Chapter I

SOLAR DYNAMO AND ACTIVITY CYCLES: OBSERVATIONS, THEORIES AND SIMULATIONS

Solar and Astrophysical Dynamos and Magnetic Activity
Proceedings IAU Symposium No. 294, 2012
A.G. Kosovichev, E.M. de Gouveia Dal Pino, & Y. Yan, eds.

doi:10.1017/S1743921313002159

Helioseismic measurements of differential rotation and meridional flow

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Abstract. Solar interior differential rotation and meridional flow play important roles in dynamo models. In this review, I briefly review results in interior rotational profiles and zonal flows obtained from helioseismology studies. Due to the new developments in recent years in interior meridional flow studies, this review focuses more on results on meridional flow. I describe new developments in the shallower poleward meridional flow, and the temporal evolution of these flows. Then I introduce a newly identified center-to-limb variation effect in helioseismology studies, and present recent results in the search of equatorward meridional flows. I also discuss how these results will effect the dynamo models.

Keywords. Sun: helioseismology – Sun: interior – Sun: rotation

1. Introduction

The Sun has sunspots appearing on its surface, and the number of sunspots has a 11-year cycle. Sunspots are regions where strong magnetic field concentrates, and the magnetic field has a 22-year cycle. Where does the magnetic field come from and how to explain its cycle rely on solar dynamo models. The $\alpha\Omega$-dynamo model is a popular theory explaining the generation of the Sun's magnetic field and its cycle, and this model largely depends on the solar interior rotational profile and the evolution of its profile with the phase of the cycle. The flux-transport dynamo includes solar meridional flow profile in the model to help determine the duration of solar cycles.

Therefore, the accurate determination of solar interior differential rotation profile and interior meridional flow profile is crucial to the modeling efforts of solar dynamo, thus leading to a better understanding of the magnetic field generation procedure inside the Sun. The solar rotational profile has been well determined by global helioseismology analysis, and the interior meridional flow, due to its smaller amplitude, is still under intensive studies and debate. In this paper, I briefly review differential rotations from helioseismology studies in Sec. 2, and focus more on recent development in meridional flow studies in Sec. 3. For the review of meridional flow studies in Sec. 3, I divide it into four subsections: poleward meridional flow, temporal variation of meridional flow, center-to-limb variation, and detection of equatorward meridional flow. I conclude the paper in Sec. 4.

2. Differential Rotation

The Sun rotates differentially. It rotates faster in low latitude and slower in high latitude, and this was recognized long time ago when tracking the rotational speed of sunspots and other visible features. With the development of global helioseismology, it was recognized that the rotational rate also varies with depth. In particular, there was a surface shear layer near the depth of 0.93 $R_\odot$; at the bottom of the convection zone,

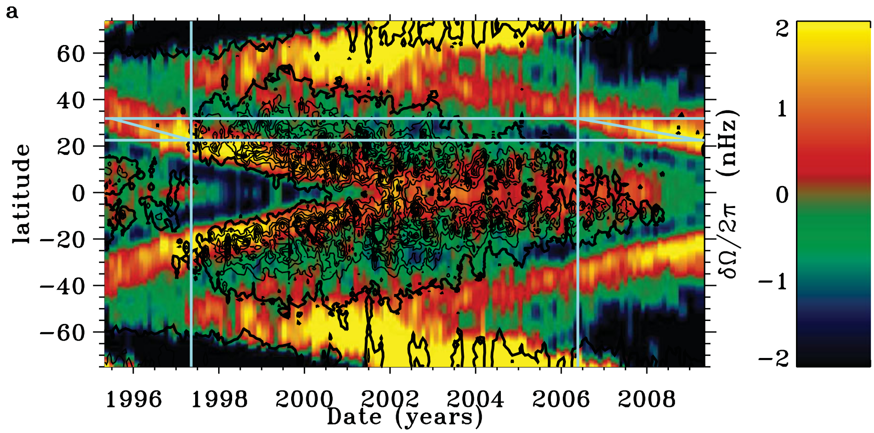

Figure 1. Background image shows torsional oscillations at the depth of 0.99 $R_\odot$ measured from MDI and GONG. Overlaid contours show the gross longitudinal magnetic field strength with 5 Gs intervals. Solid lines indicate the slower migration of faster rotation band. This figure is adapted from Howe *et al.* (2009).

