

XLII

IV

1.

2, 4, 5 8, ?
2, 4, 5 8,

2458.
8425.

2.

$$a - b - c = 1576.$$

$$a, b, c = 576?$$

$$a - b - c = 1576.$$

$$a = 576$$

$$576$$

$$576, \quad b = 576$$

$$576, \quad c = 576$$

$$576.$$

$$(a - 576) - (b - 576) - (c - 576) = 1576 - 576 + 576 + 576 = 2152,$$

$$1 = 2152.$$

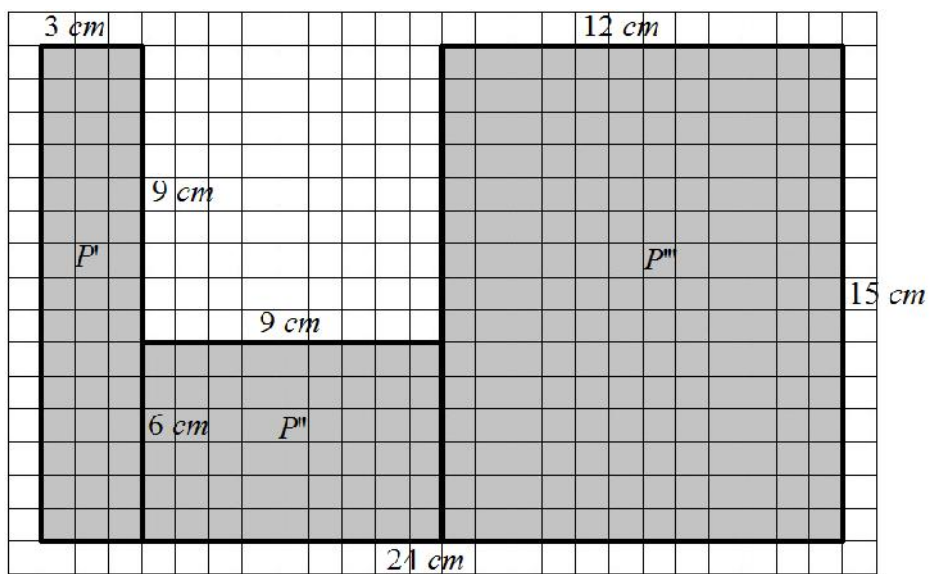
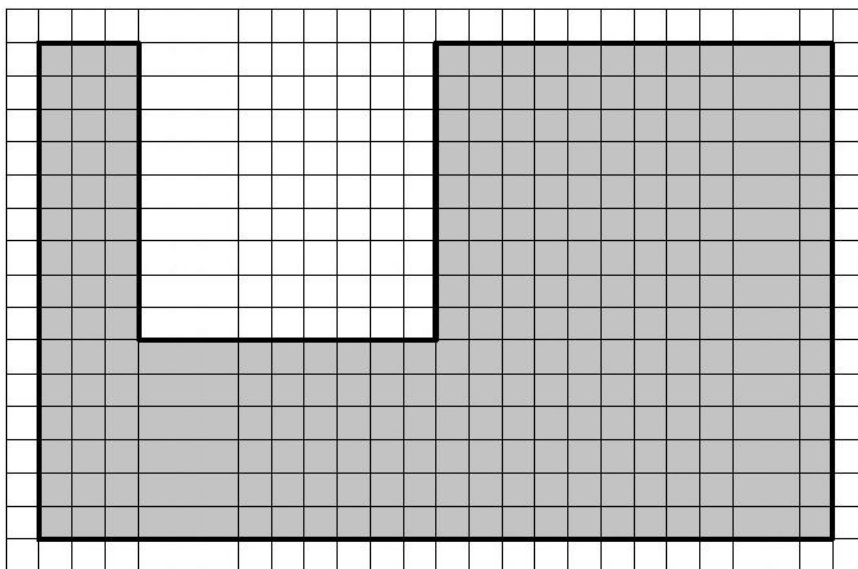
3.

26 n 2.
26.
1 2
3, 4, 6, 8, 12 24,

n
26 n
 $26 - 2 = 24,$
24 : 1, 2, 3, 4, 6, 8, 12 24,
26.
: 3 + 4 + 6 + 8 + 12 + 24 = 57.

4.

