Interpolation of Ephemerides

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Interpolation of Ephemerides

- Motivation
- Algorithms
- Software
- Tests and Results

Motivation

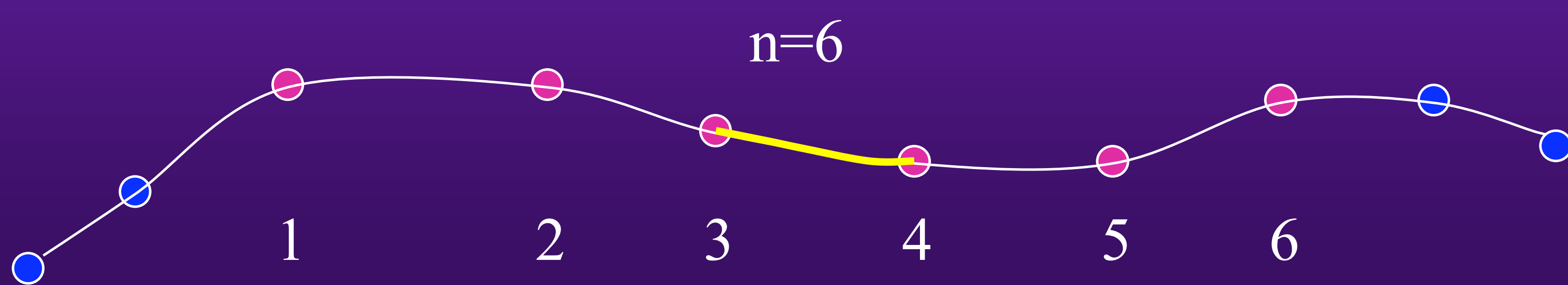
- Development of a new prediction format
- Ephemeris lists of positions (and velocities) in Earth-centered, Earth-fixed coordinates
- Need interpolation algorithm and software
- Ephemeris intervals as large as possible to minimize prediction file

Algorithms

- Ephemerides intervals may be variable within covered period (for highly eccentric orbits)
- Need simple interpolation algorithm
- Can we include given velocity into interpolation algorithm?

Lagrange Polynomial Interpolation

- Easy to program
- Use n points to interpolate with polynomial of degree n-1
- No need for equally spaced points
- Apply formula to center interval of given values only
- First derivative of the formula gives velocities
- Separate interpolation for x, y, z
- Does not explicitly give polynomial coefficients
- Not optimized for speed



Hermite Interpolation

- Similar to Lagrange
- In addition we use given first derivatives
- n=2:Two coordinates, two first derivatives
  --> Polynomial of degree 3
- n=4: Four coordinates, four first derivatives
  --> Polynomial of degree 7

Software: Interpolation Subroutine

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SUBROUTINE HERMITE(ITYP,X,Y,Z,NMAX,NVAL,XP,YP,ZP,IRCODE)
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Interpolation by a polynomial using NVAL out of NMAX given data points

     Input : ITYP   : 1: use Lagrange polynomial of degree NVAL-1
                      2: use Hermite formula:
                         Polynomial of degree 2*NVAL-1

             NVAL   : number of points to use for interpolation
             NMAX   : number of given points in list
             X(I)   : arguments of given values (I=1,...,NMAX)
             Y(I)   : functional values Y=f(X)
             Z(I)   : derivatives       Z=f'(X)
             XP     : interpolation argument

     Output: YP     : interpolated value at XP
             ZP     : first derivative of YP (ITYP=1 only)
             IRCODE: return code (0=ok, 2=error)

The function selects the NVAL values to be used for interpolation such
that the interpolated data point is located in the center interval.
(Works best for NVAL = even number, of course).
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Test and Results: Test Orbits

- Orbits given as positions (and velocities) with small enough intervals as “truth” (1 min):
  - Champ (CODE orbit, SP3 file)
  - GFO-1 (Appleby, NERC)
  - Topex (Appleby, NERC)
  - Lageos (Appleby, NERC)
  - GPS satellite (CODE precise orbits, SP3 file)
  - Moon (Ricklefs, UTX)
- Requirement: Maximum interpolation error: ~20 cm

