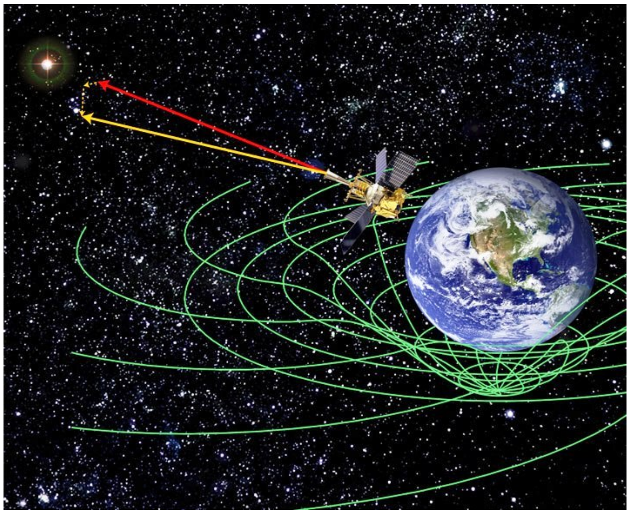

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A prediction of Albert Einstein's relativistic theory of gravity says that the pointing direction of a spinning gyroscope orbiting a massive body should slowly change over time. For Earth, this amount equals degrees/year, and this was recently confirmed by NASA's Gravity Probe-B satellite in 2011.

Einstein's theory predicts much larger shifts if the satellite orbits close to our sun, or to a dense body such as a neutron star.

The effect is called 'frame dragging' and was first predicted in 1918 by Austrian physicists Josef Lense (1890-1985) and Hans Thirring (1888-1976) using Einstein's mathematical theory of gravity published in 1915. The rate, in degrees per second, at which the gyroscope pointing angle will change is given by the formula for Ω, in degrees/sec, shown below:

$$\Omega = \frac{Rac}{r^3 + a^2 r + Ra^2}\left(\frac{360}{2\pi}\right) \quad \text{where} \quad R = \frac{2GM}{c^2} \quad \text{and} \quad a = \frac{2\,\mathrm{R}\,s^2}{5c}\left(\frac{2\pi}{T}\right)$$

and where c is the speed of light (300,000,000 m/s), Rs is the radius of the massive body in meters, M is its mass in kilograms, T is the satellite orbit period in seconds, and G is the Newtonian Gravitational constant 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}. For the GP-B satellite orbiting near Earth at an altitude of 700 km, the measured value for Ω is about 1.2 x 10^{-5} degrees/year.

Problem 1 - In the future, physicists might like to verify this effect near the sun by placing a satellite in a circular orbit at a distance of 10 million kilometers (r = 10^{10} meters). If the radius of the sun is Rs = 6.96x10^8 meters, and its rotation period is T = 24.5 days, and the mass of the sun is M = 2.0 x 10^{30} kg. To two significant figures, what is the value for the Lens-Thirring rate, Ω, in degrees/year? (Note: 1 degree = 3600 arcseconds)

Problem 2 - A neutron star is the compressed nuclear core of a massive star after it has become a supernova. Suppose the mass of a neutron star is equal to our sun, its radius is 12 kilometers, a gyroscope orbits the neutron star at a distance from its center of r = 6,000 kilometers, and its orbit period is T = 8 seconds. To two significant figures, what is Ω for such a dense, compact system in degrees/year?

Problem 1 - In the future, physicists would like to verify this effect near the sun by placing a satellite in a circular orbit at a distance of 10 million kilometers (r = 10^{10} meters). The radius of the sun is Rs = $6.96x10^8$ meters, and its rotation period is T = 24.5 days, and the mass of the sun is M = 2.0 x 10^{30} kg. To two significant figures, what is the value for the Lens-Thirring rate, Ω, in degrees/year?

$$R = \frac{2(6.67x10^{-11})(2.0x10^{30})}{(300,000,000)^2} = 2,964 \text{ m} \qquad a = \frac{2(6.96x10^8)^2}{5(300,000,000)}\left(\frac{2(3.141)}{24.5(24)3600)}\right) = 1,883 \text{ m}$$

then

$$\Omega = \frac{(2964)(1883)(3x10^8)}{\left(10^{10}\right)^3 + 1883^2(10^{10}) + (2964)(1883)^2}\left(\frac{360}{2(3.14)}\right) = 9.60x10^{-14} \text{ degrees/sec}$$

Ω = $9.6x10^{-14}$ deg/sec x (365d/1yr) x (24h /1day) x (3600 s / 1 hr) = **3.0 x 10^{-7} deg/yr**

Note, for GP-B the effect near Earth was 1.2 x 10^{-5} degrees/year because GP-B was orbiting closer to the mass of Earth than our hypothetical satellite around the sun.

Problem 2 - A neutron star is the compressed nuclear core of a massive star after it has become a supernova. Suppose the mass of a neutron star is equal to our sun, its radius is 12 kilometers, a gyroscope orbits the neutron star at a distance from its center of r = 6,000 kilometers, and its orbit period is T = 8 seconds. To two significant figures, what is Ω for such a dense, compact system in degrees/year?

$$R = \frac{2(6.67x10^{-11})(2.0x10^{30})}{(300,000,000)^2} = 2,964 \text{ meters} \qquad a = \frac{2(12,000)^2}{5(300,000,000)}\left(\frac{2(3.141)}{8.0}\right) = 0.15 \text{ meters}$$

then

$$\Omega = \frac{(2964)(0.15)(3x10^8)}{\left(6.0x10^6\right)^3 + \left(0.15\right)^2\left(6.0x10^6\right) + (4150)\left(0.15\right)^2}\left(\frac{360}{2(3.141)}\right)$$

$$\Omega = \frac{(1.33x10^{11})}{\left(2.16x10^{20}\right) + \left(1.35x10^5\right) + (93.4)}\left(\frac{360}{(6.242)}\right) = 3.65 \text{ x } 10^{-8} \text{ degrees/sec}$$

Ω = 3.65 $x10^{-8}$ deg/sec x (365d/1yr) x (24h /1day) x (3600 s / 1 hr) = **1.1 deg/yr**

Note this is nearly 100,000 times the corresponding Lens-Thirring rate near Earth.

For a detailed discussion of the derivation of the formula for Ω in the equatorial plane of a spinning body, see Wikipedia:

http://en.wikipedia.org/wiki/Frame-dragging