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Hanspeter Schaub and John L. Junkins
Texas A&M University, College Station, TX 77843

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A Variable Speed Control Moment Gyroscope (VSCMG) null motion steering law is introduced that continuously attempts to minimize the condition number of the control influence matrix. By doing so the gimbal angles are rearranged to less singular configurations without exerting a torque onto the spacecraft. By allowing the reaction wheel speeds to be variable in this steering law, more general reconfigurations are possible than what are possible with conventional CMGs. No a priori calculations of preferred sets of gimbal angles are necessary with this method. Numerical studies show that superimposing this VSCMG null motion on the VSCMG steering law can result in a drastic reduction in the required reaction wheel power consumption when operating near CMG singular gimbal configurations. The reaction wheel torque required by this VSCMG steering law is typically very small and achievable with existing CMG hardware.

Introduction

Single-axis Control Moment Gyroscopes (CMGs) are commonly used to reorient large space structures. Their largest asset is that for a small torque input, a relatively large effective gyroscopic torque output is produced onto the structure. However, the standard CMG steering laws contain singular gimbal angle configurations at which the required torque is only partially produced; in the worst case only a zero torque component is feasible in the desired direction.[1,2,3] At times where the required torque is only partially produced, the spacecraft would deviate from the prescribed trajectory. These path deviations are highly undesirable in some applications.

Reaction wheels (RWs) are another common control device often employed with smaller spacecraft. They have simpler control laws and are easier and cheaper to produce. Their drawback is that there is no torque amplification effect which makes them consume more energy than CMGs for a given rotation. Also, RWs steering laws need to be concerned with RW saturation[4,5] and not operating at the structural resonant frequency of the spacecraft.

In Ref. 6 Ford and Hall introduce the equations of motion of a spacecraft which contains several Variable Speed Control Moment Gyroscopes (VSCMGs). These control devices are essentially single-gimbal CMGs with a variable speed RW. However, in their paper the RW or CMG modes are used exclusively, not simultaneously in the control laws. Schaub et. al. developed a VSCMG steering law in Ref. 5 which makes the VSCMGs act like regular CMGs away from classical CMG singularities and gradually adds the RW control authority when approaching a CMG singularity. Except for cases where excess wheel speeds are encountered, the resulting control law always generates the required torque and does not produce any trajectory deviations. However, depending on the size of the spacecraft and the proximity to the classical CMG singularity, the required RW motor torque could be rather large and require that the RW motor be larger than its CMG counterparts. Note that conventional CMGs already have a feedback loop present on the RWs to maintain a constant rotation rate. The VSCMG would have a different feedback control law and possibly a stronger RW motor.

Naturally, the best scenario to deal with CMG singularities is to avoid them altogether. For a large class of maneuvers this can be done by initially reorienting the gimbal angles to preferred sets from which the resulting trajectory will be singularity free.[7] Calculating these preferred sets of gimbal angles is done off-line prior to the maneuver. Typically CMG null motion is used to reorient the internal gimbals without exerting a torque on the spacecraft.[8] However, this can only be performed for a limited set of angles. For example, assume that all CMGs are of the same type with equal RW spin speed. It is impossible to reconfigure a symmetric set of gimbal angles to an asymmetric set, since the internal momentum vector would change in the process. As a result, this change in momentum, no matter what steering law is used en route, would produce a torque onto the spacecraft. As is shown in Ref. 5, a much larger set of gimbal angle reconfigurations is possible by including variable reaction wheel speeds. The internal momentum vector is held constant during this VSCMG null motion by speeding the RW up or down slightly. The RW motor torques required for these types of maneuvers was found to be of the same order of magnitude as the torque required by the standard CMG reaction wheel feedback control law. Therefore, in many cases, using the VSCMG null motion only requires a change in the RW feedback control law and not a complete redesign of the CMG device itself. In other words, a sophistication of the control law, with negligible hardware changes, can enable a dramatic performance improvement.

While previous work in Refs. 5 and 9 focused on reconfiguring the gimbals while holding the spacecraft attitude constant, in this paper we investigate the use of a VSCMG null motion steering law during a maneuver itself to drive the gimbal angles away from singular configurations. As a singularity is approached, it is anticipated that a small change early on in the gimbal configuration should result in a potentially large drop in required RW torque.

Problem Statement

Assume a spacecraft with the inertia matrix $[I_s]$ has N VSCMGs embedded in it. Each VSCMG control device is constrained to gimbal about the spacecraft body fixed gimbal axis $\hat{\boldsymbol{g}}_{g_i}$ where γ_i is the corresponding gimbal angle. The RW spin speed Ω_i is assumed to be time-varying. The RW spin axis $\hat{\boldsymbol{g}}_{s_i}$ and the transverse axis $\hat{\boldsymbol{g}}_{t_i}$ are shown in Figure 1. Let the VSCMG inertias about the spin, transverse and gimbal axis be given by J_{s_i}, J_{t_i} and J_{g_i} respectively. The equations of motion were derived in Ref. 5 to be

$$[I]\dot{\boldsymbol{\omega}} = -[\tilde{\boldsymbol{\omega}}][I]\boldsymbol{\omega} - [G_s]\boldsymbol{\tau}_s - [G_t]\boldsymbol{\tau}_t - [G_g]\boldsymbol{\tau}_g + \boldsymbol{L} \tag{1}$$

where $\boldsymbol{L}$ is an external torque vector and the influence matrices $[G_s]$, $[G_t]$ and $[G_g]$ contain the respective components of the unit direction vectors of each VSCMG gimbal frame expressed in body frame components.