Test Results

- Lagrange polynomial interpolation can easily provide velocities, too (first derivative of interpolation polynomial)
- Hermite interpolation strongly depends on quality of given velocities. Does not substantially increase minimum interval

Test Results: Summary

Satellite	Interval (min) Degree 7	Interval (min) Degree 9
LEO (Champ)	2	3
GFO-1	3	4
Topex	4	5
Lageos	5	10
GPS	15	30
Moon	30	60

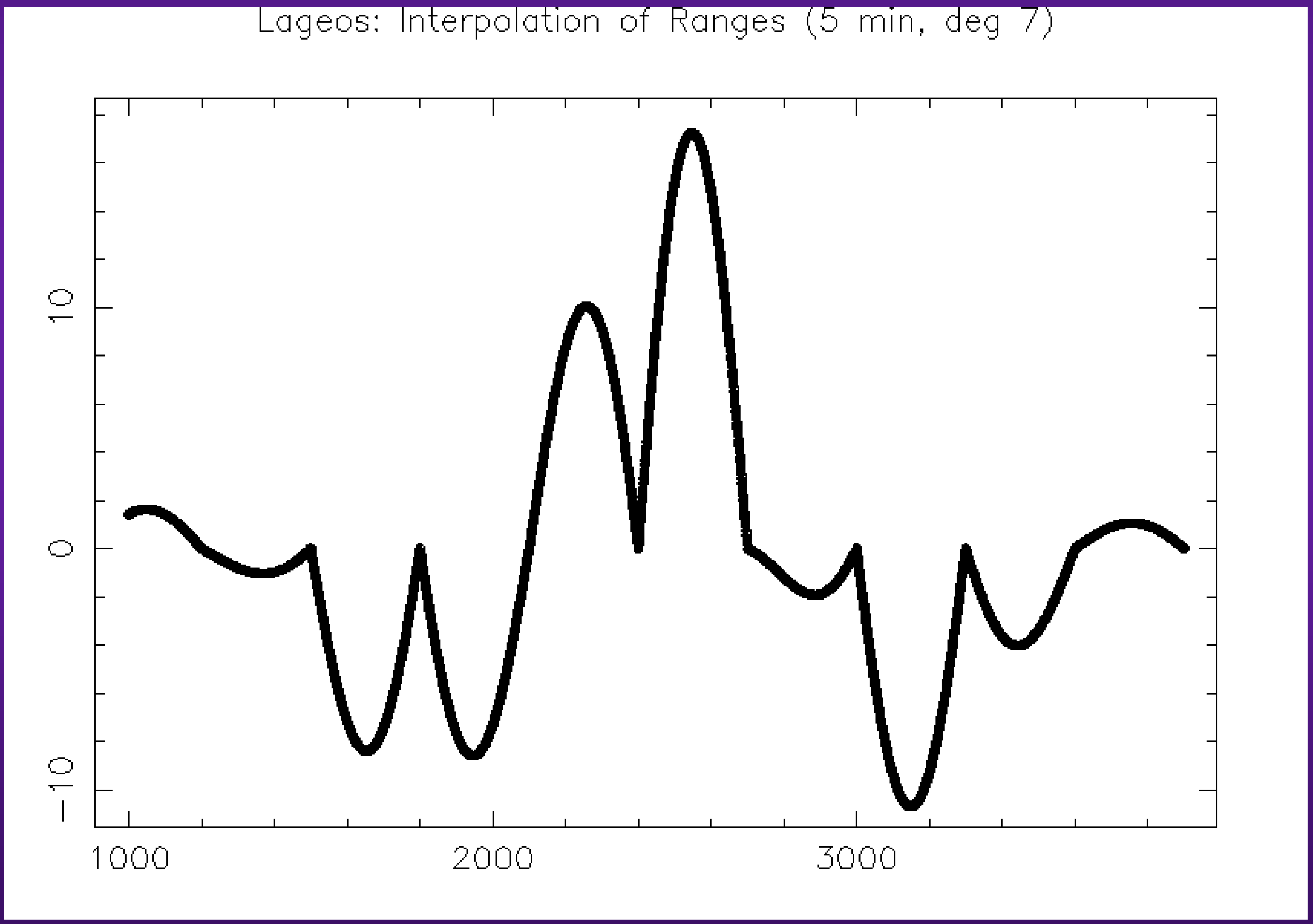
Effects on Normalpoint Computation

- Test if interpolation errors propagate into normalpoints
- Generate reference orbits (1 second sampling) for Lageos-2 and Starlette using IRV integrator
- Interpolate all positions with polynomial deg 7 and spacing according to previous interpolation tests (Lageos: 5 min, Stella: 3 min)
- Compute distances and their interpolation errors
- Average errors within normalpoint intervals
- Difference between average error and error in the bin centers is critical quantity.

