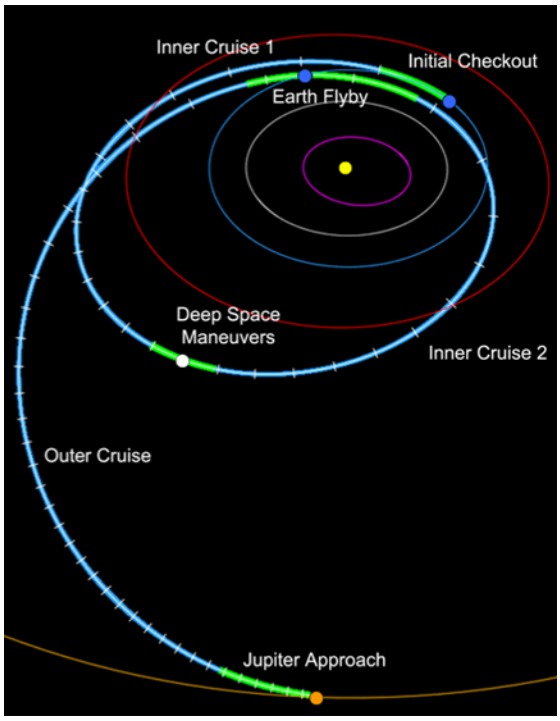
Investigating Juno's Elliptical Transfer Orbit



The Juno spacecraft was initially placed in an elliptical orbit near Earth soon after its launch on August 5, 2011. The orbit was elliptical and designed so that a Deep Space Maneuver in August 2012 would send the spacecraft into a flyby of Earth in 2013. This encounter with Earth would boost the spacecraft's speed and place it into an elliptical transfer orbit that would intersect Jupiter's orbit in 2016.

This added speed would not require extra fuel by the spacecraft making it a free resource that keeps the cost of the mission small. These kinds of 'billiard shot' gravitational assists are commonly used by NASA to place spacecraft in trajectories to the outer solar system.

An approximate equation for the transfer orbit is given by the formula: $5.15x^2 + 9.61y^2 = 49.49$. The units for x and y are given in terms of Astronomical Units where 1 AU = 150 million kilometers, which is the average orbit distance of Earth from the Sun.

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?

Problem 2 – What is the semimajor axis length in AU?

Problem 3 – What is the semiminor axis length in AU?

Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?

Problem 5 - What is the eccentricity, e, of the orbit?

Problem 6 – What are the spacecraft's aphelion and perihelion distances?

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years.If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?
Answer:
$5.15x^2 + 9.61y^2 = 49.49$ Divide both sides by 49.49 to get

$$\frac{x^2}{9.61} + \frac{y^2}{5.15} = 1$$

Problem 2 – What is the semimajor axis length in AU?
Answer: For an ellipse written in standard form: $x^2/a^2 + y^2/b^2 = 1$
Comparing with the equation from Problem 1 we get that the longest axis of the ellipse is along the x axis so the semimajor axis is $a^2 = 9.61$ so **a = 3.1 AU**

Problem 3 – What is the semiminor axis length in AU?
Answer: The semiminor axis is along the y axis so $b^2 = 85.15$ and **b = 2.3 AU**

Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?
Answer: $c = (a^2 – b^2)^{1/2}$. With a= 3.1 and b=2.3 we have **c = 2.1**.

Problem 5 - What is the eccentricity, e, of the orbit?
Answer: The eccentricity e = c/a so e = 2.1/3.1 and so **e = 0.68**

Problem 6 – What are the spacecraft's aphelion and perihelion distances?
Answer: The closest distance to the focus along the orbit is given by a – c so the perihelion distance is 3.1 – 2.1 = **1.00 AU**. The farthest distance is a + c = 3.1 + 2.1 = **5.2 AU**. *Note the perihelion distance is at Earth's orbit and the aphelion distance is at Jupiter's orbit.*

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years. If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Answer: Since a = 3.1 we have $P = 3.1^{3/2} = 5.5$ years for a full orbit. For Juno it spands 5.5/2 = 2.8 years in the elliptical transfer orbit, so it arrives at Jupiter in October 2013 + 2.8 years = October, 2013 + 2 years 8 months = **June, 2016**.