Magnetic Fields throughout Stellar Evolution
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Magnetic Fields in Stars: Origin and Impact

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Abstract. Various types of magnetic fields occur in stars: small scale fields, large scale fields, and internal toroidal fields. While the latter may be ubiquitous in stars due to differential rotation, small scale fields (spots) may be associated with envelop convection in all low and high mass stars. The stable large scale fields found in only about 10% of intermediate mass and massive stars may be understood as a consequence of dynamical binary interaction, e.g., the merging of two stars in a binary. We relate these ideas to magnetic fields in white dwarfs and neutron stars, and to their role in core-collapse and thermonuclear supernova explosions.

Keywords. Stars, magnetic fields, stellar evolution, supernovae

1. Introduction

Magnetic fields play a vital role in all stages of stellar evolution. This is already true during star formation. The magnetic support in collapsing molecular cloud cores is fundamentally affecting the fragmentation process (Price & Bate 2007). Later-on, during the accretion process, magnetic fields provide the required viscosity to bring in mass and to remove surplus angular momentum (Donati *et al.* 2007).

In this paper, we investigate the role of magnetic fields in stars once they are born. In order to do so, we distinguish various types of magnetic fields. First, it is useful to distinguish stable from dynamo fields. As stable fields we consider those which have a decay time of the order of the stellar life time or more, and which therefore do not need a dynamo action to continuously replenish them. Braithwaite & Spruit (2004) showed that combined toroidal-poloidal magnetic fields can survive in the radiative envelopes of stars for a long time, which they suggested to exist in magnetic A stars and in magnetic white dwarfs.

Other magnetic field geometries have so far been found unstable, e.g., such fields are expected to decay on their Alfvén time scale (e.g., Tayler 1973). However, inherently unstable field configurations may be present in stars over long time scales, if a dynamo process is continuously regenerating the field (Brandenburg & Subramanian). It may be expected that this regeneration process leads to some time variability. The prime example may be the Solar magnetic field, which is produced by a so called $\alpha\Omega$-dynamo, where the B-field is generated by an interplay between the differential rotation, which winds up poloidal field and generates toroidal field, and the α-effect, which generates a poloidal field from a toroidal one (Rüdiger *et al.* 2013).

While only stable and dynamo fields are long-lived and thus accessible to observations, there is some evidence for intermittent fields playing a role as well (Langer 2012). In particular during dynamical stellar merger events, which are suspected to lead to stable fields in the merger product (see below), the fields during the merger event itself are thought to be significantly stronger than thereafter. This strong intermittent component may be responsible for a removal of a large fraction of the angular momentum during the merging process.

From the observational perspective, it is also useful to distinguish various types of fields. For once, there are large scale fields, i.e. fields where the length scale over which local field maxima occur at the stellar surface is comparable to the size of the star itself. A classical example is a dipole field, i.e. a field which has only two points of maximum field strength at the stellar surface, which are located at different sides of the star. Dipole fields which have their magnetic axis inclined to the axis of rotation are in fact common amongst intermediate mass and high mass stars, although somewhat more complicated but still large scale field geometries occur as well.

In contrast to the large scale fields are small scale fields, for which the length scale of significant field variation is small compared to the stellar radius. An example for this are the Solar sunspots. The Sun also shows that stellar magnetic fields can have various components, as the small scale sunspots with field strengths of the order of 1000 G, and the global Solar dipole field with a strengths of about 1 G.

Finally, from the observational perspective, we want to distinguish toroidal magnetic fields as a third type, since toroidal fields are essentially hidden from direct observations. Still, they may strongly influence the evolution of stars, and may thus produce indirect evidence of their existence.

2. Toroidal fields: ubiquitous?

Spruit (2002) has suggested that a dynamo process can operate in differentially rotating radiative stellar envelopes. The main component of the produced magnetic field is toroidal, which is thought to counteract the differential rotation by producing a torque which transports angular momentum against the angular momentum gradient. While the model of Spruit has been criticized (Zahn, Brun & Mathis 2007), the main effect has been confirmed in simplified MHD models (Braithwaite 2006).

While the toroidal fields are not directly observable, there are currently two lines of observational evidence in their support. First, the nearly rigid rotation in the Sun beneath the Solar convection zone has been reproduced by Eggenberger *et al.* (2005) relying on the Spruit mechanism. Second, Mosser *et al.* (2012) found through oscillation measurements in a large sample of pulsating giants that the red giant cores rotate much slower than expected when only non-magnetic angular momentum transport processes are taken into account. While they claim that their results agree with the slow observed spins of white dwarfs, the evolutionary models of Suijs *et al.* (2007) which include angular momentum transport through the Spruit mechanism predict indeed white dwarf and neutron star spin periods which are close to the observed values (Fig. 1). While it can not be excluded that non-magnetic transport processes like gravity waves (Talon & Charbonnel 2008) could also reproduce the observational constraints, the results quoted above may speak in favor of the Spruit mechanism.

The only prerequisite of the Spruit mechanism is differential rotation. In case the Spruit mechanism works as expected, we may than conclude that toroidal magnetic fields are present in *all* stars, perhaps with the exception of stars with strong internal large scale fields, which perhaps rotate as rigid bodies, and of fully convective stars — where the Spruit mechanism may be overpowered by the predominance of convection.

