



# Solstice and Equinox (“Suntrack”) Season Model

*Philip Scherrer & Deborah Scherrer, Stanford Solar Center*

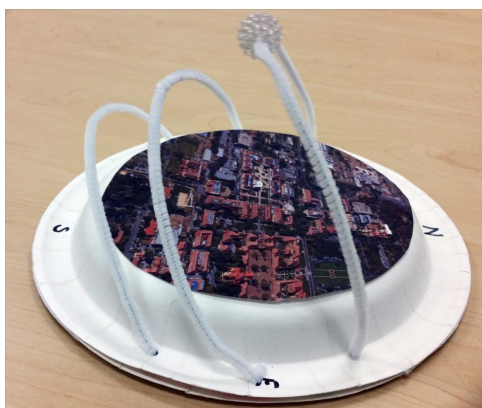
## Introduction

This physical model simulates the Sun’s tracks across the sky at summer solstice (longest track), winter solstice (shortest track), and the spring and fall equinoxes (medium track). A bead simulates the Sun moving from rising along the eastern horizon to setting on the western. The bead can be moved from track to track to demonstrate the solstices and equinoxes. The model is made to accurately represent the user’s latitude.

Many people are confused about the meaning of solstices and equinoxes. Verbal descriptions are often inadequate because they are difficult to envision or overly technical. Imagery is helpful, but some have trouble understanding the perspective of the imagery. This physical model was developed (over years) to enhance the teaching of the solstice and equinox concepts, as well as being helpful in understanding solar alignments of cultural and archaeoastronomy sites.

*A quote from Cidney Webster, National Park Service:*

My program was a guided walk through the park and the topic was the winter solstice. ... I copied a diorama used by Dr. and Mrs. Scherrer during the training that demonstrated the Sun's position in the sky at different times of the year. This became my most effective teaching tool. Everyone loved the diorama!"



Includes instructions for a simple, low-cost version made from paper plates and pipe cleaners.



<p><b>Target Audiences:</b></p> <ul style="list-style-type: none"> <li>• Youth and after school groups</li> <li>• National and state park leaders</li> <li>• Middle school and upper elementary science teachers</li> <li>• For display at public science events</li> <li>• Science museums, planetariums</li> <li>• Astronomy clubs</li> <li>• Community events</li> </ul> <p><b>Construction Time:</b> ~2+ hours (not including decoration)</p> <p><b>Age Group:</b> Can be used by 8 – adult For construction, an adult is needed</p>	<p><b>Materials Needed for complex model:</b></p> <ul style="list-style-type: none"> <li>• 30 cm (12 inch) diameter particleboard disk, 2 cm (¾ inch) thick -- purchased (often available precut at places like Home Depot™)</li> <li>• Plastic cord such as used in Weed Whacker™ machines, to be used as Sun tracks</li> <li>• Large bead to use as Sun; needs to fit on the plastic cord; white is preferred since the Sun is white. <a href="http://solar-center.stanford.edu/SID/activities/GreenSun.html">http://solar-center.stanford.edu/SID/activities/GreenSun.html</a></li> <li>• Electric drill and bit for drilling holes to insert Sun tracks</li> <li>• Materials for decoration (optional)</li> </ul> <p><b>Materials for the simple model are given below</b></p>
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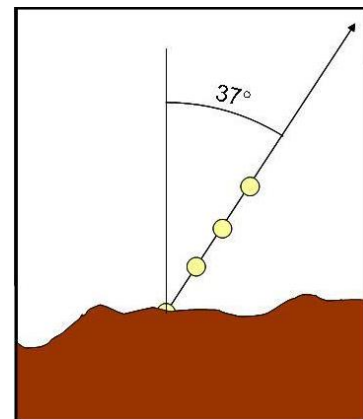
**Learner Objectives:**

- That the Sun in Earth’s sky has regular and predictable motions. These motions explain such phenomena as the seasons, equinoxes, and solstices.
- Because of the Earth’s tilt and its orbit around the Sun, the Sun appears to rise and set in a slightly different direction each day.
- A solstice is an astronomical event that occurs twice each year as the Sun reaches its farthest north or south rising and setting points on the horizon.
- The official astronomical equinox occurs twice a year, when the plane of the Earth's equator passes the center of the Sun. However, pre-technological cultures have defined the equinox as the date the Sun rises due east and sets due west of their location. This difference can cause problems for understanding astronomically aligned structures.

**Understanding Movements of the Sun in the Sky**

Most people are aware that the Sun rises in the east and sets in the west. Although ancient cultures were acutely aware of the movements of the Sun, Moon, planets, and stars, few modern people understand that the sunrise and sunset locations on the horizon change slightly each day. Nor are many people aware that, except near the equator, the Sun rises on a diagonal (as in the image at right).

On summer solstice, the Sun rises as far northeast as it ever gets, and sets as far northwest as it gets. This results in many hours of daylight, and summer. On the model, summer solstice is represented by the longest cord.