i.e., about 0.70 $R_\odot$, there was a strong velocity gradient layer, coined as tachocline. Many articles on helioseismology discussed the derivation of interior rotational profile, e.g., Thompson *et al.* (1996), Kosovichev *et al.* (1997), Schou *et al.* (1998), Howe *et al.* (2000), and others. These results changed the earlier perspective of that solar dynamo operated in the whole bulk of the convection, and it was more established that the tachocline was the location of solar dynamo operation. However, on the other hand, some shallow dynamo models (e.g., Brandenburg 2005) were also proposed suggesting the dynamo operating site at the shear layer close to the solar surface.

Torsional oscillation is another interesting phenomenon associated with the differential rotation and has been widely studied using both global and local helioseismology techniques. Torsional oscillation exhibits as latitudinal bands of faster and slower rotations, and these bands gradually migrate towards the solar equator together with the solar activity belts in both hemispheres. This phenomenon was first reported by Howard & LaBonte (1980) by analyzing photospheric rotation patterns, and later was found existing inside the Sun as revealed from helioseismology studies (Kosovichev & Schou 1997). The analyses by Howe *et al.* (2000) and by Vorontsov *et al.* (2002) found that the torsional oscillations extended far deep into the solar interior. Later studies using local helioseismological techniques also reported similar phenomenon, e.g., Haber *et al.* (2002), Zhao *et al.* (2004). To interpret the observed torsional oscillation, Schüssler (1981) and Yoshimura (1981) proposed a Lorentz force feedback, and Spruit (2003) tried to explain it by thermal driving.

To understand the unusually long and low activity minimum near the end of Cycle 23, Howe *et al.* (2009) analyzed the torsional oscillations covering the period of the minimum of Cycle 22 to the minimum of Cycle 23. The comparison of torsional oscillations between these two minima showed that the faster flow band corresponding to the new cycle had been moving more slowly toward the equator than it was during the previous cycle minimum. This resulted in a gradual increase in the length of the cycle during the 2007 – 2008 period. Figure 1 shows their result. Their follow-up analysis showed that the higher latitude branch did not appear as in the previous solar cycle, and this might indicate a very weak magnetic activity for the current solar cycle.

