

SPATIAL DISTRIBUTION OF GALAXIES: BIASED GALAXY FORMATION,
SUPERCLUSTER-VOID TOPOLOGY, AND ISOLATED GALAXIES

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ABSTRACT. To study distribution of galaxies and voids, the rectangular box under study is divided into cubic cells, and mean density of particles in cells is derived. For any density level cells can be divided into 'filled' or 'empty' ones if their density is higher or lower than a threshold density. The length and volume of the largest connected system, as well as the number of systems of connected cells are derived for observed, model and random samples. The comparison of results demonstrates that galaxy formation is biased, supercluster-void topology is sponge-like in a wide threshold density interval, and that there are no isolated galaxies in voids.

1. INTRODUCTION

Most widely used quantitative methods to describe the spatial distribution of galaxies include correlation analysis and recently introduced cluster and percolation analyses. These methods complement each other and describe different aspects of the spatial distribution of test particles.

The study of the supercluster-void topology, the comparison of the distribution of isolated galaxies and models with biased galaxy formation with observations have raised the question: how to analyze these problems in quantitative terms. By experimentation we have found that a suitable method to study these problems is the division of a box under study into small cubic cells and to investigate the distribution, clustering etc. of filled and empty cells.

In the following we give a short description of the cell method, an overview of observational data and theoretical samples used and basic results obtained. This work is a result of collaborative efforts, more detailed reports are in preparation. We thank our collaborators Mirt Gramann, Maret Einasto and Adrian Melott for permission to use our joint results prior to detailed publication. Our special thank is due to Dr John Huchra, whose compilations of redshifts of galaxies have made this investigation possible.

2. METHOD

Consider a cubic box of side L , containing N test particles, galaxies or particles from numerical simulations. Split this volume into k^3 cubic cells of side $l = L/k$. Each cell contains in the mean

$$\rho_m = N/k^3 \quad (1)$$

particles. Calculate a continuous density field by smoothing the density of each particle using a hat function centered at the particle's actual position and having the hat side equal to the grid size l . Mark cells as 'empty' or 'filled' if the density of cells is lower or equal/higher than a given threshold density.

Now we can calculate the filling factors, study clustering properties of empty and filled cells separately etc. In contrast to classical cluster analysis where individual particles are used, in the cell method we consider cells of one class, either filled or empty. If two neighboring cells have a common sidewall, we count both cells as members of a system. The advantage of the cell method lies in the fact that identical procedures can be used to study the clustering of filled and empty regions whereas classical cluster analysis is not suited to study clustering of voids. We can vary the density threshold which divides cells into filled and empty ones. This corresponds to the variation of the neighborhood radius in the classical cluster analysis.

The cell method has been used previously to study large scale distribution of galaxies by Guberman et al. (1983) and Gott, Melott and Dickinson (1986) in two and three dimensional cases, respectively. In present paper we develop this method further by using a wide variety of statistical quantities over a broad threshold density interval. In the following we use dimensionless densities

$$\delta = \rho/\rho_m. \quad (2)$$

3. DATA

Observational data are based on the Huchra's (1983) compilation of redshifts which is complete in the Northern Galactic Hemisphere up to 14.5 apparent magnitude. We have used three samples from this compilation, located in a box of side $L = 20 h^{-1}$ Mpc (h is the Hubble constant in units of 100 km/s/Mpc) and the center coordinate in supergalactic coordinates $x_0 = 0$, $y_0 = 15 h^{-1}$ Mpc, $z_0 = 0$, which is close to the center of the Local Supercluster. The samples are absolute magnitude limited. Sample Virgo A contains galaxies brighter than $M_0 = -17.5$ (absolute magnitudes are calculated for $H = 100$ km/s/Mpc). This limit corresponds to the limiting magnitude of the CfA survey at the far side of the box, i.e. data in this sample are complete. Sample Virgo B contains dwarf galaxies in absolute magnitude interval $-15.0 > M > -17.5$ and includes numerous dwarfs from H radio surveys. This sample is not complete, but it is the best sample of dwarf galaxies available. Sample

Virgo C is the sum of samples Virgo A and B.

