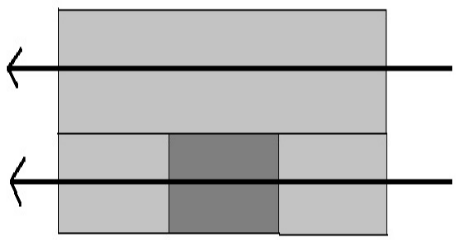
Using Opacity to Find Aerosol Density



This figure shows two light rays from the Setting sun at two different altitudes above Earth. The top ray goes through a medium with fewer aerosols and a lower optical Depth.

The bottom ray passes closer to Earth's surface where a portion of the atmosphere is denser and has a different concentration of aerosols, making the dark region a higher optical depth.

The bottom atmosphere 'slab' is divided Into three equal sections, each 3,000 kilometers in length. The total width of the atmosphere being modeled in the example is 9,000 kilometers wide.

Opacity is a term used to describe the passage of light through a material, and is defined by the basic exponential formula

$$I = I_0 e^{-\tau}$$

Opaque materials have large positive values for τ, while transparent (translucent) materials have very low values for τ.

The quantity τ is also called the optical depth of a medium, and is proportional to both the length of the path taken by light through the medium, and the density of the absorbing particles. We can write this as

$$\tau = C N x$$

where x is in units of meters,
N is in units of particles/meter^3, and
C is a constant that is different for each kind of aerosol and has the units m^2/particle.

The Sage-III mission is designed to determine the product CN by looking at the extinction of sunlight along many different paths through the atmosphere given by x as shown in the figure to the left.

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9991 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34\times10^{-17}$ km^2 /particle and $C_B=7.5\times10^{-18}$ km^2 /particle, what are the densities of the two aerosols in A) particles/kilometer^3? B) particles/meter^3?

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9994 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Answer: $I = I_0 e^{-\tau}$ and

Top layer: $I/I_0 = 0.9994$ so $\ln(0.9994) = -\tau$ and so τ **= 0.0006**

Bottom layer: $I/I_0 = 0.995$ so $\ln(0.995) = -\tau$ and so τ **= 0.005**

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Answer:

Top Layer aerosol: $0.0006 = CN_A \times (9000\text{ km})$

so $CN_A = 0.0006/9000\text{km}$

$= 6.7\times10^{-8}\text{ km}^{-1}$

Bottom Layer aerosol:

$0.005 = CN_A\,(3{,}000\text{ km}) + CN_B\,(3{,}000\text{ km}) + CN_A\,(3{,}000\text{ km})$

$0.005 = 2\,(6.7\text{x}10\text{-}8\text{ km-}1)(3{,}000\text{ km}) + CN_B\,(3{,}000\text{ km})$

$0.005 = 0.0004 + CN_B\,(3{,}000\text{ km})$

$0.0046 = CN_B\,(3{,}000\text{ km})$

So $CN_B = 0.0046/3000\text{km}$

$= 1.5\times10^{-6}\text{ km}^{-1}$

So, $\mathbf{CN_A = 6.7\times10^{-7}\ km^{-1}}$ **and** $\mathbf{CN_B = 1.5\times10^{-6}\ km^{-1}}$

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34\times10^{-17}\text{ km}^2$ /particle and $C_B = 7.5\times10^{-18}\text{ km}^2$/particle, what are the densities of the two aerosols in A) particles/kilometer^3? And B) particles/meter^3?

Answer: Aerosol A: $(1.34\times10^{-17}\text{ km}^2/\text{particle})\ N_A = 6.7\times10^{-7}\text{ km}^{-1}$

so $\mathbf{N_A = 5.0\times10^{10}}$ **particles/km^3**

Aerosol B: $(7.5\times10^{-18}\text{ km}^2/\text{particle})\ N_B = 1.5\times10^{-6}\text{ km}^{-1}$

so $N_B = \mathbf{2.0\times10^{11}}$ **particles/km^3**

Aerosol A: 5.0×10^{10} particles/km^3 x $(1\text{km}/1000\text{meters})^3$ = **50 particles/meter^3**

Aerosol B: 2.0×10^{11} particles/km^3 x $(1\text{ km}/1000\text{ meters})^3$ = **200 particles/meter^3**