$$[G_s] = [\hat{\boldsymbol{g}}_{s_1} \cdots \hat{\boldsymbol{g}}_{s_N}] \tag{2a}$$
$$[G_t] = [\hat{\boldsymbol{g}}_{t_1} \cdots \hat{\boldsymbol{g}}_{t_N}] \tag{2b}$$
$$[G_g] = [\hat{\boldsymbol{g}}_{g_1} \cdots \hat{\boldsymbol{g}}_{g_N}] \tag{2c}$$

*Post-Doctoral Research Associate, Aerospace Engineering Department, AIAA Member.

†George J. Eppright Chair Professor, Aerospace Engineering Department, Fellow AIAA.

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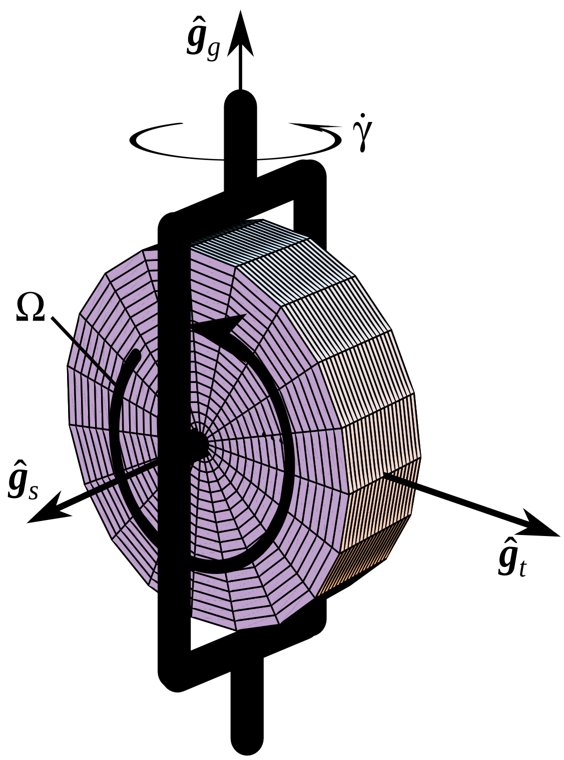


Fig. 1 Illustration of a Variable Speed Control Moment Gyroscope

The total spacecraft inertia matrix is expressed as

$$[I] = [I_s] + \sum_{i=1}^{N} [J_i] = [I_s] + \sum_{i=1}^{N} J_{s_i} \hat{\boldsymbol{g}}_{s_i} \hat{\boldsymbol{g}}_{s_i}^T + J_{t_i} \hat{\boldsymbol{g}}_{t_i} \hat{\boldsymbol{g}}_{t_i}^T + J_{g_i} \hat{\boldsymbol{g}}_{g_i} \hat{\boldsymbol{g}}_{g_i}^T \tag{3}$$

The effective torques τ_{s_i}, τ_{t_i} and τ_{g_i} are related to gimbal rates and wheel speeds by

$$\tau_{s_i} = J_{s_1} \left(\dot{\Omega}_i + \dot{\gamma}_i \omega_{t_i} \right) - \left(J_{t_i} - J_{g_i} \right) \omega_{t_i} \dot{\gamma}_i \tag{4a}$$
$$\tau_{t_i} = J_{s_i} (\Omega_i + \omega_{s_i}) \dot{\gamma}_i - (J_{t_i} + J_{g_i}) \omega_{s_i} \dot{\gamma}_i + J_{s_i} \Omega_i \omega_{g_i} \tag{4b}$$
$$\tau_{g_i} = J_{g_i} \ddot{\gamma}_i - J_{s_i} \Omega_i \omega_{t_i} \tag{4c}$$

where i ranges from 1 to N and the shorthand notation

$$\omega_s = \hat{\boldsymbol{g}}_s^T \boldsymbol{\omega} \qquad \omega_t = \hat{\boldsymbol{g}}_t^T \boldsymbol{\omega} \qquad \omega_g = \hat{\boldsymbol{g}}_g^T \boldsymbol{\omega} \tag{5}$$

was used. Note that the effect of having a variable inertial matrix is absorbed into the definition of the effective torques. The spacecraft attitude vector measured relative to the final attitude is given by the Modified Rodrigues Parameter (MRP) vector $\boldsymbol{s}$.[10,11,12,13,9] For the purpose of this paper, the desired final attitude is adopted as the inertial frame. Assuming K is a positive scalar and $[P]$ is a positive definite matrix, using Lyapunov stability theory it can be shown that the steering law constraint[5,9]

$$\sum_{i=1}^{N} \left(\hat{\boldsymbol{g}}_{s_i} J_{s_i} \dot{\Omega}_i + \hat{\boldsymbol{g}}_{g_i} J_{g_i} \ddot{\gamma}_i + \hat{\boldsymbol{g}}_{t_i} \left(J_{s_i} \left(\Omega_i + \omega_{s_i} \right) \; - J_{t_i} \omega_{s_i} \right) \dot{\gamma}_i \right) = K\boldsymbol{s} + [P]\boldsymbol{\omega} + \boldsymbol{L} \tag{6}$$

leads to a globally, asymptotically stable feedback law for rest to rest maneuvers. To express this constraint in a more compact and useable form, let us define the following *3*xN matrices.