Observed distribution of galaxies is compared with theoretical samples found from N-body calculations. Three model samples were used. The first model sample, denoted Mel, is based on an axion dominated universe, containing 64^3 particles in a 64^3 mesh (Melott 1986). The second model, denoted GR-5, is based on a cold dark matter model with nonzero cosmological constant, containing also 64^3 particles in a 64^3 mesh. Parameters of the model were tuned to have for the present epoch (expansion factor $a = 5.2$) $\Omega_\lambda = 0.8$ and $\Omega_{\text{matter}} = 0.2$ (Gramann 1986). The third model, denoted M_i , corresponds to a neutrino dominated universe and was calculated for 32^3 particles in a 32^3 mesh (Melott et al. 1983). In all models several cases were considered, without and with biased galaxy formation. The first case includes all test particles. In other cases particles in low density environment were considered as primeval ones and rejected.

For comparison three samples with randomly distributed particles were used, containing 64^3 , 32^3 and 16^3 particles and denoted by R-64, R-32 and R-16, respectively. Data on samples are given in Table 1.

TABLE 1. Data on samples used

Sample	L	Bias level	N	$\log \delta_{\text{perc}}$	C_{perc}
Virgo A	$20h^{-1}$		524	0.70	0.028
Virgo B			486	0.55	0.049
Virgo C			1010	0.65	0.036
M_0	$32h^{-2}$	2.7	4089	0.00	0.017
M_1		2.7	4139	0.00	0.017
M_2		2.7	4294	0.00	0.017
M_3		2.7	4449	0.00	0.017
Mel _{axion}	$32h^{-2}$	0	262144	0.25	0.056
		1	39944	0.35	0.039
GR-5	$40h^{-2}$	0	262144	0.55	0.031
		1.5	50823	0.35	0.039
R-64	32	0	262144	0.05	0.255
R-32		0	32768	0.12	0.254
R-16		0	4096	0.16	0.250

4. TESTS BASED ON CELL METHOD

Figs 1-3 present plots of three quantities, the relative length of the largest system, $\lambda_{\text{max}} = l_{\text{max}}/L$, the volume fraction of the largest system,

$C_{\max} = V_{\max}/V$, and the number of systems of multiplicity 4 and larger. The length of the system, l_{\max} , was calculated as the maximum length of the system in coordinates x , y , and z . The total filled volume fraction, C , was also calculated, but respective curves are not plotted here since they are quite similar to curves for largest systems, i.e. most of the volume is occupied by the largest system.

At a threshold density higher than the density of the highest peak in a respective sample, there are no filled regions, and the total volume of the box is occupied by one large system of 'empty' cells. Respectively, for filled regions $\lambda_{\max} = C_{\max} = n_4 = 0$, and for empty regions $\lambda_{\max} = C_{\max} = n_4 = 1$. With decreasing density threshold there appear filled systems which at threshold density $\delta = \delta_{\text{perc}}$ form a percolating system: the length of the largest system is just equal to the length of the box, $\lambda_{\max} = 1$. The percolating threshold density is related to the percolation parameter B (Shandarin 1983, Einasto et al. 1984) as follows

$$B = 3 / \delta_{\text{perc}}$$

At a low threshold density the whole space is occupied by one large 'filled' system and the situation 'filled' versus 'empty' regions is reversed in comparison to high threshold density situation.

Some details of the tests depend on mean density of test particles in cells. The mean density ρ_m depends on the resolution parameter k and enters as an independent parameter of the method. Some quantities are, however, almost independent of mean density. Thus percolating density only little changes with ρ_m , and critical volume fraction of 'filled' cells (the volume fraction at the percolating density) of random samples is practically constant for all mean densities considered, as seen from Table 1.

Differences between random and other samples lie in the percolating density and the critical volume fraction. All random samples percolate at a threshold density $\delta_{\text{perc}} \sim 1$, whereas all observed and simulated samples percolate at much higher threshold densities. On the contrary, all random samples have the critical volume fraction $C_{\text{perc}} = 0.25$, but all observed and model samples have $C_{\text{perc}} = 0.02 - 0.05$. Both parameters reflect the presence of a connected network of filaments in observed and model samples which make the percolation easier than for random samples.

There exist also differences between various model samples, in particular, between model samples with and without biasing galaxy formation.

