Banco de México

Working Papers

N° 2021-14

Exploring the sources of loan default clustering using survival analysis with frailty

Enrique Bátiz-Zuk
Banco de México

Abdulkadir Mohamed
Leeds University Business School

Fátima Sánchez-Cajal
Banco de México

September 2021

Documento de Investigación
2021-14

Working Paper
2021-14

Exploring the sources of loan default clustering using survival analysis with frailty*

Enrique Bátiz-Zuk†
Banco de México

Abdulkadir Mohamed‡
Leeds University Business School

Fátima Sánchez-Cajal§
Banco de México

Abstract: This paper investigates whether three microeconomic loan characteristics are sources of loan default clustering in the Mexican banking sector by employing survival analysis with frailty. Using a large sample of bank loan level data granted to micro, small and medium sized firms from January 2010 to 2018, we test whether classifying loans by the bank's systemic importance, industry or at individual firm level enhances the predictions of loans defaults. Our results show that loans granted by Domestic Systemically Important Banks contribute to the default clustering in micro and small firm loans. This is due to aggregate default rate levels and clusters that are large for these firms loans compared with loans provided to medium-sized firms. These findings have important implications for bank's expected loss management related to the correlated loan default risk.
Keywords: Credit risk; Parametric survival analysis; Accelerated Failure Time (AFT) models; Shared frailty models; IFRS 9.
JEL Classification: C53, C41, C25, G38

Resumen: Este artículo investiga si tres características microeconómicas de los préstamos son fuentes de agrupamiento de incumplimiento de préstamos en el sector bancario mexicano mediante el empleo de análisis de supervivencia con fragilidad. Utilizando una muestra grande de datos a nivel de préstamos bancarios otorgados a micro, pequeñas y medianas empresas desde enero de 2010 a 2018, se prueba si la clasificación de los préstamos por importancia sistémica del banco, sector o empresa individual mejora las predicciones de incumplimiento de los préstamos. Nuestros resultados muestran que los préstamos otorgados por los bancos con importancia sistémica a nivel doméstico contribuyen al agrupamiento predeterminado en préstamos para micro y pequeñas empresas. Esto se debe a que los niveles de la tasa de incumplimiento agregada y los agrupamientos de incumplimientos de préstamos de estas empresas son grandes en comparación con los préstamos otorgados a empresas medianas. Estos hallazgos son un insumo para la gestión de pérdidas esperadas de los bancos relacionadas con el riesgo de correlación del incumplimiento de préstamos.
Palabras Clave: Riesgo de crédito; Análisis de supervivencia paramétrico; Modelos de tiempo acelerado de fallo; modelos de fragilidad compartida; IFRS 9.

---

*We would like to thank Professor Darrell Duffie and two anonymous referees for their valuable comments and suggestions. All errors remain ours.

† Banco de México. Email: ebatiz@banxico.org.mx.

‡ Leeds University: Business School, Maurice Keyworth Building, Leeds, LS2 9JTUK.. Email: A.Mohamed@leeds.ac.uk.

§ Banco de México. Email: fatima.sanchez@banxico.org.mx.

1. Introduction

The issue of loan default clustering has been an interesting topic for loan granting financial intermediaries. For instance, the banking sector around the globe, in particular in developed countries have suffered significant losses due to default clusters during the financial crises of 2007-2009. The effect of the crises has caused banks to register large losses on mortgage default, interbank-market to freeze, and credit to consumer and business to fade away. An insight into the sources of loan default clustering is critical from the bank's perspective, especially when measuring portfolio credit risk and the valuations of securities exposed to correlated default risk as well as mitigating banks standalone solvency risk. It is also important for bank supervisors to understand the source of loan defaults triggered by inadequate or lax lending standards in order to hold senior bankers accountable for their decisions, whilst at the same time assisting the banks to avert financial difficulty. In this paper we explore whether loan characteristics such as (i) industry, (ii) firm and (iii) the bank's systemic importance are relevant sources of loans default clustering. Our choice for the three sources is motivated by the following reasons. First, our sample includes loans granted to firms that operate in various economic sectors. Hence, variations in loan default rates across industries and the number of external or internal shocks affecting these industries could be correlated and could drive the loan default clustering. Second, a firm might potentially receive more than one loan from a single or multiple banks. Therefore, when the firm defaults on one of its loan, it is likely that the firm would default on all of its remaining outstanding loans. This is due to the fact that all outstanding loans granted to an individual firm in one way or another share the same idiosyncratic risk factor and thus loan default rates might be concentrated at the firm level. A firm failure would lead to multiple loan defaults and could explain loan default clusters. Finally, systemically important banks may have incentives to grant riskier loans compared to small banks because of moral hazard motivated by the expected government bailout in the event of failure. Therefore, it is possible for large banks to apply lax lending standards especially when granting small loans. In the event of such action the aggregate loan default rate will be prone to correlation risk. By contrast, small banks may have comparative advantages lending to SMEs because of their personal interaction with their borrowers (e.g., Berger et al., 2002). They also have the expertise to use soft information

(Petersen, 2004) and in dealing with borrower's moral hazard and adverse selection. We cluster loans provided by the so-called Domestic Systemically Important Banks (D-SIBs) using the definition from the Mexican banking authority (see Appendix A for details).[1] Provided that D-SIBs may have the incentive to grant riskier loans than non-D-SIBs, the time to default correlations between these loans should help predicting loan defaults and serve as a source of default clustering.

We use proprietary loan level data collected monthly by Mexican financial authorities during January 2010 to April 2018. The data includes all domestic and foreign bank loans granted to micro and small and medium enterprises (SMEs), which are part of the Mexican banking sector. Our sample includes both public and privately owned banks as well as all foreign subsidiary banks based in Mexico. The full sample consists of approximately 1.8 million loans provided by banks and regulated multiple purpose financial societies.[2] The central bank collects information at the loan level from commercial banks, but not firms' balance sheet data. Having data from all banks at the sector level facilitates the exploration of whether grouping loans by different loan characteristics is sufficient to model correlation in loans' 'time-to-default'. Previous studies have access to data of one or a few banks (e.g., Dirick et al., 2017), but not the entire banking sector.

To analyse the determinants of loans time to default, we use survival analysis with shared frailty. The survival analysis allows the prediction of loans time to default by considering the length of time between loan origination and default. The response variable for any loan consists of a series of tracking or payment records[3] that register the beginning and ending times. Also, it includes the default status[4] at the end of each time span. Loans that are not registered as default by the end of the sample period are classified as right-censored.[5] The survival function measures the probability of loan survival beyond a specific time period controlling for loan determinants. This unique

[1] See Cantú et al. (2020, p.22) for a brief summary on the evolution of the Mexican banking sector.
[2] Berrospide and Herrerias (2015, pp.35-38) provide a brief summary describing the origin and evolution of multiple purpose financial societies in Mexico.
[3] A loan record is used to register the loan's history for each time period since its origination and it is similar to the payment history.
[4] The default status is a binary variable indicator that takes the value one if a firm defaults on its loan.
[5] Survival analysis is a technique that relies on special methods to address right-censoring, which considers the impact of loan default that has not yet occurred by the end of the sample period.

feature of the survival model has increased its popularity in the credit risk literature (See Glennon and Nigro, 2005 for a discrete time hazard application to a sample of US small firm loans and Dirick et al., 2017 for a survival application including frailties for a sample of small firm and personal loans of European banks). We use Accelerated Failure Time (AFT) with frailty to model the correlation between loans *'time to default'* and explore three sources of loan default clustering (i.e., industry, firm and bank systemic importance). Box-Steffensmeier and De Boef (2006) and Mills (2011) address the importance of using frailty relative to the standard survival analysis. A frailty in the context of survival analysis is defined as a latent random effect that enters multiplicatively on a parametric hazard function to account for both observable characteristics and unobserved heterogeneity effect (see Cleves et al., 2010). In our setting, the frailty is due to some loans being riskier/frailer than others because of shared common features, which could lead to loans' *'time to default'* correlations. In fact, frailty is likely to outperform the standard models in terms of modelling heavy tail portfolio losses.[6]

We estimate separate survival models for each loan sub-category provided to micro/SMEs and treat each firm type differently. We do not aggregate all the loans and treat them as a single group, due to strong evidence that firm size matters and credit risk attributes differ significantly among firms (e.g., Holmes et al., 2010; and Gupta et al., 2015). Specifically, these two studies suggest that there is a huge diversity of firms within micro/SMEs, and there are many differences in their capital structure, firm size, access to external finance, management style, and numbers of employees. Moreover, our sample of loan characteristics shows significant differences in aggregate average statistics related to pricing (loan interest rate) and default rate (either in intensity or loan default number and when it happens), which justify our choice of analysing the source of default clustering separately.

To identify the appropriate AFT model that best fits our data, we use the Akaike Information Criteria (AIC) statistic (see Akaike, 1974). The AIC test indicates that the lognormal distribution provides the best fit for our data. Next, we examine the

[6] We thank Professor Darrell Duffie for this remark.

determinants of loans *'time to default'* for micro, small and medium sized firms separately using micro and macroeconomic variables in line with Carling et al. (2007).[7] Also, we use frailty models to incorporate the correlations in loan time to default in the standard survival model. We test the out-sample performance of the model by computing the one-year probability to default and perform a horse race to compare the benefits of the standard and frailty model in terms of prediction. To examine the forecasting power of our model, we use the Receiver Operating Characteristics (ROC) curve, which relies on the estimate of the probability to default. The regulatory framework of current expected credit loss (CECL) requires banks to compute the probability to default (PD) of each loan over multiple risk horizons to cover the loan's life. In general, banks have the choice of using logistic regressions to estimate the one-year PD. Nevertheless, for multiple horizons, banks could estimate the PD using either a recursive method (see Section B.1 in Appendix B for details) or a separate logistic regression for each future risk horizon. The former approach could lead to a severe risk underestimation of the expected loan loss allowance, while the latter is highly inefficient and depends on the data availability to have reliable parameter estimates. By contrast, the survival analysis requires a single estimation to compute a loan level PD for multiple future risk horizons (see Appendix B) consistent with stylized facts. Moreover, survival analysis with frailties allows modeling the correlation of loan's '*time to default'*, unlike separate logistic regression models, which do not control for correlation risk. This is a critical issue in the credit risk literature that explores the sources of the default clustering over time. To overcome this limitation, we use survival analysis with frailty to estimate the PD.

Our results show that survival analysis with shared frailty in which loans are grouped by banks' systemic importance perform better than all other alternative models (i.e., as evidenced by a higher and statistically significant area under receiver operating curve (AUROC)) in terms of loan defaults predictions for micro and small firms. For loans granted to medium sized firms the frailty models do not outperform the standard survival model. This is not surprising given the lower levels of aggregate default rates in

[7] Following Carling et al. (2007), we assess the impact of: consumer confidence, economic activity and yield curve proxy. We add the inflation rate because of its possible impact on firm loan performance in emerging market.

loans granted to medium sized firms, which are characterized by a minimum or no default clustering effect. We also show that firm's and loan's characteristics at the time of loan origination, and time-varying lagged macroeconomic factors have significant impact on the loan survival times. Our ROC curve for the hold-out sample validation is concave and not steps, because of our large hold-out sample size (see Gupta et al., 2018, p.461). Our empirical evidence shows that the hazard[8] rate for SMEs loans is non-monotonic. This result is important and provides evidence supporting our hypothesis that a bank's exposure to losses due to default is not constant and varies significantly over the life of the loan. Overall, our results suggest that survival analysis with frailty is a relevant method to model loan default clustering over time.

The sources of corporate default clustering at the aggregate level have been widely studied in the literature. Standard studies in the academic literature use different types of microeconomic variables[9] along with macro variables, to control for idiosyncratic and systematic or common risk, respectively. These studies assume that the macro variables are sufficient to control for systematic risk. The seminal paper of Das et al. (2007) shows that macro variables alone are not sufficient to explain the degree of default clusters in aggregate firm default rates in the US over the period from 1974 to 2004. Hence, the authors find that frailty models are relevant techniques to incorporate the expected effect of unobservable covariates that are correlated across firms. This issue led other studies to further investigate the role of frailties in aggregate firm bankruptcy (see Duffie et al. (2007), Campbell et al. (2008), Duffie et al. (2009), Chava et al. (2011), Koopman et al. (2008), Koopman et al. (2011), Monfort and Renne (2013), and Azizpour et al. (2018)).[10] These studies document that frailty or correlated shocks may lead to corporate default clusters in time due to three factors: (i) structural changes in corporate financial markets (e.g., creation of junk bond markets); (ii) macroeconomic events or firms exposure to common risk factors that may be correlated,

[8] The hazard rate is the instantaneous rate of loan default which is known as the conditional probability that any loan defaults within a given time interval.

[9] Microeconomic data quality (i.e., public or private) and type varies across unit of analysis (i.e., firm bankruptcy or firm loan default), but it can be resumed in the following four: accounting, market, firm specific information, loan or debt characteristics.

[10] These studies examine aggregate corporate bankruptcy statistics, while ours is based on bank loans time to defaults using individual loan records. It is worth noting that there are more sources of information for large firms such as market variables (e.g., the firm's stock return), but not available for micro/SMEs.

and affect firms failure probabilities (e.g., recessions); (iii) contagion stemming from the network of contractual obligations and interconnectedness where the event of failure by a large corporate may induce other corporates into bankruptcy or financial distress. These studies do not examine the role of correlation risk to predict default clusters at the individual loan level, which is the focus of this paper.

Our study contributes to the literature on the importance of survival models with frailty in predicting loan defaults to micro/SMEs consistent with CECL standard. We provide a novel parsimonious closed-form regulatory formula to compute the regulatory PD term structure (i.e., the PD of an individual loan at any desired risk horizon) as required by CECL and IFRS 9.[11] In fact, our proposed regulatory PD formula is consistent with loan lifecycle and the annual marginal default rates, while the standard recursive PD formula used in the financial industry is inconsistent with marginal loan default rate stylized facts (see Appendix A for details).[12] Our approach is flexible and can be applied within any net cash flow modelling framework. US bank entities have struggled with the implementation of CECL as it has proved to be difficult in terms of data collection. Haldane and Madouros (2012) describe how internal models have increased opacity in the banking book of large, complex banks. This raises questions about regulatory robustness because it relies on large number of estimated parameters that are hard to validate.[13] Bank personnel have expressed concerns over the time, effort and expertise it takes to select appropriate methodology when estimating PD term structure. We show that introducing a shared frailty in modelling credit risk is important and captures the source of loan default clustering.

Our study is the first to shed light on the channels of loan default clustering at individual loan level provided to micro/SMEs. The findings of our study are important to academics interested in assessing the predictive performance of parametric survival

[11] See Appendix B for detailed estimation of the PD and Appendix C for regulatory overview.
[12] Glennon and Nigro (2005) use a large sample of US loans to small firms and show that default risk varies significantly over the life of the loan. The pattern in the annual average default rate implies that marginal default rates are time dependent. For instance, in the early years, annual average default rates increase up to a maximum, then fall towards zero as the life of the loan terminates. The shape of the hazard function linked to the log normal distribution, which is used in our paper incorporate this stylized fact.
[13] Haldane and Madouros (2012, p.7) document that a typical large international bank in the UK banking sector has to estimate several thousand default probability parameters based on a large number of models that vary for each portfolio.

models. It offers a guide for global regulators and supervisors keen to understand how to apply a simple method of estimating PD terms structure for any banking sector. Moreover, the PD formulae can be used for top-down or bottom-up stress test. The results are relevant for practitioners of small banks and show a more effective and less costly to compute loan loss allowances (see Stackhouse, 2019a and Stackhouse, 2019b). Due to the complex nature of the Mexican banking sector, our results provide empirical evidence that is useful to other banking sector regulators around the globe on the sources of the loan default clustering. The CECL standard requires banks to maintain life-of-instrument estimates of credit losses (ECL) on financial assets (i.e., both performing and non-performing) and these apply since loan origination. To the best of our knowledge, there is no single correct approach to estimate loan loss allowance in practice (Engelmann, 2018). Due to ECL's forward-looking nature and the complexity requirements imposes a natural challenge to small banks in having reliable estimates. Furthermore, the possibility of wide range estimations of ECL would make it difficult for investors to compare the loan loss estimates across banks. Our study demonstrates how our estimate of PDs using survival analysis could assist the banks to standardise the estimate of the PD for their ECL, consistent with regulatory requirements.

The rest of the paper is organized as follows. In Section 2, we summarize the literature review and provide a regulatory background on IFRS 9 and CECL Standards. In Section 3, we present both our micro/SME firm and loan default definitions along with the characteristics of our data, variable selection criteria, and methodology and model validation metrics. In Section 4, we report the results of our univariate and multivariate analysis including our main findings. In Section 5, we present our conclusions and policy recommendations.

2. Related existing studies & background of SMEs

The existing literature on credit risk modeling is broadly classified as either structural[14] or reduced-form models. Survival analysis is related to the reduced form modeling. The origin of reduced form models is based on discriminant analysis (Altman, 1968) or

[14] Structural models have their origin in the option theoretic approach (Merton, 1974).

logistic regressions (Ohlson, 1980). These models assume that corporate firm bankruptcy depends not only on a firm's structural features, but also on accounting financial information and macroeconomic variables. Most of these studies were developed and applied to predict corporate failure (i.e., firm bankruptcy) using a probability to default (PD) over a one-year horizon. Presumably, the popularity of these model's stems from the fact that they offer a simple closed form formula to estimate the PD. Gupta et al. (2014a,b) use logistic regressions to investigate the performance of firm bankruptcy of SMEs. Nevertheless, these models are applicable only to single-period default probabilities and do not consider default correlation. Even though it is possible to estimate multiple-period logit models, the estimation process is inefficient and time consuming. The seminal paper by Das et al. (2007) shows that macro variables alone cannot control for systematic risk. Hence, introducing frailty models incorporates the expected effect of unobservable covariates that are correlated across firms. This suggests that the correlation risk is relevant to examine default clusters.

Refined reduced form models use survival models (Shumway, 2001) among other approaches (see Duffie et al., 2009, and Azizpour et al. (2018) and references therein) to address multiple-period default probabilities and default clusters. Survival models deal with right-censoring issues in a simple and efficient way (i.e., with a single closed-form equation). The models are useful to compute multiple-period probabilities to default at the individual level that incorporate correlation risk using frailty. In contrast, other popular approaches suggested by Das et al. (2007), Duffie et al. (2009) and Azizpour et al. (2018) study the role of frailties using aggregate default. These approaches investigate sources of default clustering in great depth. However, there is one limitation related to these approaches. It is not clear how the methodology proposed by Das et al. (2007), Duffie et al. (2007), Duffie et al. (2009) and Azizpour et al. (2018) could be used in practice by banks to compute multiple PD's at the loan or individual level in a reliable and efficient way. Our paper closes an important gap in the literature by showing the benefit of survival analysis with frailty to model default correlations and PD estimation at the loan level. We contribute to the banking industry as our estimation framework has the edge of being efficient and simple to compute multiple PD's at the loan level. In the context of survival models, Gupta et al. (2018, p.440) highlight that a shortcoming in empirical SMEs credit risk studies is lack of discussion on the importance of frailty. We contribute to the literature by presenting an empirical comparison of survival models where correlation of firm loans 'time to default' is a critical factor to enhance SMEs loan default prediction. We use a comprehensive sample that analyzes data for the entire banking sector.

There are several methods in the literature to classify standard survival analysis used to predict corporate[15] or micro/SME failure or loan default. Within the analysis of corporate firms, some studies analyze firm survival for multiple countries (see Tsoukas (2011)), while others focus on financial sector. Studies on financial firms focus on predicting bank failures (see Pappas et al. (2017)) while others examine the occurrence of banking crisis episodes (see Evrensel (2008)). Some of the non-financial SMEs studies focus on firm bankruptcy (i.e., firm failure) (e.g., Gupta et al. (2018)), others investigate loan defaults (see Dirick et al. (2017), Glennon and Nigro (2005)). Our study is related to the latter approach and examines the predictive performance of bank loans to micro/SMEs firms.[16] Our aim is to use survival model to estimate a bank's individual loan PD and use this as input to compute loan loss allowances based on Expected Credit Loss methods that comply with IFRS9 or CECL Standards.

Survival models could be further classified into (i) the so-called 'classical survival models' which are based on either a Cox Proportional Hazards Regression Models or semi-parametric techniques (Cox (1972)) or an Accelerated Failure Time (AFT) model and (ii) 'mixture cure rate models' (Sy and Taylor (2000)).[17] This paper contributes to the classical survival literature. On one hand some studies use survival analysis to predict corporate or micro/SMEs failure and perform a horse race to compare the refined model vis-à-vis a benchmark counterpart (see Gupta et al. (2018)). Others test differences in methodologies of survival analysis (see Dirick et al. (2017)) or compare survival methods versus other statistical methods (see Bauer and Agarwal (2014), Chen and Hill (2013), Bharath, S., and Shumway, T. (2008)). This paper

[15] Corporate studies (see Pereira, 2014) have access to a broader set of information as they can incorporate market variables (e.g., firm's stock return or firm's equity price and market capitalisation) as explanatory factors. Moreover, corporates information is publicly available and subject to auditing by an external firm.
[16] In addition, we do not analyze the survival of non-bank entities to micro/SME s or any other source of firm financing.
[17] In the literature, are also known as 'long term survival' in biostatistics or 'split population' and 'mover stayer' in economics.

performs a horse race between a standard parametric survival model that may or may not include frailty grouped by different variables (i.e., refined models).

3. Empirical methods

This section discusses the definition of micro/SMEs, how default is defined for these firm loans, the source of the data-set, the selection of explanatory variables, the modelling technique and performance validation.

3.1 Definition of SMEs loan default

The definition of SMEs is not standardized around the globe. For instance, in Europe micro/SMEs definition is based on specific thresholds for the number of employees and annual sales or firm balance sheet.[18] The European definition is different from the US (see Altman et al., 2010; Gupta et al., 2018) for more details.[19] The US Small Business Administration (SBA) defines an SME as the firm having less than 500 employees and annual turnover of less than $7.5 million.[20] This definition varies across sectors (see Gupta et al., 2018) and is not convenient for Europe or Mexico, which may include very large firms. Mexico's Secretariat of Economy defines micro and SMEs based on four factors: (i) the number of Employees; (ii) firm annual turnover; (iii) industry; (iv) combined ceiling threshold, which is a function of the number of employees and annual sales. The Mexican definition of micro/SMEs is not determined by the firm's balance sheet information as in Europe.[21]

In Mexico, commercial banks have to report to the financial authorities their loan exposure characteristics independent of their size. We define micro/SME loans when the maximum exposure size is lower than 1 million USD for a given bank. This

[18] Table D1 in Appendix D shows the criteria applied by the European Union (EU) according to the EU recommendation 2003/361.
[19] Gupta et al. (2015, p. 848) describes the three criterions used in the UK to identify SMEs where the criteria depend on the average number of firm's employees, firm's balance sheet total and the annual turnover. The approach is similar in nature to the one used in the European Union.
[20] See Electronic Code of Federal Regulations, Title 13, Chapter 1 (https://www.ecfr.gov; accessed on July 7, 2020).
[21] Table D2 in Appendix D shows the criteria applied in Mexico to classify firms as micro/SMEs. Annual sales are expressed in US Dollars in parenthesis in Table D1 and Table D2 to facilitate the comparison between the EU and Mexico. As expected, the threshold considered by the EU for annual sales is higher than that of Mexico across all the firm sizes.

threshold is based on: (i) the business model, which is applied by several banks to classify firms as micro/SMEs has a size limit between approximately 540K and 1 million USD; (ii) some banks use the loan size below 810K USD as micro/SME loans; and (iii) the program of automatic guarantees of the National Development Bank "Nacional Financiera" with which banks serve micro/SMEs have a maximum exposure size of approximately 1 million USD.[22]

In this paper, a micro/SME loan default is defined as when a loan's due date is more than 90 consecutive days or when the bank has not received any payment during three consecutive months. This definition is consistent with Basel III regulatory framework for risk-weighting purposes under the standardized approach for credit risk (see BCBS (2017, p.27)). This definition is applied on a loan-by-loan level, rather than at the firm level. This definition is standard in credit risk studies and has been used in the academic survival literature by Dirick et al. (2017).