1 cm



$$L = 3 + 9 + 9 + 9 + 12 + 15 + 24 + 15 = 96 \text{ cm} .$$

$$P = P' + P'' + P''' = 3 \cdot 15 + 9 \cdot 6 + 12 \cdot 15 = 279 \text{ cm}^2.$$

$$15 \text{ cm} \quad 24 \text{ cm} \quad 9 \text{ cm}.$$

$$15 \text{ cm} \quad 24 \text{ cm}, \quad 9 \text{ cm}.$$

$$L = 2(15 + 24) + 2 \cdot 9 = 78 + 18 = 96 \text{ cm}.$$

$$15 \text{ cm} \quad 24 \text{ cm} \quad 9 \text{ cm}.$$

$$P = 15 \cdot 24 - 9 \cdot 9 = 360 - 81 = 279 \text{ cm}^2.$$

V

1. \overline{yxyxyx} 5, \overline{xyxyxy} 2,

\overline{xyxyxy} 2, $y \neq 0$,

$y \in \{2, 4, 6, 8\}$, \overline{yxyxyx} 5 $x \neq 0$

$x = 5$, : 525252, 545454,

565656 585858.

2. $AB = 2 \text{ cm}$ CD .

$AB = 10 \text{ cm}$,

CD

$AB = CD$.

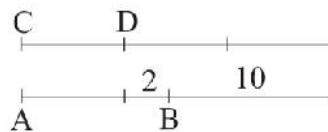
$\overline{AB} = \overline{CD} + 2$ $\overline{AB} + 10 = 3\overline{CD}$ (

).

$2\overline{CD} = 12$,

$\overline{CD} = 6 \text{ cm}$.

$\overline{AB} = 6 \text{ cm}$.



3. 12, -
 5. , -
 5. , :
 05, 23 42. :
 5. 12, -
 5. , 12 - 5 = 7 -
 : 70, 61, 52, 43, 34, 35 16.
 , 3 · 7 = 21
 .
 4. , -
 16 m²
 , 20%,
 20%.
 ?
 . 16 = 4 · 4, 16 m²
 a = 4 m = 400 cm . 20%
 a + 0,2a = 400 + 80 = 480 cm .
 20% a - 0,2a = 400 - 80 = 320 cm . -
 P = 480 · 320 = 153600 cm² .
 16 m² = 160000 cm² > 153600 cm²

VI

1. a b $\overline{78a9b}$
 18. 2 18 2 9.
 2 b ∈ {0, 2, 4, 6, 8}, 9

$$7 + 8 + 9 + a + b \quad 9.$$

$b = 0,$	$a = 3,$	78390.
$b = 2,$	$a = 1,$	78192.
$b = 4,$	$a = 8,$	78498.
$b = 6,$	$a = 6,$	78696.
$b = 8,$	$a = 4,$	78498.

2.

$$12 \cdot \left(\frac{1}{10} - \frac{1}{20} \right) = \frac{12}{20} = \frac{3}{5}$$

3. $\triangle ABC$

$\angle BAC \quad \angle ACB$

S.

$\triangle ABC \quad \angle ASC = 110^\circ \quad \angle BAC$

$\angle ABC$.

$$x = \angle SAC$$

$$y = \angle SCA, \quad \angle ASC = 110^\circ \quad \triangle ASC$$

$$x + y + 110^\circ = 180^\circ,$$

$$x + y = 70^\circ, \quad \angle BAC = 3\angle ABC,$$

$$\angle BAC = 2x$$

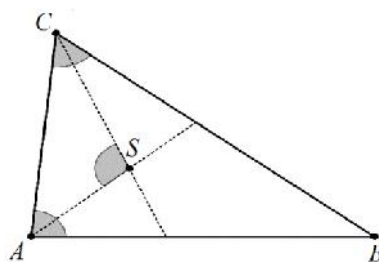
$$\angle ABC = \frac{2x}{3}, \quad \angle ACB = 2y,$$

$$2x + 2y + \frac{2x}{3} = 180^\circ, \quad x + y = 70^\circ, \quad 140^\circ + \frac{2x}{3} = 180^\circ,$$