$$[D_0] = [\hat{\boldsymbol{g}}_{s_1} J_{s_1} \cdots \hat{\boldsymbol{g}}_{s_N} J_{s_N}] \tag{7a}$$
$$[D_1] = [\hat{\boldsymbol{g}}_{t_1} J_{s_1} (\Omega_1 + \omega_{s_1}) \cdots \hat{\boldsymbol{g}}_{t_N} J_{s_N} (\Omega_N + \omega_{s_N})] \tag{7b}$$
$$[D_2] = [\hat{\boldsymbol{g}}_{t_1} J_{t_1} \omega_{s_1} \cdots \hat{\boldsymbol{g}}_{t_N} J_{t_N} \omega_{s_N}] \tag{7c}$$
$$[B] = [\hat{\boldsymbol{g}}_{g_1} J_{g_1} \cdots \hat{\boldsymbol{g}}_{g_N} J_{g_N}] \tag{7d}$$

Let $\dot{\boldsymbol{\gamma}}$, $\ddot{\boldsymbol{\gamma}}$ and $\dot{\boldsymbol{\Omega}}$ be Nx*1* vectors whose *i*-th element contains the respective VSCMG angular velocity or acceleration or RW spin rate. The stability condition in Eq. (6) then is expressed compactly as

$$[D_0]\dot{\boldsymbol{\Omega}} + [B]\ddot{\boldsymbol{\gamma}} + ([D_1] - [D_2])\,\dot{\boldsymbol{\gamma}} = \boldsymbol{L}_r \tag{8}$$

where $\boldsymbol{L}_r = K\boldsymbol{s} + [P]\boldsymbol{\omega} + \boldsymbol{L}$ is called the required control torque. The standard CMG velocity based steering law assumes that the $[B]$ and $[D_2]$ matrices are small compared to $[D_1]$ and are dropped. The resulting simplified stability condition then becomes

$$[D_0]\dot{\boldsymbol{\Omega}} + [D_1]\dot{\boldsymbol{\gamma}} = \boldsymbol{L}_r \tag{9}$$

For notational convenience, we introduce the *2*Nx*1* state vector $\boldsymbol{\eta}$

$$\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{\Omega} \\ \boldsymbol{\gamma} \end{bmatrix} \tag{10}$$

and the *3*x*2*N matrix $[Q]$

$$[Q] = \left[D_0 \,\vdots\, D_1 \right] \tag{11}$$

Eq. (9) is then conveniently written as

$$[Q]\dot{\boldsymbol{\eta}} = \boldsymbol{L}_r \tag{12}$$

To solve this redundant set of equations for $\dot{\boldsymbol{\eta}}$, Ref. 5 uses the weighted minimum norm inverse.[14] Let $[W]$ be a *2*Nx*2*N diagonal matrix whose positive entries determine the instantaneous importance of the respective VSCMG mode. Then the desired $\dot{\boldsymbol{\eta}}$ steering law is given by

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} \dot{\boldsymbol{\Omega}} \\ \dot{\boldsymbol{\gamma}} \end{bmatrix} = [W][Q]^T \left([Q][W][Q]^T \right)^{-1} \boldsymbol{L}_r \tag{13}$$

Note that there is no need here to introduce a modified pseudo-inverse as Nakamura and Hanafusa did in developing the singularity robustness steering law in Ref. 1. As long as at least 2 or more VSCMGs are used with linearly independent $\hat{\boldsymbol{g}}_{g_i}$ vectors, the square matrix $[Q][W][Q]^{-1}$ will be of full rank. Ideally the VSCMG are to perform like CMGs away from classical CMG singularities, so the RW mode weights W_{s_i} are defined to be

$$W_{s_i} = W_{s_i}^0 e^{(-\mu\delta)} \tag{14}$$

where the non-dimensional singularity indicator δ is defined as

$$\delta = \det \left(\frac{1}{\bar{h}^2} [D_1][D_1]^T \right) \tag{15}$$

with $\bar{h}$ being a nominal CMG spin axis angular momentum magnitude. The positive constant μ is a steering law parameter.

