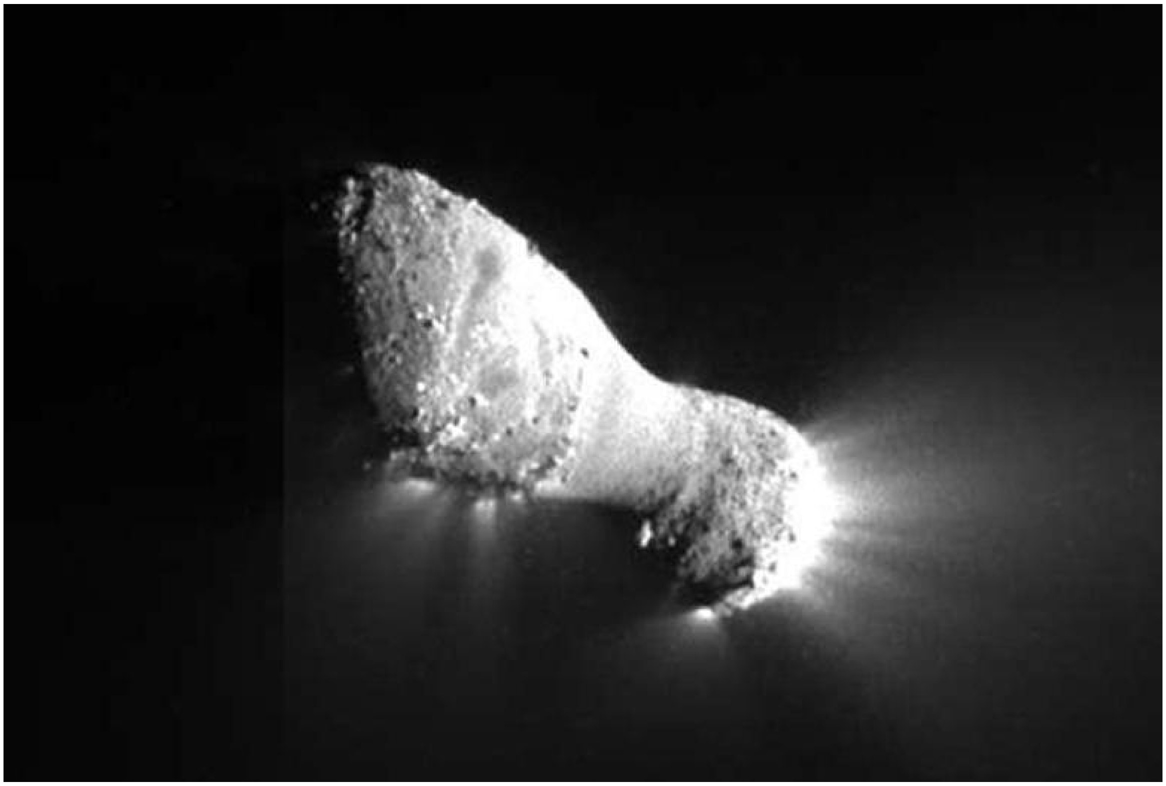
Estimating the Volume and Mass of Comet Hartley-2



Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The scale of this image is 25 meters/millimeter. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Assume that the volume of the nucleus can be approximated by a dumbbell-shaped model consisting of two spherical 'end-caps' connected by a cylindrical bar. To two significant figures, about what is the total volume of the nucleus in cubic meters? (Note: the spheres need not have the same diameters)

Problem 2 - The densities of only a few cometary nuclei have been determined from their mass and volume: Halley's Comet (0.6 gm/cm^3); Comet Tempel-1 (0.62 gm/cm^3); Comet Borrelly (0.3 gm/cm^3) and Comet Wild (0.6 gm/cm^3). The low density indicates large quantities of water and other ices make up the composition of these bodies. Assuming that the density of Comet Hartley-2 is similar to the median density of these comets, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Problem 1 - Assume that the volume of the nucleus can be approximated by a dumbbell-shaped model consisting of two spherical 'end-caps' connected by a cylindrical bar. To two significant figures, about what is the total volume of the nucleus in cubic meters? (Note: the spheres need not have the same diameters)

Answer: Although it appears that the nucleus was viewed at an oblique angle and not face-on, we will not include this perspective effect in the size estimates. Students may attempt to make this correction by, for example, assuming that the end-cap spheres are of equal diameter, with a diameter that is the average of the widths of the two ends of the nucleus.
Using a millimeter ruler, the diameter of the left end-cap is about 40mm and the right endcap is about 30 mm, so at a scale of 25 meters/mm, the actual diameters are 1,000 meters and 750 meters respectively. The diameter of the cylindrical bar is about 20 mm or 500 meters. The length of the cylinder is 2000 meters - 1000 meters - 750 meters = 250 meters based on the assumed length of 2 km and subtracting the two spheres.
The volume of a sphere is $4/3\,\pi\,R^3$ and the volume of a cylinder is $\pi\,R^2\,h$ so we have
Right endcap $V = 1.33\,(3.14)\,(750/2)^3 = 2.2 \times 10^8$ meters^3
Left endcap $V = 1.33\,(3.14)\,(1000/2)^3 = 5.2 \times 10^8$ meters^3
Cylinder $V = 3.14\,(250)^2\,(250) = 4.9 \times 10^7$ meters^3

So the total volume using this geometric model is just V = **7.9×10^8 meters^3**

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Answer: The median density is **0.6 gm/cm^3.**
Mass = Density x Volume

First convert the volume to cubic centimeters from cubic meters:

$V = 7.9 \times 10^8$ meters^3 x (100 cm/1 meter)^3 = 7.9×10^{14} cm^3.

Then, Mass = 0.6 gm/cm^3 x 7.9×10^{14} cm^3
= 4.7×10^{14} gm

Convert grams to megatons:

Mass = 4.7×10^{14} gm x (1 kg/1000 gm) x (1 ton/1000 kg) = 4.7×10^8 tons or **470 megatons**.