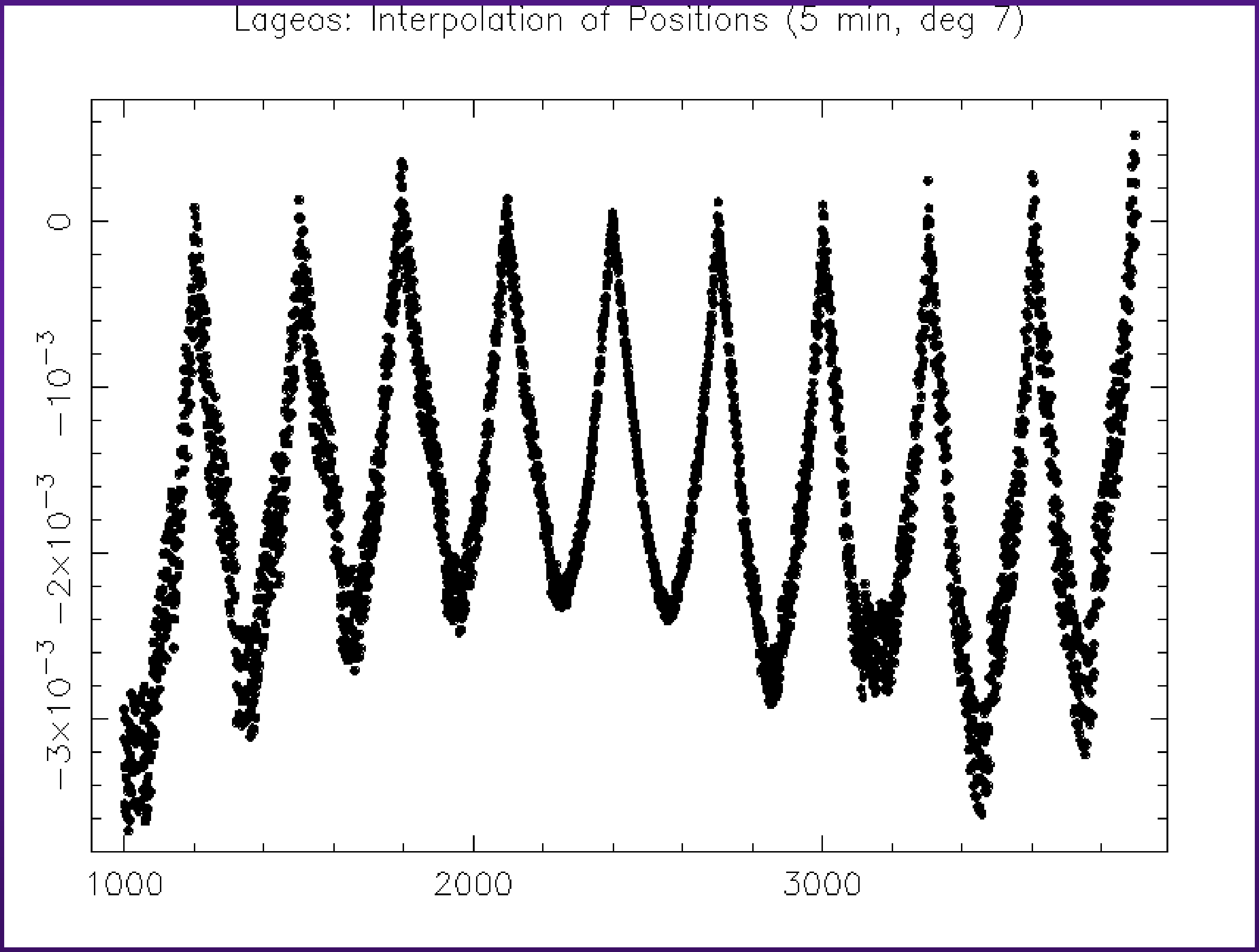
Interpolation of Distances

- A)
  - Polynomial interpolation of given positions
  - Computation of topocentric distances using interpolated positions
- B)
  - Computation of topocentric distances at given positions
  - Polynomial interpolation of these distances
  - Worse by orders of magnitude!