Assuming that $\boldsymbol{\Omega}_f$ and $\boldsymbol{\gamma}_f$ are preferred sets of VSCMG states, the VSCMG null motion introduced in Ref. 5 is shown to be globally stable and is given by

$$\dot{\boldsymbol{\eta}} = k_e \left[[\hat{W}][Q]^T \left([Q][\hat{W}][Q]^T \right)^{-1} [Q] - [I_{2N \times 2N}] \right] [A] \begin{pmatrix} \boldsymbol{\Delta\Omega} \\ \boldsymbol{\Delta\gamma} \end{pmatrix} \tag{16}$$

where the state errors $\boldsymbol{\Delta\Omega}$ and $\boldsymbol{\Delta\gamma}$ are defined as

$$\boldsymbol{\Delta\Omega} = \boldsymbol{\Omega} - \boldsymbol{\Omega}_f \tag{17}$$
$$\boldsymbol{\Delta\gamma} = \boldsymbol{\gamma} - \boldsymbol{\gamma}_f \tag{18}$$

and k_e is a positive scalar. The notation $[I_{n\times m}]$ represents a $n \times m$ identity matrix. The diagonal weight matrix $[\hat{W}]$

controls how heavily the RW and CMG modes are using during the VSCMG null motion. The diagonal matrix $[A]$ is of the form

$$[A] = \begin{bmatrix} a_{RW}[I_{N\times N}] & [0_{N\times N}] \\ [0_{N\times N}] & a_{CMG}[I_{N\times N}] \end{bmatrix} \tag{19}$$

where the parameters a_{RW} and a_{CMG} are either 1 or 0. If one is set to zero, this means that the resulting null motion will be performed with no preferred set of either $\boldsymbol{\Omega}_f$ or $\boldsymbol{\gamma}_f$. Note that this VSCMG null motion cannot reconfigure the current $\boldsymbol{\gamma}$ and $\boldsymbol{\Omega}$ states to arbitrary final states, since the vector sum of the internal VSCMG cluster momentum must remain constant. However, by setting either a_{RW} or a_{CMG} to zero it is possible to either change the gimbal angles or the RW spin speeds in a very general manner.

Singularity Avoidance

In order to drive the gimbal configuration towards a "less singular" configuration, a measure of singularity proximity is needed. A gradient type method is developed below that provides the necessary state error vectors $\Delta\boldsymbol{\Omega}$ and $\Delta\boldsymbol{\gamma}$ for the VSCMG null motion in Eq. (16). Classical CMG steering laws are found by solving

$$[D_1]\dot{\boldsymbol{\gamma}} = \boldsymbol{L}_r \tag{20}$$

Whenever the rank of the control influence matrix $[D_1]$ drops below 3 the required torque $\boldsymbol{L}_r$ may not be fully produced by $\dot{\boldsymbol{\gamma}}$. The non-dimensional singularity indicator δ could be used here as the singularity measure of the VSCMG null motion steering law. However, since using the gradient method requires taking analytical partial derivatives of the singularity measure with respect to the gimbal angles, this would lead to very complex equations which have to be derived specifically for each physical system.

Instead, the condition number κ of the matrix $[D_1]$ is used as the singularity measure. Using a singular value decomposition (SVD), the $3\times N$ matrix $[D_1]$ is decomposed as

$$[D_1] = [U][\Sigma][V]^T = \begin{bmatrix} \boldsymbol{u}_1\, \boldsymbol{u}_2\, \boldsymbol{u}_3 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 & & 0 \\ 0 & \sigma_2 & 0 & \cdots & 0 \\ 0 & 0 & \sigma_3 & & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_1 & \cdots & \boldsymbol{v}_N \end{bmatrix}^T \tag{21}$$

where $[U]$ is a 3×3, $[\Sigma]$ is a $3\times N$ and $[V]$ is a $N\times N$ matrix. Assume that the singular values have been arranged such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The non-dimensional condition number κ is then defined as

$$\kappa = \frac{\sigma_1}{\sigma_3} \tag{22}$$

As the gimbal angles approaches a singular CMG configuration, the index κ would grow very large since $\sigma_3 \to 0$. The theoretically best possible matrix conditioning would be with $\sigma_1 = \sigma_3$ where $\kappa = 1$. The goal of the VSCMG null motion would be to minimize the singularity index κ during a maneuver. Let $\kappa(t)$ be the singularity index at the current time and let $\kappa(t^+)$ be the index after a discrete gimbal angle adjustment has been made. Using a Taylor series expansion of κ in terms of the gimbal angles γ_i yields

$$\kappa(t^+) = \kappa(t) + \frac{\partial\kappa}{\partial\boldsymbol{\gamma}}^T \Delta\boldsymbol{\gamma} \tag{23}$$

Since ideally $\kappa(t^+) = 1$, using a minimum norm inverse, the desired gimbal angle correction is given by

$$\Delta\boldsymbol{\gamma} = \alpha \frac{(1-\kappa(t))}{|\frac{\partial\kappa}{\partial\boldsymbol{\gamma}}|^2} \frac{\partial\kappa}{\partial\boldsymbol{\gamma}} \tag{24}$$

where the positive scalar α scales the gradient step. As will be shown with numerical examples, Eq. (24) works well when the gimbal configuration is to be rearranged while the spacecraft attitude is held stationary. However, if Eq. (24) is used to drive the gimbal angles away from singular configurations during a maneuver, then the VSCMG null motion corrections become too "soft" as a singular configuration is rapidly approached. The $|\partial\kappa/\partial\boldsymbol{\gamma}|^2$ term in the denominator drives $\Delta\boldsymbol{\gamma}$ to zero as $\partial\kappa/\partial\boldsymbol{\gamma}$ becomes very large in the neighborhood of a singularity. To counter this softening effect, the following stiffer gimbal correction algorithm is proposed.