3.2 Data

We use proprietary data provided by the Central Bank of Mexico (see Table 1). The data includes all domestic and foreign bank loans granted to micro and small and medium enterprises (SMEs), which are part of the Mexican banking sector. The sample covers both public and privately owned banks, and all foreign subsidiary banks based in Mexico. The full sample consists of 19,204,566 individual loan records that correspond to 1,893,927 firm loans provided by banks and regulated multiple purpose financial societies during January 2010 to April 2018.[23] Of these 635,247 loans are given to 'micro' firms, 836,464 loans to 'small' firms and 422,216 loans to 'medium' sized firms.[24] The central bank collects information at the loan level from commercial banks on a monthly basis. Moreover, financial authorities assign identification (i.e., id) to each bank loan and this allows tracking of the history of each loan payment reported by the banks. The loan's interest rates are updated by the banks throughout the loan's history along with the days of payment delay. We use panel data structure in our survival analysis. Since the outliers might be a concern, we winsorize the interest rate, number of

[22] See Banxico (2015, p.21).
[23] Berrospide and Herrerias (2015, pp.35-38) provide a brief summary describing the origin and evolution of multiple purpose financial societies in Mexico.
[24] Table 1 provides detailed definitions of the variables and the data source.

employees, and gross sales at 1% level. We use the natural logarithm of number of employees, total sales, and loan size to reduce the right skewness effect in these variables. Having data from all banks at the sector level facilitates exploration whether grouping loans by different loan characteristics is sufficient to model correlation in loans' 'time-to-default'. Studies that examine bank loans survival focus on only one or a few banks and do not capture system-wide features such as the role played by the systemic importance of the originating bank versus non-D-SIBs.

3.3 Selection of variables

The dependent variable is the loan's 'time to default' or the duration of time it takes firms to default on their bank loans. Since our model computes the PD for any time t, we use days as our unit of analysis. For each loan record, we register the number of days in each month and construct a default status variable that tracks the payment history of each individual loan. We analyze only a single default and assume that firms default on their loan when the loan's due date is more than 90 consecutive days. In our multivariate survival models, we use micro variables loan specific micro variables (i.e., loan's size and its interest rate) and firms' characteristics (i.e., the firm's industry, number of employees, and annual turnover). We also use a discrete dummy variable that takes a value 1 if the loan was originated by a D-SIB. Gupta et al. (2015) documents that there is a large diversity in terms of credit risk characteristics within the broad micro/SMEs category. To mitigate this issue we examine time to default for 'micro', 'small' and 'medium' sized enterprises separately. Ignoring the difference between these loans could lead to biased estimates of firms' loan lifetimes and their survival rates.[25,26]

[25] Figure D1 in Appendix D illustrates the monthly evolution of the Kaplan-Meier survival rates for 'micro', 'small' and 'medium' enterprises. The Kaplan-Meier estimator is a nonparametric technique of survival analysis used to generate survival curves. It is also known as the product limit estimator (see Kaplan and Meier, 1958 for details). This estimator is particularly useful to compare survival patterns between two or more groups. The survival rate of 'small' enterprises has the steepest rate of decay and crosses with its 'micro' counterpart after year 3. This is regarded as the highest risk level, while 'medium' firms have the lowest risk. Figure D2 in Appendix D shows the hazard curves for the three firm loan sizes, and each of these curves exhibits a fairly similar hump-shaped functional relationship.

[26] Table F1 in Appendix F shows that the survivor functions between micro and small sized loans are statistically different. This result supports the regulatory firm size definition used in this paper to classify loans into three categories.

The most important micro variable under analysis is the loan's interest rate. It is the only time-varying micro variable for each loan record and it measures the loan price, which serves as a control for the ex-ante firm risk. As expected, following the literature on the risk taking channel of monetary policy, the interest on loans is expected to have a negative impact on survival rates indicating that survival time is shortened as interest rates increase. Holmes et al. (2010) find that a higher interest rate has a negative impact only on micro firms survival and not on small or medium-sized firms survival.

The expected signs for the number of employees are mixed. Holmes et al. (2010, p.186) find evidence that the initial plant size as measured by the number of employees has a negative impact on the survival of micro-enterprises, but a positive influence on small and medium-sized enterprises. This is because as the size of the firm increases, it approaches a minimum efficient scale level of output, and the firm may benefit from facing a reduced risk. However, it is possible that medium firms incur greater costs, as they need to recruit qualified personnel. In relation to the loan size, it could be argued that high instalment size can contribute to loan default and as such increase credit risk. However, this may not be the case as other factors, such as the collateral, may very well be linked to the loan size. Naturally credit risk varies at the industry level and we include discrete binary variables to control for this effect. We use six industries in our analysis (agriculture, commerce, construction, communications and transport, manufacturing, service).

Selection of macroeconomic variables to control for systematic risk varies widely in firm failure or firm loan default prediction studies and there is no consensus in the literature. Following Carling et al. (2007), we assess the impact of four macro variables: (i) consumer confidence, (ii) economic activity, (iii) yield curve proxy, and (iv) inflation rate. From a theoretical perspective, we expect that these variables will have an impact on the firms' loan default likelihoods. We add the inflation rate because it may have an impact on the firm's loan performance in emerging markets. Generally, in emerging economies, inflation is an important factor possibly for two reasons. Firstly, inflation has a higher level and heightened volatility. Stated differently, the price impacts on firm costs may be significant and might influence the loan survival

Table 1. Definition of variables.

Variables (unit of analysis)	**Definition**	**Source**
Time to default (in days)	Is the time elapsed (measured in days) until default occurs. This is the dependent variable of the model	Banxico
Employees (#)	Number of employees at the time of loan origination	CNBV
Total sales (in USD)	Annual sales at the time of loan origination	CNBV
Loan size (in USD)	Size of the loan at the time of loan origination	CNBV
Interest rate (%)	Interest rate of the loan	CNBV
D-SIBs	Binary discrete variable that takes 1 if the loan has been granted by one of the seven domestic systemically important banks and zero otherwise. This variable is used in most cases in the mean equation of the regression. Alternatively, as a grouping variable to model dependence among bank loans 'time-to-default' to micro/SME firms when using the so-called shared frailty models.	CNBV
Fixed Effects by industry	There are six firm sectors and fixed effects are taken into account in our regressions by including indicator variables. The six indicator variables are: (i) Agriculture, 1 if the enterprise is in the agricultural sector and zero otherwise; (ii) Commerce, 1 if the enterprise is in the commerce sector and zero otherwise; (iii) Construction: 1 if the enterprise is in the construction sector and zero otherwise; (iv) Communications & Transport (C&T), 1 if the enterprise is in the C&T sector and zero otherwise; Manufacturing, 1 if the enterprise is in the manufacturing sector and zero otherwise; Service, 1 if the enterprise is in the service sector and zero otherwise.	CNBV
Industry	This is a discrete counting variable that is used in the shared frailty model to group bank loans to micro/SMEs according to the firm industry. It takes six possible values: i for Agriculture, ii for Commerce, iii for Construction, iv for Communications and Transport; v for Manufacturing; vi for Service.	
Firm_id	Firms may receive more than one loan from the same bank or from more than one bank. This is a discrete counting variable that is used in the shared frailty model to group bank loans to micro/SMEs by firm.	
Inflation rate (%)	Computed based on the Consumer Price Index.	INEGI
Yield curve proxy (%)	Difference between the Mexican 10-year bond and the sovereign 1 month risk free rate (i.e., CETES 28)	Banxico
Consumer confidence	It is based on households plans for major purchases and their economic situation, both currently and their expectations for the immediate future.	Banxico
Economic activity	The index of global economic activity (IGAE) is a short-term indicator for GDP. The index is computed as the result of weighted information on production from all the sectors in the economy, and follows the same methodology of the National Accounting System (seasonally adjusted)	INEGI

Source: Banco de México, National Institute of Statistics and Geography and National Banking and Securities Commission.
Notes: This table lists the set of dependent variables and covariates, along with their respective definition, that we use to fit the standard survival models and the shared frailty models. The last column lists the source of our data CNBV is the acronym in Spanish for the National Banking and Securities Commission; INEGI is the acronym in Spanish for the National Institute of Statistics and Geography; and Banxico is the acronym in Spanish for the Central Bank of Mexico. All variables are available on a monthly basis. We have five time-varying variables: (i) loan's interest rate; (ii) inflation rate; (iii) yield curve proxy; (iv) consumer confidence; (v) economic activity. All microeconomic or regulatory variables are proprietary information, while all macro variables are publicly available.

period. Secondly, inflation might have a pervasive effect on SMEs' investment and project planning as interest rates tend to be higher when inflation increases. The impact of inflation on loan survival is interesting to academics and relevant to practitioners and central bankers in emerging markets. On one side, higher inflation distorts prices in the economy and may lead to high default risk. A significant increase in prices is likely to increase the cost payable on inputs to a greater extent than the corresponding profits, threatening the firm's survival. This impact might be heightened for SMEs in that these enterprises have smaller buffers to mitigate price impacts compared to large corporate.[27] On the other side, higher inflation may reduce the real value of the firm's debt service payments, and could contribute in reducing the likelihood to default.

Mexican households' expectations of the future macroeconomic development or consumers' confidence index, leads us to believe that worsening expectations are associated with increasing micro/SMEs loan default rates. The index of Mexican consumers' confidence is collected from the survey data produced by the Mexican National Institute of Statistics and Geography[28] (INEGI, by its acronym in Spanish). In the same vein, we expect higher aggregate economic activity index to reduce default risk. SMEs and micro firms may be more committed to avoid a loan default if they expect higher future economic activity.

The yield curve proxy is an important long-term indicator of future real activity. We use the difference between the Mexican 10-year bond and the sovereign one month risk free rate (i.e., CETES 28) as a proxy measure of the yield curve. In a nutshell, a steeper curve anticipates higher future economic activity, while an inversion of it is a forerunner of an economic contraction. Bauer and Mertens (2018) provide evidence that in developed economies, such as in the United States, an inverted yield curve is a reliable predictor of future recessions.[29]

We follow Carling et al. (2007) and use macro variables in levels. Regarding the number of lags, we acknowledge that it takes time for the macro variables to have an

[27] SMEs are more sensitive than large corporates to price impacts for three reasons. First, SMEs may not be able to transmit the cost increase into the final price of the good sold, as many of these firms are price takers due to intense competition. Second, SMEs do not have the flexibility that large corporates have to reduce the cost of their operations. Third, SMEs face more difficulties to leverage in order to comply with their short-term obligations.

[28] See Table 1 for details and INEGI's website for a description of this index.

[29] An inverted yield curve occurs when shorter-term rates have higher interest rates than longer-term ones.

impact on SMEs loan performance. The time to exert an impact on SME survival depends on a number of factors inherent to the jurisdiction under analysis such as the firm's sector, the type of economic shock and its intensity, as well as the institutional development of the country. Some of the previous studies use two periods' lags, while others use up to three periods (see Carling et al., 2007 and Gupta et al 2018). Based on information from the market intelligence unit at the central bank, the macro variables are expected to have a significant impact over the three month periods. Thus, a period of three months is adequate to control for the impact of the macro economy on the firm's loan performance. Based on this rationale, we lag all macroeconomic variables by three periods in order to control for possible endogeneity.

3.4 Parametric survival model

This section describes the parametric AFT survival models used in our analysis and the performance evaluation method used to assess the prediction accuracy of each loan default model.[30]

3.4.1 Survival analysis and accelerated failure time model

There are a number of events of interest that may take place when banks originate micro/SMEs' loans as a certain proportion of these may: (i) default; (ii) be repaid early; (iii) be restructured (e.g., to extend loan contract characteristics, typically referring to a loan's term extension); (iv) be transferred, either via a securitization (e.g., the bank may sell a pool of illiquid loans to any other financial entity by transforming them into a security) or sold to any other financial entity as part of a loan portfolio operation; or (v) simply mature (i.e., be fully repaid at the end of the loan term). Survival analysis assumes that all loans will default and there is no adjustment for the fraction of loans that perform.[31] We have right-censored data where default is not observed during the sample period for a large fraction of loans. In survival analysis it is assumed that censoring occurs randomly and has no relation with reasons that

[30] Section E.1 in Appendix E provides an elementary introduction to survival analysis.

[31] Mixture cure rate models adjust survival models for the fraction of loans that perform. In this regard, our probability to default estimates based on unadjusted survival curves are in any case more conservative from a risk management perspective.

possibly explain loan default. We collect data starting with loans originated from January 2010 onwards and omit outstanding loans available from previous periods.[32] Consider that each loan has a history of $k=1,\dots,m$ periods and we have $j=1,\dots,n$ loans in our data with a trivariate response (t_{0jk}, t_{jk}, d_{jk}) representing a loan record or period of observation (t_{0jk}, t_{jk}] ending in either failure ($d_{jk}=1$) or right-censoring ($d_{jk}=0$).[33] Each loan has multiple records such that loan j in time k has x_{jk} covariate vector values. In this setting, the loan's survival time is associated with macroeconomic, firm and loan characteristics not only when the loan is issued, but also during the loan's life. Let $T \in [0,\infty)$ be the loan's time to default or time until a loan either defaults or leaves the sample as a result of non-default events (e.g., loan matures). For a given survivor function, $S(t)$, the density function is obtained as $f(t) = -\frac{d}{dt}S(t)$, while the hazard function (i.e., the instantaneous rate of default) is obtained as $h(t) = \frac{f(t)}{S(t)}$. For each jth loan, it is possible to define for each kth point-in-time observation the survival time as a function of p explanatory variables as $S(t_{jk}) = S(t_{jk} \mid x_{1,jk},\dots,x_{p,jk})$ and similarly define $f(t_{jk})$ and $h(t_{jk})$. Parametric models[34] can be written as: (i) linear regression; (ii) log-time metric also known as accelerated failure time (AFT); and (iii) hazard rate (see Cleves et al. (2010, p.232)). Some models are flexible and can be defined in more than one form (e.g., the exponential or the Weibull), while others are limited to a single metric (e.g., lognormal, log-logistic or generalized gamma for the AFT metric, see Cleves et al.

[32] There was a major change in the structure and format of the regulatory layout starting mid-2009 and fully implemented until end-2009. Due to this, it was not possible to identify and match the exact date of origination (among other characteristics) for outstanding loans that were originated during previous periods to January 2010. On the other hand, it is not possible to use a left-censoring approach because this is used when the loan is never under observation and it is only known that the loan failed between the onset of risk date and the time when censoring ends (see Cleves et al. (2010, p.34). Moreover, our data are observed during the full sample period and there is no need to accommodate any type of truncation.

[33] For each loan record, we register explicitly the beginning and ending times using two variables t_0 and t_1, and we record the default status at the end of the span in variable d, where t_0 is the beginning record, t_1 is the survival or censoring time (measured in days, e.g., 28, 30 or 31 depending on the month). This counting process format where a triplet (t_0, t_1, d) is passed to STREG to record observations is used for histories where there are either right-censored, left-truncated or interval-truncation. In our sample, the first record for all our loans has $t_{0j1}=0$ and this confirms that the observation is not left-truncated. Notice that if a loan has several spells (i.e., observations), then all observations for this loan, except the first one, will be left-truncated. The log-likelihood in STATA is defined for the general case, when the observation might be left-truncated.

[34] Except for linear regression form, parametric survival models are estimated in Stata using the 'STREG' command.

(2010, p.233)). A simple and computationally efficient form to estimate the survival model is to use the Accelerated Failure Time Model (AFT) (see Cleves et al. (2010, pp. 239-241)) as shown in the following equation:

$$\ln\left(t_{jk}\right) = \beta_0 + \beta_1 x_{1jk} + \cdots + \beta_p x_{pjk} + \ln\left(\tau_{jk}\right) \tag{1}$$

where t_{jk} is the time to loan default for loan j in time span (i.e., observation) k and it is measured using days based on unit per loan record, β are the covariate coefficients to be estimated from the data using maximum likelihood technique (see Cleves et al. (2010, pp. 245-246)), x_{djk} are the time-varying or time-constant p covariates ($d=1,\ldots,p$), and $\ln(\tau_{jk})$ is a random quantity that follows an assumed parametric distribution with density $f()$. In this setting, $\exp(-(\beta_0 + \beta_1 x_{1jk} + \cdots + \beta_p x_{pjk}))$ is called the acceleration parameter and the covariates accelerate (i.e., $\exp(-(\beta_0 + \beta_1 x_{1jk} + \cdots + \beta_p x_{pjk})) > 1$) or decelerate (i.e., $\exp(-(\beta_0 + \beta_1 x_{1jk} + \cdots + \beta_p x_{pjk})) < 1$) the effect of time. In the case of an acceleration (deceleration) the coefficient is negative (positive), time passes faster (slower) and loan default is expected to occur sooner (later).[35] The survival function depends on the distribution assumed for τ_{jk}.[36] For example, if we assume that τ_{jk} is distributed as lognormal with parameters β_0 and σ, (i.e., $\tau_{ji} \sim \text{lognormal}(\beta, \sigma)$. The AFT can be defined[37] as:

$$S\left(t_{jk} \middle| x_{1jk}, \ldots, x_{pjk}\right) = 1 - \Phi\left(\frac{\ln\left(t_{jk}\right) - \left(\beta_0 + \beta_1 x_{1jk} + \cdots + \beta_p x_{pjk}\right)}{\sigma}\right) \tag{2}$$

where $\Phi()$ is the CDF for the standard Gaussian (normal) distribution and σ is known as a strictly positive ancillary parameter.

[35] See Cleves et al. (2010, p. 240).

[36] Stata can fit up to six parametric models: exponential, Weibull, Gompertz, log-normal, log-logistic and generalized gamma (see Cleves et al. (2010, pp.245-282) for details). As an example, if: (i) $f()$ is the normal density, we obtain the lognormal regression model; (ii) $f()$ is the logistic density, then the loglogistic regression arises; (iii) $f()$ is the extreme-value density, the exponential and the Weibull regression models appear; (iv) if $f()$ is log-gamma density, then the generalized gamma regression model is obtained.

[37] Since covariates accelerate time by a factor $\exp(-(\beta_0 + \beta_1 x_{1jk} + \cdots + \beta_p x_{pjk}))$, Cleves et al. (2010, p.270) show that the survival function can be derived as shown in eq. (2).

3.4.2 Accelerated failure time model with shared frailty model

In a shared or constant frailty model it is assumed that the frailties are time-invariant and loans are allowed to share the same frailty value.[38] This assumption is relevant because sharing a frailty value is a source that creates dependence between those loans that share frailties, whereas conditional on the frailty those loans are independent. For data consisting of *n* groups with the *i*th group comprised of n_i loans (*i=1,...,n*), in the multivariate survival model, shared frailty is introduced as an unobservable multiplicative effect α on the hazard, so that conditional on the frailty:

$$h\left(t_{ijk}\left|x_{1jk},...,x_{njk},\alpha\right.\right)=\alpha\, h\left(t_{ijk}\left|x_{1jk},...,x_{njk},\alpha\right.\right) \tag{3}$$

where α is some random positive quantity assumed to have mean one (for purposes of model identifiability) and variance θ. Gutierrez (2002. p. 24) shows that the individual survival function conditional on the frailty is $S(t_{jk}\mid x_{1jk},...,x_{njk},\alpha)=\{S(t\mid x_{1jk},...,x_{njk})\}^{\alpha}$ where survival function from a survival model may include ancillary parameters. In particular, when α is distributed as gamma[39] with mean one and variance θ, the survival function becomes:

$$S_{\theta}\left(t_{jk}\left|x_{1jk},...,x_{pjk}\right.\right) \;=\; \left[1 \;-\; \theta\; ln\{S\left(t_{jk}\left|x_{1jk},...,x_{pjk}\right.\right)\}\right]^{-1/\theta} \tag{4}$$

For example, in the case of the log-normal AFT regression, the conditional hazard and conditional survival function for an individual loan are given by:

$$h\left(t_{ijk}\left|x_{1ijk},...,x_{nijk},\alpha_i\right.\right)=\alpha_i\,\frac{\frac{1}{t_{jik}\sigma}\,\phi\left(\frac{\ln t_{ijk}-\left(\beta_0+\beta_1 x_{1ijk}+\cdots+\beta_p x_{pijk}\right)}{\sigma}\right)}{\sigma t_{ijk}\left[1-\Phi\left(\frac{\ln t_{ijk}-\left(\beta_0+\beta_1 x_{1ijk}+\cdots+\beta_p x_{pijk}\right)}{\sigma}\right)\right]} \tag{5}$$

$$S\left(t_{ijk}\left|x_{1ijk},...,x_{nijk},\alpha_i\right.\right)=\left\{1-\Phi\left(\frac{\ln t_{ijk}-\left(\beta_0+\beta_1 x_{1ijk}+\cdots+\beta_p x_{pijk}\right)}{\sigma}\right)\right\}^{\alpha_i} \tag{6}$$

where $\phi()$ and $\Phi()$ are the density function and cumulative distribution function for the standard Gaussian (normal) distribution. We test if the loan time to default is correlated in loans grouped into: (i) domestic systemically importance of the originating

[38] According to Winkle (2010), Clayton (1978) was the first to study frailty, although he did not use the notion. Frailty has been studied in great Depth by several different authors (see Winkle (2010) for detail).
[39] The frailties α_i are assumed to be identically and independently distributed random variables with a common density function f(α, θ), where θ is the parameter of the frailty distribution.

bank; (ii) firm industry or (iii) individual firm level. To control for these correlations, we use a shared frailty term (i.e., random intercept that adjusts the level in regression) in the Accelerated Failure Time Model (AFT). A shared frailty model is the analog of a regression model with random effects where the frailties are shared among groups of individuals and are randomly distributed across groups. It is important to highlight that a limitation of this shared frailty model is that dependence or association is always positive. Moreover, if we randomly select any pair of life times, the correlation between them will be always the same. The correlated frailty model is an extension of the shared frailty model where individuals in a group share only a part of the frailty and it allows inclusion of an additional correlation parameter to address both positive and negative dependence. Also, the design of the shared frailty model imposes a limitation as it assumes that the unobserved factors are the same within the cluster (Wienke, 2010). Heterogeneity may vary during the loan lifetime.

