 1(a). Without any adaptation, the open loop control is still asymptotically stable. However, the transient attitude errors don't match those of the desired LCLD well at all. With adaptation turned on, the performance matches that of the ideal LCLD very closely.

Figure 1(b) shows the magnitude of the MRP attitude Error vector $\boldsymbol{\sigma}$ on a logarithmic scale. Again the large transient errors of the open loop, adaptation-free control law are visible during the first 20 seconds of the maneuver along with the good final convergence characteristics. The ideal LCLD performance is indicated again through the dotted line. Two versions of the adaptive control law are compared here which differ only by whether or not the external disturbance is adaptively estimated too. On this figure both adaptive laws appear to enforce the desired LCLD very well for the first 40 seconds of the maneuver. After this the adaptive law without disturbance learning starts to decay at a slower rate, slower even than the open loop solution. Including the external disturbance adaptation clearly improves the final convergence rate. Note however that neither adaptive case starts to deviate from the ideal LCLD case until the MRP attitude error magnitude has decayed to roughly 10^{-3}. Using Eq. (1), this corresponds to having a principal rotation error of roughly 0.23 degrees. With external disturbance adaptation, the tracking error at which the LCLD deviations appear is about two orders of magnitude smaller.

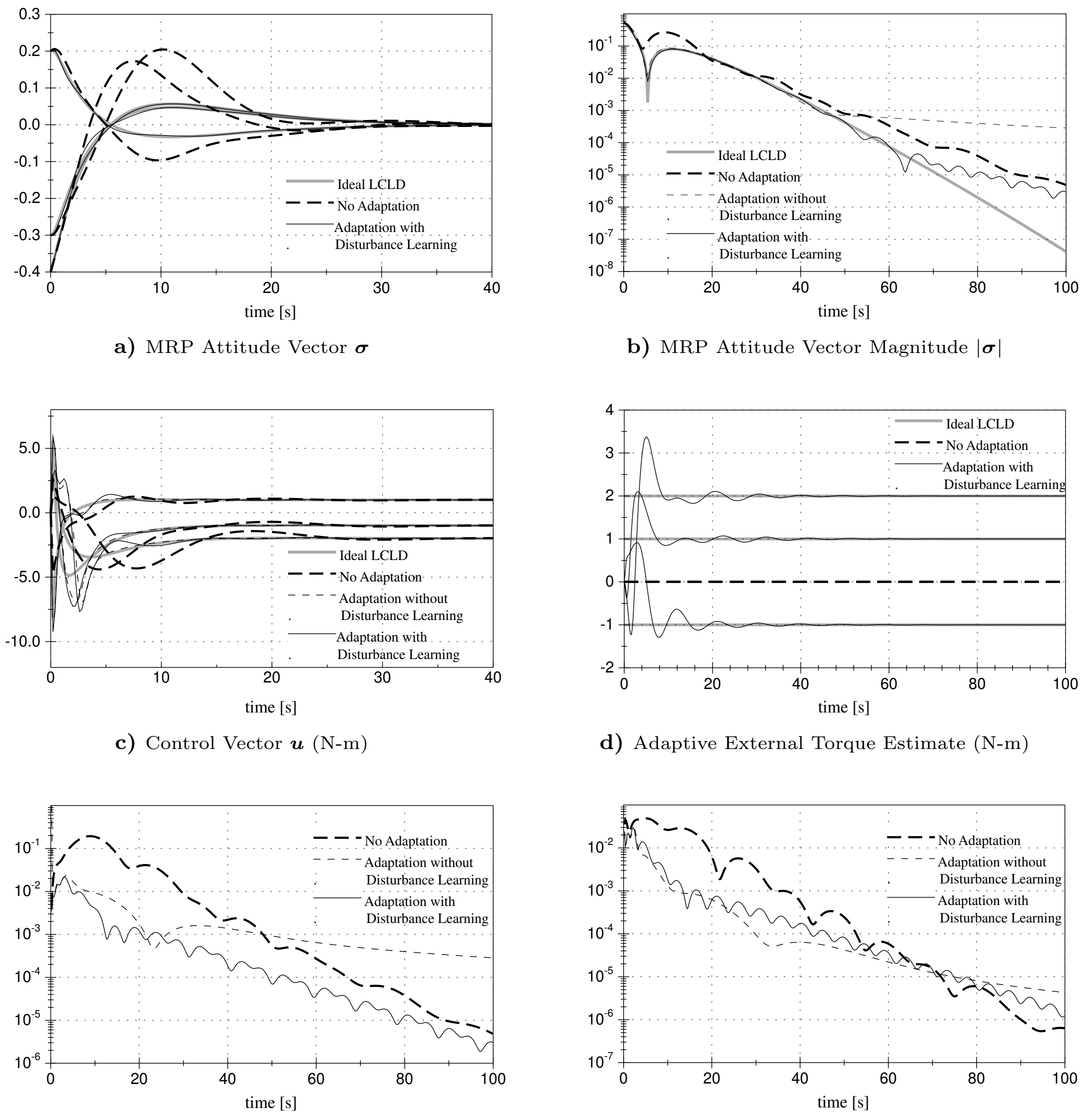
The performance of the adaptive control law can be greatly varied by choosing different learning rates. However, since *large* initial inertia matrix and external disturbance model errors are present, the adaptive learning rates were reduced to avoid radical transient torques. The control torque vector components u_i for various cases are shown in Figure 1(c). The open loop torques don't approach the ideal LCLD torque during the transient part of the maneuver. The torques required by either adaptive case are very similar. The difference is that the case with external disturbance learning is causing some extra oscillation of the control about the LCLD case. However, note that with the chosen adaptive learning rates neither control law exhibits any radical transient torques about the ideal LCLD torque profile. Figure 1(d) illustrates that the adaptive external disturbance estimate $\boldsymbol{F}_e$ indeed asymptotically approaches the true external disturbance $\boldsymbol{F}_e^*$. By reducing the external disturbance adaptive learning rate γ_{F_e} the transient adaptive estimate errors are kept within a reasonable range.