$$\Delta\boldsymbol{\gamma} = \alpha(1-\kappa(t))\frac{\partial\kappa}{\partial\boldsymbol{\gamma}} \tag{25}$$

Numerical studies show that the VSCMG null motion driven by this $\Delta\boldsymbol{\gamma}$ during a maneuver is more successful in keeping the gimbal angles away from singular configurations.

If $[D_1]$ is reasonably well conditioned, it is not desirable to have the VSCMG be active at this point and drive the gimbal angles to an even better conditioned configuration. Doing so would only unnecessarily waste fuel and energy. To stop the VSCMG null motion at some pre-determined singularity index κ_{db}, a deadband is introduced. Whenever $\kappa \leq \kappa_{db} > 1$, then we set $\alpha = 0$.

Using Eq. (22), the partial derivatives of κ with respect to the gimbal angles are found to be

$$\frac{\partial\kappa}{\partial\gamma_i} = \frac{1}{\sigma_3}\frac{\partial\sigma_1}{\partial\gamma_i} - \frac{\sigma_1}{\sigma_3^2}\frac{\partial\sigma_3}{\partial\gamma_i} \tag{26}$$

The partial derivatives of the singular values with respect to the gimbal angles are given by[15]

$$\frac{\partial\sigma_j}{\partial\gamma_i} = \boldsymbol{u}_j^T \frac{\partial[D_1]}{\partial\gamma_i}\boldsymbol{v}_j \tag{27}$$

The result in Eq. (27) may have to be modified if $\sigma_1 = \sigma_3$. However, this event will never be encountered if $\kappa_{db} > 1$ is adopted. Using Eq. (7b), the partial derivative of $[D_1]$ with respect to γ_i is readily found to be

$$\frac{\partial[D_1]}{\partial\gamma_i} = [\boldsymbol{0}\cdots\boldsymbol{0}\ \boldsymbol{\chi}_i\ \boldsymbol{0}\cdots\boldsymbol{0}] \tag{28}$$

where the i-th column vector $\boldsymbol{\chi}_i$ is defined to be

$$\boldsymbol{\chi}_i = \frac{\partial\hat{\boldsymbol{g}}_{t_i}}{\partial\gamma_i}J_{s_i}\left(\Omega_i + \omega_{s_i}\right) + \hat{\boldsymbol{g}}_{t_i}J_{s_i}\left(\Omega_i + \frac{\partial\hat{\boldsymbol{g}}_{s_i}}{\partial\gamma_i}^T\boldsymbol{\omega}\right) \tag{29}$$

Since the partial derivatives of the gimbal frame axes are given by

$$\frac{\partial\hat{\boldsymbol{g}}_{s_i}}{\partial\gamma_i} = \hat{\boldsymbol{g}}_{t_i} \tag{30a}$$

$$\frac{\partial\hat{\boldsymbol{g}}_{t_i}}{\partial\gamma_i} = -\hat{\boldsymbol{g}}_{s_i} \tag{30b}$$

the vector $\boldsymbol{\chi}_i$ is expressed compactly as

$$\boldsymbol{\chi}_i = -\hat{\boldsymbol{g}}_{s_i}J_{s_i}\left(\Omega_i + \omega_{s_i}\right) + \hat{\boldsymbol{g}}_{t_i}J_{s_i}(\Omega_i + \omega_{t_i}) \tag{31}$$

Substituting Eqs. (28) and (31) into Eq. (27) and carrying out the vector algebra, the partial derivatives of the singular values with respect to the gimbal angles are given by

$$\frac{\partial\sigma_j}{\partial\gamma_i} = \left(\boldsymbol{u}_j^T\boldsymbol{\chi}_i\right)[V_{ij}] \tag{32}$$

Note that these singular value sensitivities can be computed very quickly given the vectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ obtained from a numerical SVD of the local matrix $[D_1]$. Therefore the $\Delta\boldsymbol{\gamma}$ vector can be easily computed and fed to the VSCMG null motion in Eq. (16).

Numerical Simulations

To illustrate the use of the VSCMG null motion to drive the gimbal angles away from singular configurations, the following numerical simulation was performed. A rigid spacecraft is reoriented from large initial displacements to coincide with the target attitude through the use of four VSCMGs. The VSCMGs are arranged in the standard CMG pyramid configuration shown in Figure 2. The spacecraft and VSCMG properties are given in Table 1.