3.5 Performance evaluation

To examine the prediction performance of the estimated AFT survival models, we use the nonparametric receiver operating characteristics (ROC) curve along with the Gini-coefficient (GC) and Kolmogorov-Smirnov (K-S) statistic computed from the area under the ROC curve.[40,41] The GC can be used to evaluate the consistency of predictions in the developed model. The K-S statistic is applied to measure the distance between the failed and non-failed distributions at the optimal cut-off points. The ROC curve is the area beneath the plot of detecting true positive rate (i.e., sensitivity defined as the case where an observed loan default is classified by the model as expected default) and false negative rate (i.e., 1-specificity, defined as the case where an observed loan default is classified as expected non-default) for an entire range of possible

[40] The Gini coefficient can be calculated as GC=2(AUROC-0.5). A GC of 1 shows that the predictions are fully consistent, while a GC of 0 means that predictions are fully inconsistent. In turn, following Anderson (2007), the K-S statistic is approximately equal to 0.8×GC. A K-S equal to 0.8 suggests that the distance between the two distributions is at its maximum, whereas a K-S equal to 0 suggests that the two distributions are fully overlapped.

[41] It is also possible to perform calibration exercises using metrics such as the log score or Brier score. Calibrated probabilities are important for risk assessment and pricing. Calibration is required when the estimated value of PDs which are grouped into buckets diverges with the observed default rate. The supervisor will approve the rating buckets if there is heterogeneity between risk buckets and homogeneity within risk buckets. Calibration is important to ensure that the model yields valid joint probabilities across calendar time. The Mexican bank supervisory agency (i.e., CNBV) is the entity in charge of performing this exercise. In this paper, we focus specifically on prediction benefits.

probability values or cut-points. The computations compare the estimated probability to default, over the next year, with the value of each cut-point. In contrast to binary regression models, in the survival context, forecasting assessment is possible for any future point-in-time of the survival curve (see Heagerty et al., 2000). According to Gupta et al. (2018, p.447), the AUROC is the most popular and widely used method in the banking industry for evaluating a predicted PD ability to forecast the event of interest, namely the loan default over the next year.

In the AUROC the larger area, the better is the prediction. The size of the AUROC varies between zero and one. An AUROC size between 1 and 0.90 is regarded as excellent; an AUROC between 0.9 and 0.8 is very good; an AUROC between 0.8 and 0.7 is good; an AUROC between 0.7 and 0.6 is fair, while an AUROC between 0.6 and 0.5 is poor (see Hosmer et al. (2013)). An AUROC of 1 denotes a model with perfect prediction accuracy while an AUROC of 0.5 suggests that the model is random and its prediction accuracy is negligible. To test the statistical equality of any two ROC areas related to the same sample, we use the Chi-squared statistic which is based on an algorithm designed and developed by DeLong et al. (1988).[42] Our hold-out sample size is large and our ROC curve is concave. For instance, Gupta et al., (2018) find step ROC curve instead of concave due to a small hold-out sample.

Madorno et al. (2013) proposed a very simple way to compute the PD for any future point-in-time or risk horizon as a function of the conditional survival curve as:

$$PD\left(t_{jk}+b\middle|x_{1jk},\dots,x_{njk}\right)=1-\frac{S\left(t_{jk}+b\middle|x_{1jk},\dots,x_{njk}\right)}{S\left(t_{jk}\middle|x_{1jk},\dots,x_{njk}\right)} \tag{7}$$

where b is measured in days and the survival model may or may not include a shared frailty model. If the model includes a shared frailty model, then $S(t_{jk}+b\,|\,x_{1jk},\dots,x_{njk})$ and $S(t_{jk}\,|\,x_{1jk},\dots,x_{njk})$ as defined in eq.(2) should be substituted by $S_\theta(t_{jk}+b\,|\,x_{1jk},\dots,x_{njk})$ and $S_\theta(t_{jk}\,|\,x_{1jk},\dots,x_{njk})$ as defined in eq.(4). For instance, if we are interested to compute the one-year PD, then b=365. In contrast to the traditional one-year logistic model, parametric survival models depend explicitly on the time t and this makes the models more versatile because they do not require the future value of the explanatory variables to be known in order to forecast any desired risk horizon.

[42] This statistic is available in Stata with command 'roccomp'.

4. Results and discussions

Our analysis starts with discussion of the descriptive statistics of our covariates and the correlations. Next, we discuss the criteria used for selecting an appropriate distribution for the parametric AFT survival models. Our multivariate discussions include AFT survival model with and without frailty (i.e., sign and statistical significance of coefficients), the model's fit and its predictive performance.

4.1. Descriptive statistics and correlation

Analysis of summary statistics provides essential steps in understanding the nature of the covariates and the potential measurement problems that may arise in the estimation process due to extreme values.[43] Table 2 presents the steady-state (i.e., long-term average) default rate for micro, small and medium enterprises' loans. The event of interest is firm loan defaults and default characteristics of our sample. Table 2 reveals that the loan default rate is inversely related to the firm size (i.e., smaller firms have higher default rates). The default rate of micro firms (7.61%) is almost twice that of small firms (4.23%). Since we intend to assess the role of frailty grouped by D-SIBs, industry and individual firm level to predict micro/SMEs loans, we examine how the loan default rates vary when grouping defaults for each of these groups. Table 3 shows the firm's default rate assuming that when the firm defaults on one of its loans, it also defaults on all of its outstanding loan obligations. The results of Table 2 show that firm default rate is inversely related to firm size, but there are two differences in Table 3 with respect to the loan default rate reported in Table 2. First, firm default rates are greater than loan default rates, independent of firm size. Second, the difference in firm default rates between the three types of firms is smaller compared to the loan default rates as shown in Table 2.

[43] Table D3 in Appendix D provides bank and regulated multiple purpose financial societies (MPFS) market share based on the number of loan originations in Mexico. The 'micro', 'small' and 'medium'-size aggregrate share of MPFS as a group increases with firm size: 1.27% for 'micro', 2.95% for 'small', and 4.15% for 'medium'-sized loans, respectively. Table D4 in Appendix D shows the ownership structure of Mexican foreign bank subsidiaries.

Table 2. Loan default rate

Enterprise Size	*Loans Failed*	*Loans Non-failed*	*Loan Number*	*Loan Default rate*
Micro	48,373	586,874	635,247	7.61%
Small	35,349	801,115	836,464	4.23%
Medium	7,803	414,413	422,216	1.85%

Source: Banco de México, authors' calculations.
Notes: This table displays the sub-classification of default rate statistics among bank loans to 'micro', 'small' and 'medium' enterprises for analysis period January 2010 to April 2018.

Table 3. Firm default rate

Enterprise Size	*Firms Failed*	*Firms Non-failed*	*Firm Number*	*Firm Default rate*
Micro	34,746	160,822	195,568	17.77%
Small	18,624	108,041	126,665	14.70%
Medium	3,088	19,862	22,950	13.46%

Source: Banco de México, authors' calculations.
Notes: This table reports the sub-classification of default rate statistics among bank loans to 'micro', 'small' and 'medium' enterprises grouped by firm id for analysis period January 2010 to April 2018. A firm may have more than one loan with one or many banks. To compute these statistics, we assume that if a firm defaults on one of its loans, then the firm files for bankruptcy and defaults on all its outstanding obligations with the banking sector. In other words, this table displays a proxy of firms that survived and of those which failed during the sample period.

Table 4 reveals that the loan default rate for D-SIBs or large banks is higher compared to small banks and consistent regardless of the firm size. For instance, the default rate of micro firm loans originated by D-SIBs (9.50%) is almost three times that of loans originated by non-D-SIBs (3.17%). Unfortunately, we cannot conclude whether loans originated by D-SIBs are riskier or safer, because we don't know when default occurred. It could happen that loans granted by D-SIBs are safer in the short run, but not in the long run as default may take some time to occur. Also, bank loans in Mexico are plain vanilla in the sense that there is a period of similar monthly payments.[44] Also, the loan default rate varies widely by industry as shown in Table 5. For example, the default rate of commerce micro firm loans (6.81%) is smaller than that of construction (10.05%). Although the analysis described in Tables 2 to 5 is useful, these default rates

[44] In Mexico, there are no balloon bank loan types to micro, small and medium sized enterprises, which are characterized by an initial period of low or no monthly payments, followed by a final period where the firm is required to pay off the full balance in a lump sum.

do not indicate when the default occurs. To analyse time-to-default, we use survival analysis.

Table 4. Loan default rate grouped by D-SIBs

Enterprise Size	*D-SIBs*				*Non-D-SIBs*			
	Loans Failed	*Loans Non-failed*	*Loan number*	*Loan Default rate*	*Loans Failed*	*Loans Non-failed*	*Loan number*	*Loan Default rate*
Micro	42,364	403,453	445,817	9.50%	6,009	183,421	189,430	3.17%
Small	26,725	410,739	437,464	6.11%	8,624	390,376	399,000	2.16%
Medium	5,721	260,874	266,595	2.15%	2,082	153,539	155,621	1.34%

Source: Banco de México, authors' calculations.
Notes: This table shows the sub-classification of default rate statistics among bank loans to 'micro', 'small' and 'medium' enterprises grouped by the bank's domestic systemically importance for analysis period January 2010 to April 2018. The D-SIBs group comprises the seven largest banks of the Mexican banking sector. As of April, 2018, the Mexican banking sector comprised 49 banks. There are two additional banks that have recently been authorized to enter the banking sector, but these do not form part of our sample. It is important to point out that the Mexican banking sector is highly concentrated in D-SIBs. As of April2018, the so-called D-SIBs group controls 78.4 percent of the banking sector assets, 81.9 percent of the bank's loan portfolio, 82.5 percent of the bank's customer' deposits and 79.4 percent of the total bank's equity in the system. A breakdown of banks market share based on the number of loans granted is available in table 3D in Appendix D.

Table 6 presents the descriptive statistics by mean, standard deviation, minimum and maximum values for the dependent variable (i.e., firm loan 'time to default') and the micro and macro covariates. It is clear that the average time to default decreases when the firms are larger in size. The average number of employees increases with firm size and it is around 4 for micro, 15 for small and 60 for medium companies. The average annual sales amount is approximately 61K USD for micro, 1.8M USD for small and 6.8M USD for medium. The mean loan is approximately 66K USD for micro, 114K USD for small and 148K USD for medium firms. As expected, the average loan's interest rate decreases with firm size and it is 14.06% for micro, 12.81% for small and 10.82% for medium. Most micro and SME bank loans are channelled to satisfy the financial needs of the commerce, service and manufacturing sectors, whereas construction, agriculture and communications attract only a few of the bank loans. Most of the loans have been granted by the D-SIBs and it is approximately 70% for micro, 52% for small and 63% for medium firms. Maximum and minimum values for all micro variables are consistent and provide good boundaries. The average inflation rate for the period under study is 3.94%. During the same period, the difference between the 10 years and 1 month bonds is 2.06% (i.e., the average yield curve proxy). On average, the consumer confidence index is 90, while the economic activity index is 102%. It is evident from our univariate analysis that micro firms are different from small and medium enterprises as expected.[45]

Table 5. Loan default rate by industry

Enterprise Size	*Agriculture*				*Commerce*			
	Loans Failed	*Loans Non-failed*	*Loan number*	*Loan Default rate*	*Loans Failed*	*Loans Non-failed*	*Loan number*	*Loan Default rate*
Micro	2,509	26,971	29,480	8.51%	17,875	244,492	262,367	6.81%
Small	1,202	39,210	40,412	2.97%	14,647	334,117	348,764	4.20%
Medium	421	17,795	18,216	2.31%	3,022	209,139	212,161	1.42%

Enterprise Size	*Construction*				*Communications and transport*			
	Loans Failed	*Loans Non-failed*	*Loan number*	*Loan Default rate*	*Loans Failed*	*Loans Non-failed*	*Loan number*	*Loan Default rate*
Micro	3,370	30,157	33,527	10.05%	1,940	23,224	25,164	7.71%
Small	3,415	59,689	63,104	5.41%	1,154	33,292	34,446	3.35%
Medium	1,090	32,349	33,439	3.26%	164	12,359	12,523	1.31%

Enterprise Size	*Manufacturing*				*Services*			
	Loans Failed	*Loans Non-failed*	*Loan number*	*Loan Default rate*	*Loans Failed*	*Loans Non-failed*	*Loan number*	*Loan Default rate*
Micro	6,604	81,636	88,240	7.48%	18,016	185,433	203,449	8.86%
Small	6,663	174,806	181,469	3.67%	9,035	163,613	172,648	5.23%
Medium	1,890	111,322	113,212	1.67%	1,279	31,449	32,728	1.67%

Source: Banco de México, authors' calculations.
Notes: This table shows the sub-classification of default rate statistics among bank loans to 'micro', 'small' and 'medium' enterprises grouped by industry for the analysis period January 2010 to April 2018.

4.2. Multivariate parametric survival analysis models

In this section, we discuss our parametric multivariate survival analysis models. We use the Akaike Information Criterion (AIC) for micro, small and medium loan enterprises and test among different parametric distributions to identify the distribution that

[45] Table D5 to D7 in Appendix D report on the pairwise correlation matrix for 'micro', 'small' and 'medium'-sized enterprises, respectively.

Table 6. Descriptive statistics of micro and macro variables

Firm size	**Micro firms**[a]				**Small firms**[b]				**Medium firms**[c]			
Variable	*Mean*	*STD*	*Min*	*Max*	*Mean*	*STD*	*Min*	*Max*	*Mean*	*STD*	*Min*	*Max*
Dependent variable												
Time to default (in years)	1.14	1.14	0.078	8.33	0.77	0.90	0.08	8.33	0.54	0.73	0.078	8.33
Independent variables												
Employees (#)	4	3	1	10	15	12	1	50	60	47	1	250
Total sales (in USD)	61,028	71,884	4,679	343,410	1,814,817	1,648,796	95,881	8,585,239	6,820,844	4,874,416	46,624	21,500,000
Loan size (in USD)	66,973	102,643	10,000	999,888	114,056	148,004	10,000	999,985	148,905	198,743	10,000	999,993
Loan's interest rate (%)	14.065	3.792	3.290	27.980	12.813	3.472	3.290	23.190	10.822	3.433	3.300	21.300
Agriculture	0.046	0.210	0	1	0.048	0.214	0	1	0.043	0.203	0	1
Commerce	0.413	0.492	0	1	0.417	0.493	0	1	0.502	0.500	0	1
Construction	0.053	0.224	0	1	0.075	0.264	0	1	0.079	0.270	0	1
Communication & Transport	0.040	0.195	0	1	0.041	0.199	0	1	0.030	0.170	0	1
Manufacturing	0.136	0.343	0	1	0.215	0.411	0	1	0.268	0.443	0	1
Service	0.312	0.463	0	1	0.203	0.402	0	1	0.077	0.267	0	1
D-SIBs	0.702	0.457	0	1	0.523	0.499	0	1	0.631	0.482	0	1
Inflation (%)	3.94	1.07	2.13	6.77	3.94	1.07	2.13	6.77	3.94	1.07	2.13	6.77
Yield curve proxy (%)	2.06	1.00	-0.17	3.42	2.06	1.00	-0.17	3.42	2.06	1.00	-0.17	3.42
Consumer confidence index	90.10	5.02	68.49	100.01	90.10	5.02	68.49	100.01	90.10	5.02	68.49	100.01
Economic activity index	102.07	6.48	89.69	112.78	102.07	6.48	89.69	112.78	102.07	6.48	89.69	112.78

Source: Banco de México, authors' calculations.
Notes: (a) the number of bank loans to 'micro' firms is 635,247; (b) the number of bank loans to 'small' firms is 836,464; and (c) the number of bank loans to 'medium' firms is 422,216. This table reports the descriptive statistics of all micro and macro variables of bank loans to 'micro', 'small' and 'medium' enterprises for the analysis period January 2010 to April 2018, following the default definition discussed in Section 3.2. To compute descriptive statistics, we used values of the time to default for the last record for each loan. This is because there are multiple records for each loan and this implies that we cannot use population mean time to default (Cleves et al. (2010, pp. 91-92)). Censoring implies that the estimate of our mean in Table 6 for the time to default is downward biased. Cleves et al. (2010, Sec. 8, p. 92) shows that it is not possible to estimate the mean or median survival time using standard non-survival calculations with censored observations. Among the microeconomic variables, only the interest is time-varying. All other variables are time constant and their value is determined at the time of loan origination. All macroeconomic variables are time-varying. A full description of each variable is available in Table 1.

provides the best fit.[46] Finally, for each of the three types of firms' loans, we estimate five parametric models (i.e., M1 to M5) and assess the models' fit and their predicting performance.

4.2.1. Choosing between non-nested parametric models

There are six parametric distributions for the AFT model, which includes: (i) exponential, (ii) Weibull, (iii) Gompertz, (iv) log-normal, (v) log-logistic and (vi) generalized gamma. The most flexible modelling for the common types of hazard functions is the generalized gamma, but it cannot be used with shared frailty models and hence we focus on the remaining five parametric distributions. To determine the appropriate distribution for our data, we use the AIC test criteria.[47] Typically, the preferred model is the one with the lowest AIC. To the best of our knowledge, there is no study addressing the validity of applying AIC in the context of a shared frailty model.

Table 8D in Appendix D shows the log likelihoods and AIC values for micro, small and medium-sized loan enterprises. Based on the AIC criterion, the lognormal is the best model for the data regardless of the firm size and thus we use lognormal distribution in our study.

4.2.2. Model-building strategy

Among model-building strategy is including all theoretically motivated covariates without considering their significance in the univariate analysis (see Carling et al., 2007). Some studies refine the model and omit insignificant predictors (p-value>0.05). An alternative strategy is to perform univariate regression for each variable and retain covariates that have p-values of less than 0.25 (see Gupta et al., 2018, p.456). This strategy is useful when dealing with many variables. In our study, the number of covariates is manageable and there is no need for such an approach. We follow a simple approach to develop our multivariate model and variable selection. We use a sequential

---

[46] Some of the distributions analysed in this paper include/nest other distributions as special cases. For example, the generalized gamma includes the exponential, Weibull and log-normal as special case. For nested models, the Wald or Likelihood Ratio (i.e. LR) statistic can be used to test the fit of the distribution that includes other distributions as special cases.

[47] Akaike (1974) designed for non-nested models a test that punish each model's log likelihood to take into account the number of parameters being estimated and then comparing log likelihoods.

approach as described below. First, we search the distribution that provides the best parametric fit for our data based on the AIC (see Akaike, 1974) and we test a very simple model that serves as a benchmark. Second, we use a simple AFT model that incorporates only microeconomic variables (M1). Third, following Carling et al. (2007), we add a set of macro variables to assess the contribution of controlling for common risk (M2). We use M2 as a benchmark to test the benefit of loan default prediction when introducing shared frailty. Consistent with Carling et al. (2007), we acknowledge the importance of macroeconomic variables for the default risks. It is worth noting that we label these two models (i.e., M1 & M2) as standard AFT models, which do not incorporate frailties. Fourth, we analyse three different models (M3 to M5) where we group shared frailty by (i) industry (M3), (ii) firm (M4) and (iii) bank's domestic systemically importance (M5). Similar to the previous studies, we use a horse race approach in terms of model forecasting to assess the benefit of using a shared frailty model at the system-wide level.[48] Finally, we examine both models' goodness of fit and their predictive performance using a hold-out sample area under ROC curve.

To assess the forecasting power of the five parametric models we use ROC curve using a hold-out sample validation approach. We start by estimating an AFT model based on the period from January 2010 to December 2015 as our sample period. Then, we use the period from January 2016 to April 2018 as our hold-out sample. Finally, utilizing the information stemming from each sample, we estimate the PD over a one-year horizon and compute the AUROC, GC and K-S for this period (i.e., for the out-of-sample). For completeness, we also assess the fit of our models using AIC values and use the chi-square statistic using the Wald test to determine whether at least one of the parameters is statistically significant.

4.2.3. AFT parametric survival models for micro/SMEs loans

Tables 7 to 9 report the results of the AFT models (i.e., M1 to M5) estimated for bank loans to micro, small and medium sized enterprises respectively. In our estimation framework, we have not included the number of employees and total firm sales in the same analysis. This is because the labour variable is a key variable affecting the

[48] For completeness, we previously tested for multicollinearity and we find no evidence of multicollinearity.

production of the firm (total sales) in the production function. Hence, we report the results for the case where only employees are included as an independent variable.[49] The tables show the results for the AFT models with and without a shared frailty model using firm and loan characteristics, and macroeconomic variables. Since we use a lognormal distribution, we report the estimated standard deviation in logarithms[50] (i.e., ln(sigma) or $\ln(\sigma)$), and also the estimated shared frailty variance parameter (i.e., ln(theta) or $\ln(\theta)$).[51] For each variable, we assess the coefficient in terms of sign, size, significance and robustness.

It is evident from Tables 7 to 9 that the interest rate has a negative impact on the time to default, but for medium sized firms the interest rate is not significant in M4 of Table 9. These results suggest that the impact of the interest rate on default time is consistent for all firms, regardless of their sizes. In fact, the coefficients for the interest rate remains robust (i.e., the coefficient size remains similar across different specifications) in all specifications except for M4 in Table 9. A possible explanation is that there are many firms in the sample, which results in a large number of groups compared to the number of units when classified by D-SIBs (i.e., two groups) or firm industry (i.e., six firm sectors). This also leads to a relatively large frailty variance estimate (i.e., theta estimate is the largest in Table 7 and it is 1.65 for micro, 2.02 for small (Table 8) and 2.33 for medium firms in Table 9). The size of ln(theta) grouped by firm has an impact on the value of all estimated coefficients irrespective of firm size.

Tables 7 to 9 also demonstrate that firms characteristics (i.e., number of employees at loan origination) have a significant, albeit mixed impact on the time to default of micro and small firms. This is consistent with the findings of previous studies (see Gupta et al., 2015). In contrast to the evidence reported for developed countries (see Holmes et al., 2010), the number of employees has a negative influence on micro,

---

[49] In unreported results, we have analysed total sales as an independent variable instead of the number of employees. The results do not differ and are available from the corresponding author.

[50] For the natural logarithm of sigma, Stata provides its coefficient estimate, standard error, Z-Statistic and its corresponding p-value. To compute the sigma estimate, we have to compute the exponential of the reported coefficient. Ancillary parameters such as σ are usually restricted to be strictly positive, and a Delta method is required to compute the coefficient's standard error. Expressing and modelling sigma in logarithm mean that it can assume any value on the real line and there is no need to use unconventional methods (e.g., Delta-method) which are not free of criticism, especially to perform inference in the boundaries of the parameter space.

[51] The shared frailty is gamma distributed with mean 1 and finite variance theta for all survival models that include this feature. Moreover, the theta coefficient reported in Tables 7 to 9 is expressed in natural logarithm. To compute the theta estimate, we have to compute the exponential of the reported coefficient.

but a positive for small firms and insignificant for medium enterprises. This suggests a considerable difference in the factors affecting the time to default of micro enterprises vis-à-vis small and medium firms. Regarding the loan size at loan origination, the coefficient expected sign is ambiguous and may depend to a large extent on the loans collateral or its guarantor. This information is the key determinant of the loss given default.[52] We find that loan size is negative and significant for micro and small enterprises, but positive for medium firms.