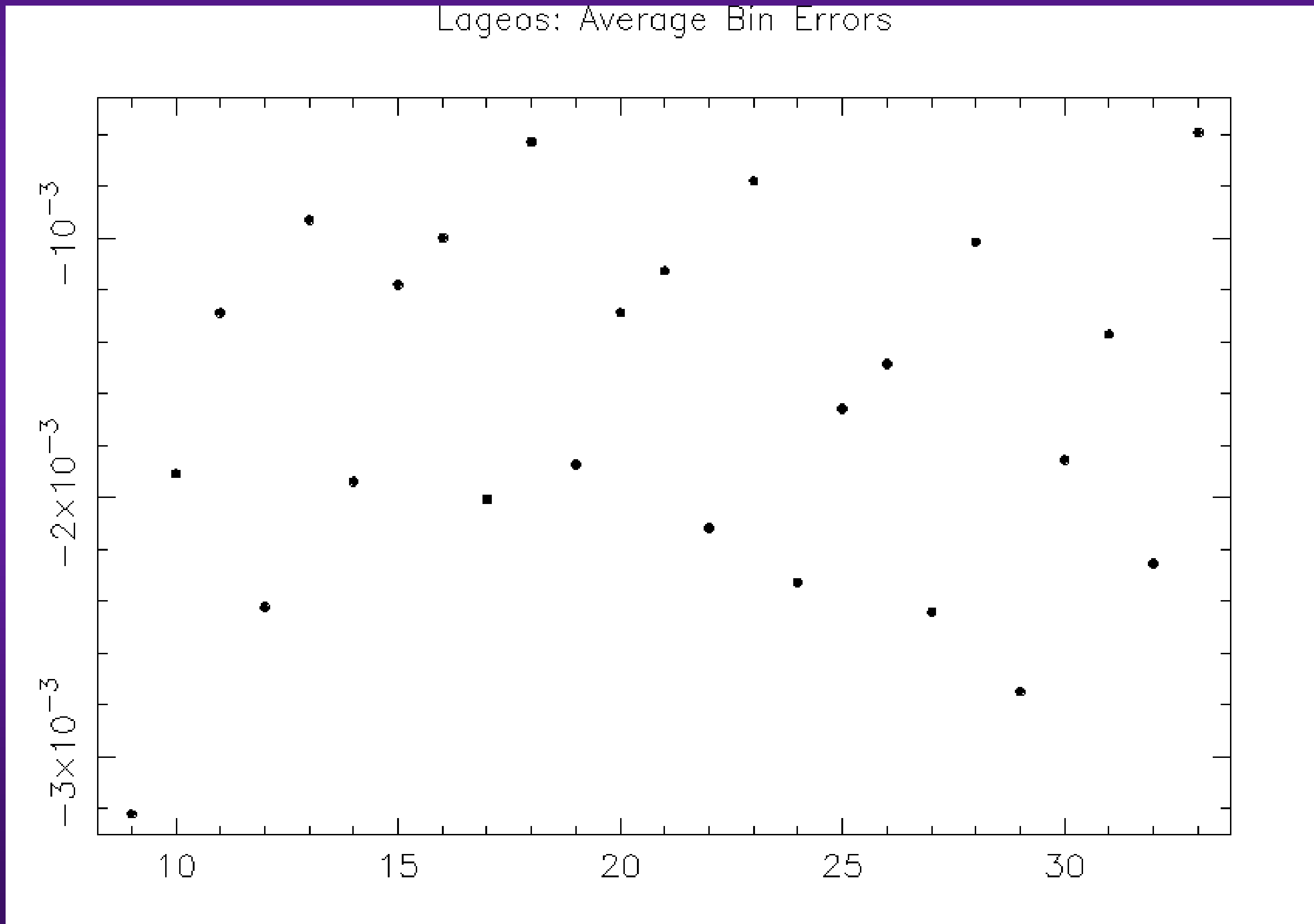
Range Interpolation Error via Interpolated Ranges (m)