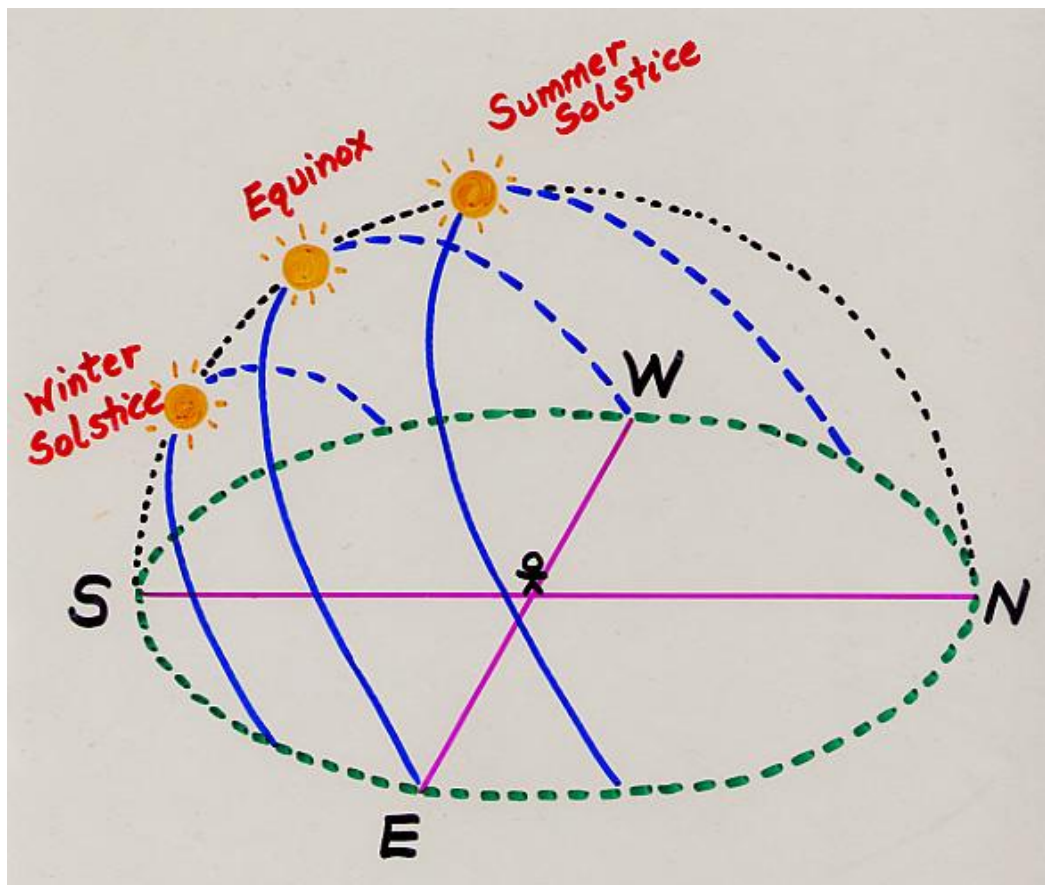
*Sunrise at equinox at latitude 37 degrees facing due east*



Every day from summer solstice the Sun rises and sets a bit further south. Three months later it rises due east and sets due west, which we call the equinox. Now, we have roughly equal hours of daylight and nighttime, and it is the start of fall. Your model shows this as the medium-length cord that runs from due east to due west.

The Sun continues its southward march until, on the winter solstice, it rises as far southeast as it gets and sets as far southwest. The Sun's track is low in the sky and we have fewer hours of daylight than nighttime. It is winter, dark and cold. Your model shows this as the shortest cord.

After the winter solstice, the Sun starts working its way back up to the northeast and the pattern repeats. Note that, because of the tilt of the Earth, the Sun is never directly overhead, except if one is at a latitude of less than 23.5 degrees.



### Solstices:

Summer solstice occurs on the day the Sun has reached its highest/northernmost point in Earth's sky, marking the beginning of summer (in the northern hemisphere). This is around 21 June. Winter solstice occurs on the day the Sun reaches its lowest/southernmost point in the sky, marking the beginning of winter (in the northern hemisphere). Winter solstice is around 22 December. For more details, check out <http://scijinks.jpl.nasa.gov/solstice/>.



Winter solstice was of particular interest to many pre-technological cultures. Because the Sun rose and set at a very southern/low point, people were concerned that it might continue its southward journey, hence leaving the Earth in darkness. Ceremonies were held to encourage the Sun to resume its journey northward, or to celebrate the change to longer days. Many solstice and winter celebrations have descended from these events.

### Equinoxes:

The equinoxes are astronomically defined as either of the two moments in the year when the Sun is exactly above the equator (i.e. when the plane of the Earth's equator passes the center of the Sun). The Vernal Equinox, marking the beginning of spring in the Northern Hemisphere, occurs about March 21, when the Sun moves north across the celestial equator. The Autumnal Equinox falls about September 23, as the Sun crosses the celestial equator going south.

The above definitions are modern, and the word “equinox” derives from the concept of equal day/night. However, pre-technological cultures had no way of timing the exact length of the day and night, nor knowing the exact moment when the ecliptic and celestial equator intersected, nor did they know about refraction from the Earth’s atmosphere. But they were quite capable of knowing on which day of the year the Sun rose exactly east and set exactly west. Times of sunset and sunrise vary with an observer's location (longitude and latitude), so the dates when the Sun rises due east and sets due west likewise depend on location. Depending on location, the dates of Sun rising and setting due east and west can be as much as 2-3 days from the officially designated equinox. This is important to keep in mind when studying solar aligned archaeoastronomy sites such as Stonehenge, Chaco Canyon, Chichen Itza, and Majorville Medicine Wheel (aka Canada’s Stonehenge). For more information on this, see the book *Canada’s Stonehenge* by Gordon R. Freeman (<http://canadastonehenge.com/>).