It is evident from Tables 7 to 9 that macro variables have a significant impact on the time to default of SMEs. From Tables 7 to 9 the sign of the coefficient for the yield curve proxy is negative for micro and small enterprises and insignificant for medium firms. This is contrary to the positive expected relationship based on economic theory. Part of the reason is that the expected sign can be explained using the following argument. According to Sánchez (2018), the flat yield curve in Mexico prevailing during the past year may not necessarily suggest that a recession is forthcoming, due to two reasons. The first is that the yield curve proxy and its predictive power have not been widely researched in Mexico. Specifically, part of the problem is that the yield curve is complete and available only for the past ten years. Second, economic cycles in Mexico depend to a large extent on the economic activity in the US. Therefore, strong growth expectations in the US may partly restrain or neutralize the internal weakness weakness trends in the Mexican economic growth.

As expected, Tables 7 to 9 show that changes in the three-month lagged consumer confidence have a positive and significant impact irrespective of firm size. In our model, positive expectations of future macroeconomic developments are associated with decreasing default rates (i.e., higher survival rate). by contrast, a change in economic activity has a mixed effect: it is positive and significant for micro firms, negative and significant on small firms and insignificant (i.e., M2, M3, and M5) for medium firms. In principle, we would expect the evolution of the real sector of the economy in the short term to be positively related to the survival times regardless of firm size. The impact of the inflation rate is mixed for micro and SMEs time to default.

[52] In the expected loss approach, individual loan loss allowance is equal to PD×LGD×Exposure. Thus, collateral should not be included as a determinant of the PD if the ultimate objective is to use this approach. Instead, collateral should be included as a determinant of LGD. For an application of survival analysis in modeling LGD see Zhang and Thomas (2012).

Table 7. Multivariate AFT survival models for bank loans to micro-sized firms

Variable (Expected sign)	*Standard AFT*		*AFT + shared frailty*		
	M1	M2	M3	M4	M5
Interest rate (-)					
β	-0.0361[a]	-0.0340[a]	-0.0334[a]	-0.0198[a]	-0.0333[a]
SE	0.0009	0.0009	0.0009	0.0009	0.0009
ln(number of employees) (+)					
β	0.0379[a]	-0.0125[a]	-0.0103[a]	0.0387[a]	-0.0113[a]
SE	0.0037	0.0034	0.0035	0.0046	0.0035
ln(loan size) (+/-)					
β	0.0028	-0.0179[a]	-0.0144[a]	-0.0330[a]	-0.0140[a]
SE	0.0039	0.0036	0.0036	0.0039	0.0036
Yield curve proxy (+)					
β		-0.0763[a]	-0.0758[a]	-0.0467[a]	-0.0798[a]
SE		0.0045	0.0047	0.0041	0.0047
Consumer confidence index $_{t-3}$ (+)					
β		0.0210[a]	0.0229[a]	0.0260[a]	0.0220[a]
SE		0.0006	0.0006	0.0006	0.0006
Economic activity index $_{t-3}$ (+)					
β		0.0275[a]	0.0254[a]	0.0070[a]	0.0249[a]
SE		0.0005	0.0005	0.0006	0.0005
Inflation $_{t-3}$ (-)					
β		-0.0329[a]	-0.0315[a]	-0.0227[a]	-0.0355[a]
SE		0.0040	0.0041	0.0035	0.0041
D-SIBs (+)					
β	0.2109[a]	0.2490[a]	0.2716[a]	0.1604[a]	
SE	0.0094	0.0086	0.0084	0.0095	
Fixed effects industry (+/-)	Yes	Yes	No	Yes	Yes
Constant					
β	7.9400[a]	3.6763[a]	2.2950[a]	4.5718[a]	2.7220[a]
SE	0.0510	0.1079	0.1167	0.1031	0.1182
Lognormal standard deviation parameter (sigma or σ)					
ln (sigma)	0.1526[a]	0.0575[a]	-0.4377[a]	-0.2304[a]	-0.3300[a]
SE	0.0032	0.0033	0.0203	0.0044	0.0200
Shared frailty variance parameter (theta or θ)					
Industry					
ln (theta)			0.6252[a]		
LR- Chi² statistic			1272.0		

Table 7. (Continued)

Variable (Expected sign)	*Standard AFT*		*AFT + shared frailty*		
	M1	M2	M3	M4	M5
Firm id					
ln (theta)				1.6531[a]	
LR- Chi^2 stat				35000.0	
D-SIBs					
ln (theta)					0.2886[a]
LR- Chi^2 stat					976.4
Model's Goodnes of Fit					
Chi^2 statistic	2,839.7[a]	9,305.1[a]	9,064.8[a]	5,295.1[a]	8,951.5[a]
LogLikelihood	-155,054	-151,821	-151,501	-134,439	-151,725
AIC	310,130	303,672	303,024	268,910	303,480
N_{1-F} (# of loans)	635,247	635,247	635,247	635,247	635,247
N_{2-F} (# of observations)	8,708,445	8,708,445	8,708,445	8,708,445	8,708,445
D_F (# of defaults)	48,373	48,373	48,373	48,373	48,373
Model's performance measures					
AUROC-H	0.5365	0.5545	0.5543	0.5649	0.6208
Gini Coefficient	0.0731	0.109	0.1086	0.1298	0.2415
Kolmogorov Smirnov	0.0585	0.0872	0.0869	0.1039	0.1932
N_{1-O} (# of loans)	230,358	230,358	230,358	230,358	230,358
N_{2-O} (# of observations)	2,553,261	2,553,261	2,553,261	2,553,261	2,553,261
D_O (# of defaults)	12,050	12,050	12,050	12,050	12,050

Source: Banco de México, authors' calculations.

Notes: a (b) [c] significant at 1% (5%) [10%] level (two-sided test). This table displays the multivariate estimates for bank loans to 'micro' firms for the five AFT models (i.e., M1 to M5), with and without shared frailty model, using both micro (i.e., firm and loan characteristics) and macroeconomic variables. Some macro variables (i.e., consumer confidence index, economic activity index, inflation) have a 3 month time period lag as highlighted by the subscript 't-3'. All estimations follow the same loan default definition as discussed in section 3.2. Models M1 and M2 are estimated based on eq. (1), while models M3 to M5 are estimated using eq.(3). LR is the likelihood ratio. The Chi-squared statistic values reported in Model's goodness of fit are obtained using the Wald test, while the Chi-squared values reported for the shared frailty parameter were obtained using the likelihood ratio test. The subscript F for the number of loans (N_{1-F}), number of observations (N_{2-F}) and number of defaults (D_F) is used to define that these statistics were computed for the full sample. AUROC-H represents hold-out sample area under ROC curves. The subscript O for the number of loans (N_{1-O}), number of observations (N_{2-O}) and number of defaults (D_O) is used to define that these statistics are computed for the hold-out-sample.

Table 8. Multivariate AFT survival models for bank loans to small-sized firms

Variable (Expected sign)	*Standard AFT*		*AFT + shared frailty*		
	M1	M2	M3	M4	M5
Interest rate (-)					
β	-0.0418[a]	-0.0446[a]	-0.0391[a]	-0.0084[a]	-0.0409[a]
SE	0.0011	0.0011	0.0011	0.0012	0.0012
ln(number of employees) (+)					
β	0.0151[a]	0.0133[a]	0.0082[a]	-0.0109[a]	0.0090[a]
SE	0.0027	0.0027	0.0027	0.0038	0.0028
ln(loan size) (+/-)					
β	-0.0308[a]	-0.0365[a]	-0.0212[a]	-0.0393[a]	-0.0213[a]
SE	0.0035	0.0036	0.0036	0.0038	0.0037
Yield curve proxy (+)					
β		-0.0757[a]	-0.0613[a]	-0.0424[a]	-0.0687[a]
SE		0.0051	0.0052	0.0047	0.0053
Consumer confidence index $_{t-3}$ (+)					
β		0.0058[a]	0.0085[a]	0.0144[a]	0.0072[a]
SE		0.0007	0.0007	0.0006	0.0007
Economic activity index $_{t-3}$ (+)					
β		-0.0011[c]	-0.0007	-0.0275[a]	-0.0019[a]
SE		0.0006	0.0006	0.0007	0.0006
Inflation $_{t-3}$ (-)					
β		-0.0203[a]	-0.0072[c]	0.0130[a]	-0.0069
SE		0.0042	0.0043	0.0039	0.0044
D-SIBs (+)					
β	0.2605[a]	0.2668[a]	0.2825[a]	0.1857[a]	
SE	0.0074	0.0074	0.0071	0.0080	
Fixed effects industry (+/-)	Yes	Yes	No	Yes	Yes
Constant					
β	8.5561[a]	8.4697[a]	6.0377[a]	9.1860[a]	6.7435[a]
SE	0.0509	0.1325	0.1406	0.1300	0.1474
Lognormal standard deviation parameter (sigma or σ)					
ln (sigma)	0.0757[a]	0.0684[a]	-0.5158[a]	-0.2246[a]	-0.4136[a]
SE	0.0035	0.0035	0.0210	0.0049	0.0222
Shared frailty variance parameter (theta or θ)					
Industry					
ln (theta)			0.8747[a]		
LR- Chi² stat			1296.9		

Table 8. (Continued)

Variable (Expected sign)	Standard AFT		AFT + shared frailty		
	M1	M2	M3	M4	M5
Firm id					
ln (theta)				2.0240[a]	
LR- Chi^2 stat				50000.0	
D-SIBs					
ln (theta)					0.6211[a]
LR- Chi^2 stat					1395.8
Model's Goodness of Fit					
Chi^2 statistic	2,770.6[a]	3,219.7[a]	2,939.0[a]	3,706.4[a]	2,197.7[a]
LogLikelihood	-117,416	-117,192	-116,765	-92,277	-117,100
AIC	234,854	234,414	233,552	184,586	234,230
N_1 (# of loans)	836,464	836,464	836,464	836,464	836,464
N_2 (# of observations)	7,773,333	7,773,333	7,773,333	7,773,333	7,773,333
D (# of defaults)	35,349	35,349	35,349	35,349	35,349
Model's performance measures					
AUROC-H	0.5126	0.5188	0.5019	0.5485	0.5807
Gini Coefficient	0.0252	0.0377	0.0039	0.0969	0.1614
Kolmogorov Smirnov	0.0202	0.0302	0.0031	0.0775	0.1291
$N_{1\text{-}O}$ (# of loans)	349,920	349,920	349,920	349,920	349,920
$N_{2\text{-}O}$ (# of observations)	2,767,682	2,767,682	2,767,682	2,767,682	2,767,682
D_O (# of defaults)	12,354	12,354	12,354	12,354	12,354

Source: Banco de México, authors' calculations.

Notes: a (b) [c] significant at 1% (5%) [10%] level (two-sided test). This table displays the multivariate estimates for bank loans to 'small' firms for the five AFT models (i.e., M1 to M5), with and without shared frailty model, using both micro (i.e., firm and loan characteristics) and macroeconomic variables. Some macro variables (i.e., consumer confidence index, economic activity index, inflation) have a 3 month time period lag as highlighted by the subscript 't-3'. All estimations follow the same loan default definition as discussed in section 3.2. Models M1 and M2 are estimated based on eq. (1), while models M3 to M5 are estimated using eq.(3). LR is the likelihood ratio. The Chi-squared statistic values reported in Model's goodness of fit are obtained using the Wald test, while the Chi-squared values reported for the shared frailty parameter were obtained using the likelihood ratio test. The subscript F for the number of loans ($N_{1\text{-}F}$), number of observations ($N_{2\text{-}F}$) and number of defaults (D_F) is used to define that these statistics were computed for the full sample. AUROC-H represents hold-out sample area under ROC curves. The subscript O for the number of loans ($N_{1\text{-}O}$), number of observations ($N_{2\text{-}O}$) and number of defaults (D_O) is used to define that these statistics are computed for the hold-out-sample.

Table 9. Multivariate AFT survival models for bank loans to medium-sized firms

Variable (Expected sign)	*Standard AFT*		*AFT + shared frailty*		
	M1	M2	M3	M4	M5
Interest rate (-)					
β	-0.0564[a]	-0.0595[a]	-0.0485[a]	0.0026	-0.0515[a]
SE	0.0023	0.0024	0.0024	0.0027	0.0025
ln(number of employees) (+)					
β	-0.0080	-0.0069	-0.0083	-0.0275[b]	-0.0023
SE	0.0072	0.0072	0.0069	0.0122	0.0070
ln(loan size) (+/-)					
β	0.0049	-0.0009	0.0216[a]	0.0246[a]	0.0195[a]
SE	0.0063	0.0064	0.0062	0.0069	0.0063
Yield curve proxy (+)					
β		-0.0215[c]	-0.0024	-0.0168	-0.0014
SE		0.0114	0.0113	0.0107	0.0114
Consumer confidence index $_{t-3}$ (+)					
β		0.0047[a]	0.0073[a]	0.0094[a]	0.0067[a]
SE		0.0016	0.0015	0.0015	0.0015
Economic activity index $_{t-3}$ (+)					
β		0.0014	0.0021[c]	-0.0427[a]	0.0016
SE		0.0013	0.0012	0.0016	0.0012
Inflation $_{t-3}$ (-)					
β		0.0358[a]	0.0546[a]	0.0862[a]	0.0535[a]
SE		0.0098	0.0097	0.0096	0.0098
D-SIBs (+)					
β	0.1176[a]	0.1289[a]	0.1155[a]	0.0267	
SE	0.0156	0.0157	0.0151	0.0176	
Fixed effects industry (+/-)	Yes	Yes	No	Yes	Yes
Constant					
β	8.6013[a]	8.0296[a]	5.2253[a]	10.1571[a]	5.4420[a]
SE	0.0989	0.2825	0.2971	0.2864	0.3027
Lognormal standard deviation parameter (sigma or σ)					
ln (sigma)	0.1603[a]	0.1575[a]	-0.4600[a]	-0.1359[a]	-0.4294[a]
SE	0.0070	0.0072	0.0446	0.0107	0.0456
Shared frailty variance parameter (theta or θ)					
Industry					
ln (theta)			0.9998[a]		
LR- Chi² stat			424.4		

Table 9. (Continued)

Variable (Expected sign)	Standard AFT		AFT + shared frailty		
	M1	M2	M3	M4	M5
Firm id					
ln (theta)				2.3314[a]	
LR- Chi² stat				17000.0	
D-SIBs					
ln (theta)					0.9007[a]
LR- Chi² stat					234.2
Model's Goodness of Fit					
Chi^2 statistic	1,082.2[a]	1,149.3[a]	738.7[a]	1,186.1[a]	914.2[a]
LogLikelihood	-28,508.47	-28,474.93	-28,377.74	-20,137.52	-28,390.85
AIC	57,039	56,980	56,777	40,307	56,812
N_1 (# of loans)	422,216	422,216	422,216	422,216	422,216
N_2 (# of observations)	2,722,788	2,722,788	2,722,788	2,722,788	2,722,788
D (# of defaults)	7,803	7,803	7,803	7,803	7,803
Model's performance measures					
AUROC-H	0.7495	0.7980	0.7889	0.6089	0.7936
Gini Coefficient	0.4991	0.5961	0.5778	0.2179	0.5873
Kolmogorov-Smirnov	0.3993	0.4769	0.4623	0.1743	0.4698
N_{1-O} (# of loans)	174,802	174,802	174,802	174,802	174,802
N_{2-O} (# of observations)	836,329	836,329	836,329	836,329	836,329
D_O (# of defaults)	2,035	2,035	2,035	2,035	2,035

Source: Banco de México, authors' calculations.

Notes: a (b) [c] significant at 1% (5%) [10%] level (two-sided test). This table displays the multivariate estimates for bank loans to 'medium' firms for the five AFT models (i.e., M1 to M5), with and without shared frailty model, using both micro (i.e., firm and loan characteristics) and macroeconomic variables. Some macro variables (i.e., consumer confidence index, economic activity index, inflation) have a 3 month time period lag as highlighted by the subscript 't-3'. All estimations follow the same loan default definition as discussed in section 3.2. Models M1 and M2 are estimated based on eq. (1), while models M3 to M5 are estimated using eq.(3). LR is the likelihood ratio. The Chi-squared statistic values reported in Model's goodness of fit are obtained using the Wald test, while the Chi-squared values reported for the shared frailty parameter were obtained using the likelihood ratio test. The subscript F for the number of loans (N_{1-F}), number of observations (N_{2-F}) and number of defaults (D_F) is used to define that these statistics were computed for the full sample. AUROC-H represents hold-out sample area under ROC curves. The subscript O for the number of loans (N_{1-O}), number of observations (N_{2-O}) and number of defaults (D_O) is used to define that these statistics are computed for the hold-out-sample.

For instance, inflation has a negative impact on micro and small firms time to default, suggesting that higher inflation rates shorten their default times. However, inflation has a positive impact on time to default for medium firms. This result is interesting and unique for an emerging economy. Possibly, this is due to the fact that micro and small firms are more sensitive than medium enterprises to price shocks. Also, the tables show that the D-SIB bank indicator is significant and positive on the loan time to default for micro and SMEs.

The AIC values for the AFT models in Tables 7 to 9 indicate that shared frailty models have better fit, compared to the standard AFT models irrespective of the firm size. It is worth pointing out that the model with shared frailty by firm has the lowest AIC among frailty models regardless of the firm size indicating its better fit. The chi-squared statistic, which tests the null hypothesis that all parameters are equal to zero, is rejected for all models.

The out-of-sample AUROC varies in all specifications depending on the firm size. Table 7 shows that for micro firms, the model with the best statistically significant[53] prediction performance has a shared frailty grouped by D-SIBs (e.g., M5: AUROC is 62%) which is regarded as a fair performance[54]. All other AUROC are below the 56% threshold and have a similar performance. The models that include shared frailty by firm (e.g., M4: AUROC is 56%) and industry (e.g., M3: AUROC is 55%) are similar to the standard benchmark AFT model (e.g., M2: AUROC is 55%) and model that includes only micro variables (e.g., M1: AUROC is 54%). Perhaps, low values of AUROC might be due to lack of more financial information such as balance sheet data.

Table 8 shows a similar result for bank loans to small firms. The model with the best[55] prediction performance is a shared frailty model grouped by D-SIB (e.g., M5: AUROC is 58%) and outperforms other models. Shared frailty model grouped by firm (e.g., M4: AUROC is 55%) has a slightly better performance than the remaining three

[53] Table D9 in Appendix D reports significant pairwise differences at the 1% level between M5:AUROC and all other models (i.e. M1 to M4) for bank loans to micro-sized firms.
[54] This result may be driven by the lack of time-varying firm balance sheet information.
[55] Table D10 in Appendix D reports significant pairwise differences at the 1% level between M5: AUROC and all other models (i.e. M1 to M4) for bank loans to small-sized firms.

models (e.g., M1: AUROC is 51%, M2: AUROC is 52%, M3: AUROC is 50%).[56] Overall, results lead to the same conclusion as in the micro firm case (i.e., the best model in terms of prediction accuracy has a shared frailty by D-SIB).

Table 9 shows results for bank loans to medium enterprises. In contrast to the micro and small enterprises, there is a tie between the three models with similar performance (i.e., AUROC approximately equal to 80%).[57] One of the models is the standard AFT with micro and macro covariates (i.e., M2), while the remaining are shared frailty models grouped by D-SIB and industry (i.e., M3 and M5). The estimated AUROC of 80% for any multivariate model suggests that the classification performance is excellent. Part of the reason that explains this improvement is that the number of defaults is significantly lower compared to micro and small enterprises, which leads to lower classification errors. The shared frailty model based on D-SIBs does not outperform the standard AFT for the following two reasons. First, the full sample average loan default rate for medium enterprises is 1.85% compared to 4.23% and 7.61% for small and medium enterprises (see Table 2). Second, low levels of loan default rates are characterized by moderate or absence of loan default clustering effect. Thus, there is no need to consider the correlation between loans 'times to default'.

Table 7 and 8 show a positive significant coefficient for the D-SIBs variable in models M1 to M4. This result leads to conclude that D-SIBs have more expertise to originate loans that take longer to default compared to non-DSIBs. Moreover, in the long run, we also conclude that the number of defaults is relatively high compared to non-DSIBs (see Figure D4 in Appendix D). Models M1 to M4 omit the presence of default clustering attributed to the variability of the frailty across the two bank types (i.e., D-SIBs and non-D-SIBs). Model M5 addresses a significant source of risk that is ignored in models M1 to M4. Moreover, the parameter for the frailty is significant at the 1% level. In this context, model M5 in Table 7 and 8 shows that ignoring the variability

[56] In our sample, there is a large number of firms that have only one loan, while there is at the same time, a few firms that have more than one thousand loans. Having a large number of firms with a very different number of loans may create more variability than desired and this may lead to a deterioration of predictions. Lando and Nielsen (2010) have argued to aggregate firms to account for parent-subsidiary relationships when model frailties as these may be one entity with a similar risk rather than a number of companies. Unfortunately, we don't have information to group firm loans based on parent subsidiary relationships.

[57] Table D11 in Appendix D reports that the pairwise difference in AUROC among M2, M3 and M5 is not significant for bank loans to medium-sized firms.

of the frailty may lead to high costs in terms of prediction accuracy. The shared frailty coefficient captures the effect of loan defaults cluster in time.[58] Together these results indicate that it is important to incorporate a shared frailty by bank type to address loan unobserved heterogeneity.

Some of the previous studies (see Gupta et al., 2018, p.461) compute both in-samples and hold-out sample AUROC when the sample size of the hold-out sample is small. Typically, the shapes of ROC curves are steps rather than concave when the number of outcome events is very low. Furthermore, the estimates of AUROC might be misleading when drawing inferences regarding out-of-sample predictive ability of the forecasting model.[59] Our AUROC are smooth and concave due to the large hold-out sample size, suggesting that our AUROC results are robust.[60]

5. Conclusions and policy recommendations

This paper investigates the source of loan default clustering using proprietary loan level data provided by Mexican financial authorities. We focus on industry, firm level and bank systemic importance as possible channels of loan default clustering for the following reasons. First, there is a significant variation in loan default rates across industries. Hence, the numbers of external or internal shocks affecting these industries are likely to be correlated and drive the loan default clustering. Second, all outstanding loans granted to an individual firm in one way or another share the same idiosyncratic risk factor and thus loan default rates might be concentrated at the firm level. Therefore, a firm failure is expected to result in multiple loan defaults. Third and finally, is that systemically important banks have incentives to grant riskier loans compared to small banks because of the expected government bailout in the event of failure. Consequently,

[58] In model M5, we exclude the D-SIB variable in the AFT equation because it is perfectly correlated with the variable used to estimate the shared frailty parameter.

[59] Figure D1 in Appendix D shows a table that displays the area under the ROC curve for all our hold-out sample estimated models.