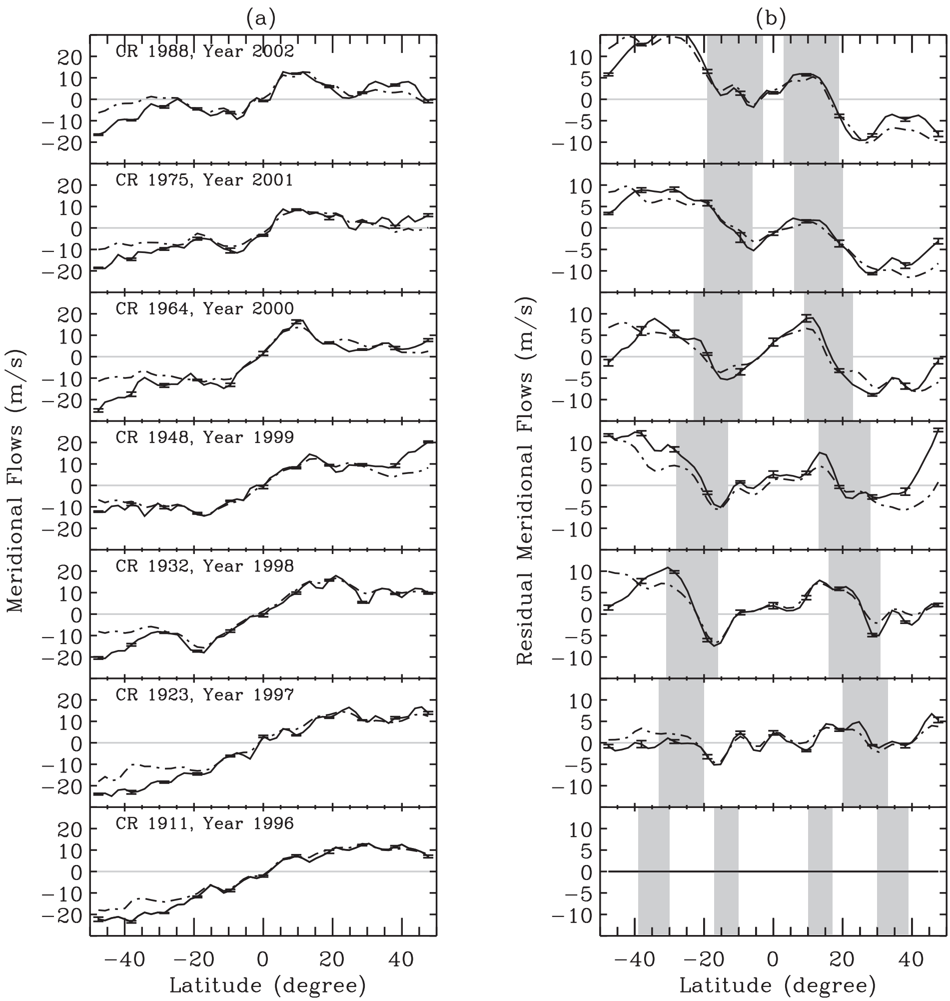

Figure 2. (a) Meridional flow obtained from 3 – 4.5 Mm (solid curves) and 6 – 9 Mm (dash–dotted curves) for different Carrington rotations. (b) Residual meridional flows after the flows of CR1911 have been subtracted from each rotation. Shaded regions indicate the locations of magnetic activity belts. This figure is adapted from Zhao & Kosovichev (2004).

3. Meridional Flow

3.1. *Poleward Meridional Flow*

Due to the relatively small amplitude of meridional flow speed, typically around 20 m s^{-1}, the determination of meridional flow speed was not easy in the earlier years. After some confusions, a poleward meridional flow was finally observed through analyzing Doppler velocities in the photosphere observed at Stanford Solar Observatory (Duvall 1979). This result was confirmed by later analyses using Doppler observations (Hathaway *et al.* 1996), sunspot tracking (Howard & Gilman 1986), magnetic feature tracking (Komm *et al.* 1993), and correlation tracking of MDI magnetograms (Meunier 1999) and Dopplergrams (Švanda *et al.* 2007). Therefore, meridional flow with a poleward speed of approximately 20 m s^{-1} has been well established in the solar photospheric level.

For meridional flow in the solar interior, local helioseismology offers us an opportunity to investigate. By use of time-distance helioseismology and employing MDI Doppler

https://doi.org/10.1017/S1743921313002159 Published online by Cambridge University Press

velocity data, Giles *et al.* (1997) reported that the poleward meridional flow penetrated into the solar convection to a depth of at least 0.04 $R_\odot$ with a peak speed of around 10 – 20 m s^{-1}. The poleward flow in the interior help to redistribute angular momentum within the Sun. Later analyses using the same or different local helioseismology techniques confirmed the poleward flow in the Sun's shallow interior, e.g., by ring-diagram analysis (González Hernández *et al.* 1999, Haber *et al.* 2002, Basu *et al.* 2004), by time-distance helioseismology (Zhao *et al.* 2004, Zhao *et al.* 2012b), and by analysis of acoustic frequency shifts (Braun & Fan 1998, Krieger *et al.* 2007). Therefore, it has also been widely accepted that the poleward meridional flow extends from the photosphere to at least 30 Mm deep into the Sun. Figure 2a shows some examples of the poleward flow in the shallow areas inside the Sun. Very recently, Woodard *et al.* (2012) developed a new method of inferring interior meridional flow using global oscillation eigenfunctions, and their results were also in agreement with results reported earlier in shallow regions as well.