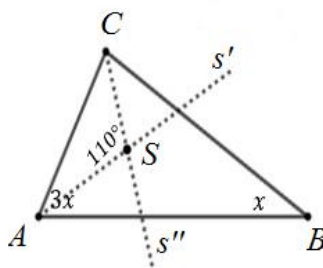
$$\frac{2x}{3} = 40^\circ, \quad \dots \quad x = 60^\circ, \quad y = 70^\circ - x = 10^\circ,$$

$$\angle BAC = 120^\circ, \quad \angle ACB = 20^\circ \quad \angle ABC = 40^\circ.$$

$$r = 3x, \quad S = x \quad x = 180^\circ - 4x.$$



$$\begin{aligned} \frac{r}{2} + \frac{x}{2} + 110^\circ &= 180^\circ, \\ r + x &= 140^\circ, \\ 3x + 180^\circ - 4x &= 140^\circ, \\ x &= 40^\circ, \\ r &= 120^\circ, s = 40^\circ, x = 20^\circ. \end{aligned}$$



4.

$$\begin{aligned} 3 \cdot (90^\circ - r) &= 180^\circ - r, \\ 270^\circ - 3r &= 180^\circ - r, \\ 2r &= 90^\circ, \\ r &= 45^\circ. \end{aligned}$$

VII

1.

	30%	280	
2 kg			240
			?
		1 kg	x
240		$\frac{240}{x}$ kg	
1 kg	70%		0,7x
		280	$\frac{280}{0,7x} = \frac{400}{x}$ kg
			x = 80
		1 kg	80
			0,7 · 80 = 56

2.

	a, b, c	$ab = -12,$
$ac = -20$	$bc = 60.$	$abc.$

$$(abc)^2 = 120^2.$$

1) $abc = 120,$

$$c = \frac{abc}{ab} = \frac{120}{-12} = -10.$$

2) $abc = -120,$

$$c = \frac{abc}{ab} = \frac{-120}{-12} = 10.$$

$$ab = -12, \quad ac = -20 \quad bc = 60,$$

:

$$a = \frac{abc}{bc} = \frac{120}{60} = 2, \quad b = \frac{abc}{ca} = \frac{120}{-20} = -6$$

$$a = \frac{abc}{bc} = \frac{-120}{60} = -2, \quad b = \frac{abc}{ca} = \frac{-120}{-20} = 6$$

3.

12

$\frac{3}{8}$

3

$\frac{1}{4}$

$\frac{1}{3}$

$\frac{1}{24}$

?

x

$$\frac{3}{8}x + 12$$

$$\frac{1}{4}x + 3$$

$$\frac{1}{3}(\frac{3}{8}x + 12 + \frac{1}{4}x + 3) = \frac{5}{24}x + 5$$

$$\frac{1}{24}x$$

$$\frac{3}{8}x + 12 + \frac{1}{4}x + 3 + \frac{5}{24}x + 5 + \frac{1}{24}x = x,$$

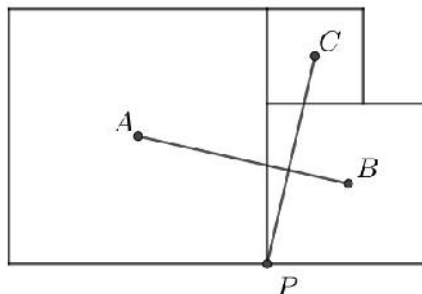
$$x = 160 \text{ m.}$$

4.

A, B C .

P

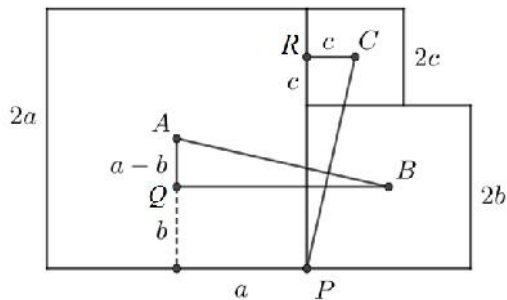
$$\frac{AB}{PC} = \frac{AB}{PC} .$$



).