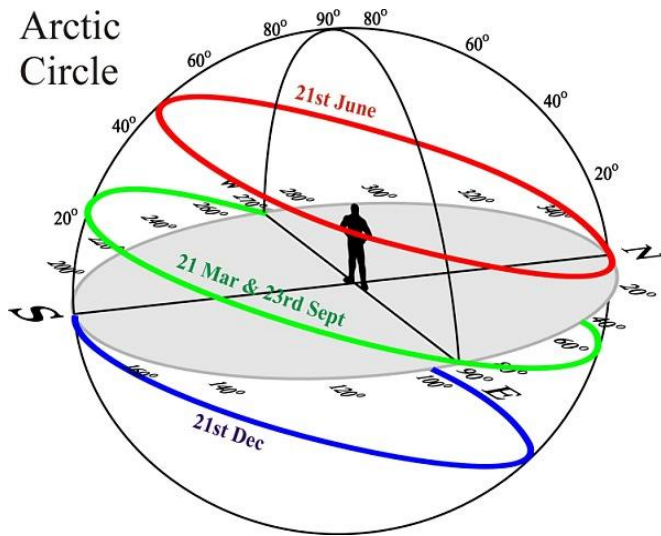
Good, and more complete, explanations of the apparent movements of the Sun are available at: [http://facstaff.gpc.edu/~pgore/Earth&Space/GPS/motion\\_of\\_objects\\_in\\_sky.html](http://facstaff.gpc.edu/~pgore/Earth&Space/GPS/motion_of_objects_in_sky.html), <http://physics.weber.edu/schroeder/ua/SunAndSeasons.html>, and <http://www.umass.edu/sunwheel/pages/moonteaching.html>.



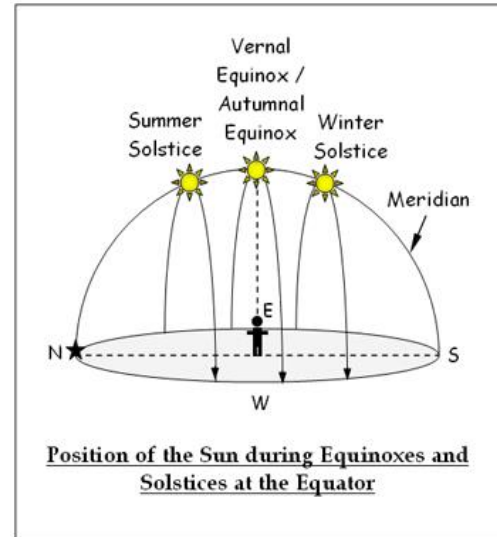
Equinox at the Sun Dagger site in Chaco Canyon



Equinox at Chichen Itza



Sun's track at the Arctic Circle



Position of the Sun during Equinoxes and Solstices at the Equator

mydarksky.org

Sun's track at the equator

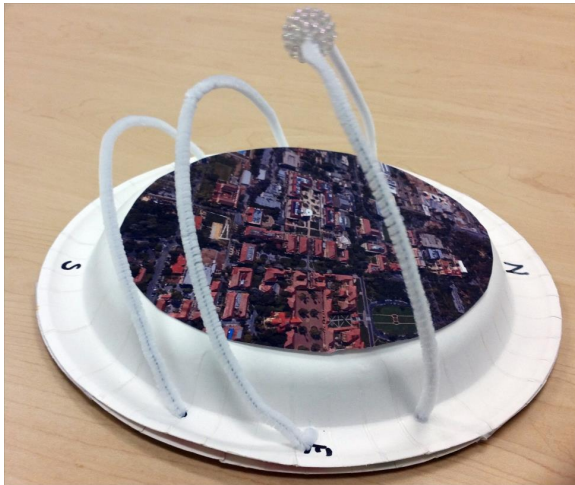


Sun's track at 37 degrees latitude



## Assembly Instructions

### Simple Model



#### Materials Needed:

- 2 sturdy 10" paper plates
- 5 long (12") pipe cleaners
- White bead large enough to be threaded onto the pipe cleaner
- Adhesive tape, maybe glue
- Sharp scissors
- Protractor
- Satellite image (e.g. from Google Earth™) or crayons for decoration (*optional*)

#### Construction Time:

30-45 minutes

1. Take one sturdy paper plate, turn it over, mark **N**, **S**, **E**, **W** directions on the edges.
2. With your protractor, measure 30 degrees above and below **E** and **W**, & mark these with a pen.



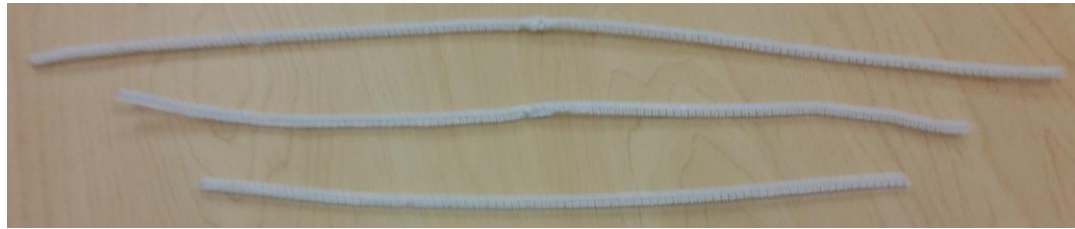
3. Punch holes large enough for the pipe cleaners to fit through at **E** and **W** plus the marks you made 30 degrees above and below (6 holes).
4. Prepare your pipe cleaners:
  - Take two 12" pipe cleaners and cut 2½" off each of them. Tightly twist the top ½" of each together to form one long 18" pipe cleaner. You might use a little drop of glue to hold them together.
  - Take two 12" pipe cleaners and cut 4½" off each one. Tightly twist the top ½" of each together to form one long 15" pipe cleaner.
  - Leave the last pipe cleaner at 12".