[60] Tables 9D, 10D and 11D in Appendix D report a matrix with the pairwise chi-squared statistics to test whether the AUROC of any pairwise models are statistically different for micro, small and medium-sized firms, respectively. For micro-sized firms, even though some AUROC differ in magnitude by less than 2%, the chi-squared test and the corresponding p-value suggest that all pairwise AUROC models are statistically different, except for the pairwise comparison between M2 and M3. This result supports the finding that the model with shared frailty by D-SIBs (i.e. M5) has the best prediction ability. For small-sized firms, the chi-squared test is significant in all cases and this support the finding that a shared frailty model by D-SIBs is superior to any of its contenders.

they are likely to apply lax lending standards when granting small loans, which lead to aggregate loan default rates to be subject to correlation risk. We use a survival model with shared frailty to investigate the channels of loans default clustering. Our results show that bank's systemic importance is a key source of default clustering for loans granted to micro and small enterprises. To examine the forecasting power of our model, we use the Receiver Operating Characteristics (ROC) curve and find that survival model with shared frailty are better in terms of forecasting micro and small loan defaults than the standard survival model.

Our study provides two important policy recommendations. The first is that financial authorities should use a regulatory formula based on survival analysis to compute the term structure of default probability as a benchmark to ameliorate the regulatory burden on non-D-SIBs. Survival analysis techniques provide an ideal framework to implement either IFRS9 or CECL frameworks for the entire banking sector consistent with actual aggregate loan default rates term structure. This issue is also important for banks to recognize loss in a timely and accurate way, as well as reducing the stand-alone or solvency risk of failure. Moreover, it is important for investors to compare loan loss provisions based on a similar benchmark to promote the transparency of the bank balance sheet. This is important to promote confidence in both banks' asset quality and capital adequacy.

The second is related to our findings which suggest that a PD regulatory formula should include a shared frailty by a bank's systemic importance for those loans characterized by large average value of aggregate loan default rate (e.g., micro and small). In fact, a key post-crisis aim is developing policy tools to mitigate systemic risk. Including frailty would lead to a loan loss provision model that captures default clustering attributed to the variability of the frailty across the two bank types. This would be consistent with the fact that D-SIBs in our sample originate loans characterized by default clusters that occur in the long term at a relative intense rate. Finally, it is worth emphasizing that more evidence at the international and loan type level is desirable to assess whether default clusters stem from a different pattern. It would be interesting to identify if in the case of a developed banking sector, loans granted by large banks default sooner at a relative intense rate compared to small banks.

References

Abad, J., Suarez, J., 2018. The procyclicality of expected credit loss provisions. Center for Economic Policy Research Discussion Paper 13135. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3244507; accessed on February 21, 2020.

Agenor, P., Zilberman, R., 2015. Loan loss provisioning rules, procyclicality and financial volatility. Journal of Banking and Finance 61, 301-315.

Akaike, H., 1974. A new look at the statistical model identification. IEEE Transaction on Automatic Control 19, 716-723.

Altman, E.I., 1968. Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. Journal of Finance 23, 589-609.

Altman, E.I., Sabato G., 2007. Modelling credit risk for SMEs: Evidence from the US market. Abacus 43, 332-357.

Altman, E.I., Sabato, G., Wilson, N., 2010. The value of non-financial information in small and medium-sized enterprise risk management. The Journal of Credit Risk 2, 95-127.

Anderson, R., 2007. The Credit Scoring Toolkit: Theory and Practice for Retail Credit Risk

Management and Decision Automation (1st edn.). OUP, Oxford.

Azizpour, S., Giesecke, K., Schwenkler, G., 2018. Exploring the sources of default clustering. Journal of Financial Economics 129, 154–183.

Basel Committee on Banking Supervision (BCBS), 2011. Global systemically important banks: assessment methodology and the additional loss absorbency requirement. In: Rules Text, Basel Committee on Banking Supervision (BCBS).Basel Committee on Banking Supervision (BCBS), 2012. A framework for dealing with domestic systemically important banks. Available at: https://www.bis.org/publ/bcbs224.pdf; accessed on February 21, 2020.

Basel Committee on Banking Supervision (BCBS), 2013. Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement. In: Rules Text, Basel Committee on Banking Supervision (BCBS).

Basel Committee on Banking Supervision (BCBS), 2014. Supervisory framework for measuring and controlling large exposures. In: Standards, Basel Committee on Banking Supervision (BCBS).

Basel Committee on Banking Supervision (BCBS), 2017. Basel III: Finalising post crisis reforms. Available at: https://www.bis.org/bcbs/publ/d424.pdf; accessed on February 21, 2020.

Banco de México (Banxico), 2015. Reporte sobre las condiciones de competencia en el otorgamiento de crédito a las pequeñas y medianas empresas. Available at: http://www.banxico.org.mx/publicaciones-y-discursos/publicaciones/informes-periodicos/reporte-sobre-las-condiciones-de-competencia-en-lo/reporte-condiciones-competenc.html; accessed on February 21, 2020.

Batiz-Zuk, E., López-Gallo, F., Martínez-Jaramillo, S., and Solórzano-Margain, J., 2016. Calibrating limits for large interbank exposures from a system-wide perspective. Journal of Financial Stability 27, 198–216.

Bauer, J., Agarwal, V., 2014. Are hazard models superior to traditional bankruptcy prediction approaches? A comprehensive test. Journal of Banking and Finance 40, 432-442.

Bauer, M., Mertens, T., 2018. Information in the yield curve about future recessions Economics Letter, Federal Reserve Bank of San Francisco. Available at: https://www.frbsf.org/economic-research/files/el2018-20.pdf; accessed on February 21, 2020.

Berger, A., Miller, N., Petersen, M., Rajan, R., Stein, J., 2002. Does function follow organizational form? Evidence from the lending practices of large and small banks. NBER Working Papers 8752, National Bureau of Economic Research.

Berrospide, J. and Herrerias, R., 2015. Finance companies in Mexico: Unexpected victims of the global liquidity crunch. Journal of Financial Stability 18, 33–54.

Bharath, S., Shumway, T., 2008. Forecasting default with the Merton distance to default model. Review of Financial Studies 21, 1339-1369.

Box-Steffensmeier, J.M., De Boef, S., 2006. Repeated events survival models: The conditional frailty model. Statistics in Medicine 25(20), 3518-3533.

Cantú, C., Lobato, R., López, C., López-Gallo, F., 2020. A loan-level analysis of financial resilience in Mexico. Journal of Banking and Finance, doi: https://doi.org/10.1016/j.jbankfin.2020.105951

Campbell, J., Hilscher, J., Szilagyi, J., 2008. In search of distress risk. The Journal of Finance 63(6), 2899-2939.

Carling, K., Jacobson, T., Lindé, J., Roszbach, K., 2007. Corporate credit risk modeling and the macroeconomy. Journal of Banking and Finance 31, 845-868.

Chae, S., Sarama, R., Vojtech, C., Wang, J., 2018. The impact of the current expected credit loss standard (CECL) on the timing and comparability of reserves. Finance and Economics Discussion Series Working Paper 020. Available at: https://www.federalreserve.gov/econres/feds/files/2018020r1pap.pdf; accessed on February 21, 2020.

Chava, S., Jarrow, R., 2004. Bankruptcy prediction with industry effects. Review of Finance 8(4), 537-569.

Chava, S., Stefanescu, C., Turnbull, S., 2011. Modeling expected loss. Management Science 57 (7), 1267–1287.

Chen, J., Hill, P., 2013. The impact of diverse measures of default risk on UK stock returns. Journal of Banking and Finance 37, 5118-5131.

Clayton, D., 1978. A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. Biometrika 65, 141-151.

Cleves, M., Gutierrez, R., Gould, W., Marchenko, Y., 2010. An Introduction to Survival Analysis Using Stata. Stata Press, Texas.

Cox, D., 1972. Regression models and life-tables. Journal of the Royal Statistical Society. Series B (Methodological) 34, 187-220.

Covas, F., Nelson, W., 2018. Current expected credit loss: Lessons from 2007-2009. Banking Policy Institute Working Paper. Available at: https://bpi.com/wp-content/uploads/2018/07/CECL-Lessons-2007-2009-WP-July-12-2018.pdf; accessed on February 21, 2020.

Das, S., Duffie, D., Kapadia, N., Saita, L., 2007. Common failings: How corporate defaults are correlated. The Journal of Finance 62 (1), 93-117.

DeLong, E.R., DeLong, D.M., Clarke-Pearson, D.L., 1988. Comparing the areas under two or more correlated receiver operating characteristic curves: A nonparametric approach. Biometrics 44, 837-845.

Dirick, L., Claeskens, G., Baesens, B., 2017. Time to default in credit scoring using survival analysis: A benchmark study. Journal of the Operational Research Society 68, 652-665.

Duffie, D., Eckner, A., Horel, G., Saita, L., 2009. Frailty correlated default. The Journal of Finance 64(5), 2089-2123.

Duffie, D., Saita, L., Wang, K., 2007. Multi-period corporate default prediction with stochastic covariates. Journal of Financial Economics 83, 635-665.

Engelmann, B., 2018. Calculating lifetime expected loss for IFRS9: which formula is correct?. Working Paper, ssrn.com/abtract=3238632.

Financial Accounting Standards Board (FASB), 2016. FASB Issues new guidance on accounting for credit losses. FASB news release. Available at: https://www.fasb.org/jsp/FASB/FASBContent_C/NewsPage&cid=1176168232900; accessed on February 21, 2020.

Glennon, D., Nigro, P., 2005. Measuring the default risk of small business loan: A survival analysis approach. Journal of Money Credit and Banking 37, 923-947.

Gupta, J., Gregoriou, A., Ebrahimi, T., 2018. Empirical comparison of hazard models in predicting SMEs failure. Quantitative Finance 18(3), 437-466.

Gupta, J., Gregoriou, A., Healy, J., 2015. Forecasting bankruptcy for SMEs using hazard function: To what extent does size matter? Review of Quantitative Finance and Accounting 45, 845-869.

Gupta, J., Wilson, N., Gregoriou, A. Healy, J., 2014a. The value of operating cash flow in modelling credit risk for SMEs. Applied Financial Economics 24(9), 649-660.Gupta, J., Wilson, N., Gregoriou, A. Healy, J., 2014b. The effect of internationalisation on modelling credit risk for SMEs: Evidence from UK market. Journal of International Financial Markets Institutions and Money 31, 397-413.

Gutierrez, R., 2002. Parametric frailty and shared frailty survival models. The Stata Journal 2, 22-44.

Gutierrez, R., Carter, L., Drukker, D., 2001. On boundary-value likelihood-ratio tests. Stata Technical Bulletin 60, 15-18. Reprinted in Stata Technical Bulletin Reprints (10), 269-273. Stata Press, Texas, TX.

Haldane, A., G., and Madouros, V., 2012. The dog and the frisbee. Revista de Economía Institucional, 14 (27), 13-56.

Holmes P., Hunt A., Stone, I., 2010. An analysis of new firm survival using a hazard function. Applied Economics 42,185-195.

Hosmer, J., Lemeshow, S., Sturdivant, R., 2013. Applied Logistic Regression. (3rd ed.) Wiley, Hoboken, NJ.

Kaplan, E., Meier, P., 1958. Nonparametric estimation from incomplete observations. Journal of the American Statistical Association 53(282), 457-481.

Koopman, S.J., Lucas, A., Monteiro, A., 2008. The multi-stage latent factor intensity model for credit rating transitions. J. Econom. 142 (1), 399–424.

Koopman, S.J., Lucas, A., Schwaab, B., 2011. Modeling frailty-correlated defaults using many macroeconomic covariates. J. Econom. 162 (2), 312–325.

Lando, D. and Nielsen, M., 2010. Correlation in corporate defaults: Contagion or conditional independence? Journal of Financial Intermediation 19(3), 355-372.

Levin, G., López, C., López-Gallo, F., 2016. The impact of expected losses provisioning on credit growth: The case of Mexico. Mimeo.

Loudis, B., Ranish, B., 2019. CECL and the credit cycle. Finance and Economics Discussion Series 2019-061. Washington: Board of Governors of the Federal Reserve System. Available at: https://doi.org/10.17016/FEDS.2019.061; accessed on February 21, 2020.

Liu, J., 2004. Macroeconomic determinants of corporate failures: Evidence from the UK. Journal of Applied Economics 36(9),939-945.

Madorno, F., Mecatti, F., Figini, S., 2013. Survival models for credit risk estimation. Working paper.

Mills, M., 2011. Introducing Survival and Event History Analysis. Sage, London.

Monfort, A., Renne, J.-P., 2013. Default, liquidity, and crises: an econometric framework. Journal Financial Economics 11 (2), 221–262.

Petersen, M., 2004. Information: Hard and soft. Working paper, Northwestern University and NBER.

Pappas, V., Ongena, S., Izzeldin, M., Fuertes, A., 2017. A survival analysis of Islamic and conventional banks. Journal of Financial Services Research 51(2), 221-256.

Pereira, J., 2014. Survival analysis employed in predicting corporate failure: A forecasting model proposal. International Business Research 7(5), 9-20.

Sánchez, M., 2018. ¿Qué presagian las tasas de interés? El Financiero, Bloomberg. Available at: http://elfinanciero.com.mx/opinion/manuel-sanchez-gonzalez/que-presagian-las-tasas-de-interes; accessed on February 21, 2020.

Shumway, T, 2001. Forecasting bankruptcy more accurately: A simple hazard approach. Journal of Business 74,101-124.

Stackhouse, J., 2019a. Managing CECL change: How banks can prepare for the new accounting standard. Federal Reserve Bank of St. Louis. Available at: https://www.stlouisfed.org/on-the-economy/2019/september/cecl-how-banks-can-manage-change-prepare; accessed on February 21, 2020.

Stackhouse, J., 2019b. The CECL model: Accounting changes coming for banks. Federal Reserve Bank of St. Louis. Available at: https://www.stlouisfed.org/on-the-economy/2019/august/cecl-model-accounting-changes-coming-banks; accessed on February 21, 2020.

Sy, J., Taylor, J., 2000. Estimation in a Cox proportional hazards cure model. Biometrics 56, 227-236.

Tinoco, M., Wilson, N., 2013. Financial distress and bankruptcy prediction among listed companies using accounting, market and macroeconomic variables. International Review of Financial Analysis 30, 394-419.

Tsoukas, S., 2011 Firm survival and financial development: Evidence from a panel of emerging Asian economies. Journal of Banking & Finance 35: 1736-1752.

Wienke, A., 2010. Frailty models in survival analysis. CRC press.

Zhang, J. and Thomas, L., 2012. Comparisons of linear regression and survival analysis using single and mixture distributions approaches in modelling LGD. International Journal of Forecasting 28, 204-215.

Internet Appendix

Appendix A

This section discusses the D-SIB framework in Mexico. The Mexican financial banking and securities supervisor (CNBV) uses a score to measure any bank's systemic importance. The CNBV considers 15 banks characteristics or proxy variables, which they group into four classes, as shown below:

(i) bank size (1);

(ii) degree of interconnectedness (8);

(iii) substitutability/financial institution infrastructure (4); and

(iv) complexity (2).

Where the number in parenthesis refers to the number of variables in each of the four categories and each class has the same weight (i.e., 25%). Since the number of variables used as a proxy for each class varies, the weight of each variable decreases proportionally to the increasing number of proxy variables used for each of the class. Table A1 provides the definition of each variable and its corresponding weight (β_j). For any bank i (where $i=1,\ldots,k$) and for each bank's variable j (where j=1,…, 15), CNBV computes a score (SBV) based on the following specification:

$$\mathrm{SBV}_{i,j} = \frac{\mathrm{BV}_{i,j}}{\sum_{s=1}^{k} \mathrm{BV}_{i,s}} \times 10,000 \tag{A.1}$$

Where $BV_{i,j}$ is bank's i with variable value j. The bank's i score is intended to reflect its systemic importance (*SDSIBs*), which is computed as follows:

$$SDSIBs_i = \sum_{j=1}^{n} \beta_j SBV_{i,j} \tag{A.2}$$

where β_j is the bank's variable weight, n is the total number of variables available for each bank and SBV is bank's i value and its characteristic j. According to Mexican rules, any bank is identified as D-SIB when its score (SDSIBs) is greater than 350. In Mexico, D-SIBs must have a capital supplement that may vary in size (as a share of regulatory capital ratio) from 0.60% to 1.5% in addition to the 10.5% of the minimum regulatory capital ratio. The size of the buffer varies with the degree of systemic importance. Table A2 shows how the size of the buffer increases depending on the level of the bank's systemic importance score.

CNBV updates the list of D-SIBs each year. In this study, we use as reference banks that are shown in the 2016 list. Based on the list, CNBV identified the following seven banks as D-SIBs: BBVA Bancomer MX (BBVA, Spain), Citibanamex (Citigroup, USA), Banorte, Banco Santander MX (Banco Santander, Spain), HSBC Mexico (HSBC, UK), Scotiabank (Scotiabank, Canada), and Inbursa. Five out of these seven D-SIBs are foreign banks. The list remained the same until 2020 except for Inbursa, which was removed from the list. D-SIBs have a maximum period of four years to comply with this requirement and must have at least 25% of the capital increase annually. For further details, see https://www.dof.gob.mx/nota_detalle.php?codigo=5516120&fecha=14/03/2018, accessed on 3 November 2020.

Table A1. Variable definitions

	Definitions	Proxy for	Weight (β_j)	Time
1	Bank asset size (including both on- and off-balance sheet items).	Size	25%	At quarter end*
2	Aggregate size of bank deposits and loans held by the bank with other financial institutions.	Interconnection	3.13%	Average of the past 4 quarters
3	Size of bank holdings consisting of debt securities, commercial paper and bank certificates of deposit.	Interconnection	3.13%	Average of the past 4 quarters
4	Positive exposure of securities with other financial institutions: (Repo Debtors and Securities Loans).	Interconnection	3.13%	Average of the past 4 quarters
5	Debtors as a result of operations settlements.	Interconnection	3.13%	Average of the past 4 quarters
6	Creditors for liquidation of operations	Interconnection	3.13%	Average of the past 4 quarters

Table A1. (continued)

7	Loan size from bank and non-bank financial entities.	Interconnection	3.13%	Average of the past 4 quarters
8	Negative exposure of securities with other financial entities (creditors for repos and securities lending).	Interconnection	3.13%	Average of the past 4 quarters
9	Outstanding size of debt issues, commercial paper and certificates of deposit.	Interconnection	3.13%	At quarter end*
10	Assets size held in custody	Infrastructure	6.25%	Average of the past 4 quarters
11	Payments in national currency	Infrastructure	6.25%	Average of the past 4 quarters
12	Market makers or clearing partners.	Infrastructure	6.25%	At quarter end*
13	Participation in selected portfolios.	Infrastructure	6.25%	At quarter end*
14	Exposure size of trading and available-for-sale securities.	Complexity	12.50%	Average of the past 4 quarters
15	Exposure value in derivatives.	Complexity	12.50%	Average of the past 4 quarters

Source: Official Gazette of the Mexican Federation. For details, see https://www.dof.gob.mx/nota_detalle.php?codigo=5516120&fecha=14/03/2018, accessed on 3 November 2020.

Notes: *= At quarter end in either June or December. The information to compute these bank balance sheet variables is obtained from the regulatory report R01 "Minimum Catalogue". This report is fully described in Annex 36 of the "General provisions applicable to credit institutions", which was published in the Official Gazette of the Federation on 2 December 2005. According to this law, any bank's positive (i.e., purchase) and negative positions (i.e., sale) related to bank operations with derivatives, will be published by Banco de México. For further details, see https://www.dof.gob.mx/nota_detalle.php?codigo=5516120&fecha=14/03/2018, accessed on 3 November 2020. For an international reference, see Basel Committee on Banking Supervision (BCBS) (2012) for a list of principles and a framework for national authorities to identify and deal with D-SIBs.

Table A2. Degree of bank's systemic importance and size of capital supplement

Range of values to classify any bank's systemic importance score (SDSIBs, see eq. 1.2 in this appendix)	Degree of systemic importance	Size of additional capital supplement as a share of regulatory capital based on the degree of systemic importance
(350 ; 825]	I	0.60
(825 ; 1,300]	II	0.90
(1,300; 1,775]	III	1.20
(1,775 - 2,250]	IV	1.50
Greater than 2,250	V	1.50

Source: Official Gazette of the Mexican Federation. For details, see https://www.dof.gob.mx/nota_detalle.php?codigo=5516120&fecha=14/03/2018, accessed on 3 November 2020.

Notes: This table shows the degree of bank's systemic importance and size of additional capital supplement as a share of regulatory capital as defined in the Official Gazette of the Mexican Federation..

Appendix B

In this section we illustrate with a simple example the importance of modeling the *PD* term structure and how this affects the bank's expected loan loss allowance. We also show how a regulatory model based on a parametric survival function can be used as a simple mechanism to assist banks to implement either the International Financial Reporting Standard (IFRS 9) or the Current Expected Credit Loss (CECL) standards.

The impact of *PD* term structure on bank's loan loss allowance

Our approach can be explained in four steps. First, we describe a simple way to compute any loan *PD* term structure based on a one-year logistic regression model. We show how practitioners in the industry may use a simple one-year marginal loan *PD* estimate to compute cumulative *PD* for any desired risk horizon h to obtain a *PD* term structure. Specifically, we show how to compute a loan's annual marginal marginal *PD* over six risk horizons (i.e., h={1,2,3,4,5,6}) using logistic regression. Second, we show how to use aggregate actual default rates as a proxy for the *PD* term structure. We use our loan sample and compute four *PD* term structures for different loan types between January 2010 and April 2018. We show that four of the *PD* term structures based on actual default rates are inconsistent with the term structure estimated using logistic regression. Third, we use a very simple example and show that the underestimation of the expected loan loss is likely to be large using either a one-year or a lifetime logistic regression approach. Finally, we explain how a formula based on survival analysis provides a simple and consistent approach in line with actual practice and current CECL regulatory requirements.

B.1. Step 1: PD-term structure based on a one-year Logistic regression estimate

Typically, banks estimate the one-year (i.e., h=1) default likelihood $PD_{j,h}$ based on panel data or cross section data at the loan level j taking loan history k into account using a logistic model as follows:

$$PD_{jk,1} = \frac{1}{1+\exp(\beta_0 + \beta_1 x_{1,jk} + \ldots + \beta_p x_{p,jk})} \tag{B.1}$$

Where β are coefficients, $x_{p,jk}$ are 'p' explanatory variables, h refers to the loan's future risk horizon measured as a time fraction of a year (e.g., $h=1$ means one year in the future) and there are n loans in the portfolio (i.e., $j=1,\ldots n$) and each loan has several periods of history ($k=1,\ldots T$) where T is the most recent loan observation available. The dependent variable $Y_{jk,h}$ is a binary variable that takes value 1 if the loan defaults in one year and 0 otherwise. For simplicity of exposition, we assume that banks will use cross section data and we get rid of the k sub-index. If the bank uses annual data, the binary variable is related to the current period, while the explanatory variables are lagged by at least one period (i.e., one-year lag). Based on annual data, the banks traditionally estimate the logistic model for a fixed time horizon of one year, such that the fitted PD is annual (i.e., PD^{Annual}). To estimate PD for any future risk horizon, companies in the financial industry[61] typically use the cumulative probability as follows:

$$PD_{j,h}^{Cumulative} = 1 - \left(1 - PD_{j,1}^{Annual}\right)^{h} \tag{B.2}$$

Where h is the loan's risk horizon (i.e., tenure) and it takes values for future years such as 1, 2, 3, 5, 7 or 10 ($h \geq 1$). Specifically, h is measured as a time fraction of a year, so that h=1 is one-year, while h=1.5 is one year and a half. Interestingly, this formula assumes that the annual marginal probability grows at a decreasing rate over time.[62] This implies that the one-year marginal default probabilities for future risk horizons decrease with higher values of h. For instance, assume that based on borrowers' characteristics and macro variables, the bank estimates $PD_{j,1}^{Annual}$=1%. Then, the loan's cumulative probabilities are:

$$PD_{j,h=1}^{Cumulative} = 1 - \left(1 - PD_{j,1}^{Annual}\right)^{1} = PD_{j,1}^{Annual} = 1.00\% \tag{B.3}$$

[61] For an example, see the so-called "Starmine quantitative model" developed by Thomson Reuters available at: https://training.refinitiv.com/docs/attachments/videos/1639/attachment_1639.pdf; accessed on December 10, 2020.