3.2. *Temporal Variations of Meridional Flow*

Solar meridional flow does not stay the same for different phases of a solar cycle. This has already been realized in earlier studies by, e.g., Snodgrass (1987), Komm *et al.* (1993), and Hathaway *et al.* (1996), in photospheric flow fields. Similar analyses and results have already been done in more recent years by Meunier (2005) and Švanda *et al.* (2008). For the interior meridional flow structure, Chou & Dai (2001) studied the flow as functions of both latitude and depth for the period of 1994 through 2000, and found that a new component of meridional flow, centered at about latitude of 20°, was created in each hemisphere as the solar activity increased from 1997 to 2000. Beck *et al.* (2002) did similar studies but with more continuous data coverage. They also found one extra time-varying component that formed a banded structure migrating towards the solar equator. This time-varying component of meridional flow consists of a flow diverging from the dominant latitude of magnetic activity. Both studies targeted at a very deep interior of the Sun.

The near-surface helioseismological studies confirmed the temporal variations of the meridional flows, too. Using ring-diagram analysis, Haber *et al.* (2002) reported that the gradient of the near-equator meridional flow steepened with the development of the

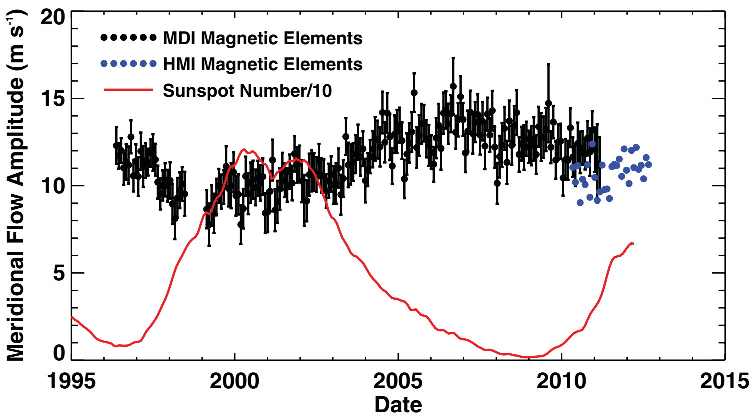

Figure 3. Meridional flow amplitude variation (black and blue) measured from MDI and HMI magnetic elements. The scaled (by 1/10) smoothed sunspot number is shown in red to indicate phases of the solar cycle. This figure is an updated version of Figure 4 in Hathaway & Rightmire (2010). Courtesy: David Hathaway.

solar cycle towards the solar maximum. Employing time-distance technique, Zhao & Kosovichev (2004) removed the mean meridional flow profile of 1996, a solar minimum year, as a reference, and found that the residual meridional flow converged towards the solar activity belts. Figure 2b illustrates these results. This observational phenomenon was later confirmed by other studies. The converging meridional flow toward the activity belts was believed caused by thermal driving, as suggested by Spruit (2003) and further modeled by Rempel (2006) after taking into consideration Lorentz force feedback.

Recent analysis (Hathaway & Rightmire, 2010) using magnetic feature tracking found that for Solar Cycle 23, the meridional flow speed was faster at the cycle minimum and slower during the maximum years. And more interestingly, the speed was substantially faster during the minimum years of Cycle 23 than it was during the minimum years of Cycle 22 (see Figure 3 for these results). The authors suggested that the faster meridional flow speed in the late years of Cycle 23 helped to explain why there was an extremely long and weak minimum. However, on the other hand, flux-transport dynamo model by Nandy *et al.* (2011) claimed that a slower meridional flow speed was needed to explain the extended minimum of Cycle 23, at odds with the observational evidences presented by Hathaway & Rightmire (2010).