18"

15"

12"



5. Take the short, 12" pipe cleaner and bend  $\frac{1}{2}$ " of each end at a right angle. Thread each end through the lower holes on both the **E**ast and **W**est sides of your plate. On the opposite side of the paper plate (side without your markings), tape down **ONLY** the pipe cleaner end on the **W**est side. Leave the end on the **E**ast side loose. On the side with your markings, bend the pipe cleaner slightly down towards the South end of the plate.

Tape down bent end of pipe cleaner on East side only

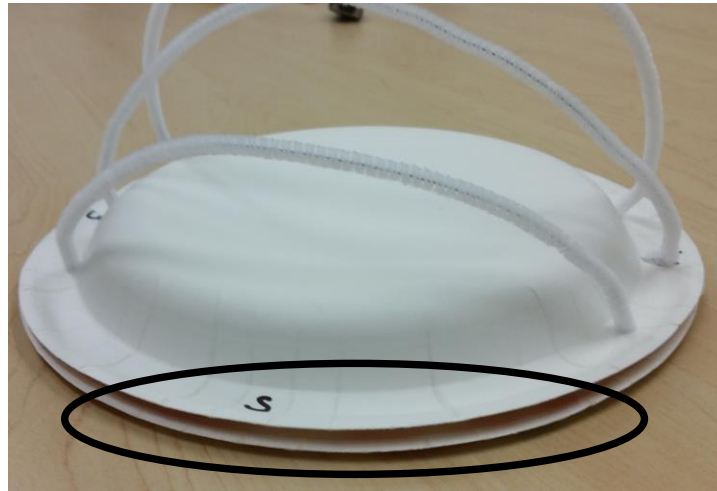


6. Take your 15" pipe cleaner, bend down its ends, and thread it through the holes at the **E**ast and **W**est markings of your plate. On the opposite side of the plate, tape **ONLY** the pipe cleaner end on the **W**est side. Bend the pipe cleaner towards the **S**outh.
7. With your 18" pipe cleaner, do the same in the holes 30 degrees north of your **E**ast and **W**est holes. You should now have 3 Sun tracks, all bent towards the **S**outh.



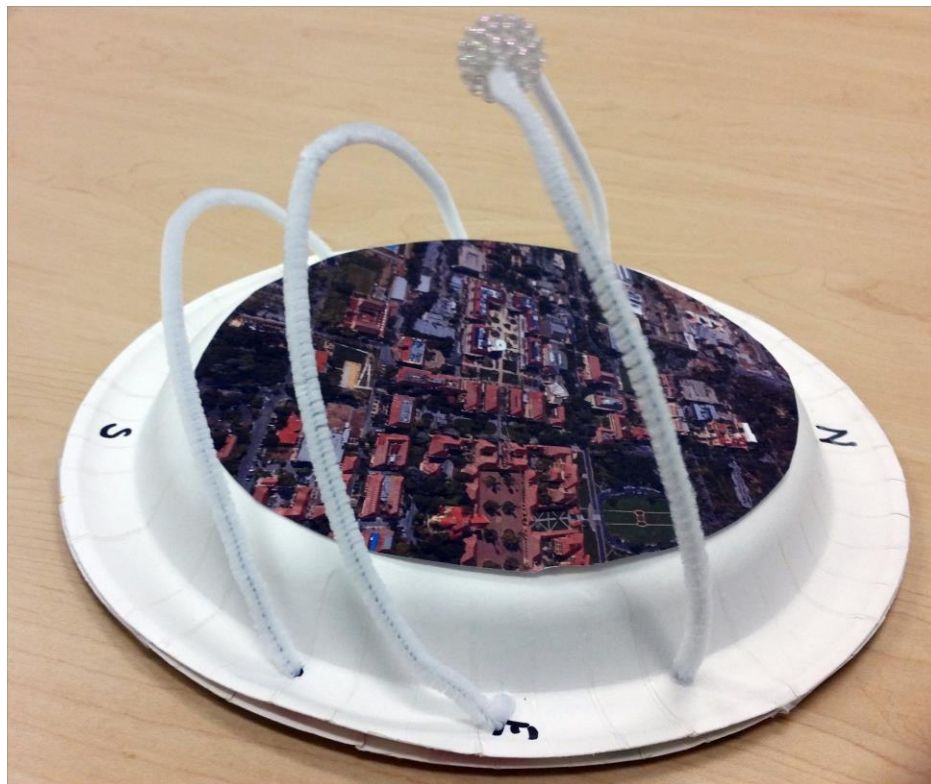


8. Insert your second plate into the first, for stability, and tape the 2 plates together around the edges, taking care that you can still pull out the pipe cleaner ends from the **West** side.



Stack and tape plates together for stability

9. Decorate the surface of your plate, if you wish. We used an satellite image from Google Earth™ showing the Stanford campus.
10. Thread your bead onto one of the pipe cleaners to show the Sun rising and setting at either the equinox (East-West) or the summer or winter solstice.