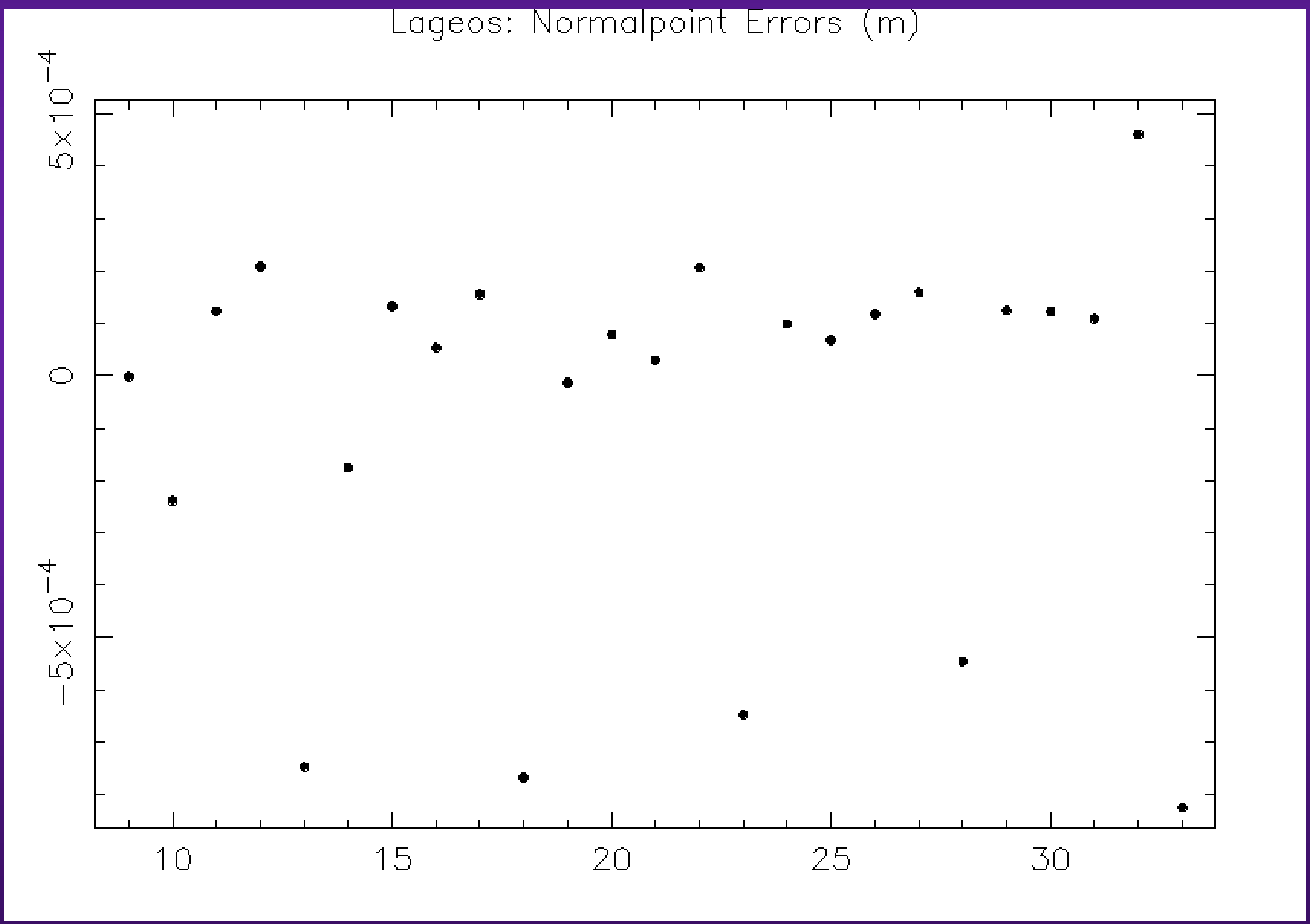
Range Interpolation Error via Interpolated Positions (m)