The meridional flow speed not just changes with the phase of the solar cycle, but also changes from cycle to cycle. By analyzing the long-term observations made by Mt. Wilson Observatory, Ulrich (2010) compared the meridional flow profiles averaged from Solar Cycle 22 and from Cycle 23, and found significant differences in the two averaged profiles. It turns out that the flow speed is generally faster in Cycle 23 than in Cycle 22. More interestingly, it shows that there existed a counter flow cell in latitude higher than about 65° for Cycle 22, but there was no such a cell in Cycle 23. Dikpati *et al.* (2010) suggested that this observational fact might be the cause of the longer duration of Cycle 23 through simulations of flux-transport model with an inclusion of the observational results of counter-flow cell in high latitude during Cycle 22, as reported by Ulrich (2010).

3.3. *Center-to-Limb Variation*

Recently, a systematic center-to-limb variation effect was identified in helioseismology analysis, and this effect would have a strong influence in the helioseismology-derived subsurface meridional flow speed (Zhao *et al.* 2012c). This systematic effect can be briefly introduced as follows. HMI has different observables, namely, Doppler velocity, continuum intensity, line-core intensity, and line-depth, and in principle all these observables are suitable to perform helioseismic analysis. However, the time-distance measured acoustic travel times in the North-South direction, supposedly corresponding to the solar interior

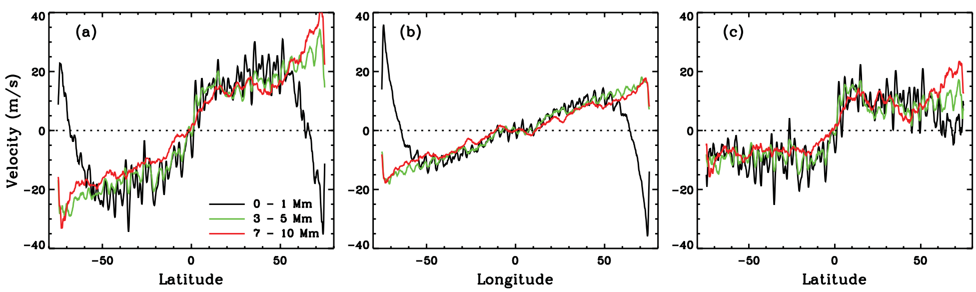

Figure 4. (a) Meridional flow velocity for different depths, obtained before removal of the systematic effect. (b) Antisymmetrized east-west velocity obtained from inversion along the equatorial area, representing the center-to-limb variation. (c) Meridional flow for different depths after removal of the systematic effect. This figure is adapted from Zhao *et al.* (2012c).

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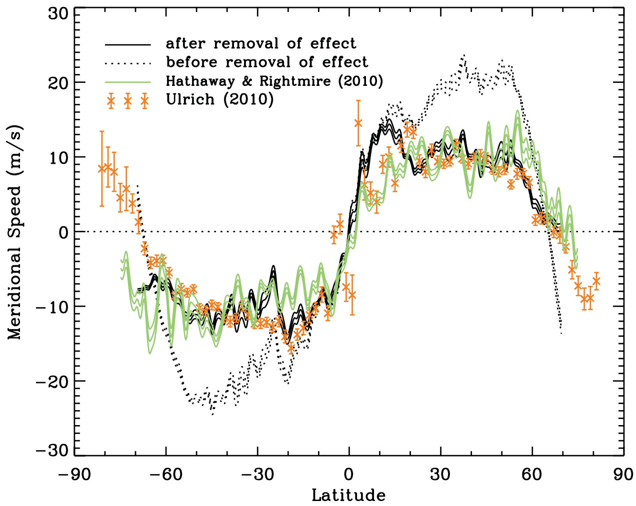

Figure 5. Comparison of the meridional flow speed obtained from magnetic feature tracking (Hathaway & Rightmire 2010), from photospheric Doppler measurement (Ulrich 2010), and obtained from time-distance analysis at a depth of 0 – 1 Mm before and after removal of the center-to-limb effect. This figure is adapted from Zhao *et al.* (2012c).