5. BIASED GALAXY FORMATION

Early comparisons of the observed distribution of galaxies (Joeveer, Einasto and Tago 1977, Einasto, Joeveer and Saar 1979) with numerical simulations (Zeldovich 1978) demonstrated a striking difference between theory and observations: in the real world the space between superclusters is empty whereas in simulations there exist a rarefied field population between densely populated structures. Quantitatively

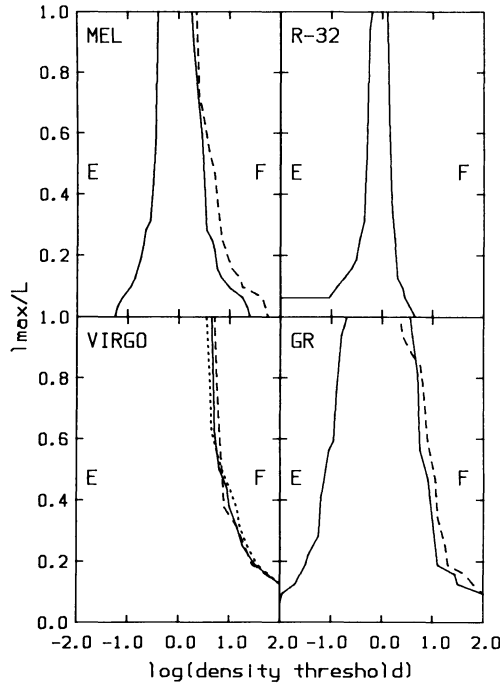


Fig. 1. The plot of the length of largest system in units of the box size, $\lambda = l_{\max}/L$, versus relative threshold density. Panel VIRGO describes observed samples, panels MEL and GR axion-dominated models (GR model includes non-zero cosmological constant), and panel R-32 - a Poisson sample. Curves corresponding to filled and empty regions are labeled by F and E, respectively. Bright and faint galaxy samples are plotted by dashed and dotted curves, and the total sample by solid line (panel VIRGO). In panels MEL and GR unbiased samples are plotted by solid lines, biased samples by dashed lines.

this difference is seen in the relative fraction of populations of isolated particles (Einasto, Klypin and Shandarin 1983, Einasto et al. 1984). This difference was explained by Zeldovich, Einasto and Shandarin (1982) by the absence of galaxy formation in low density regions.

The term 'biased galaxy formation' was suggested by Kaiser (1984) who independently noticed that galaxy formation occurs basically in high-density regions. Biased galaxy formation has been recently widely discussed (Efsthathiou et al. 1985, Bardeen et al. 1986, Kaiser 1986, Melott and Fry 1986), however detailed physical mechanisms which lead to the bias are not fully understood (Rees 1985, Silk 1985).

Tests provided with the cell method give further quantitative evidence for biased galaxy formation. As seen in Figs 2 and 3, unbiased model samples have some similarity to random samples. This similarity