Modeling noon on the Summer Solstice at the Stanford University campus.





## Assembly Instructions

For sturdy Solstice and Equinox Model  
("Suntrack")



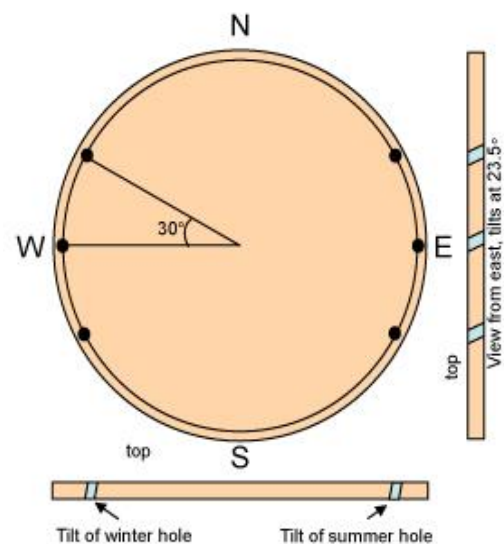
- 1. Mark the north, south, east, and west** directions on the wooden disk. (Disk shown is for the northern hemisphere. For the southern hemisphere, simply swap the directions.)
- 2. Plan your slanted holes** in the base at east, west, northeast, southeast, northwest, and southwest as below. The diameter of the holes should be sufficient to hold tightly, but allow removal of, ends of the plastic cord.

Six (6) holes will need to be drilled 1.3 cm ( $\frac{1}{2}$  inch) in from the edge. The angle at which they are drilled is important, in fact critical if you expect to be exact. However, even improperly drilled holes will still result in a model that conveys the key concepts.

You will drill your holes all the way through the disk. You will be placing your plastic Weed Eater™ cord into them. So you'll need to cut the cord long enough to display the solar path as well as fit down into the disk.

At the latitude of San Francisco, roughly 37 degrees, the solstice rise and set points are almost exactly 30 degrees north and south of due east and west. That is, at summer solstice the Sun rises at 30 degrees north of due east, and sets 30 degrees north of due west. For winter solstice, it rises at 30 degrees south of due east and sets 30 degrees south of due west. Consult Appendix B, which gives a table of this separation for other latitudes. Mark places on your disk for due east and west, then mark 30 degrees (adjust for your latitude) north and south of each of these.

You will be drilling the holes where you have placed the marks. However, don't drill yet! The holes will need to lean the right amount. The equinox holes (at due east and west) are the easiest. In our 37 degree example, for a 30 cm (12 inch) diameter base, drill the holes in 1.3 cm ( $\frac{1}{2}$  inch) from the edge. Simply tip the drill to the south by your latitude (e.g. 37 degrees). Other than the latitude adjustment, the cord has no other tilt for the equinoxes. However, your other holes will have an





additional tilt. For the solstices, the cord should be on a plane parallel to the equinox plane, but displaced to the rise/set points. So, as with the equinoxes, the tip of N-S is also 37 degrees (or your latitude). But it also is tipped in at winter solstice and out at summer by about 20 degrees ( $1 - 9:26/12:00 * 90$ ). Details on how these calculations are performed are given in Appendix A, below.

Once you have planned your holes, you may drill them.

### 3. Cut your cord:

Below are the cord lengths for a 37 degree latitude. Details for computing cord lengths for other latitudes are given in Appendix A.

<i>Season/Sun track</i>	<i>Hours Sun is up</i>	<i>Length of cord above disk</i>	<i>Length of cord</i>
Winter solstice	9.5	35 cm (13.7 inches)	40 cm (15.7 inches)
Spring/fall equinox	12.0	44 cm (17.3 inches)	50 cm (19.3 inches)
Summer solstice	14.5	53 cm (20.9 inches)	58 cm (22.9 inches)

Begin by cutting the short strip of plastic cord for the "winter" solstice solar track. Push, but do not glue, it into holes at southeast and southwest. Then, cut the medium length strip of plastic cord for the "equinox" solar track. Push, but do not glue, that into holes at east and west. Cut the long length strip of plastic cord for "summer" solstice solar track. Push, but do not glue, it into holes at northeast and northwest. Thread the Sun bead onto one of the tracks and move it to simulate different tracks at equinoxes and solstices.

4. **Decorate** as you so choose. Decoration is not necessary to understand the concepts, but we have found that investing the time in building and decorating this model enhances the understanding of the concepts. Possibilities for decoration include a Medicine Wheel, or divide into summer, winter, and spring/fall and decorate accordingly, or have a miniature human model standing in the center observing the Sun's progress. Or, be as creative as you wish!





## Going Further

Ancient Observatories – Timeless Knowledge: <http://solar-center.stanford.edu/AO/>

*The Real Reasons for Seasons*, GEMS guide, Sun-Earth Connections, Grades 6-8.  
<http://lhsgems.org/GEMSSeasons.html>

Cherilynn Morrow, *Kinesthetic Astronomy*.  
[http://www.space-science.org/education/extra/kinesthetic\\_astronomy/](http://www.space-science.org/education/extra/kinesthetic_astronomy/)

Gorden Freeman, *Canada's Stonehenge: Astounding Archaeological Discoveries in Canada, England, and Wales*. 2008. ISBN-13: 978-0978452612 ISBN-10: 0978452615

<http://spaceplace.nasa.gov/seasons/en/>

[http://www.nasa.gov/audience/foreducators/postsecondary/features/F\\_Planet\\_Seasons.html](http://www.nasa.gov/audience/foreducators/postsecondary/features/F_Planet_Seasons.html)

[http://www.esa.int/Our\\_Activities/Space\\_Science/Seasons\\_on\\_other\\_planets](http://www.esa.int/Our_Activities/Space_Science/Seasons_on_other_planets)

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[http://facstaff.gpc.edu/~pgore/Earth&Space/GPS/motion\\_of\\_objects\\_in\\_sky.html](http://facstaff.gpc.edu/~pgore/Earth&Space/GPS/motion_of_objects_in_sky.html),  
<http://physics.weber.edu/schroeder/ua/SunAndSeasons.html>, and  
<http://www.umass.edu/sunwheel/pages/moonteaching.html>.