meridional flow, obtained from these different observables gave sharply different results, contradictory to the expectation that they should be the same or very close. Moreover, the measured travel times along East-West direction, which correspond to the solar interior rotation and are expected to be flat along a same latitude due to that the rotation speed is the same at the same latitude, also show a systematic variation. Interestingly, the travel-time differences between the North-South travel times and the East-West travel times for different observables gave similar results. This demonstrates that there exists a systematic center-to-limb variation in helioseismology analysis, and this effect must be removed before inferring interior meridional flow speed from the measured acoustic travel times. Please refer to Zhao *et al.* (2012c) for a more detailed description of this systematic effect.

After removal of this systematic effect, the meridional flow speed in the shallow interior is about 5 m s^{-1} slower than previously obtained without removing this effect. Figure 4 illustrates the effect of this systematic variation on the inferred meridional flow speed. And, the newly obtained meridional flow is in a better agreement with the measurements using other methods, e.g., direct Doppler measurement and magnetic field tracking. Figure 5, adapted from Zhao *et al.* (2012c), shows an example of how these results compare to each other.

This systematic effect was firstly found using time-distance analysis, but it exists in other local helioseismology analysis techniques, e.g., ring-diagram analysis. Recent evidence also showed that this effect also exists in the normal mode analysis when trying to derive the interior meridional flow profile (Schou *et al.* 2012). Despite the importance of this center-to-limb variation effect in meridional flow inference, its physical basis is not yet quite clear. Very recently, Baldner & Schou (2012) suggested that this effect might be due to the highly asymmetrical nature of solar granulation that results in a net radial flow to the oscillation modes. But their results do not fully account for the measured acoustic travel time shifts, and more observations may be required to examine the validity of their interpretation.

