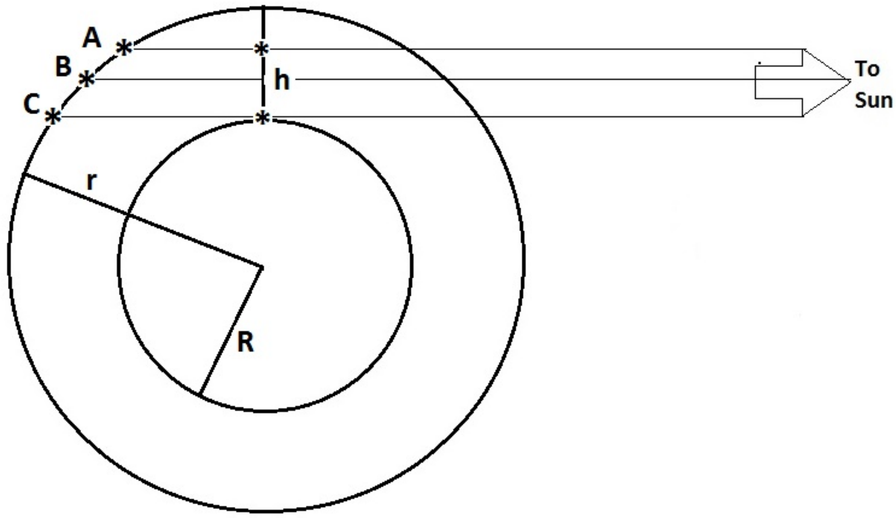
SAGE-III Sunset and Sunrise Geometry



The SAGE-III instrument on the International Space Station orbits Earth at a distance of r = 6,730 km from the center of Earth. The radius of Earth is R = 6,378 km. The time for one complete orbit is about 90 minutes. As it travels from Point A to C in the figure, the height of the sun, h, above the edge of Earth decreases to zero and astronauts observe a sunset. Each time SAGE-III observes a sunrise or sunset, its instruments measure the brightness of the sun. From this sun-dimming information scientists can determine the aerosol content of the stratosphere above an altitude of 10 km.

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Answer: $l^2 = r^2 - (R+h)^2$ so

Length = $2l = 2\,(r^2 - R^2 - 2Rh - h^2)^{1/2}$

Since R = 6378 and r = 6730 we have by simplifying

$L = 2\,(6730^2 - 6378^2 - 2(6378)h - h^2)^{1/2}$

Factor out 6730^2

Then $L = 2\,(6730)(1 - 0.90 - 0.00028h - (h/6730)^2)^{1/2}$

But h/6730 is never more than 100/6730 = 0.015 so we can ignore the h^2 term entirely!

So, $\mathbf{L = 13460(0.10 - 0.00028h)^{1/2}}$

Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

Answer: h = 0 at sunset so $L = 13460\,(0.10)^{1/2}$ = **4256 km**.

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Answer: The ISS will travel about 40 km in its orbit. Since the circumference of the circular orbit is $C = 2\,\pi\,(6730 \text{ km}) = 42280$ km, and this takes 90 minutes, the sunset range of 40 km will be traversed in

$$\frac{40 \text{ km}}{42280 \text{ km}} \times 90 \text{ minutes} \times (60 \text{ sec}/1 \text{ minute}) = \mathbf{5 \text{ seconds}}.$$