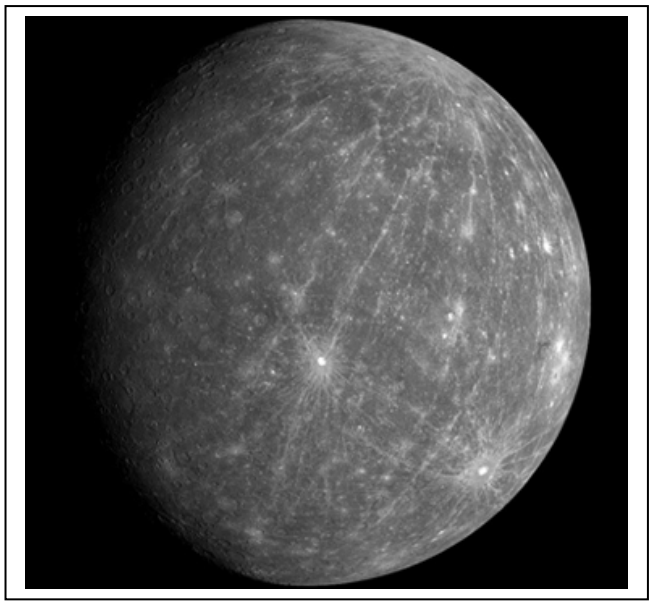

According to Kepler's Third Law, the time it takes a satellite to go once around its planet is given by the formula

$$\frac{R^3}{T^2} = 1.69x10^{-12} M$$

where M is the mass of the planet in kilograms, T is the orbit period in seconds, and R is the radius of the orbit in meters.

For example, the International Space Station orbits Earth with R = 6738 km, T= 92.5 minutes so the mass of Earth is just M = 5.9 x10^{24} kg.

The MESSENGER spacecraft orbits the planet Mercury, but has changed its orbit several times since it arrived in April 2011. The following problems explore how these orbit changes affect the estimate for the mass of Mercury using the Kepler formula.

Problem 1 - On April 25, 2011 the orbit period of MESSENGER was 12 hours and 2 minutes, and its distance was 10,124 km from the center of Mercury. To three significant figures, what was the estimated mass of Mercury?

Problem 2 - On September 14, 2011 the orbit was changed to a distance of 10,085 kilometers and a period of 11hours 58 minutes. To three significant figures, what was the mass of Mercury?

Problem 3 - On May 25, 2012 the final orbit had a period of 8.0 hours and a distance of 7,715 kilometers. To three significant figures, what was the mass of Mercury?

Problem 4 - Explain what the formula is telling us about the properties of the orbit of a satellite and the mass of the body?

Problem 1 - On April 25, 2011 the orbit period of MESSENGER was 12 hours 2 minutes and its distance was 10,124 km from the center of Mercury. To three significant figures, what was the estimated mass of Mercury?

Answer: We first need to convert R and T to meters and seconds so that R = 10,124,000 meters and T = 12h x (3600 s/1hr) +120 sec = 43,320 seconds. Then from the formula:

$$M = 5.92x10^{11}\frac{(10124000)^3}{(43320)^2} = \textbf{3.27 x } \mathbf{10^{23}}\textbf{ kg}$$

Problem 2 - On September 14, 2011 the orbit was changed to a distance of 10,085 kilometers and a period of 11 hours 58 minutes. To three significant figures, what was the mass of Mercury?

Answer: R = 10,085,000 meters and T = 43,080 seconds and so

$$M = 5.92x10^{11}\frac{(10085000)^3}{(43080)^2} = \textbf{3.27 x } \mathbf{10^{23}}\textbf{ kg}$$

Problem 3 - On May 25, 2012 the final orbit had a period of 8.0 hours and a distance of 7715 kilometers. To three significant figures, what was the mass of Mercury?

Answer: R = 7715000 meters and T = 28,800 seconds and so

$$M = 5.92x10^{11}\frac{(7715000)^3}{(28800)^2} = \textbf{3.27 x } \mathbf{10^{23}}\textbf{ kg}.$$

Problem 4 - Explain what the formula is telling us about the properties of the orbit of a satellite and the mass of the body?

Answer: Although the orbit properties change, the mass of Mercury remains the same because the spacecraft is orbiting the same body and the values for R ,T and M must lead to consistent solutions.

Note: From more careful orbital studies, astronomers use the adopted mass of Mercury of 3.31 x 10^{23} kg.