$AQB \quad CRP$ (

$A, B \quad C \quad 2a, 2b$
 $2c,$
 $2a = 2b + 2c, \dots a = b + c,$



$$\overline{AQ} = a - b = c = \overline{CR}$$

$$\overline{RP} = c + 2b = a - b + 2b = a + b = \overline{QB}.$$

, $AQB \quad CRP$
 , $\overline{AB} = \overline{PC}.$, $\overline{AB} : \overline{PC} = 1.$

VIII

1. $n \quad \frac{5n+23}{n+3}$

$$\frac{5n+23}{n+3} = \frac{5n+15+8}{n+3} = \frac{5(n+3)+8}{n+3} = \frac{5(n+3)}{n+3} + \frac{8}{n+3} = 5 + \frac{8}{n+3}.$$

, $\frac{8}{n+3} \quad , \quad \frac{8}{n+3} \quad n+3 \quad 8.$
 , $n+3 \in \{-8, -4, -2, -1, 1, 2, 4, 8\}, \quad n \in \{-11, -7, -5, -4, -2, -1, 1, 5\}.$
 , $n \quad , \quad n=1 \quad n=5.$

2. $26 \text{ cm},$

$$97 \text{ cm}^2.$$

$a \quad b.$

$$a + b = 13 \quad a^2 + b^2 = 97.$$

$$b = 13 - a,$$

$$a^2 + (13 - a)^2 = 97,$$

$$a^2 + 13^2 - 2 \cdot 13 \cdot a + a^2 = 97,$$

$$2a^2 - 26a + 72 = 0,$$

$$a^2 - 13a + 36 = 0,$$

$$a^2 - 4a - 9a + 36 = 0,$$

$$a(a - 4) - 9(a - 4) = 0,$$

$$(a - 4)(a - 9) = 0.$$

$$0,$$

$$0,$$

$$a = 4$$

$$a = 9.$$

$$a = 4$$

$$a - 4 = 0$$

$$a - 9 = 0,$$

$$b = 13 - a = 9,$$

$$a = 9$$

$$b = 13 - 9 = 4.$$

,

$$4 \text{ cm}$$

$$9 \text{ cm}.$$

3.

$$5800$$

$$50$$

$$40$$

$$14\%$$

$$60$$

$$15\%$$

$$80$$

$$1934$$

?

$$x$$

$$y$$

$$40x + 50y = 5800, \quad 4x + 5y = 580.$$

$$0,86x$$

$$0,85y$$

$$60 \cdot 0,86x + 80 \cdot 0,85y = 5800 + 1934,$$

$$51,4x + 68y = 7734.$$

$$4x + 5y = 580$$

$$x = \frac{580 - 5y}{4},$$

$$51,4 \cdot \frac{580 - 5y}{4} + 68y = 7734,$$

$$y = 72.$$

,

$$x = \frac{580 - 5 \cdot 72}{4}$$

$$x = \frac{580 - 5 \cdot 72}{4} = 55.$$

$$55$$

$$72$$

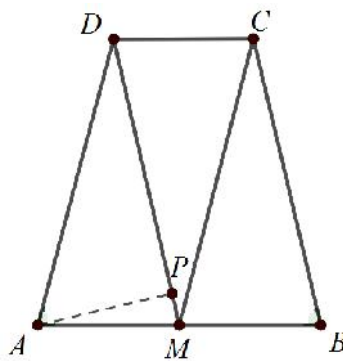
4.

$$75^\circ,$$

2:1,

30 cm.

$\overline{AB} : \overline{CD} = 2:1$
 $\overline{AB} = 2\overline{CD}$. M \overline{AB} .
 $\overline{CD} = b$, $\overline{AB} = 2b$
 $\overline{AM} = \overline{MB} = b$. $AB \parallel CD$ $\overline{CD} = \overline{MB}$
 $MBCD$
 $AMCD$
 $\overline{AD} = \overline{MC}$ $\overline{BC} = \overline{MD}$,
 AMD, MBC CDM
 30 cm ,



b , 75°
 $180^\circ - 2 \cdot 75^\circ = 30^\circ$.

AP AMD .
 APD , $\overline{AP} = \frac{1}{2}\overline{AD} = 15\text{ cm}$.
 $P = 3P_{AMD} = 3 \cdot \frac{30 \cdot 15}{2} = 675\text{ cm}^2$.

IX

1. n $a = \frac{10^{2n}-1}{3(10^n+1)}$.
 $a = 567$, n .

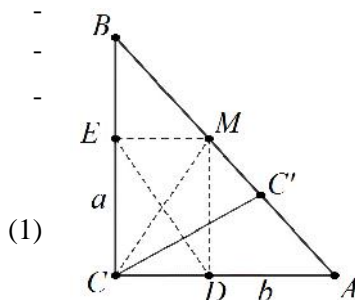
$10^n + 1 \neq 0$
 $a = \frac{10^{2n}-1}{3(10^n+1)} = \frac{(10^n+1)(10^n-1)}{3(10^n+1)} = \frac{10^n-1}{3}$.