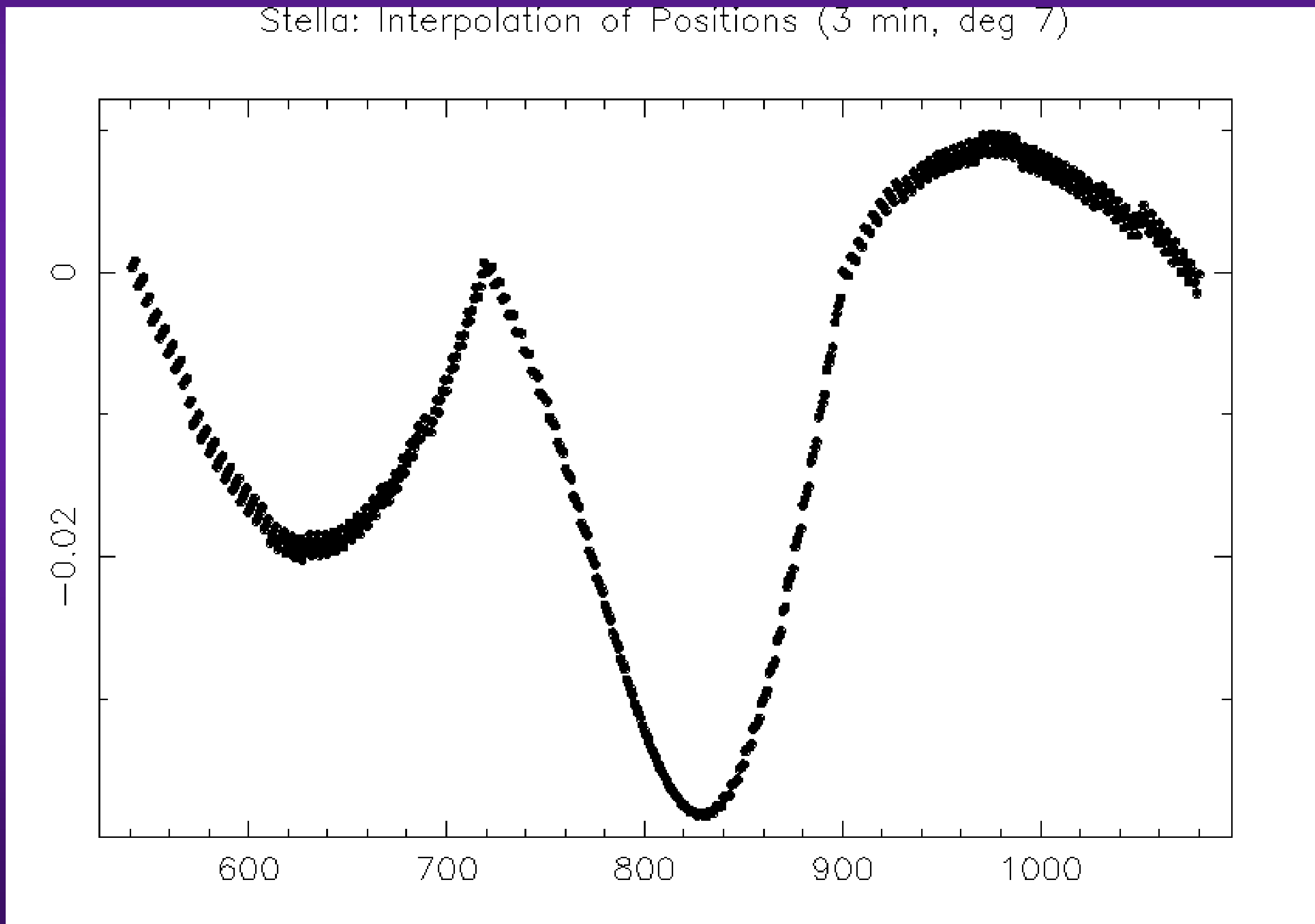
Bin Average of Range Errors (m) (2 min bin width)