[62] Using the exponential rule we can show that the first derivative with respect to h is $\frac{\partial}{\partial h}[1-(1-PD^{Annual})^{h}] = -\ln(1-PD^{Annual})(1-PD^{Annual})^{h}$, which is always positive, suggesting that the function is increasing. Moreover, using the exponential function we can show that the second derivative with respect to h is $\frac{\partial^2}{\partial h^2}[1-(1-PD^{Annual})^{h} = -\ln^2(1-PD^{Annual})(1-PD^{Annual})^{h}$ which is always negative, suggesting that the first derivative is a decreasing function of h.

$$PD_{j,h=2}^{Cumulative} = 1-(1-0.01)^2 = 0.0199 = 1.99\% \quad \text{(B.4)}$$

$$PD_{j,h=3}^{Cumulative} = 1-(1-0.01)^3 = 0.0297010 = 2.9701 \approx 2.97\% \quad \text{(B.5)}$$

$$PD_{j,h=4}^{Cumulative} = 1-(1-0.01)^4 = 0.039404 = 3.9404 \approx 3.94\% \quad \text{(B.6)}$$

$$PD_{j,h=5}^{Cumulative} = 1-(1-0.01)^5 = 0.0490099 = 4.9009 \approx 4.90\% \quad \text{(B.7)}$$

$$PD_{j,h=6}^{Cumulative} = 1-(1-0.01)^6 = 0.058519 = 5.8519 \approx 5.85\% \quad \text{(B.8)}$$

The marginal probability for the interval *h-1* to *h* is obtained as:

$$PD_{j,h}^{\text{Marginal}} = \Delta_h PD_j^{\text{Cummulative}} = PD_{j,h}^{\text{Cummulative}} - PD_{j,h-1}^{\text{Cummulative}} \quad \text{(B.9)}$$

Applying this formula to our case leads to:

$$PD_{j,1}^{\text{Marginal}} = PD_{j,1}^{\text{Cummulative}} - PD_{j,0}^{\text{Cummulative}} = PD_{j,1}^{Annual} - PD_{j,0}^{Annual} = 1\% - 0\% = 1\% \quad \text{(B.10)}$$

$$PD_{j,2}^{\text{Marginal}} = PD_{j,2}^{\text{Cummulative}} - PD_{j,1}^{\text{Cummulative}} = 1.99\% - 1\% = 0.99\% \quad \text{(B.11)}$$

$$PD_{j,3}^{\text{Marginal}} = PD_{j,3}^{\text{Cummulative}} - PD_{j,2}^{\text{Cummulative}} = 2.9701\% - 1.99\% = 0.9801\% \quad \text{(B.12)}$$

$$PD_{j,4}^{\text{Marginal}} = PD_{j,4}^{\text{Cummulative}} - PD_{j,3}^{\text{Cummulative}} = 3.9404\% - 2.9701\% = 0.9703\% \quad \text{(B.13)}$$

$$PD_{j,5}^{\text{Marginal}} = PD_{j,5}^{\text{Cummulative}} - PD_{j,4}^{\text{Cummulative}} = 4.9010\% - 3.9404\% = 0.9606\% \quad \text{(B.14)}$$

$$PD_{j,6}^{\text{Marginal}} = PD_{j,6}^{\text{Cummulative}} - PD_{j,5}^{\text{Cummulative}} = 5.8519\% - 4.9010\% = 0.9509\% \quad \text{(B.15)}$$

Overall, this simple model has the following property: $PD_{j,1}^{\text{Marginal}} \geq PD_{j,2}^{\text{Marginal}} \geq PD_{j,3}^{\text{Marginal}} \geq PD_{j,4}^{\text{Marginal}} \geq PD_{j,5}^{\text{Marginal}} \geq PD_{j,6}^{\text{Marginal}}$. This implies that the probability to default decreases as the risk horizon *h* increases.

B.2. Step 2: *PD* term structure based on aggregate actual default rates

It is important for inference purposes to have a sufficiently long period to analyze the loan's lifecycle. Figure B1 shows annual actual aggregate loan default rates for the

Mexican banking sector, while Figures B2 to B4 show the breakdown for firm type based on our sample of bank loans to micro, small and medium-sized firms, respectively. These annual aggregate default rates serve as a proxy for the annual average marginal probability to default of any bank loan. It is clear from these figures that annual actual aggregate default rates reach a maximum when the loan tenure has 3 years. This suggests that the method widely used by some firms to compute the cumulative *PD* for future risk horizons is misleading. Clearly, the *PD* term structure estimated using a one-year logistic regression decreases as h increases and its value is smaller than the *PD*'s implied by the annual aggregate actual default rates. The varying nature of the actual aggregate default rates for bank loans suggests that marginal probabilities increase significantly during the first three years as compared to the subsequent three years. In other words, the logistic *PD* term structures do not resemble actual default rates in which the *PD* first increases with h up to a maximum and then decreases to zero once the loan is fully repaid. Naturally, this could lead to a significant risk underestimation as measured by expected loan loss allowance.

B.3. Step 3: example to assess risk underestimation based on expected loss

Assume that a bank offers on 1 January 2021 a loan of $1,000,000 to a firm j which is expected to be paid in five annual instalments of $210,000 at year end.[63] Table B1 shows the expected cash flows. For simplicity, assume that loan's $LGD_{j,h}$ is equal to one for all risk horizons (i.e., loan's recovery rate is zero) and the discount rate (ρ_h) is zero (i.e., the time value of the cash flows is not considered). Assume that the loan's one-year $PD_{j,1}$ estimate using the logistic regression model is 1%, while the actual default rates are as shown in Figure B1. It is possible to assess the effect of the PD term structure on the expected loan loss allowance of any bank's loan using the following three approaches:

1. One year expected loan loss based on logistic regression (e.g., Mexican regulatory hybrid approach).
2. Lifetime expected loan loss based on logistic regression (e.g., CECL approach).

[63] For simplicity, we do not use any fixed or time varying interest rate. Instead, we provide the cash flow that will be paid at year end. Note that this assumption implies that $EAD_{j,h}$ is constant.

3. Lifetime expected loan loss using actual aggregate default rates term structure (e.g., CECL approach).

Typically, the CECL standard requires banks to maintain life-of-instrument estimates of credit losses (ECL) on financial assets (i.e., both performing and non-performing) and these requirements apply from loan origination (i.e., $h=0$). To estimate the lifetime expected loss for any firm loan j, we require three inputs: (i) the loan's probability to default ($PD_{j,h}$), (ii) the loss given default ($LGD_{j,h}$) and (iii) the Exposure at Default ($EAD_{j,h}$) as shown below:

$$E\left[L_{j,h}\right]=\frac{\sum_{h=1}^{H} PD_{j,h}\times LGD_{j,h}\times EAD_{j,h}}{\left(1+\rho_h\right)^h}=\sum_{h=1}^{H} PD_{j,h}\times LGD_{j,h}\times EAD_{j,h} \qquad \text{(B.16)}$$

Where h={1,2,...6} refers to the horizon, the PD is one of the term structure component and the focal point of our estimate. To compute the expected loss for a one-year horizon (i.e., $H=1$), the corresponding expected loss formula is:

$$E\left[L_j\right]=PD_{j,1}\times LGD_{j,1}\times EAD_{j,1} \qquad \text{(B.17)}$$

Table B1 shows results for the three approaches. Panel A in Table B1 shows the results for the one-year $PD_{j,1}$ estimate based on logistic regression, where the loan loss allowance is simply the expected loss for a one-year risk horizon (i.e., $PDj,_1$*Cash $\text{Flow}_{i,1}$). Since the $PD_{j,1}$ estimate is assumed to be 1%, the expected loan loss allowance is $2,100 (i.e., 0.01*210,000).

Panel B in Table B1 shows the CECL approach using the lifetime expected loss approach using logistic PD regression. The estimation of lifetime expected loss can be best explained in three steps. Step 1: compute the cumulative PD term structure (i.e., $PD_h^{Cumulative}$). Step 2: compute the marginal PD for each risk horizon h (i.e., $PD_h^{Marginal}$). Step 3: compute the loan's expected loss for each risk horizon h (PD_h*Cash Flow_h). Since the discount rate is zero, the time value of money is irrelevant and the expected loan loss allowance is simply the sum of expected loss cash flows $10,292. This value is almost five times higher than the value reported in Panel A.

Panel C of Table B1 shows the CECL using actual aggregate default rate term structure as a proxy for the loan's *PD* term structure. In particular, we use as PD estimates the values reported in Figure B1. Although the one year $PD_1^{Marginal}$ estimate is roughly 1%, there are at least four risk horizons (i.e., h={2,3,4,5}), where the *PD* estimate is significantly higher than 1% (e.g., $PD_2^{Marginal}$=5.40%; $PD_3^{Marginal}$=6.90%; $PD_4^{Marginal}$=3.10%; $PD_5^{Marginal}$=4.00%). Using these *PD* estimates yields an expected loan loss allowance of $42,840. This value is more than 20 times the value of the one-year expected loan loss approach and approximately four times the value of the lifetime expected loan loss using a *PD* term structure from logistic regression. These results indicate that logistic regression models significantly underestimate risk.

In principle, it is possible to estimate a logistic regression for different risk horizons (e.g., we could estimate six models using the following set of binary dependent variables $Y_h=\{Y_1, Y_2, Y_3, Y_4, Y_5, Y_6\}$, where default status varies with the risk horizon). However, this would be an inefficient approach because of the number of *PD* formulas involved (i.e., one formula to represent a different risk horizon). By contrast, survival models are designed to fit a *PD* term structure pattern consistent with Figures B1 to B4 using one single formula for all different horizons. Moreover, survival models are flexible because of the possibility to incorporate the effect of frailty factors to assess correlation between loan defaults. Overall, survival analysis provides a very efficient method as the *PD* depends explicitly on the risk horizon. Thus, we only need to estimate one single model to implement a simple formula to reflect any desired future risk horizon. The survival analysis captures the patterns of aggregate actual default rate term structure better than logistic regressions.

B.4. Step 4: using survival models to compute marginal *PD* without frailty

The survival function may be estimated from loan level data (using loan records) as follows:

$$\ln\left(t_{jk}\right)=\beta_0+\beta_1 x_{1,jk}+\cdots+\beta_n x_{n,jk}+\ln\left(\tau_{jk}\right) \tag{B.18}$$

Where t_{jk} is the time to default for loan j in time span (i.e., observation) k and it is measured using days as time unit per loan record; β are the vector of covariate coefficients to be estimated from the data using maximum likelihood technique; X_{jk} are the time-varying or time-constant vector of covariates (i.e., micro and macro variables); $\ln(t_{jk})$ is a random quantity that follows an assumed parametric distribution with density $f()$. If the model is an AFT that follows a lognormal distribution, then:

$$S\left(t_{jk} \middle| x_{1,jk},\ldots,x_{n,jk}\right) = 1 - \Phi\left(\frac{\ln\left(t_{jk}\right) - \left(\beta_0 + \beta_1 x_{1,jk} + \cdots + \beta_n x_{n,jk}\right)}{\sigma}\right) \tag{B.19}$$

To estimate PD, we can use Madorno et al. (2013) as follows:

$$PD\left(t_{jk} + h \middle| x_{1,jk},\ldots,x_{n,jk}\right) = 1 - \frac{S\left(t_{jk} + h \middle| x_{1,jk},\ldots,x_{n,jk}\right)}{S\left(t_{jk} \middle| x_{1,jk},\ldots,x_{n,jk}\right)} \tag{B.20}$$

Where t is measured in days and h is the risk horizon (measured as number of days). To estimate this model, the dependent variable t_j is the time to loan default and this variable is decomposed into two variables: one that registers the number of days within each loan record (i.e., time span) and another binary variable indicates the loan performance status (1 for default; 0 otherwise). To compare the marginal probabilities we compute:

$$PD_{jk,1}^{\text{Marginal}} = PD\left(t_{jk} + 365 \middle| x_{1,jk},\ldots,x_{n,jk}\right) = 1 - \frac{S\left(t_{jk} + 365 \middle| x_{1,jk},\ldots,x_{n,jk}\right)}{S\left(t_{j} \middle| x_{1,jk},\ldots,x_{n,jk}\right)} \tag{B.21}$$

$$PD_{jk,2}^{\text{Marginal}} = PD\left(t_{jk} + 730 \middle| x_{1,jk},\ldots,x_{n,jk}\right) = 1 - \frac{S\left(t_{jk} + 730 \middle| x_{1,jk},\ldots,x_{n,jk}\right)}{S\left(t_{jk} + 365 \middle| x_{1,jk},\ldots,x_{n,jk}\right)} \tag{B.22}$$

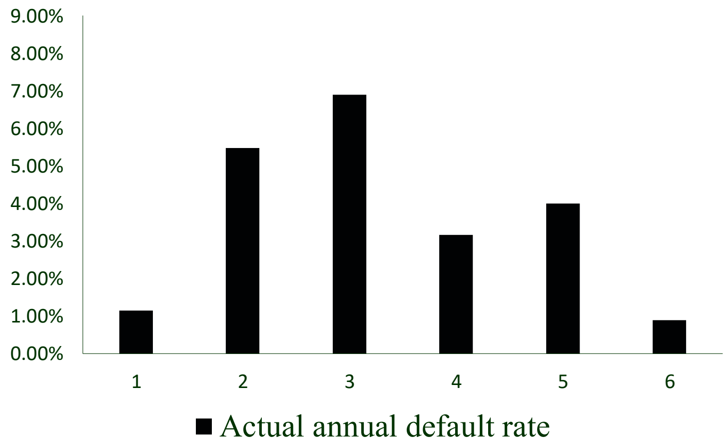
$$PD_{jk,3}^{\text{Marginal}} = PD\left(t_{jk} + 1095 \middle| x_{1,jk},\ldots,x_{n,jk}\right) = 1 - \frac{S\left(t_{jk} + 1095 \middle| x_{1,jk},\ldots,x_{n,jk}\right)}{S\left(t_{jk} + 730 \middle| x_{1,jk},\ldots,x_{n,jk}\right)} \tag{B.23}$$

$$PD_{jk,4}^{\text{Marginal}} = PD\left(t_{jk} + 1460 \middle| x_{1,jk},\ldots,x_{n,jk}\right) = 1 - \frac{S\left(t_{jk} + 1460 \middle| x_{1,jk},\ldots,x_{n,jk}\right)}{S\left(t_{jk} + 1095 \middle| x_{1,jk},\ldots,x_{n,jk}\right)} \tag{B.24}$$

$$PD_{jk,5}^{\text{Marginal}} = PD\left(t_{jk} + 1460 \middle| x_{1,jk}, \ldots, x_{n,jk}\right) = 1 - \frac{S\left(t_{jk} + 1825 \middle| x_{1,jk}, \ldots, x_{n,jk}\right)}{S\left(t_{jk} + 1460 \middle| x_{1,jk}, \ldots, x_{n,jk}\right)} \tag{B.25}$$

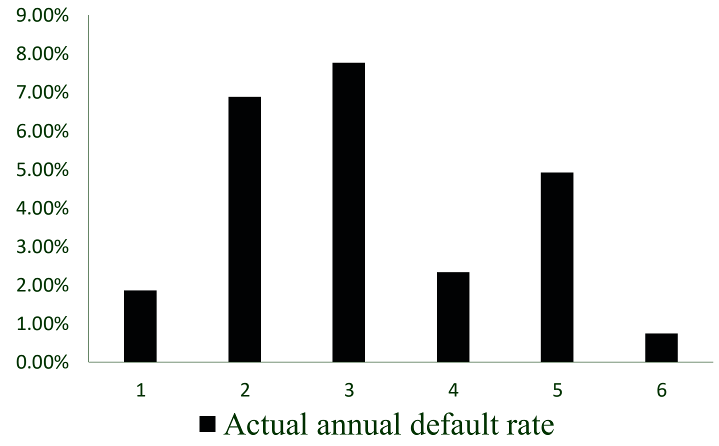
In this paper, we do not test the prediction benefits for all tenures. Standard practice in credit risk studies is assessing the performance only for one-year horizon (see Gupta et al., 2018). This is due to the fact that the sample is not sufficiently large to validate our out-of-sample forecasts for risk horizons greater than one-year.

Figure B1. Actual default rate term structure for bank loans to micro and SMEs over the sample period: January 2010 to April 2018.



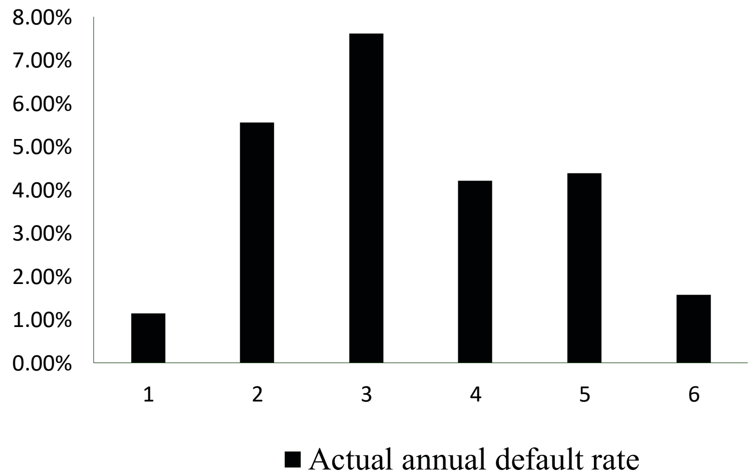
Source: Banco de México, authors' calculations.
Notes: This figure shows the actual annual default rates for bank loans to micro/SMEs for varying tenures over the sample period. The default rate per tenures is defined as the ratio between the marginal sum of loan defaults over the total number of loans outstanding during each tenure. The labels on the horizontal axis indicate the end of year.

Figure B2. Actual default rate term structure for bank loans to micro firms over the sample period: January 2010 to April 2018.



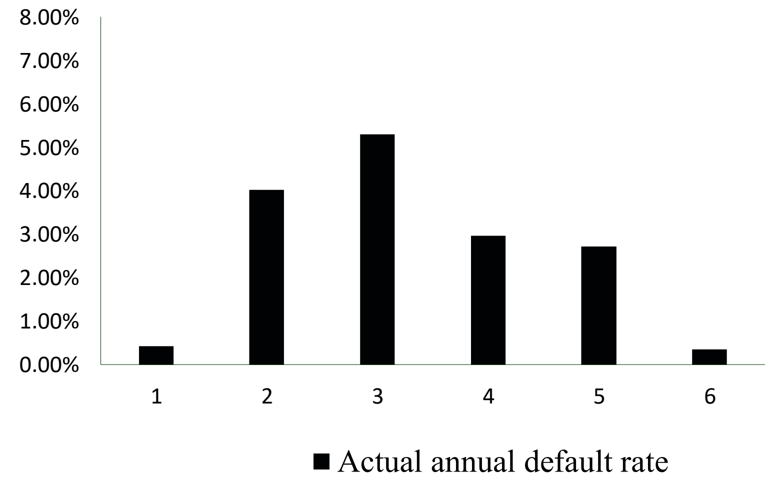
Source: Banco de México, authors' calculations.
Notes: This figure shows the actual annual default rates for bank loans to micro firms for varying tenures over the sample period. The default rate per tenure is defined as the ratio between the marginal sum of loan defaults over the total number of loans outstanding during each tenure. The labels on the horizontal axis indicate the end of year.

Figure B3. Actual default rate term structure for bank loans to small firms over the sample period: January 2010 to April 2018.



Source: Banco de México, authors' calculations.
Notes: This figure shows the actual annual default rates for bank loans to small firms for varying tenures over the sample period. The default rate per tenures is defined as the ratio between the marginal sum of loan defaults over the total number of loans outstanding during each tenure. The labels on the horizontal axis indicate the end of year.

Figure B4. Actual default rate term structure for bank loans to medium firms over the sample period: January 2010 to April 2018.



Source: Banco de México, authors' calculations.
Notes: This figure shows the actual annual default rates for bank loans to medium firms for varying tenures over the sample period. The default rate per tenure is defined as the ratio between the marginal sum of loan defaults over the total number of loans outstanding during each tenure. The labels on the horizontal axis indicate the end of year.

Appendix C

Appendix C presents an overview of the regulatory framework implemented by two world authorities the US and Europe. We also highlight the interesting research literature on the effects of procyclicality and how our approach contributes to the regulatory implementation framework. Finally, we explain the current regulatory approach in Mexico.

C.1 Regulatory background: IFRS9 and CECL standards

The majority of banking sectors around the globe will adopt the expected credit loss (ECL) accounting standards and implement International Financial Reporting Standard 9 *Financial Instruments* (IFRS 9). The approach is promoted by the International Accounting Standards Board (IASB) and US Generally Accepted Accounting Principles (US GAAP). In fact, the Current Expected Credit Losses (CECL) is promoted by the US Financial Accounting Standards Board (FASB).[64] It is important to point out that the US is implementing a more conservative version of ECL compared to the approach proposed in IFRS 9 by the European Union.[65] The key difference between IFRS 9 and CECL is that loan loss allowances under CECL must be computed based on the expected credit losses (ECL) model for the lifetime of each loan irrespective of any sign of credit risk increase or impairment. In contrast, IFRS 9 allowances need to cover only one year of expected losses for loans that have not experienced significant deterioration in terms of credit risk. The lifetime ECL has to be computed for any loan that shows a sign of deterioration on an individual or collective basis. In a nutshell, IFRS 9 proposes a dual-measurement model while CECL is a single-measurement model.

The introduction of IFRS 9 and CECL represents a significant regulatory challenge for entities, as the proposed standards introduce a major methodological change in the way in which financial institutions compute the loss allowance for their credit exposures. IFRS 9 and CECL will supersede the International Accounting Standard 39, and banks will no longer compute their loss allowance based on a

[64] IFRS 9 was issued by IASB in July, 2014. IFRS 9 will be used by a large majority of countries. A detailed list can be found here: https://www.ifrs.org/use-around-the-world/use-of-ifrs-standards-by-jurisdiction/

[65] CECL was issued by the Financial Accounting Standards Board (FASB) on 16 June 2016 (see FASB (2016) for details). It will be effective beginning in 2020 and will initially apply only for the largest publicly traded banks. Community banks and credit unions will not start until January 2023.