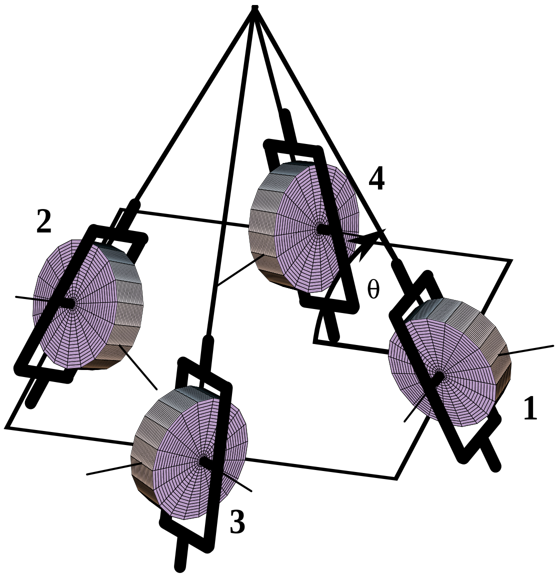


Fig. 2 VSCMG Pyramid Configuration

Table 1 Spacecraft and VSCMG Properties

Parameter	Value	Units
I_{s_1}	15053	kg-m^2/sec
I_{s_2}	6510	kg-m^2/sec
I_{s_3}	11122	kg-m^2/sec
N	4	
θ	54.75	degrees
J_s	0.70	kg-m^2
J_t	0.35	kg-m^2
J_g	0.35	kg-m^2

Two simulations are performed. One simulation uses only the VSCMG steering law in Eq. (13). The second simulation superimposes onto this steering law the VSCMG null motion given Eq. (16) with the stiff gradient multiplier in Eq. (25) to continuously reconfigure the gimbal angles away from singular configurations. The numerical simulation parameters are given in Table 2. The vector $\boldsymbol{\Omega}_f$ is chosen to be the same as the initial $\boldsymbol{\Omega}(t_0)$ vector, which results in the null motion trying to keep the RW spin speeds as close to their original values as possible. The values of the diagonal angular velocity feedback gain matrix $[P]$ were chosen such that each mode of the linearized closed loop dynamics is critically damped.[9,5] The null motion weights $\hat{W}_{s_i}$ and $\hat{W}_{g_i}$ are set equal in this simulation. Setting $\hat{W}_{s_i}$ equal to zero would have yielded a pure CMG null motion. This is typically the preferred setting. By having $\hat{W}_{s_i} = \hat{W}_{g_i}$ in this simulation, the null motion utilizes the RW mode very little. However, setting $\hat{W}_{s_i}$ equal to zero would restrict the types of null motion (therefore what types of gimbal angle reconfigurations) are possible.[5,9]

The resulting numerical simulations are illustrated in Figure 3. Note that three figures have a different time scale of 0 to 150 seconds. This is done to magnify what occurs during this time interval and since nothing interesting happens for these parameters after 150 seconds. Results obtained

Table 2 Numerical Simulation Parameters

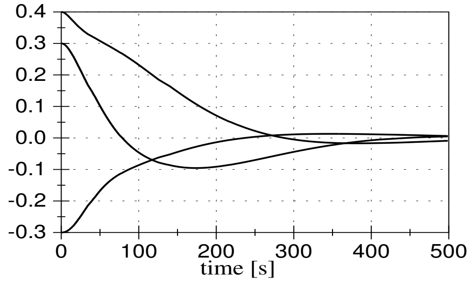
Parameter	Value	Units
$\boldsymbol{s}(t_0)$	$[0.4\ 0.3\ -0.3]$	
$\boldsymbol{\omega}(t_0)$	$[0.4\ 0.3\ -0.3]$	rad/sec
$\boldsymbol{\gamma}(t_0)$	$[45\ -45\ 45\ -45]$	deg
$\dot{\boldsymbol{\gamma}}(t_0)$	$[0\ 0\ 0\ 0]$	rad
$\Omega_i(t_0)$	628	rad/sec
Ω_f	628	rad/sec
$[P]$	[725 477 623]	kg-m^2/sec
K	35	kg-m^2/sec^2
$K_{\dot{\gamma}}$	1.0	sec^{-1}
$W_{s_i}^0$	40	
W_{g_i}	1.0	
μ	100	
$\hat{W}_{s_i}$	1.0	
$\hat{W}_{g_i}$	1.0	
κ_{db}	3	

from the simulation that only utilized the steering law in Eq. (13) are indicated by a dashed line, results obtained from the simulation with VSCMG null motion added are indicated with a solid line. Figures 3(a) and 3(b) are valid for both simulations and show that the closed loop dynamics is indeed asymptotically stable for both simulations.