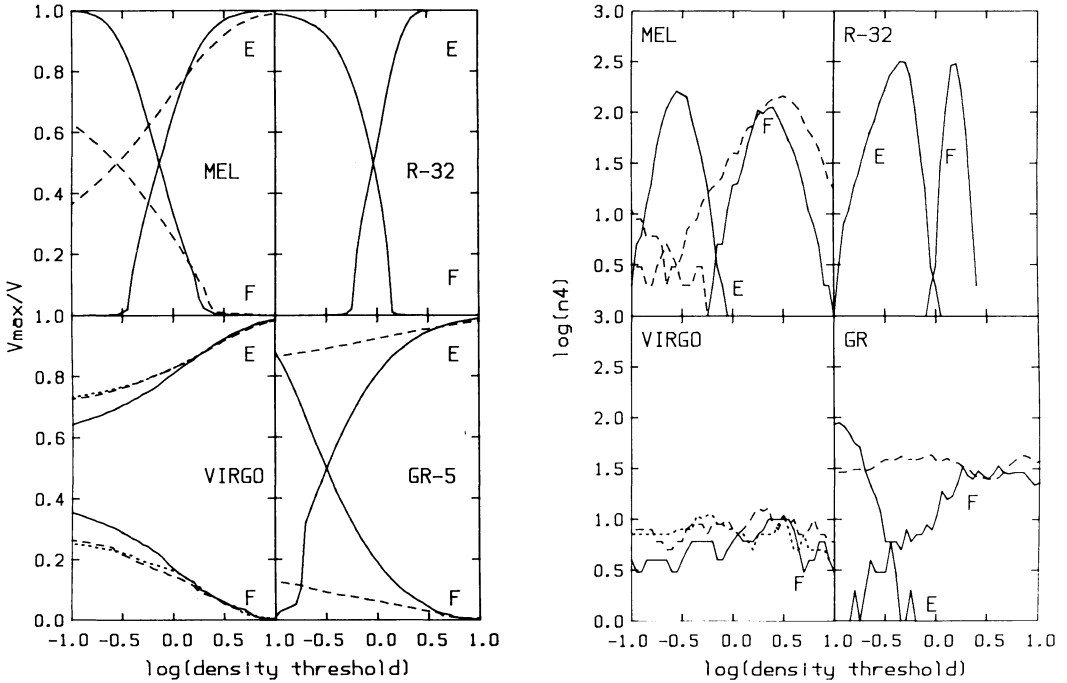


Fig. 2 (left). The plot of the volume fraction of the largest system, $C_{\max} = V_{\max}/V$, versus relative threshold density. Designations as in Fig. 1.

Fig. 3 (right). The plot of the number of systems of multiplicity 4 and larger, n_4 , versus relative threshold density. Designations as in Fig. 1.

is, however, only a qualitative one. E.g. the unbiased model sample GR-5 has mean density $\rho_m = 8$, as in the random sample R-64, but the volume fraction curve of this sample is similar to that of random samples between R-32 and R-16, which have much smaller mean density. This similarity is due to the fact that unbiased model samples have a population of low density particles distributed more or less randomly.

Model samples can be brought into agreement with observations if particles from low density regions are removed, i.e. by introducing a bias in galaxy formation. Changing the biasing threshold it is possible to bring model curves plotted in Figs 2 and 3 (as well as the curve for total filling factor C) into agreement with observations. Filling factors C and C_{\max} are especially sensitive to the biasing level, thus this test can be used to derive 'observational' biasing level of model samples.

Observed samples are centered on the core of the Virgo supercluster which has higher than an average filling factor. Model samples

correspond to much larger boxes which must have lower filling factors. Thus we have used a biasing parameter which supplies a C_{\max} curve by a factor of two lower than the observed curve.

6. SUPERCLUSTER-VOID TOPOLOGY

There has been some discussion concerning the supercluster-void topology. Zeldovich (1983) argued from theoretical considerations that primeval matter at some low density must have cellular topology, i.e. low density regions are from all sides surrounded by high density regions. Also from theoretical considerations Gott, Melott and Dickinson (1986) demonstrated that at median density the topology is of sponge-type, i.e. both high and low density regions form penetrating percolating systems. From observational point of view both possible topologies were discussed by Joeveer, Einasto and Tago (1978) with no conclusive results. More detailed observational data became available in early 80-ies. These data suggest that there are no isolating surfaces between voids (Einasto and Miller 1983, Einasto et al. 1984).

The cell method used here indicates that at low density unbiased model samples have cellular topology which confirms theoretical predictions by Zeldovich. At densities close to median density all samples have sponge-topology conforming to the Gott, Melott and Dickinson study. Observed and biased model samples have sponge-topology in a wide range of threshold densities. At high threshold densities all samples have the topology of isolated islands in a continuous ocean of voids.

Recently interest to supercluster-void topology was renewed by the discovery of large empty bubbles by de Lapparent, Geller and Huchra (1986), surrounded apparently by continuous sheets of galaxies. Data available on nearby voids do not support the view that surfaces surrounding bubbles are actually continuous. If sheets of galaxies around bubbles observed in the de Lapparent et al. survey do not have structure radically different from nearby sheets observed in the Local Supercluster then these distant sheets should also have holes which make percolation of voids possible.

7. ISOLATED GALAXIES

Currently the most popular galaxy formation scenario is the biased cold dark matter model (White et al. 1986). One particular problem with this scenario is the presence or absence of isolated galaxies. As suggested by Dekel and Silk (1986), giant galaxies should be formed only at highest density peaks of the initial density field whereas dwarf galaxies should be formed either everywhere or at lower density peaks. In both cases there should exist a population of isolated dwarf galaxies.

