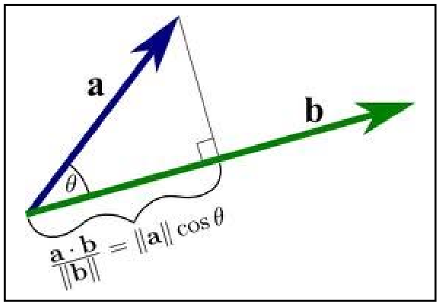
Point A: (4.0, 3.0)
Point B: (6.0, 10.0)

Vector Notation:

A = +4.0**x** +3.0**y**
B = +6.0**x** + 10.0**y**



The Van Allen Belt Probes spacecraft determine their orientation in space by using the direction of the local magnetic field of Earth. This is like using a compass to figure out where you should go next!

The VABP spacecraft measure the intensity of the magnetic field along, and perpendicular to their direction of motion at a given time. They can then use these measurements to find the angle between their motion (vector **A**) and the magnetic field (vector **B**). They compare their measurements against a mathematical model of Earth's magnetic field to find the field's true direction, then they rotate the spacecraft until the angles match up.

To do these calculations we have to determine the projection of the magnetic field onto the direction of the spacecraft motion, and to the axis perpendicular to this motion. This involves using the Vector Dot Product:

$$\mathbf{A}\ \mathbf{B} = ||A||\ ||B||\ \cos(\theta)$$

which gives the <u>component</u> of vector **A** along the <u>direction</u> of the unit vector **B** where ||B||=1.

Problem 1 – The spacecraft moves from point A (0,2) to point B (2,1). Write these points in vector notation and calculate the spacecraft motion vector **C** = **B** – **A**.

Problem 2 – Direction vectors are like motion vectors except they have unit length. What do you have to do to the motion vector, **C**, to give it a magnitude of exactly 1.0?

Problem 3 – What is the slope of the vector, **C**?

Problem 4 – The predicted magnetic field is given by the vector **B** = +4**x** + 3**y** in units of nanoTeslas. What is the projection of the magnetic field along the direction of motion of the spacecraft?

Problem 5 – The spacecraft measures a magnetic field strength of 1 nanoTesla along the spacecraft motion. What is the angle, θ, between the spacecraft motion and the local magnetic field?

Problem 1 – The spacecraft moves from point A (0,2) to point B (2,1). Write these points in vector notation and calculate the spacecraft motion vector **C** = **B** – **A**.

Answer: **A** = +2**y**, **B**= +2**x** + 1y, therefore **C = +2x – 1y**

Problem 2 – Direction vectors are like motion vectors except they have unit length. What do you have to do to the motion vector, **C**, to give it a magnitude of exactly 1.0?

Answer: You have to rescale the vector **C** so that its magnitude is 1.0. Since |C| = $(5)^{1/2}$, the direction vector is **D** = $+2/(5)^{1/2}$**x** – $1/(5)^{1/2}$**y,** therefore |D| = 1.0

Problem 3 – What is the slope of the vector, **C**? Answer: **-1/2**

Problem 4 – The predicted magnetic field is given by the vector **M** = +4**x** + 3**y** in units of nanoTeslas. What is the projection of the magnetic field along the direction of motion of the spacecraft?

Answer: Use the dot product with **M** = 4**x**+3**y**, and **B** = (2**x**-1**y**) where $||B||=(5)^{1/2}$.

M dot B/||B|| = (4x + 3y) dot (2x – 1y) / $(5)^{1/2}$

The projection is $(8-3)/(5)^{1/2}$ or $5^{1/2}$ **nanoTeslas.**

Problem 5 – The spacecraft measures a magnetic field strength of 1 nanoTesla along the spacecraft motion. What is the angle, θ, between the spacecraft motion and the local magnetic field?

Answer: Use A B = ||A|| ||B|| cos(θ)

1 nano Tesla = M dot B/||B|| = ||M|| cos θ

Therefore 1 nanoTesla / ||B|| = cos θ and ||B|| = $5^{1/2}$

$\cos(\theta) = 1/5^{1/2}$
$= 0.447$

Therefore θ = **63 degrees**.