

Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Suppose that the shape of the comet nucleus can be approximated by the following function

$$y(x) = -1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03$$

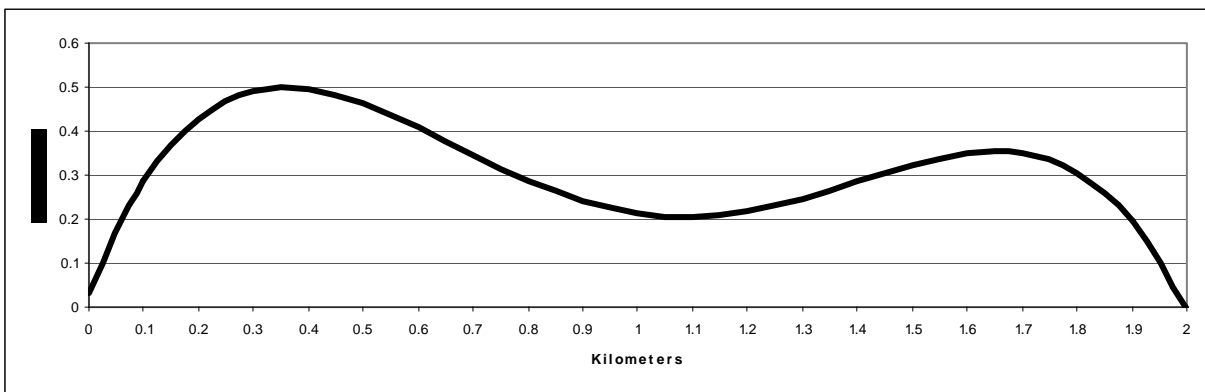
rotated about the x-axis between $x=0$ and $x=2.0$, where all units are in kilometers.

- A) Graph this function;
- B) Perform the required volume integration by using the method of circular disks.
- C) To two significant figures, what is the total volume of the nucleus in cubic meters?

Problem 2 - Assuming that the density of Comet Hartley-2 is 0.6 grams/cm^3 , what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: $1000 \text{ kg} = 1 \text{ metric ton}$)

Problem 1 - Answer:

A) Graph:



B)

$$V = \int_0^2 \pi y(x)^2 dx \quad \text{then} \quad V = \pi \int_0^2 (-1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03)^2 dx$$

Expand integrand and collect terms (be careful!):

$$V = \pi \int_0^2 (1.49x^8 - 12.30x^7 + 41.94x^6 - 76.00x^5 + 77.55x^4 - 42.28x^3 + 9.46x^2 + 0.18x + 0.0009) dx$$

Integrate each term:

$$V = \pi \left[0.17x^9 - 1.54x^8 + 5.99x^7 - 12.67x^6 + 15.51x^5 - 10.57x^4 + 3.15x^3 + 0.09x^2 + 0.0009x + c \right]_0^2$$

Now evaluate V(x) at the two limits to get V = V(2) - V(0): Note that the answer for V will be sensitive to the accuracy of the polynomial coefficients, here given to 4 decimal place accuracy:

$$V = (3.14)[0.1655(2)^9 - 1.5375(2)^8 + 5.9914(2)^7 - 12.6667(2)^6 + 15.51(2)^5 - 10.57(2)^4 + 3.1533(2)^3 + 0.09(2)^2 + 0.0009(2)]$$

$$V = 3.14[0.157]$$

So **V = 0.49 cubic kilometers.**

Problem 2 - Mass = Density x Volume; First convert the volume to cubic centimeters from cubic kilometers: $V = 0.49 \text{ km}^3 \times (10^3 \text{ meters}/1 \text{ km})^3 \times (100 \text{ cm}/1 \text{ meter})^3 = 4.9 \times 10^{14} \text{ cm}^3$. Then, **Mass = $0.6 \text{ gm}/\text{cm}^3 \times 4.9 \times 10^{14} \text{ cm}^3 = 2.9 \times 10^{14} \text{ gm}$.** Convert grams to megatons: **Mass = $2.9 \times 10^{14} \text{ gm} \times (1 \text{ kg}/1000 \text{ gm}) \times (1 \text{ ton}/1000 \text{ kg}) = 2.9 \times 10^8 \text{ tons}$ or **290 megatons.****