‘backward looking’ incurred-loss (IL) approach. Under IFRS 9 and CECL, banks have to compute their loss allowance based on the so-called ‘forward-looking’ expected credit loss model. The main objective of this rule is to ensure that there is a timely and early recognition of ECLs. Naturally, ECLs in this new framework increase when there is the perception that economic forecasts will deteriorate. Also, ECLs decrease with the perception that the economic outlook will improve and become more favourable.

C.1 Procyclicality

There is no doubt that adopting IFRS9 or CECL Standards will affect banks’ income, lending and capital distributions. A line of research in the literature is investigating whether standard will have a ‘procyclical’ impact where lending is reduced in downturns (see Agenor et al., 2015). The evidence on procyclicality between CECL and IL model is mixed. Loudis and Ranish (2019) find that CECL is slightly less procyclical than IL model, while Abad and Suarez (2018) and Covas and Nelson (2018) find conflicting results. Covas and Nelson (2018) find evidence suggesting that introducing CECL might have a significant negative impact on banks’ credit growth during the financial crisis, while Levin et al. (2016) find evidence of a negative impact on credit growth for Mexico. The main challenge for this line of research is that it is difficult to model the impact of introducing CECL or IFRS 9 on banks, due to the range of approaches banks can choose to implement.

We do not contribute to this strand of the literature. Instead, we focus on investigating methodologies designed to implement IFRS9 or CECL from a regulatory system-wide perspective for the banking sector. The parametric survival model applied in this paper is flexible and can be used to accommodate or implement either IFRS 9 or CECL Standards at the loan level.[66]

The main practical challenge for credit entities is that IFRS 9 and CECL Standards do not prescribe any specific model to estimate ECLs. In the absence of a regulatory formula to implement IFRS 9 or CECL, banks’ modelling assumptions will make it difficult to compare provisions across banks and times. Investors and market

[66] For a practical discussion of variability in CECL implementation see Chae et al. (2018).

participants will not be able to disentangle whether variations in provision stem from underlying common and idiosyncratic risk factors or modelling assumptions.

C.3 Mexican regulatory approach

As of today, Mexican banks compute their loan loss allowance based on an expected loss model over a one-year horizon. To compute the PD for each loan, Mexican banks use a regulatory formula based on a logistic model. This is a hybrid approach that is not designed to comply with IFRS9 or CECL Standards as it cannot be used to compute loan loss allowances for the lifetime of the loan. This paper contributes to the methodological implication by demonstrating a possible way to overcome this limitation.

Appendix D

This appendix comprises 11 Tables and 4 Figures. We provide a summary content list of the Tables and Figures included in this appendix.

Table D1. Classification of enterprises size in the Europe Union

Enterprise size	*Number of employees*	*Annual sales (millions of dollars)*[2]	*Balance Sheet Total*
Micro	< 10	≤ € 2 m ($2.4 m)	≤ € 2 m ($2.4 m)
Small	< 50	≤ € 10 m ($11.9 m)	≤ € 10 m ($11.9 m)
Medium	< 250	≤ € 50 m ($59.8 m)	≤ € 43 m ($51.5 m)

Source: This classification of enterprises is based on the European Union approach as defined in the EU recommendation 2003/361 (available at: https://ec.europa.eu/growth/smes/business-friendly-environment/sme-definition_en; accessed on January 9, 2020).

Notes: This table shows the classification of enterprises into 'micro', 'small' and 'medium'-sized enterprises. To facilitate the comparison with Mexican Standards as described in Table D2 in Appendix D, we define in brackets the amounts in USD using Bloomberg's X-Rate, e.g., X-Rate = 0.8357 euro per dollar for September 13, 2018. Following the European approach, a firm is defined as: 'micro' if it has less than 10 employees and annual sales less or equal to approximately € 2 million (i.e., 2.4 million USD); 'small' if it has greater than or equal to 10 but less than 50 employees and less or equal to approximately € 10 million (i.e., 11.9 million USD); 'medium' if it has greater than or equal to 50 but less than 250 employees and annual sales of less than approximately € 50 million (i.e., 60 million USD).

Table D2. Classification of enterprises size in Mexico

Enterprise size	*Sector*	*Number of employees*	*Annual sales (millions of dollars)*	*Combined Ceiling*
Micro	All	< 11	< 0.2	1.2
Small	Agricultural and Commerce	≤ 30	≤ 5.6	8.1
	Manufacturing, Services, Construction and Communications and Transport	≤ 50	≤ 5.6	10.1
Medium	Commerce, Agricultural and Services	≤ 100	≤ 14.1	22.7
	Manufacturing, Construction and Communications and Transport	≤ 250	≤ 14.1	37.7

Source: This classification of enterprises is based on the guidelines published on June 30, 2009 in "Diario Oficial de la Federación" by the Mexican Secretariat of Economy (available at: http://dof.gob.mx/nota_to_imagen_fs.php?codnota=5096849&fecha=30/06/2009&cod_diario=221134; accessed on January 9, 2020).

Notes: This table shows the classification of enterprises into 'micro', 'small' and 'medium'-sized enterprises. To facilitate the comparison with European Standards as described in Table D1, we express Annual Sales in USD using Bloomberg's X-Rate, i.e., we use X-Rate = 17.7836 pesos per dollar for September 13, 2018. In practice, to implement the so-called combined ceiling threshold, authorities must compute for each company a score value based on a Combined Ceiling (CC) formula which is a weighted average of the firm's number of employees and its annual sales. This formula is inscribed in law and was determined by authorities as: "CC = Number of employees*10%+Annual Sales*90%". The outcome of the CC formula must be either equal or lower than the CC threshold. In this paper, we do not use the combined ceiling approach designed by the Mexican Secretariat of Economy because –to the best of our knowledge- there is no publicly available document or study that explains how the calibration of the weights in the formulae was done and why it is required. According to the Mexican regulation, the idea of using a combined ceiling to classify firms is based on two arguments. The first is that the Mexican Secretariat wants to avoid discrimination against labor-intensive companies. The second is to prevent firms that have relatively large annual sales from engaging in SME loans. In Section 3.1 we fully describe the criteria that we use in this paper to classify firms. In a nutshell, we follow an approach very similar to the popular European approach, which is based solely on the number of employees and the annual sales. We depart slightly in that we take into account the firm sector and we use the Mexican thresholds for number of employees and annual sales. Since we analyse banks' micro/SME loans, there is one additional criterion related to the loan size that we take into account to distinguish loans to micro/SMEs from corporate or large firm loans. In Mexico, commercial banks have to report to the financial authorities their loan exposure characteristics independent of their size. We consider that a micro/SME loan for any entity has a maximum exposure size vis-à-vis a given bank group lower than 1 million USD. This threshold is based on the following factors (see Banxico, 2015, p.21): (i) the business model applied by several banks to classify firms as micro/SMEs has a size limit between approximately 540K and 1 million USD; (ii) some banks apply a differentiated methodology in the analysis of loan granting, when the loan size is below 810K USD; and (iii) the programme of automatic guarantees of the National Development Bank 'Nacional Financiera' with which banks serve micro/SMEs has a maximum exposure size of approximately 1 million USD. Finally, for simplicity, we consider SMEs' credit exposures that are at least 10K USD.

Table D3. Market share based on the number of originated loans in Mexico.

	Micro		Small		Medium	
	Market Share	Aggregate Sum	Market Share	Aggregate Sum	Market Share	Aggregate Sum
Panel A: Banks						
D-SIBs						
Mexican owned						
Banorte	6.71	6.71	12.05	12.05	7.00	7.00
Inbursa	1.57	8.28	0.66	12.72	0.51	7.52
Foreign bank subsidiaries						
Banco Santander MX	13.29	21.57	7.84	20.55	6.94	14.45
BBVA MX	35.88	57.45	14.91	35.46	36.34	50.79
Citibanamex	4.97	62.42	9.58	45.05	2.67	53.46
HSBC México	7.63	70.05	7.06	52.10	9.58	63.04
Scotiabank	0.13	70.18	0.20	52.30	0.10	63.14
Non-DSIBs						
Mexican owned						
ABC Capital	0.10	70.28	0.27	52.57	0.32	63.46
Accendo Banco	0.00	70.28	0.00	52.57	0.01	63.47
Actinver	0.06	70.34	0.04	52.60	0.03	63.50
Afirme	3.58	73.92	7.57	60.18	4.77	68.27
Autofin	0.01	73.93	0.05	60.22	0.13	68.40
Azteca	0.02	73.95	0.02	60.24	0.01	68.41
Bajío	20.36	94.31	30.70	90.94	21.07	89.47
Bancrea	0.02	94.32	0.07	91.01	0.05	89.52
Bancoppel	0.01	94.33	0.01	91.02	0.01	89.54
Bankaool	0.16	94.49	0.06	91.08	0.04	89.58
Banregio	1.70	96.19	1.86	92.95	1.32	90.90
Bansi	0.12	96.32	0.15	93.09	0.22	91.12
Base	0.01	96.33	0.04	93.13	0.08	91.20
Bicentenario	0.02	96.35	0.01	93.14	0.00	91.21
Cibanco	0.02	96.37	0.02	93.16	0.02	91.23
Famsa	0.16	96.53	0.57	93.73	0.95	92.18
Finterra	0.03	96.56	0.08	93.81	0.05	92.23
Inmobiliario Mexicano	0.02	96.58	0.07	93.88	0.08	92.31
Interacciones	0.14	96.72	0.01	93.89	0.01	92.32
Invex	0.01	96.73	0.02	93.91	0.04	92.36
IXE	0.48	97.21	1.02	94.93	0.45	92.81
Mifel	0.16	97.37	0.74	95.68	1.24	94.05
Monex	0.06	97.43	0.26	95.93	0.34	94.39
Multiva	0.07	97.50	0.23	96.17	0.27	94.67
Intercam	0.01	97.51	0.00	96.17	0.01	94.68
Ve por Más	1.14	98.65	0.64	96.81	0.53	95.21

Table D3. (continues)

Wal-Mart	0.06	98.71	0.14	96.96	0.43	95.63
Foreign bank non-DSIBs subsidiaries						
Banco Credit Suisse	0.00	98.71	0.00	96.96	0.00	95.63
Bank of America MX	0.00	98.72	0.00	96.96	0.00	95.63
MUFG Bank Mexico	0.00	98.72	0.00	96.96	0.00	95.63
Deutsche Bank MX	0.00	98.72	0.00	96.96	0.00	95.63
Sabadell	0.00	98.72	0.00	96.96	0.02	95.65
Mizuho Bank	0.00	98.72	0.00	96.96	0.00	95.65
Panel B: Regulated Multiple Purpose Financial Societies						
Start Banregio	0.01	98.73	0.08	97.05	0.19	95.85
Arrendadora y Factor Banorte	0.00	98.73	0.02	97.06	0.00	95.85
Sociedad Financiera Inbursa	0.01	98.74	0.09	97.16	0.14	95.99
Sofom Bajío	0.02	98.76	0.05	97.21	0.08	96.07
Mifel	0.00	98.76	0.00	97.21	0.00	96.08
Sofom Inbursa	0.11	98.87	0.16	97.37	0.14	96.22
Finanmadrid México	0.00	98.87	0.00	97.37	0.00	96.23
Arrendadora Banamex	0.00	98.87	0.00	97.37	0.00	96.23
Arrendadora Ve por Más	0.15	99.02	0.36	97.73	0.20	96.42
Arrendadora Afirme	0.00	99.03	0.00	97.74	0.01	96.43
Santander Vivienda	0.00	99.03	0.00	97.74	0.00	96.43
Metrofinanciera	0.00	99.03	0.00	97.74	0.00	96.43
Navistar Financial	0.18	99.21	0.29	98.03	0.19	96.62
NR Finance México	0.06	99.27	0.25	98.28	0.13	96.75
Mercader Financial	0.00	99.27	0.01	98.29	0.08	96.84
Caterpillar Crédito	0.00	99.27	0.01	98.30	0.02	96.86
Factoring Corporativo	0.24	99.51	0.00	98.30	0.00	96.86
Cetelem	0.00	99.51	1.41	99.71	3.01	99.86
Ford Credit	0.11	99.63	0.18	99.89	0.04	99.91
Portafolio de Negocios	0.00	99.63	0.00	99.90	0.00	99.91
Value Arrendadora	0.00	99.63	0.03	99.92	0.04	99.95
ION Financiera, S.A.PI.	0.00	99.63	0.00	99.93	0.00	99.95
Sofoplus	0.00	99.63	0.00	99.93	0.00	99.95
FC Financial	0.36	100.00	0.05	99.97	0.02	99.97
Finactiv	0.00	100.00	0.02	99.99	0.02	99.99
Financiera Bepensa	0.00	100.00	0.01	100.00	0.01	100.00

Source: Banco de México, authors' calculations.

Notes: This table shows the bank's market share based on the number of originated loans to 'micro', 'small' and 'medium'-sized firms.

Table D4. Foreign bank ownership

Foreign banks	International Owner (Country)
Forein banks D-SIBs	
Banco Santander MX	Banco Santander (Spain)
BBVA MX	BBVA (Spain)
Citibanamex	Citigroup (USA)
HSBC México	HSBC (UK)
Scotiabank	Scotiabank (Canada)
Foreign bank non-DSIBs	
Banco Credit Suisse	Credit Suisse Group (Switzerland)
Bank of America MX	Bank of America Corporation (USA)
MUFG Bank Mexico	MUFG Bank, Ltd (Japan)
Deutsche Bank MX	Deutsche Bank AG (Germany)
Sabadell	Banco Sabadell Group (Spain)
Mizuho Bank	Mizuho Financial Group (Japan)

Source: Banco de México, authors' calculations.
Notes: This table shows foreign bank ownership in Mexico.

Table D5. Correlation matrix for bank loans to 'micro'-sized firms

Variable		1	2	3	4	5	6	7
Interest rate	1	1.0000						
ln(number of employees)	2	-0.0686	1.0000					
ln(loan size)	3	-0.3457	0.0293	1.0000				
Yield curve proxy	4	-0.2057	-0.1430	0.0285	1.0000			
Consumer confidence index$_{t-3}$	5	-0.1143	0.0938	0.0826	0.0659	1.0000		
Economic activity index$_{t-3}$	6	-0.1280	0.1966	0.0381	-0.4713	-0.0680	1.0000	
Inflation$_{t-3}$	7	0.2036	0.0071	-0.0713	-0.5744	-0.2626	0.1545	1.0000
D-SIBs	8	0.1934	-0.1327	-0.1191	0.0844	-0.1364	-0.1668	0.0493
Agriculture	9	-0.0506	-0.0223	0.0902	-0.0158	-0.0001	-0.0009	-0.0040
Commerce	10	-0.0612	0.0778	-0.0695	0.0182	0.0064	0.0305	-0.0061
Construction	11	-0.0435	0.0204	0.0919	0.0101	0.0091	-0.0202	-0.0111
Communications and transport	12	0.0198	-0.0125	-0.0100	-0.0182	-0.0353	0.0084	0.0234
Manufacturing	13	-0.0300	0.0102	0.0481	0.0164	0.0124	-0.0216	-0.0093
Service	14	0.1229	-0.0847	-0.0429	-0.0216	-0.0055	-0.0098	0.0107
		8	9	10	11	12	13	14
D-SIBs	8	1.0000						
Agriculture	9	-0.1092	1.0000					
Commerce	10	0.0707	-0.1850	1.0000				
Construction	11	0.0239	-0.0521	-0.1980	1.0000			
Communications and transport	12	-0.0090	-0.0448	-0.1704	-0.0479	1.0000		
Industry	13	-0.0641	-0.0877	-0.3335	-0.0938	-0.0807	1.0000	
Service	14	0.0142	-0.1485	-0.5645	-0.1589	-0.1367	-0.2675	1.0000

Source: Banco de México, authors' calculations.
Notes: This table shows the correlation matrix for bank loans to 'micro'-sized firms.

Table D6. Correlation matrix for bank loans to 'small'-sized firms

Variable		1	2	3	4	5	6	7
Interest rate	1	1.0000						
ln(number of employees)	2	0.0041	1.0000					
ln(loan size)	3	-0.3335	0.0819	1.0000				
Yield curve proxy	4	-0.2125	-0.0122	0.0104	1.0000			
Consumer confidence index$_{t-3}$	5	-0.1055	-0.0156	0.0701	0.0659	1.0000		
Economic activity index$_{t-3}$	6	-0.0047	-0.0531	0.0469	-0.4713	-0.0680	1.0000	
Inflation$_{t-3}$	7	0.2002	0.0313	-0.0118	-0.5744	-0.2626	0.1545	1.0000
D-SIBs	8	0.1837	-0.0745	-0.1319	-0.0236	-0.1116	-0.0198	0.0635
Agriculture	9	-0.0795	-0.0398	0.0889	0.0246	0.0032	-0.0546	-0.0058
Commerce	10	0.0404	-0.1763	-0.1309	-0.0286	-0.0062	0.0673	0.0025
Construction	11	-0.0520	0.0623	0.0784	0.0291	-0.0033	-0.0564	-0.0103
Communications and transport	12	-0.0134	0.0080	0.0208	-0.0320	-0.0305	0.0191	0.0327
Manufacturing	13	-0.0593	0.1521	0.0549	0.0536	0.0435	-0.0487	-0.0419
Service	14	0.0943	0.0371	-0.0048	-0.0362	-0.0214	0.0241	0.0335
		8	9	10	11	12	13	14
D-SIBs	8	1.0000						
Agriculture	9	-0.0231	1.0000					
Commerce	10	0.0927	-0.1905	1.0000				
Construction	11	-0.0009	-0.0644	-0.2416	1.0000			
Communications and transport	12	-0.0406	-0.0467	-0.1753	-0.0592	1.0000		
Manufacturing	13	-0.0760	-0.1181	-0.4432	-0.1497	-0.1086	1.0000	
Service	14	-0.0030	-0.1136	-0.4263	-0.1440	-0.1045	-0.2642	1.0000

Source: Banco de México, authors' calculations.
Notes: This table shows the correlation matrix for bank loans to 'small'-sized firms.

Table D7. Correlation matrix for bank loans to ‘medium’-sized firms

Variable		1	2	3	4	5	6	7
Interest rate	1	1.0000						
ln(number of employees)	2	0.0796	1.0000					
ln(loan size)	3	-0.0844	0.1585	1.0000				
Yield curve proxy	4	-0.2564	0.1641	0.1135	1.0000			
Consumer confidence index$_{t-3}$	5	-0.1703	0.0840	0.1205	0.0659	1.0000		
Economic activity index$_{t-3}$	6	-0.0646	-0.1658	-0.1397	-0.4713	-0.0680	1.0000	
Inflation$_{t-3}$	7	0.2611	-0.1112	-0.0516	-0.5744	-0.2626	0.1545	1.0000
D-SIBs	8	-0.0538	-0.0159	-0.3295	-0.0088	-0.0315	0.0662	0.0141
Agriculture	9	-0.0313	-0.0128	0.1060	0.0230	0.0054	-0.0609	-0.0003
Commerce	10	-0.0696	-0.2652	-0.2725	-0.0997	-0.0468	0.1845	0.0243
Construction	11	0.0059	0.0466	0.0934	0.0443	-0.0070	-0.1029	-0.0110
Communications and transport	12	0.0327	-0.0256	0.0356	-0.0089	-0.0091	-0.0001	0.0130
Manufacturing	13	-0.0133	0.2788	0.1676	0.0854	0.0595	-0.1161	-0.0418
Service	14	0.1493	0.0131	0.0345	-0.0117	-0.0023	-0.0024	0.0269
		8	9	10	11	12	13	14
D-SIBs	8	1.0000						
Agriculture	9	-0.0385	1.0000					
Commerce	10	0.1424	-0.2134	1.0000				
Construction	11	-0.0075	-0.0623	-0.2947	1.0000			
Communications and transport	12	-0.0219	-0.0371	-0.1757	-0.0513	1.0000		
Manufacturing	13	-0.1121	-0.1285	-0.6082	-0.1775	-0.1058	1.0000	
Service	14	-0.0300	-0.0615	-0.2912	-0.0850	-0.0507	-0.1753	1.0000

Source: Banco de México, authors’ calculations.
Notes: This table shows the correlation matrix for bank loans to ‘medium’-sized firms.

Table D8. Assessing parametric distribution fit using Akaike Information Criterion

	Distribution	Log-likelihood	*Covariates*[a]	*Parameters*[b]	AIC
Section A: bank loans to micro-sized firms					
	Exponential	-164857.13	13	1	329,742.26
	Gompertz	-159804.38	13	2	319,638.76
	Weibull	-154033.21	13	2	308,096.42
	Loglogistic	-153367.42	13	2	306,764.84
	Lognormal	-151821.74	13	2	303,673.48
Section B: bank loans to small-sized firms					
	Exponential	-129607.49	13	1	259,242.98
	Gompertz	-125086.09	13	2	250,202.18
	Weibull	-119553.23	13	2	239,136.46
	Loglogistic	-118879.64	13	2	237,789.28
	Lognormal	-117192.23	13	2	234,414.46
Section C: bank loans to medium-sized firms					
	Exponential	-31375.08	13	1	62,778.16
	Gompertz	-30432.05	13	2	60,894.10
	Weibull	-29047.31	13	2	58,124.63
	Loglogistic	-28920.51	13	2	57,871.02
	Lognormal	-28474.93	13	2	56,979.86

Source: Banco de México, authors' calculations.
Notes: (a) this is the number of model covariates excluding the intercept term; (b) this is the number of model-specific distributional parameters. This table presents the Akaike Information Criterion (AIC) which is used to discriminate between different parametric distributions in the context of non-nested models for 'micro', 'small', and 'medium'-sized enterprises. Section A reports AIC results for survival of bank loans to micro-sized firms; Section B reports AIC results for survival of bank loans to small-sized firms; Section C reports AIC results for survival of bank loans to medium-sized firms. The standard AFT model that we use for testing the distribution parametric fit corresponds to M2 in Tables 15 to 17. The preferred model or the model that provides the best fit is the one with the lowest AIC. The AIC is defined as -2lnL+2(k+c) where lnL is the log likelihoods, k is the number of model covariates excluding the intercept term and c is the number of model-specific distributional parameters. Per the AIC criterion, the Lognormal model is selected. We do not include the Generalized Gamma in our analysis because this distribution cannot be used with shared frailty models.

Table D9. Matrix of pairwise Chi-squared statistic to test equality of ROC areas for the five AFT estimated models (M1-M5) for bank loans to 'micro' firms

(1)	(2)	(3)	(4)	(5)	(6)	(7)
AUROC	Model	M1	M2	M3	M4	M5
0.5365	M1					
0.5545	M2	15.28[a]				
0.5543	M3	11.66[a]	0.01			
0.5649	M4	28.46[a]	48.62[a]	22.18[a]		
0.6208	M5	267.31[a]	943.91[a]	595.34[a]	596.81[a]	

Source: Banco de México, authors' calculations.
Notes: a (b) [c] significant at 1% (5%) [10%] level (two-sided test). This table presents the pairwise Chi-squared statistic to test whether the areas under the ROC of any two curves are equal for bank loans to micro sized firms. The first column shows the areas under the ROC curves for the five AFT models (M1-M5) estimated in Table 7. The second column identifies the model under analysis. Columns (3) to (7) show the pairwise Chi-squared statistic to test equality of ROC areas for the five estimated models (M1-M5). All models are estimated through maximum likelihood estimation. M1 and M2 are estimated using eq.(1), while Models M3 to M5 are estimated using eq.(3). The one-year *PD* used as an input for the AUROC is computed using eq.(5). The survival function used in the *PD* computation for Models M1 and M2 is based on eq.(1), while Models M3 to M5 are based on eq.(3).