Correlation and conventional cluster analyses are not sensitive to the presence or absence of a relatively small population of isolated galaxies. The cell method used here is more appropriate for this purpose. We have found that the most sensitive test is the number of small systems with cell multiplicity 1 to 3.

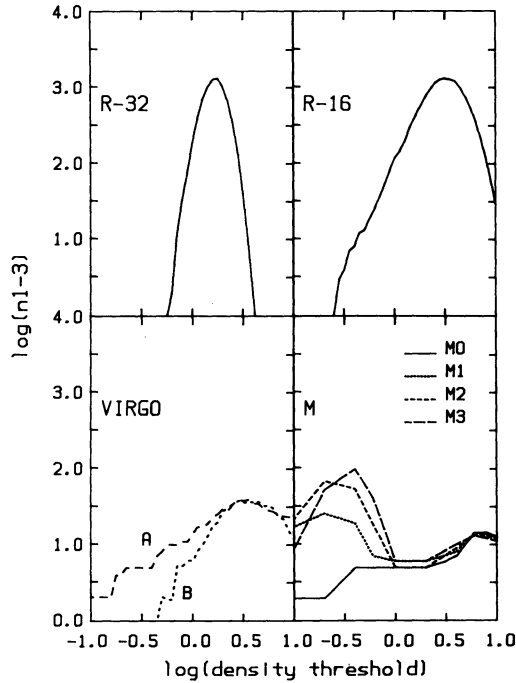


Fig. 4. Number of systems of multiplicity 1-3 versus relative threshold density. Panels R-32 and R-16 describe Poisson samples, panel VIRGO - observed samples, and panel M - a neutrino-dominated model. Bright and faint galaxy samples in panel VIRGO are plotted by dashed and dotted curves, respectively.

Cluster analysis (Einasto et al. 1984) demonstrates that there exists a small population of relatively isolated galaxies in observed as well as in model samples. Traditional cluster analysis gives, however, no answer to the question, are these galaxies outlying members of larger systems or do they form a population of truly isolated galaxies, more or less randomly spread over the whole space.

If the spatial distribution of galaxies is studied by the cell method, then isolated galaxies create a population of systems of low multiplicity, 1, 2 or 3 (due to density smoothing one galaxy increases the density in 1, 2 or 3 neighboring cells). If 'isolated' galaxies are actually outlying members of larger systems then at low threshold densities these systems of low multiplicity join up with larger systems. On the other hand, if these galaxies are really isolated, then at these same threshold densities they create an excess of systems of low multiplicity. This effect is seen in panel d of Fig. 4. Curve labeled M_0 is based on the Melott neutrino dominated model (Melott et al. 1983) with biased galaxy formation. All test particles from low density regions are removed, and voids contain no test particles. Samples M_1 ,

M_2 , and M_3 contain all particles of sample M_0 plus 50, 205 and 410 randomly located additional particles (1%, 5% and 10% of the original population). An excess of systems of low multiplicity in samples M_1 , M_2 and M_3 is clearly visible. Sample M_0 , as well as both observational samples have no excess of systems of low multiplicity in respective density interval.

This test demonstrates that galaxies at low density tail are related to other galaxy systems and that voids are really empty. This result is valid for both bright and faint galaxy samples.

8. FINAL REMARKS

The cell method used here complements other quantitative methods used earlier and is no substitute for them. Real world has a complex structure and there exist no single statistics which characterizes all aspects of the distribution of galaxies. Tests applied in this paper, suggest that

(i) there is strong observational evidence for biased galaxy formation,

(ii) at a broad relative density interval superclusters and voids have sponge-like topology, i.e. both filled and empty regions form infinite percolating and intertwined systems,

(iii) voids are really empty and contain neither bright nor faint galaxies.

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DISCUSSION

LACHIEZE-REY: A smooth variation of the filling factor with scale is an indication of scale invariance. So the fit between the observations and biased galaxy formation is the consequence of the fact that the real distribution of galaxies, and the one predicted by biased galaxy formation, both obey some scaling law. A fractal model exhibits such a characteristic.