Summer solstice sunrise at Big Horn Medicine Wheel  
Image courtesy of Tom Melham



## Appendix A

### Details for calculating the lengths and angles and drilling holes

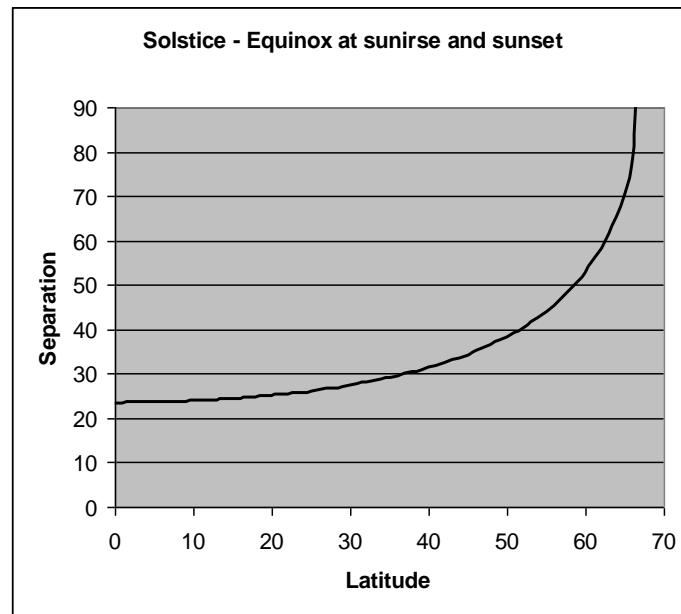


Figure 1

Figure 1 (above) shows the separation between the solstice and equinox rise and set locations. For a latitude of about 37 degrees the separation is 30 degrees. This means the sunrise on the summer solstice (June 21) is at 30 degrees north of east and the sunrise at the winter solstice (December 21) is 30 degrees south of due east. The offset from due east also applies at sunset from due west. The formula is  $\sin(\text{offset}) = \sin(\text{declination})/\cos(\text{latitude})$ . At the solstices the declination of the Sun is the inclination of the Earth at 23.5 degrees. (The formulas in this note can be found or derived using spherical trigonometry. See e.g. *Textbook on Spherical Astronomy*, W. M. Smart (Author), R. M. Green (Editor).

The planes determined by the path of the Sun across the sky at each season are parallel to each other, with the separation determined by the declination (angular distance between the Sun and the plane of the equator). Actually this is not exactly true since the declination changes a small amount during each day. This change is small at the solstices so the sunrise and sunset points seem to sit still for a few days at those times. The daily change is largest at the equinoxes when the sunrise and sunset points move at about 0.4 degrees per day, which is almost the Sun's diameter. At the equator the Sun's path is vertical at rise and set times, so the plane of its path is 90 degrees to the horizon. This angle decreases with latitude so at 37 degrees it is tipped 53 degrees from the horizon (37 degrees from the vertical).

The length of the day is 12 hours at all latitudes at the equinox but gets longer (or shorter) the closer it gets to the summer (or winter) solstice for higher latitudes. At 66.5 degrees latitude the day is 24 hours long at the summer solstice and 0 hours long at the winter solstice. At latitude of about 37 degrees the day is about 14.5 hours long at the summer solstice and 9.5 hours at the winter solstice. The formula for the altitude of the Sun is:



$$\sin(\text{alt}) = \sin(\text{lat}) * \sin(\text{dec}) + \cos(\text{lat}) * \cos(\text{dec}) * \cos(t)$$

so for the rise/set times when the altitude is 0 this can be rearranged to be:

$$\cos(t) = \sin(\text{lat}) * \sin(-\text{dec}) / \cos(\text{lat}) * \cos(\text{dec})$$

where t is fraction of a day from noon, dec is declination and lat is latitude. So at the equator where lat is 0.0, cos(t) is 0.0 which means it is 90 degrees, or 1/4 of a day, so the length of the day is 1/2 of a day for all declinations. For e.g. 37 degrees latitude at the summer solstice when the declination is +23.5 degrees, the length of the day (2\*t) is about 14.5 hours.