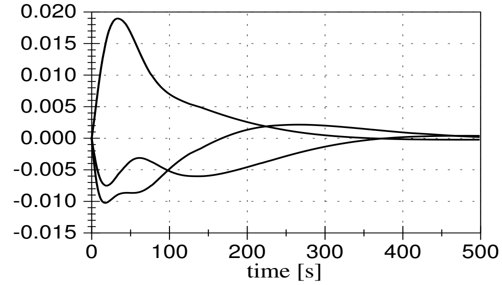
Figures 3(c) and 3(d) shows the singularity indices κ and δ for both simulations. Without the singularity-avoiding null motion added, the gimbal angles approach a singularity twice. During the second approach the non-dimensional determinant δ actually reaches zero and remains zero for a finite duration. Therefore it would be impossible to precisely perform this maneuver with the conventional CMG steering law. Some modifications would have to be used to produce and *approximate* required torque in the neighborhood of this singular configuration. However, the VSCMG steering law is easily able to handle this singularity by temporarily using its RW modes. During both periods where $\delta \to 0$, the condition number κ grows very large as seen in Figure 3(c). If the same maneuver is performed with the singularity avoiding VSCMG null motion added, the condition number κ is reduced from the outset and remains relatively low throughout the maneuver. Note that this index could have been reduced even more, but it remains essentially around the given condition number deadband value of 3. The trade off of lowering this deadband value is that the VSCMG null motion ends up reconfiguring the gimbals more often (i.e. using more energy).

One drawback of the VSCMG steering law as proposed in Ref. 5 is that for it to be able to drive through singular configurations, a change in $\boldsymbol{\Omega}$ (i.e. large RW motor torque) is required. For this maneuver the associated $\boldsymbol{\Omega}$ changes are illustrated in Figure 3(e). Note that the time scale in this and some other Figures is changed to better illustrate the "interesting" regions. The weights were better tuned than in Ref. 5 resulting in the RW speeds having a relatively small change percentage wise. Using the VSCMG null motion in References 5 and 9 to reconfigure the gimbal angles a priori it was found that the associated RW $\boldsymbol{\Omega}$ changes were rather small. The same is observed here where the null motion is performed during the maneuver itself as seen in Figure 3(f).

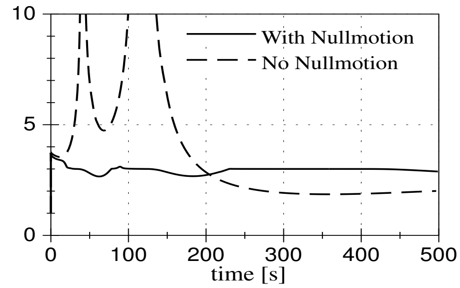
The equivalent RW motor torque vector magnitudes $|\boldsymbol{u}_s|$ are plotted in Figure 3(g). Note that classical CMGs already have an active RW control motor that simply maintains a constant wheel speed. The additional effort required by the VSCMG null motion is visible as small "humps" of the solid line at the beginning of the maneuver and before 100 seconds. What is very encouraging is that the magnitude of these humps is very small and still easily feasible with the standard existing RW torque motors. Conversely, the standard VSCMG steering law requires periodically RW torques that are much larger and would require some reengineering of the RW control motors.



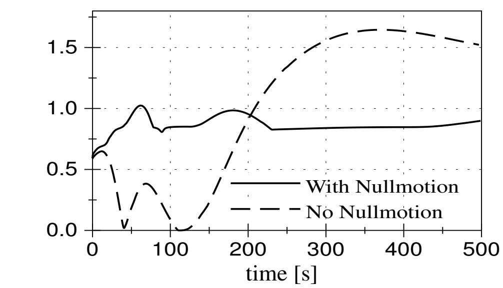
a) MRP Attitude Vector $\boldsymbol{s}$



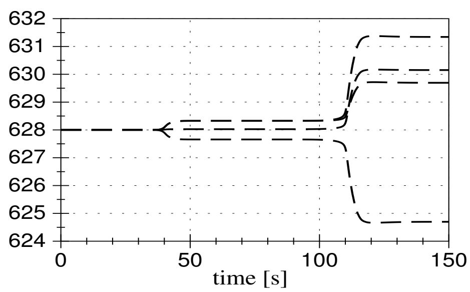
b) Angular Velocity Vector $\boldsymbol{\omega}$ (rad/s)



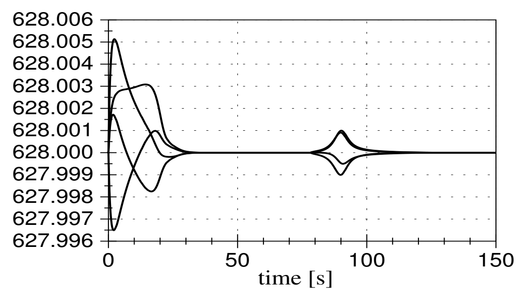
c) Singularity Index κ



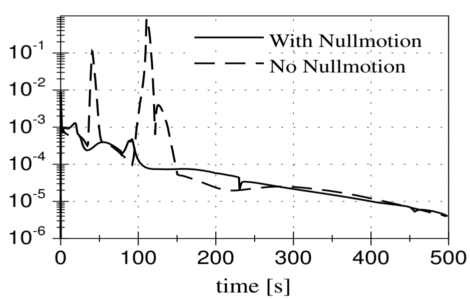
d) Singularity Index δ



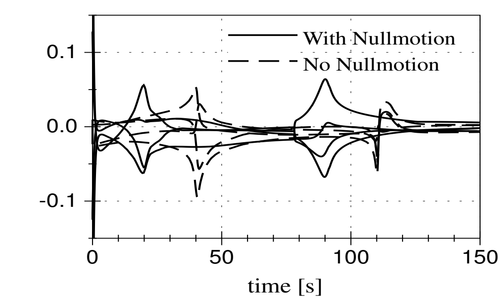
e) $\boldsymbol{\Omega}$ Vector Without Null Motion (rad/s)



f) $\boldsymbol{\Omega}$ Vector With Null Motion (rad/s)



g) Torque Vector $\boldsymbol{u}_s$ Magnitude (N-m)



h) Gimbal Rates Vector $\dot{\boldsymbol{\gamma}}$ (rad/s)