Table D10. Matrix of pairwise Chi-squared statistic to test equality of ROC areas for the five AFT estimated models (M1-M5) for bank loans to 'small' firms

(1)	(2)	(3)	(4)	(5)	(6)	(7)
AUROC	Model	M1	M2	M3	M4	M5
0.5126	M1					
0.5188	M2	18.02[a]				
0.5019	M3	21.50[a]	151.12[a]			
0.5485	M4	74.88[a]	74.53[a]	168.93[a]		
0.5807	M5	427.37[a]	583.04[a]	598.57[a]	72.15[a]	

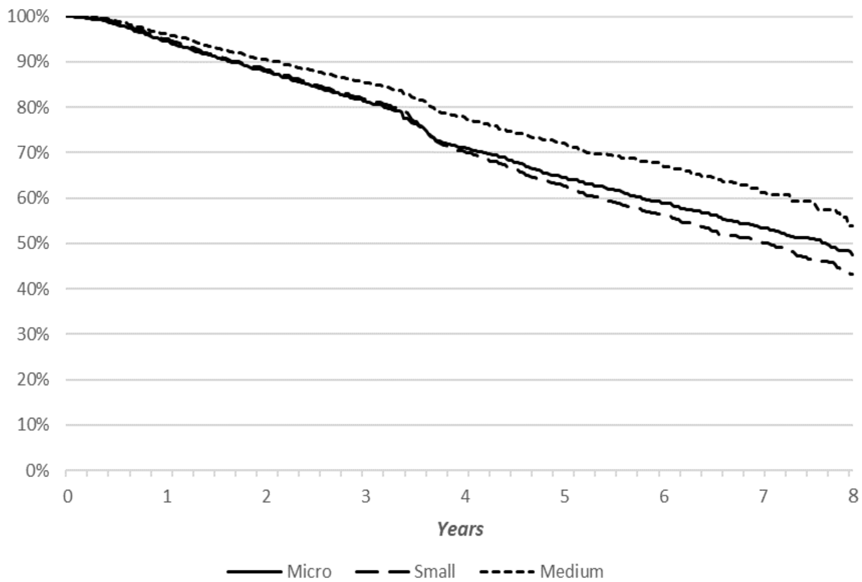
Source: Banco de México, authors' calculations.
Notes: a (b) [c] significant at 1% (5%) [10%] level (two-sided test). This table presents the pairwise Chi-squared statistic to test whether the areas under the ROC of any two curves are equal for bank loans to small sized firms. The first column shows the areas under the ROC curves for the five AFT models (M1-M5) estimated in Table 8. The second column identifies the model under analysis. Columns (3) to (7) show the pairwise Chi-squared statistic to test equality of ROC areas for the five estimated models (M1-M5). All models are estimated through maximum likelihood estimation. M1 and M2 are estimated using eq. (1), while Models M3 to M5 are estimated using eq.(3). The one-year *PD* used as an input for the AUROC is computed using eq.(5). The survival function used in the *PD* computation for Models M1 and M2 is based on eq.(1), while Models M3 to M5 are based on eq.(3).

Table D11. Matrix of pairwise Chi-squared statistic to test equality of ROC areas for the five AFT estimated models (M1-M5) for bank loans to 'medium' firms

(1)	(2)	(3)	(4)	(5)	(6)	(7)
AUROC	Model	M1	M2	M3	M4	M5
0.7495	M1					
0.7980	M2	196.29[a]				
0.7889	M3	38.71[a]	3.86[b]			
0.6089	M4	178.13[a]	442.85[a]	386.47[a]		
0.7936	M5	88.58[a]	3.00[c]	0.68	476.62[a]	

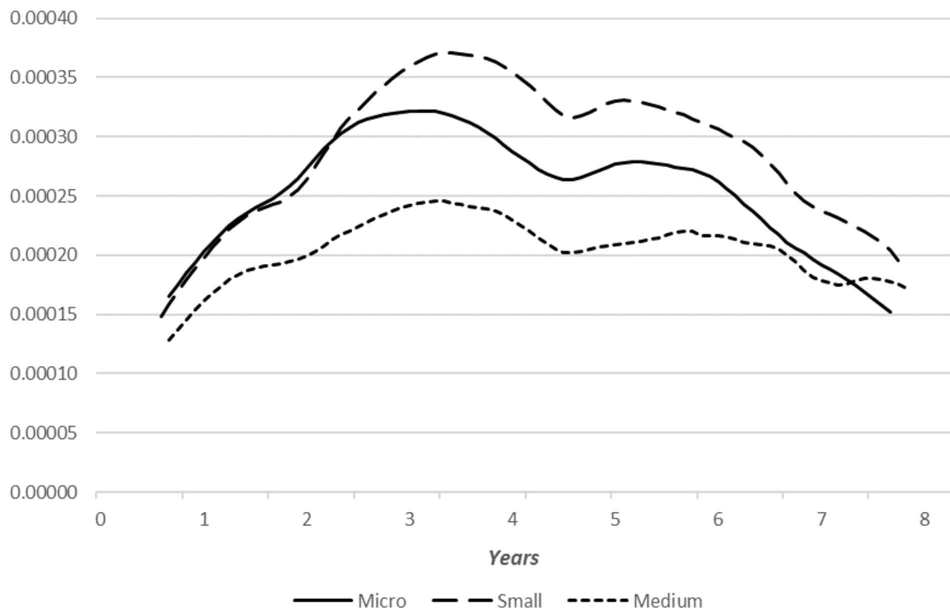
Source: Banco de México, authors' calculations.
Notes: a (b) [c] significant at 1% (5%) [10%] level (two-sided test). This table presents the pairwise Chi-squared statistic to test whether the areas under the ROC of any two curves are equal for bank loans to medium sized firms. The first column shows the areas under the ROC curves for the five AFT models (M1-M5) estimated in Table 9. The second column identifies the model under analysis. Columns (3) to (7) show the pairwise Chi-squared statistic to test equality of ROC areas for the five estimated models (M1-M5). All models are estimated through maximum likelihood estimation. M1 and M2 are estimated using eq. (1), while Models M3 to M5 are estimated using eq.(3). The one-year *PD* used as an input for the AUROC is computed using eq.(5). The survival function used in the *PD* computation for Models M1 and M2 is based on eq.(1), while Models M3 to M5 are based on eq.(3).

Figure D1. Kaplan-Meier survival function



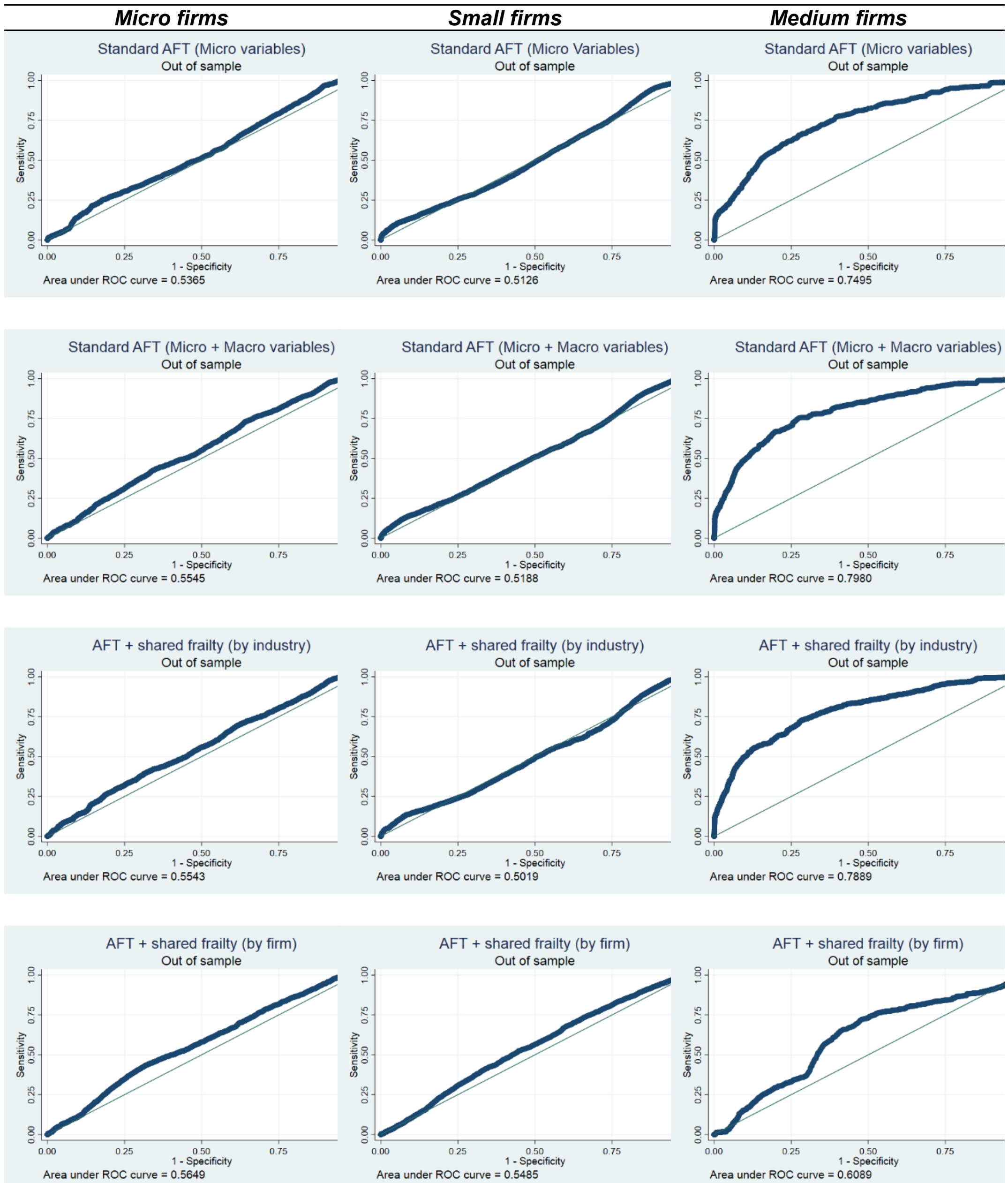
Source: Banco de México, authors' calculations.
Notes: This figure reports the evolution of the estimated survivor curve using the Kaplan-Meier estimator. This graph was generated in Stata using the command "sts graph" along with the option "survival". The Y-axis shows the survival rate in percent. Here 'Years' represents the lifetime of firm loans in years. This figure reports the estimator for bank loans to 'micro', 'small' and 'medium' enterprises using the default definition discussed in Section 3.2.

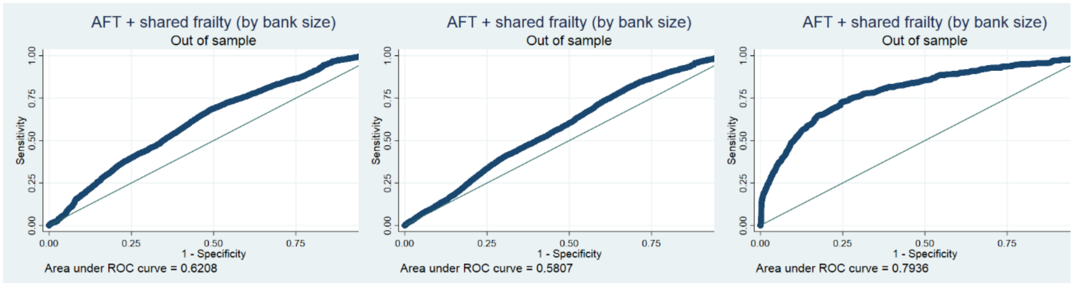
Figure D2. Smoothed hazard function



Source: Banco de México, authors' calculations.
Notes: This figure reports the evolution of the smoothed hazard curve estimate. This graph was generated in Stata using the command "sts graph" along with the option "hazard". The Y-axis is the hazard rate. Here 'Years' represents the lifetime of firm loans in years. Both figures 'Years' represents the lifetime of firm loans in years. This figure reports the estimator for bank loans to 'micro', 'small' and 'medium' enterprises using the default definition discussed in Section 3.2.

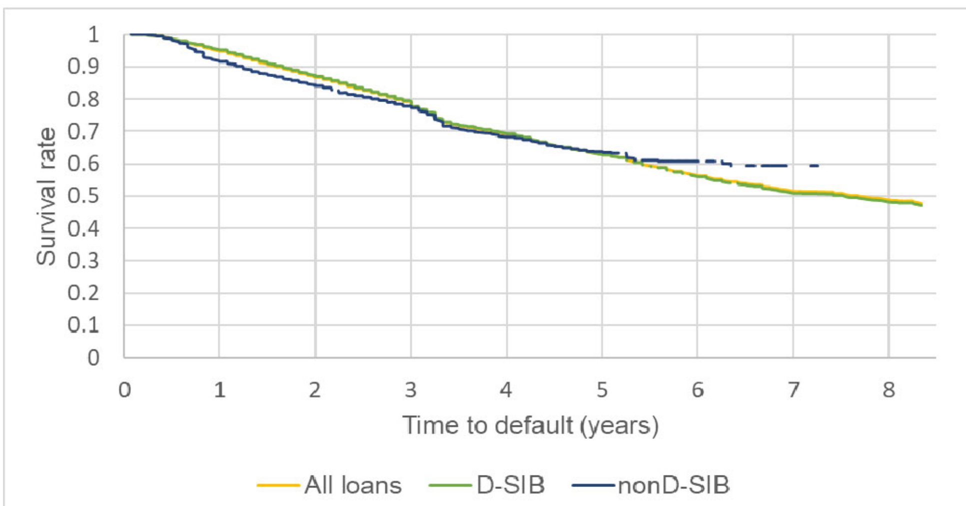
Figure D3. Table of area under ROC curves





Source: Banco de México, authors' calculations.
Notes: This figure reports the out-of-sample area under the ROC curve for bank loans to 'micro', 'small' and 'medium'-sized firms for the five econometric models (M1 to M5) under study. This statistic and the corresponding graph is available in Stata with command 'roctab'

Figure D4. Kaplan-Meier loan survival function by bank type for micro loans



Source: Banco de México, authors' calculations.
Notes: This figure reports the evolution of the estimated survivor curve using the Kaplan-Meier estimator by bank type. This graph was generated in Stata using the command "sts graph" along with the option "survival" by bank type. The Y-axis shows the survival rate in percent. Here 'Years' represents the lifetime of firm loans in years. This figure reports the estimator for bank loans to 'micro' enterprises using the default definition discussed in Section 3.2.

Appendix E.

Appendix E presents core elements of the basic survival function and a detailed derivation of the survival function with frailties.

E.1 Basic survival or hazard model

We use survival analysis to model the time to default of bank firm loans. Assume that *T* is a nonnegative random variable, which denotes the time to loan default of any firm while *t* represents any specific realized value of *T*. Since *T* is a random variable, we could in principle follow a classical probabilistic approach and refer to its cumulative distribution function (CDF) as $F(t) = \Pr(T > t)$ or to its probability density function $f(t)$. However, in survival analysis, we focus on the survival function $S(t)$ defined as the reverse CDF of *T* as:

$$S(t) = 1 - F(t) = \Pr(T > t) \tag{E.1}$$

The survivor function shows the probability of surviving beyond time *t*. This function equals one at *t*=0 and goes to zero as *t* approach to infinity. There is a one to one relationship between any survivor and its corresponding hazard function $h(t)$ $h(t)$ defined as follows:

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr(t + \Delta t > T > t | T > t)}{\Delta t} = \frac{f(t)}{S(t)} = \frac{d(-\ln S(t))}{dt} \tag{E.2}$$

The hazard function can be understood as the limiting probability that the firm loan default occurs within a specific time interval, given that the loan has survived up to the start of that time interval divided by the width of the time interval. The hazard rate can vary from zero (meaning no risk at all) to infinity (which means that it will eventually fail at that instant with certainty). The shape of the survivor function and its hazard rate depends on the survival model under analysis.

Following Cleves et al. (2010) there are three approaches to estimate a survival model: (i) non-parametric; (ii) semi-parametric; and (iii) parametric. In this paper, we use parametric models because we aim to apply a simple model for the banking industry conditional on a set of covariates using a tractable closed form expression. Typically, every survival function has a corresponding hazard function.

As time passes by the hazard function can: (i) remain constant; (ii) increase; (iii) decrease; (iv) adopt hump shapes or other non-monotonic shapes such as serpentine shapes. The shape of the hazard function is of essential interest in empirical applications. When the hazard remains constant for all time *t*, we say that the process exhibits duration independence. In our setting, the time to loan default does not depend on time spent in the initial state. There could be positive duration dependence (e.g., if the hazard is monotonically increasing for every time *t*) or negative duration dependence (e.g., if the hazard is monotonically decreasing for every time *t*). In the credit risk literature, Carling et al. (2007) and Gupta et al. (2018) provide empirical evidence suggesting duration dependence and this implies that any binary model specification of default risk is inappropriate to obtain consistent default-risk estimates (see Carling et al. (2007, p.847) and Shumway (2001)).

E.2 Detailed derivation of survival function with frailties

The shared frailty model is a generalization of the AFT model where individuals are allowed to share the same frailty value, which generates dependence between those individuals who share frailties. In other words, the frailty can be used to model intra-group default correlation.[67] Gutierrez (2002) shows that the survival function conditional on the frailty is:

$$S\left(t_{jk}\left|\alpha, x_{1,jk},\ldots,x_{n,jk}\right.\right)=\left(S\left(t_{jk}\left|x_{1,jk},\ldots,x_{n,jk}\right.\right)\right)^{\alpha} \tag{E.3}$$

In the AFT metric, Gutierrez (2002) shows that the relationship in eq.(E.3) holds in a frailty model when the frailty 'α' is distributed as gamma with mean one and finite variance θ. The conditional survival function with a shared frailty is expressed as:

$$S_{\theta}\left(t_{jk}\left|x_{1,jk},\ldots,x_{n,jk}\right.\right)=S_{\theta}^{0}\left(\exp\left(-\left(\beta_0+\beta_1 x_{1,jk}+\cdots+\beta_n x_{n,jk}\right)\right)t_{jk}\right) \tag{E.4}$$

$$S_{\theta}\left(t_{jk}\left|x_{1,jk},\ldots,x_{n,jk}\right.\right)=\int_0^{\infty}\left(S\left(t_{jk}\left|x_{1,jk},\ldots,x_{n,jk}\right.\right)\right)^{\alpha} g(\alpha)d\alpha \tag{E.5}$$

---

[67] In Stata, the frailty 'α' can follow any of two possible density functions $g(\alpha)$, either a gamma distribution or an inverse Gaussian distribution. Any of these two specifications can be used and there is no criterion that underscores the superiority of one of them over the other. For any shared frailty model, estimation of theta is jointly with the estimation of β and the ancillary parameters.

where $S_\theta^0(t_{jk}) = S_\theta(t_{jk} \mid x_{1,jk} = 0, ..., x_{n,jk} = 0)$, and the subscript θ is used to underscore the dependence of the survival function on the frailty variance θ. The final specification of $S_\theta(t_{jk} \mid x_{1,jk}, ..., x_{n,jk})$ depends on both the assumed distribution for τ_j and the assumed distribution for the frailty. To test for heterogeneity and random effects, namely, whether θ is statistically different from zero, we use a likelihood ratio (*LR*) test available in Stata.[68] If the null hypothesis is not rejected, the standard survival model described in Section 3.4.2 is adequate. In our study, we use the AFT with shared frailty that follows a gamma distribution.

[68] The *LR* test of theta=0 is a boundary test, and this implies that the null distribution is not the usual Chi-squared with one degree of freedom, but rather it is a 50:50 mixture of a Chi-squared with zero degrees of freedom (point mass at zero) and a Chi-squared distribution with one degree of freedom (for further technical details, see Gutierrez et al. (2001)).

Appendix F.

Appendix F presents the results of comparing the equality of loan survivor functions across two firm types (i.e., small and micro) to assess whether a regulatory loan firm classification, as used in this paper, is appropriate. It is clear from Figure D1 that the survival curve of bank loans to medium-sized firms differs from the survival curve for micro and small size firms. However, it is not immediately clear whether the survival curve for bank loans to micro firms differs from small size firms. STATA provides several nonparametric tests to compare survival curves between two or more groups. We use the log-rank, Wilcoxon and Tarone-Ware (see Cleves et al. (2010, p.122)). Essentially, these are global tests that compare the overall survivor functions. These tests compare at each default time the expected versus the observed number of loan defaults for each group and combine these comparisons over all observed loan default times. Each test is different only with respect to how they weight each of the individual comparisons that occur at each default time to form one overall test statistic. It is important to point out that these tests do not test the equality of the survivor functions at a specific point in time. Table F1 show that the survival curve between micro and small firm loans are significantly different independently of the test under consideration.

Table F1. Tests to compare survival curves

Test	Firm type	Events Observed	Events Expected	Total	Sum of Ranks	chi2(1)	Pr>chi2
Log-rank Test	micro	36,102	33,750	62,241	-	363	0.0000
	small	26,139	28,491	62,241	-		
Wilcoxon Test	micro	36,102	33,750	62,241	9.83E+08	652.67	0.0000
	small	26,139	28,491	62,241	-9.83E+08		
Tarone-Ware Test	micro	36102	33749.99	62,241	1514964.6	557.36	0.0000
	small	26139	28491.01	62,241	-1514964.6		

Source: Banco de México, authors' calculations.

Notes: This table shows three test results to compare survival experience between bank loans to 'micro' and 'small'-sized firms using formal tests of hypothesis. The null hypothesis is that the two survival curves are equal. Since the relative survival experience of the firm bank loans may be characterized by the groups' hazard functions, the null hypothesis can be expressed in the hazards as H_o: $h_{\text{Micro}}(t)=h_{\text{Small}}(t)$.

Using statistical tests to distinguish between groups may not be appropriate in our context. We outline three reasons for classifying bank loan firms into three categories that should be considered even when the outcome of statistical classification results do not suggest that survival curves are different. First, there is a large body of the literature that argues that firm size matters (see Holmes et al., 2010; Gupta et al., 2015 and Gupta et al. 2018 and the references therein). This international evidence may be more relevant than using the results of a statistical test to assess whether there is a difference in survival curves between bank loans to micro, small and medium-sized firms. Second, if we use a forward-looking perspective, we are not certain that the survival curve of any two bank loans will remain the same in the future. For instance, it is likely that any crisis may affect micro firms more than small firms. If the sample does not include a severe crisis period, then we support the view that classification tests could be inappropriate. Therefore, it is appropriate to emphasize that the period that we have analyzed is characterized by the absence of a strong crisis. Third, the Mexican law differentiates firms according to their size, industry and sales into 'micro', 'small' and 'medium' sized enterprises. Other countries have similar regulatory frameworks. It is desirable to have PD formulae that are congruent with the law to achieve consistency with the bank's regulatory framework.