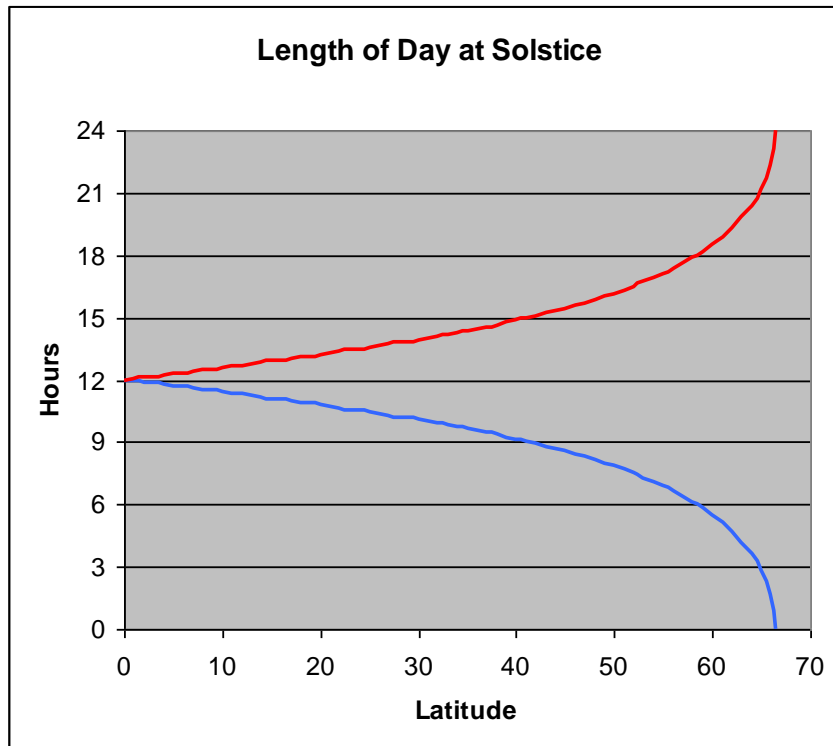


Figure 2. The length of the day on the winter (blue) and summer (red) solstice as a function of latitude.

These calculations and the model are for the geometrical location of the center of the Sun. The length of the day is a little longer as seen on the ground because of refraction when looking at the Sun through the atmosphere, which displaces the Sun by about its diameter at the rise and set times. Hence we actually see the Sun for about 2 minutes after it sets (geometrically). Also, since when even the first or last part of the Sun is visible we say it is still not set or has started to rise, this adds another 1/2 of the Sun's diameter (1 minute) morning and evening. So at the equator the apparent day is about 6 minutes longer than 12 hours with that amount slowly increasing with latitude.

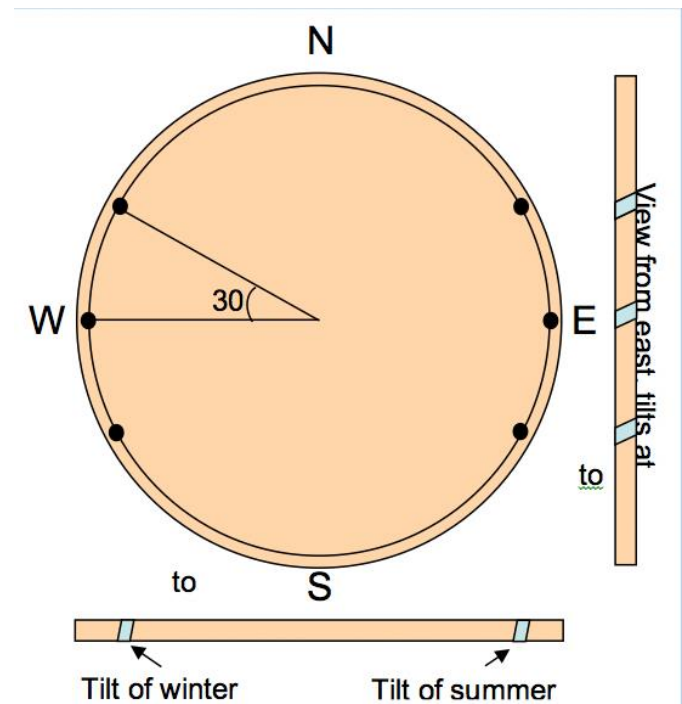
The details described above give us the information needed to make the Sun location model. The following is for a base of 30 cm (12 inches) (D) diameter and 2 cm (3/4 inch) thick (T). First, we can calculate the length of the cords. If the holes for the cords are e.g. 1.3 cm (1/2 inch) (d) from the edge of the base, then the diameter of the circle that includes the holes for the sunrise and sunset points is D –