Fig. 3 Comparison of Maneuvers With and Without VSCMG Null Motion

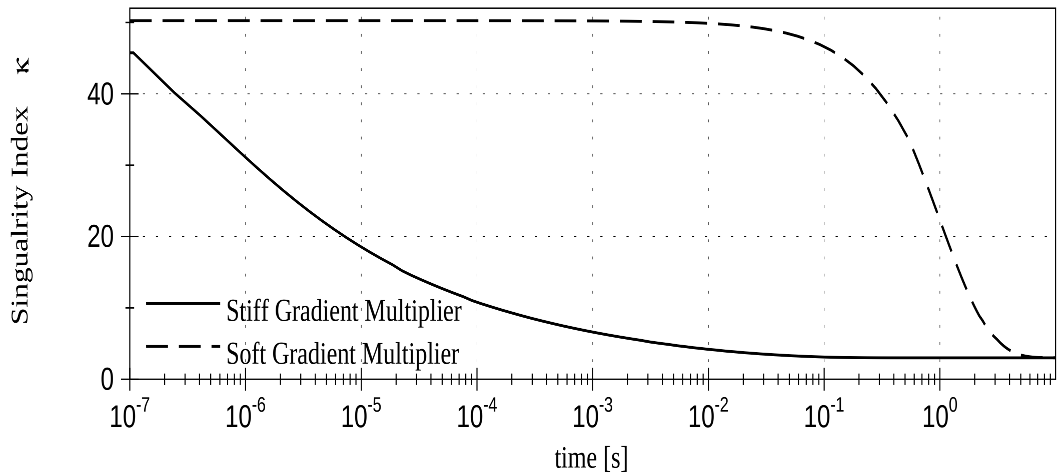


Fig. 4 Comparison of "Soft" vs "Stiff" Gradient Step Multiplier

The associated gimbal rates for both simulations are shown in Figure 3(h). While the added VSCMG null motion does require periodically higher gimbal rates to reconfigure the gimbals, the overall control effort for the CMG mode is about the same. Again, the biggest difference in control effort between adding the VSCMG null motion or not to the VSCMG steering law manifests itself in the required RW control effort.

Had the softer gradient multiplier in Eq. (24) been used instead of the stiffer one in Eq. (25), then the gimbals would not reconfigure fast enough to avoid the singularity. As the VSCMG null motion starts to reconfigure the gimbals with the singularity approaching, the gradient multiplier starts to go zero. This effectively turns off the VSCMG null motion until the singularity passes by. However, this soft gradient multiplier was found to be superior when trying to reconfigure the gimbals while holding the spacecraft attitude steady. Assume that a spacecraft is at rest, but the internal gimbal configuration is close to a singularity. Since no maneuver is being performed, the steering law won't drive the gimbals closer to the singularity. However, the stiffer gradient multiplier causes the VSCMG null motion to reduce the singularity index extremely sharply at first, and then decay slowly after this. The problem with this is that in order to keep the gimbal rates within reasonable limits, the null motion gain k_e had to be substantially reduced. The resulting self-reconfiguration was very sluggish and inefficient. The softer gradient multiplier in Eq. (24) throttles back the initial VSCMG null motion and decays the condition number κ in a more useful manner. This behavior is illustrated in Figure 4. The same spacecraft and simulation parameters were used as in the previous simulations. The only difference between both simulations is the use of the softer and stiffer gradient multiplier. To compare both initial κ time histories, the time axis is shown in a logarithmic scale. Clearly the stiffer multiplier causes κ to decay unacceptably fast initially. Therefore, if the VSCMG null motion is to be used on a steady spacecraft to reduce the condition number of the $[D_1]$ matrix, the softer gradient multiplier in Eq. (24) provides better performance.

Conclusion

A variable speed CMG null motion steering law is introduced that drives the gimbal angles away from singular configurations. This is done by continuously using redundant degrees of freedom to minimize the condition number κ of the control influence matrix through a gradient method. This null motion can be applied to a spacecraft performing a reorientation maneuver or to a stationary spacecraft. Numerical studies show that adding this VSCMG null motion to the variable speed CMG steering law can drastically reduce the reaction wheel power consumption, while maintaining a singularity-free steering law. To reduce the condition number κ, two gradient multipliers are introduced. It is found that the stiffer gradient multiplier provides superior performance than the softer multiplier during a maneuver. The softer multiplier is better suited when trying to reconfigure the gimbals to a less singular configuration while holding the spacecraft attitude steady.

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