The purpose of the adaptive control is to enforce the desired LCLD. The previous figures illustrate that the resulting overall system remains asymptotically stable. Figure 1(e) illustrates the absolute performance error between the actual motion $\boldsymbol{\sigma}(t)$ and the desired linear reference motion $\boldsymbol{\sigma}_r(t)$. This figure demonstrates again the large performance error that results from using the open loop control law with the incorrect system model. Adding adaptation improves the transient performance tracking by up to two orders of magnitude. Without including the external disturbance learning, the final performance error decay rate flattens out. This error will decay to zero. However, with the given learning gains, it does so at a slower rate than if no adaptation is taking place. Adding the external disturbance learning greatly improves the final performance error decay since the system is obtaining an accurate model of the actual constant disturbance. If the initial model estimates were more accurate, then more aggressive adaptive learning rates could be used, resulting in even better LCLD performance tracking. This simulation illustrates though that even in the presence of *large* system uncertainty it is possible to track the desired LCLD very well.

Figure 1(f) shows the absolute performance error in attitude rates. Both cases with adaptation added show large reductions in attitude rate errors compared to the the non-adaptive case.

Conclusion

The open loop control laws presented in terms of the MRP vector $\boldsymbol{\sigma}$ allows for the closed loop dynamics to achieve any desired linear form. Choosing the MRPs as attitude parameters results in a formulation which is globally non-singular. This greatly simplifies the process of finding proper feedback gains P and K which match various performance requirements. To achieve a desired LCLD if modeling errors are present, the open loop law is augmented with an adaptive learning law. For the regulator problem discussed in this paper, this adaptive control tracks the desired LCLD asymptotically and is able to learn a constant external disturbance perfectly. A key feature of this adaptive control law is that it does not require any previous knowledge of the rigid body inertias or the external disturbance. This is direct result of



a) MRP Attitude Vector $\boldsymbol{\sigma}$

b) MRP Attitude Vector Magnitude $|\boldsymbol{\sigma}|$

c) Control Vector $\boldsymbol{u}$ (N-m)

d) Adaptive External Torque Estimate (N-m)

e) Performance Error $|\boldsymbol{\sigma} - \boldsymbol{\sigma}_r|$

f) Performance Error $|\dot{\boldsymbol{\sigma}} - \dot{\boldsymbol{\sigma}}_r|$

Fig. 1 Rigid Body Stabilization While Enforcing LCLD in the Presence of Large Inertia and External Disturbance Ignorance

using a *kinematic* differential equation as the desired closed loop dynamics.

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