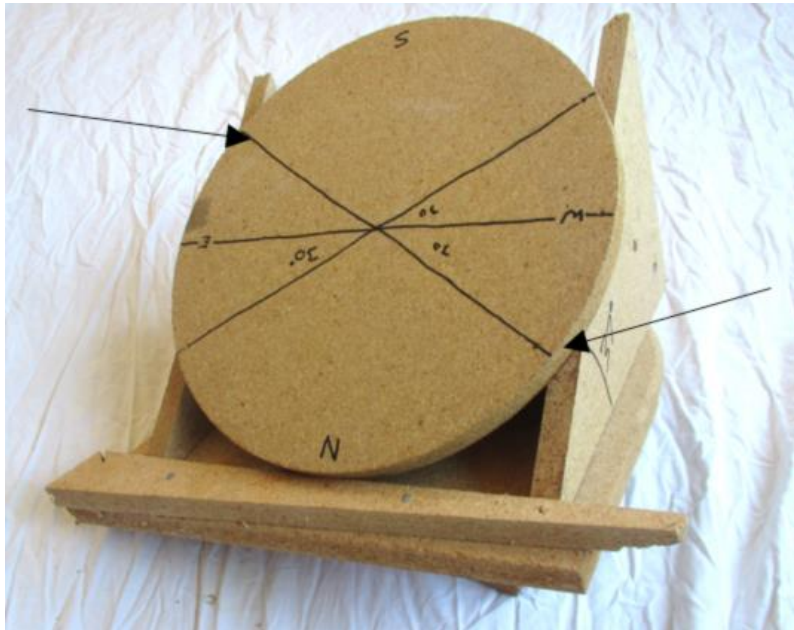


$2d = 28$  cm (11 inches) for a circumference of 88 cm (34.6 inches). We will make the full day of Sun travel then 88 cm (34.6 inches). For the equator this means we need a cord length of half that, 44 cm (17.3 inches). Since the summer solstice day is 14.5/12.0 of a day longer than the equinox day, the cord for this is 53 cm (20.9 inches). Then the winter solstice day length is  $88 \text{ cm} - 53 = 35$  cm ( $34.6 - 20.9 = 13.7$  inches). Since we plan to drill holes for the cord all the way through the base we need to make the cords a bit longer. The cords will be inclined at 37 degrees from the vertical so the length will be  $T/\cos(37) = 1.25T$  at each hole. For the equator we need to add  $2.5T = 5$  cm (1.87 inches) to the day part of the cord so the equator cord needs to be 49 cm (19.2 inches) long. The solstice cords will also be tilted in or out in the plane of the cord. The hole should be tangent to the cord where it enters the base. For 14.5 hours of 24 total this angle is 19 degrees. The path through the base will then be  $1/\cos(19) = 1.06$  times the length at the equinox. So the extra length of the solstices is about  $1.06 * 1.25 * T = 1.32T$  which is 2.5 cm (1 inch) for each hole. Thus the winter cord will be 40 cm (15.7 inches) and the summer cord will be 58 cm (22.9 inches).

In summary, for a 30 cm (12 inch) diameter base, drill the holes in 1.3 cm ( $\frac{1}{2}$  inch) from the edge. Tilt toward the south by 37 degrees and for the winter (summer) holes tilt in (out) by 19 degrees in the plane tipped 37 degrees.

To accomplish this accurately is not easy. Since we made many models, we built a jig to hold the base. It has a flat board with vertical boards with slopes of 37 degrees fastened to it. Then a wedge is added with a tilt of 19 degrees. Positioning the jig on a drill press table with the wedge removed, the equinox holes can be drilled. Then add the wedge with the thick part to the west and drill the solstice holes for the summer sunset and winter sunrise then change the wedge to have the thick side on the east for the summer sunrise and winter sunset. Move the jig to keep the base orientation with north at the low point at all times. Our jig is shown below. (The wedge should be on top of the latitude boards but for these angles the difference is small, about 1 degree. For latitudes higher than 37 degrees, put the wedge on top.)





Shown with base board oriented for drilling summer sunset and winter sunrise holes (arrows). For summer sunrise and winter sunset reverse direction of 19 degree wedge. For equinox holes remove the wedge. For all holes keep north down.



## Appendix B

### Table of Solstice-Equinox Separation and Day Length vs. Latitude

Latitude	Separation	Day length	
		Winter	Summer
0	23.50	12.00	12.00
1	23.50	11.94	12.06
2	23.52	11.88	12.12
3	23.53	11.83	12.17
4	23.56	11.77	12.23
5	23.60	11.71	12.29
6	23.64	11.65	12.35
7	23.69	11.59	12.41
8	23.75	11.53	12.47
9	23.81	11.47	12.53
10	23.88	11.41	12.59
11	23.97	11.35	12.65
12	24.06	11.29	12.71
13	24.16	11.23	12.77
14	24.26	11.17	12.83
15	24.38	11.11	12.89
16	24.51	11.05	12.95
17	24.64	10.98	13.02
18	24.79	10.92	13.08
19	24.94	10.85	13.15
20	25.11	10.79	13.21
21	25.28	10.72	13.28
22	25.47	10.65	13.35
23	25.67	10.58	13.42
24	25.88	10.51	13.49
25	26.10	10.44	13.56
26	26.34	10.37	13.63
27	26.59	10.29	13.71
28	26.85	10.22	13.78
29	27.12	10.14	13.86
30	27.42	10.06	13.94
31	27.72	9.98	14.02
32	28.05	9.90	14.10
33	28.39	9.81	14.19
34	28.75	9.73	14.27
35	29.13	9.64	14.36
36	29.53	9.54	14.46
37	29.95	9.45	14.55
38	30.40	9.35	14.65
39	30.87	9.25	14.75
40	31.37	9.15	14.85
41	31.89	9.04	14.96
42	32.45	8.93	15.07





43	33.04	8.81	15.19
44	33.66	8.69	15.31
45	34.33	8.56	15.44
46	35.03	8.43	15.57
47	35.78	8.29	15.71
48	36.58	8.15	15.85
49	37.43	8.00	16.00
50	38.34	7.84	16.16
51	39.32	7.67	16.33
52	40.37	7.49	16.51
53	41.50	7.30	16.70
54	42.72	7.10	16.90
55	44.04	6.88	17.12
56	45.49	6.65	17.35
57	47.07	6.40	17.60
58	48.80	6.12	17.88
59	50.73	5.82	18.18
60	52.89	5.49	18.51
61	55.33	5.11	18.89
62	58.14	4.69	19.31
63	61.44	4.19	19.81
64	65.45	3.59	20.41
65	70.65	2.82	21.18
66	78.63	1.66	22.34
66.5	90.00	0.00	24.00