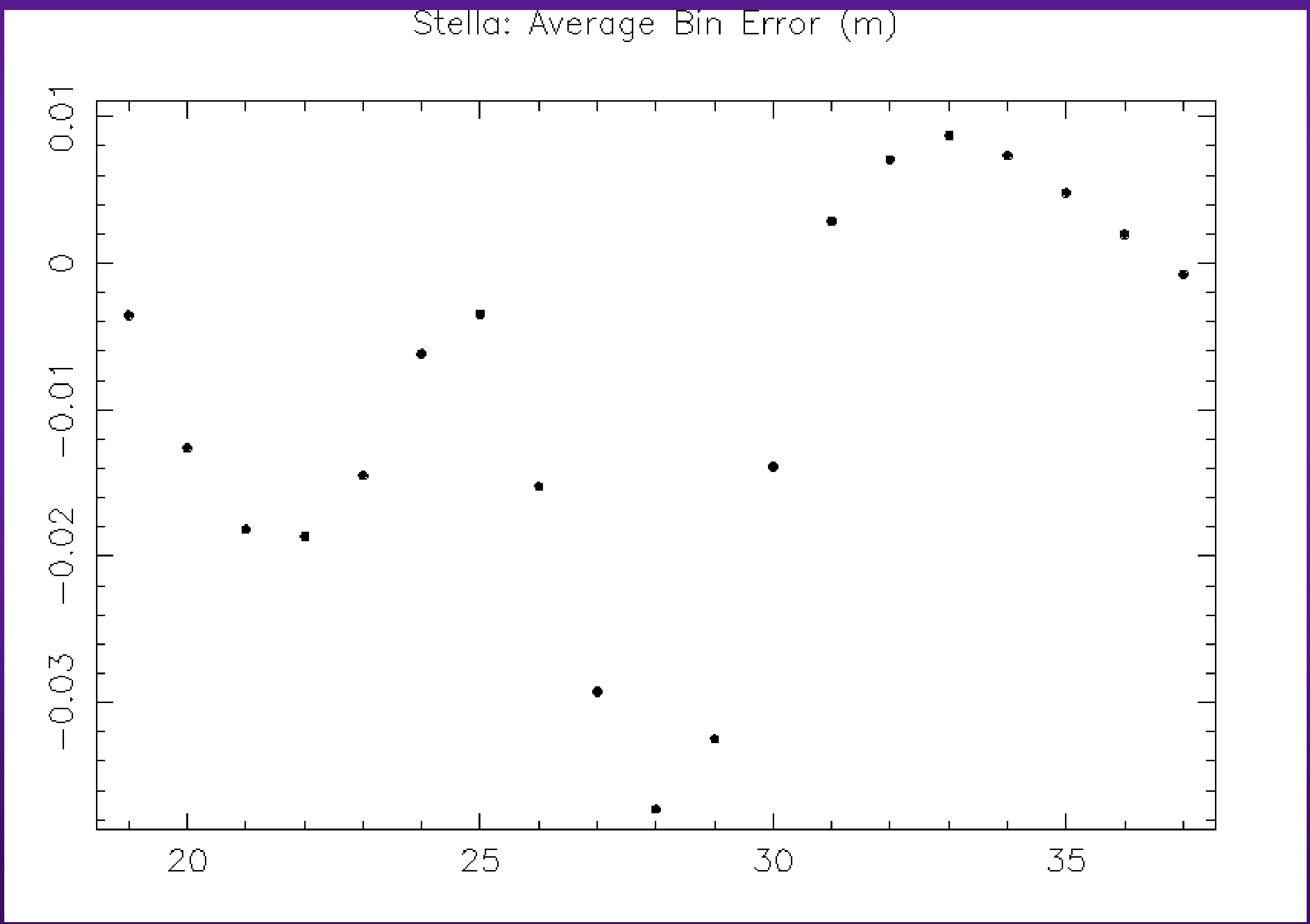
Normalpoint Error
(Difference between bin average and range error in bin center)



Range Interpolation Error via Interpolated Positions (m)



Bin Average of Range Errors (30 sec bin width)



Normalpoint Error

(Difference between bin average and range error in bin center)