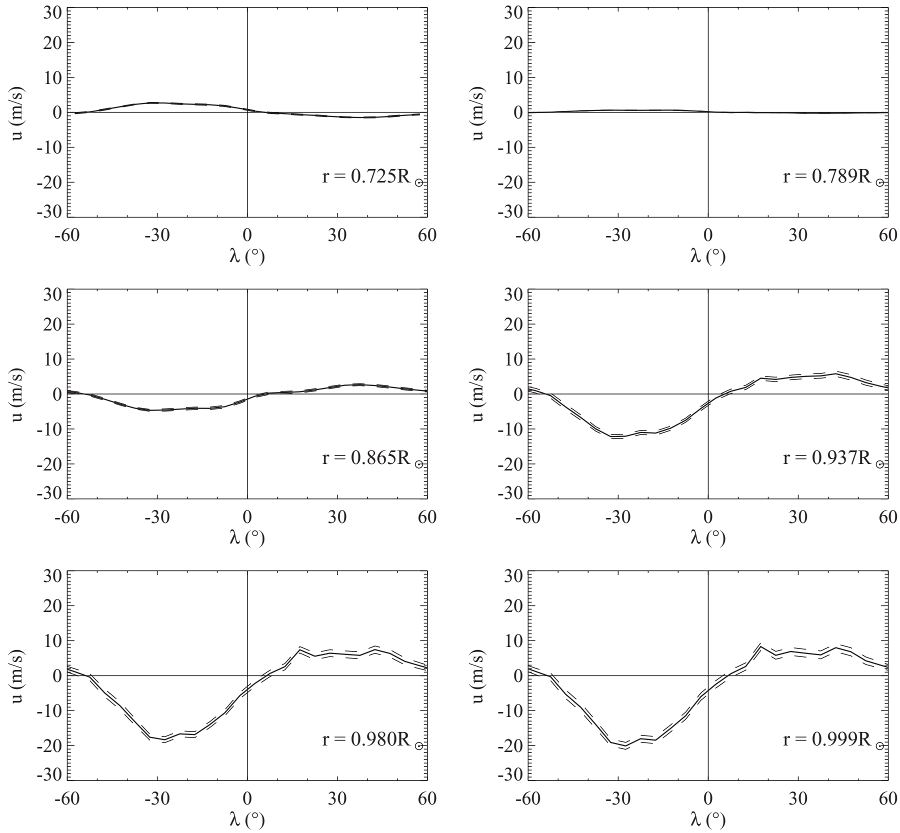

Figure 6. Meridional circulation inversion results, as a function of latitude λ, for 6 different depths. The inversion was performed with a constraint of mass conservation inside the solar convection zone, and an assumption of that meridional flow speed is 0 below 0.70 $R_{\odot}$. Positive velocities are northward in these plots. This figure is adapted from Giles (1999).

3.4. *Equatorward Meridional Flow*

As introduced in Sec. 3.1, poleward meridional flow extends from the photosphere to at least 30 Mm in depth. Due to mass conservation, an equatorward meridional flow must exist somewhere in the solar interior to balance the mass flows. And, the equatorward flow also plays a crucial role in transporting magnetic field from the polar area back to lower latitude, as demonstrated by Dikpati & Gilman (2009).

The search for the equatorward meridional flow has a decade-long history, and only very recently have some evidences been found. Giles (1999) analyzed MDI full-disk Dopplergrams by use to time-distance helioseismology technique. Briefly, equatorward flow was not able to be found in their inversions if only using the acoustic travel times measured from their analysis. However, if an inversion was done by a combination of acoustic travel-time measurements and a mass conservation constraint, then a 2 m s^{-1} return flow was found near the base of the convection zone, i.e., at a depth of about 200 Mm. Together with the poleward flow in the shallower interior, the meridional flow inside the Sun forms a single circulation cell. Figure 6 shows the interior meridional flow profile inferred from their study. And in fact, many flux-transport models employed such a single circulation cell (e.g., Dikpati & Charbonneau 1999), or a cell with even deeper equatorward flow below the convection zone (Nandy & Choudhuri, 2002).

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A later analysis on sunspots drift rate gave a similar result of deep return meridional flow near the bottom of convection zone. Hathaway et al. (2003) analyzed the drift of the centroid of the sunspot area toward the equator observed during the years of 1874 and 2002, and found that the drift rate was consistent with a equatorward flow of a speed of 1.2 m s^{-1}. These authors also found that the speed of this flow was correlated with the amplitude of the following solar cycle, hence could be used as a predictor of strength of solar cycles.

However, more recent analyses seemed to contradict with these older claims. By analyzing acoustic frequency shifts observed by MDI, Mitra-Kraev & Thompson (2007) derived meridional flow as a function of depth through nearly the entire convection zone, but without a determination of latitudinal dependence. They found a fairly shallow equatorward flow at a depth of 40 Mm or so. This disagrees with results of Giles (1999), and is also at odds with results obtained using a same analysis technique (Braun & Fan 1998, Krieger *et al.* 2007).

By tracking the movement of supergranules in the photosphere, Hathaway (2012a) determined that supergranules were advected by the zonal flows at depths equal to the widths of the convective cells. Furthermore, interior meridional flow profile was derived using the depths determined from the zonal analysis (Hathaway 2012b). His result showed that the poleward meridional flow speed drops fast with the depth, and the meridional flow returns equatorward at depths greater than 50 Mm, just below the surface shear layer. The equatorward flow speed is roughly 4.6 m s^{-1} at the depth of ~70 Mm. However, these results are controversial due to his method of determining the depth of supergranules. The basic idea of using supergranules as tracers of studying the solar interior does