$10^n - 1$ n 9 ,
 $a = \frac{10^n-1}{3}$ n 3 ,
 $a = \frac{10^n-1}{3} = 567$, $3n$, $n = 567 : 3 = 189$.

2. $\triangle ABC$

C M AB E
 D a, b M BC AC ,
 $\triangle ABC$
 $\overline{DE} \geq \frac{ab}{\sqrt{a^2+b^2}}$. (*)

?
 $CDME$
 $\frac{CM}{DE}$
 $\triangle ABC$
 C ().



$\overline{DE} = \overline{CM} \geq \overline{CC'}$. (1)
 $\triangle ACB \quad \triangle AC'C$
 $\overline{AB} : \overline{AC} = \overline{BC} : \overline{CC'}$, $\dots \overline{AB} \cdot \overline{CC'} = \overline{AC} \cdot \overline{BC}$.

$\overline{AB} = \sqrt{a^2 + b^2}$,
 $\sqrt{a^2 + b^2} \cdot \overline{CC'} = ab$,
 $\overline{CC'} = \frac{ab}{\sqrt{a^2 + b^2}}$. (2)

(1) (2) (*)
 $\overline{CM} = \overline{CC'}$, \dots

3. $x + y = 0$ $x^2 + y^2 = \frac{1}{2}$, $x^8 + y^8$.
 $x + y = 0$ $x = -y$, $x^2 = y^2$.
 $x^2 + y^2 = \frac{1}{2}$ $2y^2 = \frac{1}{2}$, $y^2 = \frac{1}{4}$.

$x^2 = y^2 = \frac{1}{4}$,
 $x^8 + y^8 = (x^2)^4 + (y^2)^4 = (\frac{1}{4})^4 + (\frac{1}{4})^4 = \frac{1}{256} + \frac{1}{256} = \frac{2}{256} = \frac{1}{128}$.
 $x + y = 0$ $(x + y)^2 = 0$, $\dots x^2 + 2xy + y^2 = 0$,
 $xy = -\frac{x^2 + y^2}{2}$, $x^2 + y^2 = \frac{1}{2}$, $xy = -\frac{1}{4}$.

$$\begin{aligned}
 x^8 + y^8 &= (x^4 + y^4)^2 - 2x^4y^4 \\
 &= ((x^2 + y^2)^2 - 2x^2y^2)^2 - 2(xy)^4 \\
 &= ((x^2 + y^2)^2 - 2(xy)^2)^2 - 2(xy)^4 \\
 &= \left(\left(\frac{1}{2}\right)^2 - 2\left(-\frac{1}{4}\right)^2\right)^2 - 2\left(-\frac{1}{4}\right)^4 \\
 &= \left(\frac{1}{4} - \frac{1}{8}\right)^2 - \frac{2}{256} \\
 &= \left(\frac{1}{8}\right)^2 - \frac{1}{128} \\
 &= \frac{1}{64} - \frac{1}{128} = \frac{1}{128}.
 \end{aligned}$$

4.

AB

C D (C A D). $\overline{AC} = m,$

$m.$

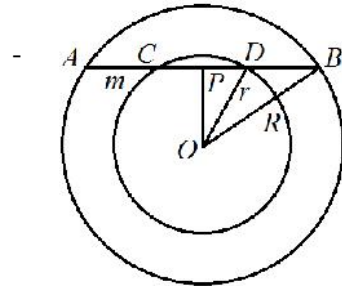
P

O

AB ($CD = m$) $\overline{CD} = m$

$$\overline{PD} = \frac{m}{2} \quad \overline{PB} = \frac{m}{2} + m = \frac{3m}{2}.$$

r R



$$d^2 + \overline{PB}^2 = R^2, \dots d^2 + \frac{9m^2}{4} = R^2,$$

$$d^2 + \overline{PD}^2 = r^2, \dots d^2 + \frac{m^2}{4} = r^2.$$

$$, d^2 + \frac{9m^2}{4} - d^2 - \frac{m^2}{4} = R^2 - r^2,$$

$$2m^2 = R^2 - r^2.$$

$$P = f(R^2 - r^2) = 2m^2f.$$