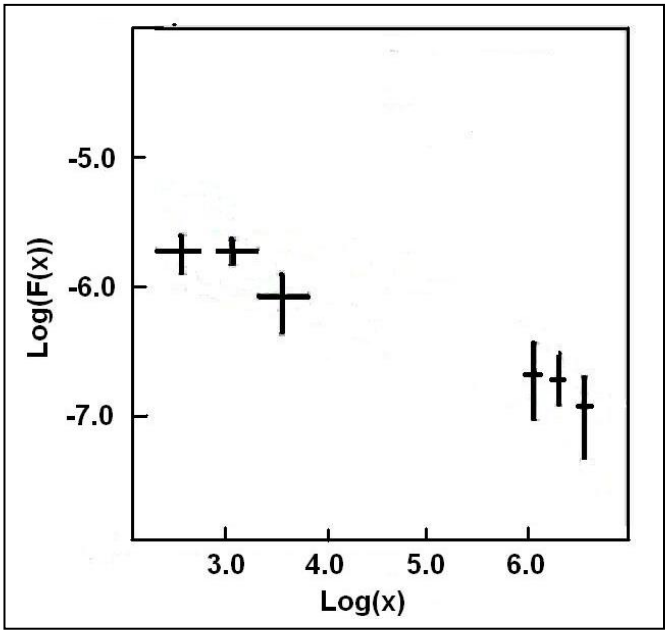
*M82, or the Cigar Galaxy, is a starburst galaxy about 12 million light-years away from Earth. In the galaxy's center, stars are being born 10 times faster than they are inside the entire Milky Way galaxy. The high stellar birth and death rate made M82 a good test case for the theory that cosmic rays are generated in supernovae explosions. In this false-color image, X-ray data recorded by the Chandra X-ray observatory is blue; infrared light recorded by the Spitzer infrared telescope is red; Hubble space telescope observations of hydrogen line emission is orange, and the bluest visible light is yellow-green. (Credit: NASA/JPL-Caltech, STScI, CXC, Uof A, ESA, AURA, JHU)*




The Fermi Gamma-Ray Space Telescope has recently confirmed that the nearby galaxy, Messier 82, is the major source of high-energy gamma-rays seen at Earth: over 12 million light years away!

This galaxy is a mini-quasar with an active core in which a super-massive black hole is absorbing matter and turning it into energy at a ferocious rate.

The gamma-rays arrive at Earth, at a rate of about one or two per hour, and span a range of energies (x given in MeV) that are shown in the Log-Log plot to the left. F(x) is related to the number of gamma-rays detected per second over an area of 1 square centimeter.

**Power-Laws**

A surprising number of physical phenomena can be mathematically represented, at least over a part of their range, in terms of a power-law function $\mathbf{F(x) = ax^n}$. We are going to explore some interesting, and convenient, features of power-law functions in analyzing data.

**Problem 1** - Show that the graph of Log(F(x)) vs Log(x) is a straight line.

**Problem 2** - The graph to the left shows the gamma-ray energy spectrum measured by Fermi. The data points are presented as crosses (called error bars), with the measured value being at the center of the cross. The size of each error bar shows the acceptable range of the measurement. Using a ruler, what linear equation passes through the crosses for the entire collection of data?

**Problem 3** - From your answer to Problem 2, what is the 'best fit' power-law, F(x), defined by the linear equation you derived from the data?

**Problem 4** - Evaluate F(4.0) and F(5.0) to determine the number of gamma rays/sec/cm$^2$ at energies of 10,000 and 100,000 MeV.

**Problem 1** - Show that the graph of Log(F(x)) vs Log(x) is a straight line.
Answer: $F(x) = ax^n$ so $Log(F) = Log(a) + n\,Log(x)$ . This is of the form y(x) = b + mx which is the equation for a line.

**Problem 2** - Using a ruler, what linear equation passes through the crosses for the entire collection of data? Answer: Students may 'fit' several different lines to the data. One possibility is shown below and has the form Y = mX + b. Using the 'Point-Slope Form' of a linear equation where Y = Log(F(x)) and X = Log(x):

$Y - Y1 = \frac{Y2 - Y1}{X2 - X1}(X - X1)$ where X1 = 2.0, Y1 = -5.6, X2 = 7.0 and Y2 = -7.0 we have

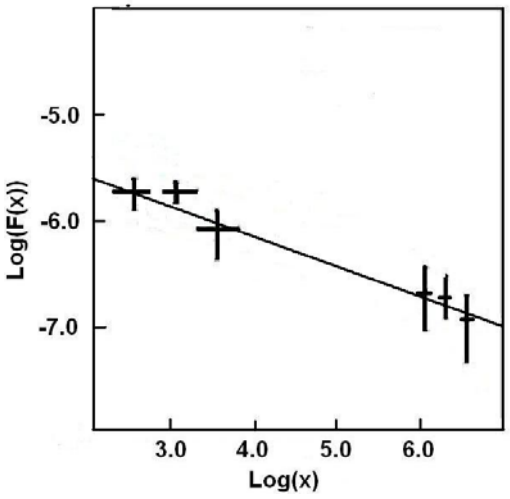
Y = -0.28 X - 5.0

**Problem 3** - From your answer to Problem 2, what is the 'best fit' power-law, F(x), defined by the linear equation you derived from the data? Answer: From the above linear example, Y = Log(F(x)) and X = Log(x), so Log(F(x)) = -0.28Log(x) - 5.0 and

$10^{Log(F(X))} = 10^{-0.28Log(x)-5.0}$ becomes $F(x)=10^{-5}\,x^{-0.28}$ or **$F(x)=0.00001x^{-0.28}$**

**Problem 4** - Evaluate F(4.0) and F(5.0) to determine the number of gamma rays/sec/cm$^2$ at energies of 10,000 and 100,000 MeV. Answer:
$F(10{,}000) = 0.00001(10{,}000)^{-0.28} = 7.58\text{x}10^{-7}$ gamma rays/sec/cm$^2$ (Log(F) = -6.1)
$F(100{,}000) = 0.00001(100{,}000)^{-0.28} = 3.98\text{x}10^{-7}$ gamma rays/sec/cm$^2$ (Log(F) = -6.4)

Note: that these two points, plotted on the Log-Log graph are (+4.0, -6.1) and (+5.0, -6.4) and 'fill-in' the gap between the data points. This is an example of interpolation.




Graph adapted from Abdo, A. A. et al, 2009, 'Detection of Gamma-Ray Emission from the Starburst Galaxies M-82 and NGC-253 with the Large Area Telescope on Fermi', (http://arxiv.org/PS_cache/arxiv/pdf/0911/0911.5327v1.pdf)

Note x is the gamma-ray energy in millions of electron volts (MeV), so '3.0' is 1,000 MeV, and F(x) is in units of MeV/cm$^2$/sec.