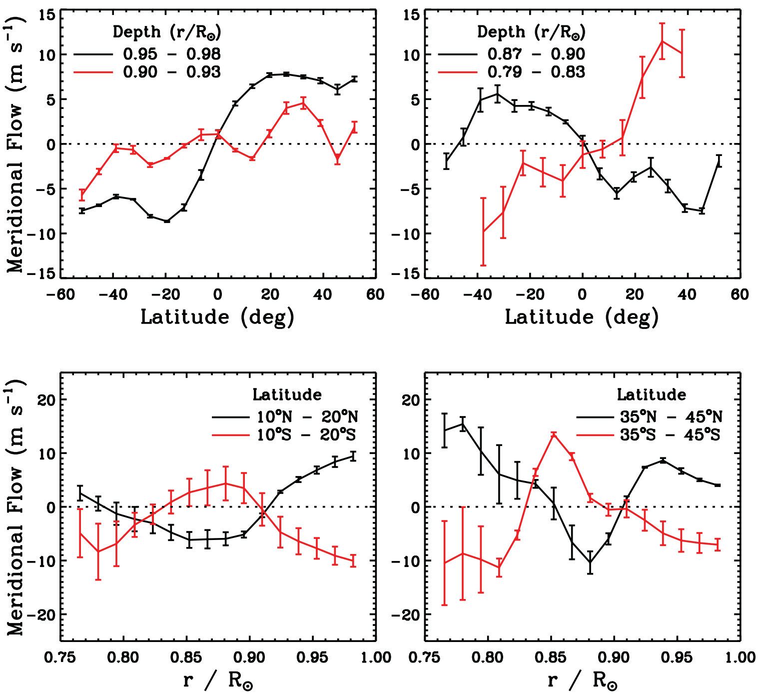

Figure 7. Deep meridional flow profile obtained from acoustic travel time measurements after a center-to-limb variation correction. Upper panels show the flow velocity as functions of latitude averaged from several depths, and lower panel show the velocity as functions of depth averaged from selected latitudinal bands.

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not agree with explaining supergranules as wave-like features by Gizon *et al.* (2003) and Schou (2003). Also, results from a similar supergranulation tracking analysis seemed to be able to be well explained by wave phenomenon Gizon & Duvall 2003. All these contradictory results demand a better understanding of the nature of supergranulation.

Given that a systematic center-to-limb variation was recently discovered, it is more useful to repeat deep interior measurements using time-distance helioseismology analysis like what Giles (1999) has done. Our most recent analysis, after combining the measurement technique developed by Giles (1999) and the center-to-limb effect correction approach proposed by Zhao *et al.* (2012c), has detected an equatorward flow located below the depth of 60 Mm or so with a highest speed of approximately 10 m s^{-1} (Zhao *et al.* 2012a). This equatorward flow extends from about 0.91 $R_\odot$ to 0.82 $R_\odot$, and below that, there exists another poleward flow. Therefore, these new observations provide an evidence of double-cell meridional flow profile with an equatorward flow located in the middle of the convection zone, and poleward flows located near the surface and near the bottom of the convection zone. Figure 7 shows some results from that study. These results were obtained from a well accepted method after correction of newly identified systematic effect, hence are more robust and less controversial. But, perhaps, it is fair to say, all these efforts together make it a convincing case that equatorward flow exists in a much shallower depth than the bottom of the convection zone.

4. Conclusion

Solar differential rotation and meridional flow are crucial to solar dynamo models, and rapid developments in both global and local helioseismology have helped dynamo models progress. By including results from helioseismology studies and other observational facts, dynamo models have been successful in explaining mean-field magnetic field, solar cycles, and predicting the duration and strength of a cycle. The very recent result of the detection of shallow equatorward flows and the double-cell meridional circulation structure, as introduced in Sec. 3.4, poses new challenges to dynamo models, especially to the flux-transport dynamo model. Although numerical simulations of the solar interior dynamics have already revealed the possibility of a multi-cell circulation in the convection zone (e.g., Miesch *et al.* 2006, Käpylä *et al.* 2012), some numerical experiments using flux-transport dynamo model showed the double-cell circulation was not able to reproduce the magnetic butterfly diagram (Jouve et al. 2008). New observations as introduced in Sec. 3.4 require a re-examination of solar dynamo models.

Acknowledgements

I thank Drs. R. Howe and D. Hathaway for providing their figures to be included in this paper.

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