3. Low mass stars

We define low mass stars as such stars which have convective envelopes during core hydrogen burning. The Sun is a low mass star. Magnetic activity in low mass stars is investigated since may decades, and many papers in these proceedings give the status of

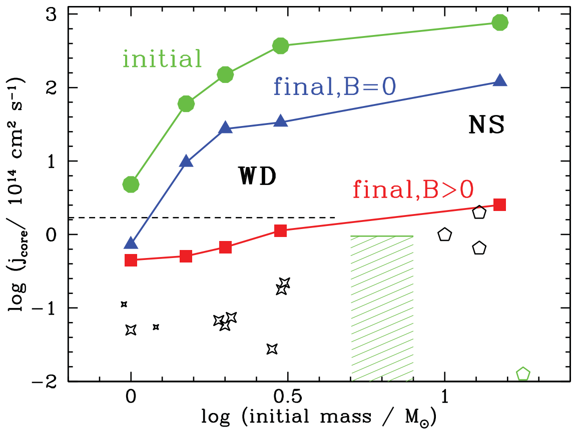

Figure 1. Average core specific angular momentum versus initial mass for the low and intermediate mass models of Suijs *et al.* (2008) and for the 15M$_\odot$ model of Heger, Woosley & Spruit (2005) evolving from the zero age main sequence to their end stages (full drawn lines). The upper line corresponds to the initial models. Filled triangles mark the final models of the non-magnetic sequences, and filled squares the final models of the magnetic sequences. The dashed horizontal line indicates the spectroscopic upper limit on the white dwarf spins obtained by Berger et al. (2005). Star symbols represent asteroseismic measurements from ZZ Ceti stars, where smaller symbols correspond to less certain measurements. The green hatched area is populated by magnetic white dwarfs. The three black open pentagons correspond to the youngest galactic neutron stars, while the green pentagon is thought to roughly correspond to magnetars. See Suijs *et al.* (2008) for details.

the current research. We therefore restrict ourselves here to address the magnetic fields in low mass stars just for comparison to those in more massive stars.

The present conclusion is that *all* low mass stars show magnetic fields. I.e., the presence of a convective envelope is sufficient to develop a field. While rotation is a necessary ingredient to the $\alpha\Omega$-dynamo, and faster rotators tend to show higher magnetic activity, even slow rotators as our Sun possess an appreciable magnetic field. It is clear that these fields lead to an efficient angular momentum loss over the lifetime of these stars, to the extent that the their spin rate is a function of their age.

4. Intermediate mass stars

Intermediate mass stars prove mostly to be non-magnetic. Only about 10% of the core hydrogen burning stars in this mass range show a strong large scale field of more than a few hundred Gauss, while the remaining 90% appear to have fields which are weaker than about one Gauss. While the fields in the magnetic fraction of intermediate mass main sequence stars appear, partly, to have a rather complex morphology (Donati *et al.* 2006), their structure appears to be simple compared to the fields of low mass stars (Donati & Landstreet 2009).

The magnetic intermediate mass stars are generally slow rotators. I.e., while the field strengths of low mass stars are larger for faster rotators, the situation is almost the reverse for intermediate mass stars. While their cores are convective and could produce a magnetic field deep down (Brun *et al.* 2005), it appears unlikely that this field is transported deeply into the radiative envelope or even to the stellar surface (Charbonneau & MacGregor 2001, MacGregor & Cassinelli 2003, MacDonald & Mullan 2004). Since also their envelops are radiative, it appears not to be possible to explain their magnetic fields through a dynamo process.

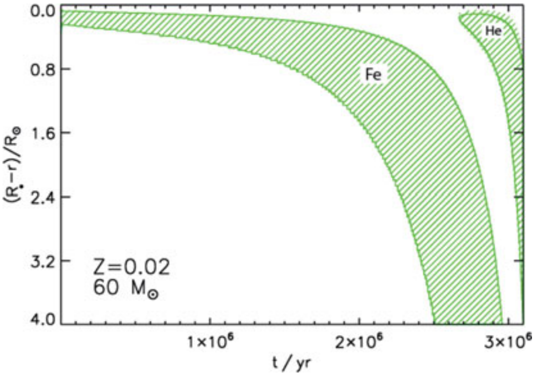

Figure 2. Evolution of the radial extent of the subsurface helium and iron convective regions (hatched) as function of time, from the zero age main sequence to roughly the end of core hydrogen burning, for a 60 $M_{\odot}$ star (Cantiello *et al.* 2009). The top of the plot represents the stellar surface. Only the upper 4 $R_{\odot}$ of the star are shown in the plot, while the stellar radius itself increases during the evolution. The star has a metallicity of $Z = 0.02$, and its effective temperature decreases from 48 000 K to 18 000 K during the main sequence phase.

As mentioned above, Braithwaite & Spruit (2004) found stable magnetic field configurations to be able to exist in these stars. However, while the suggested field geometries are quite compatible to those observed, this does not allow any conclusion on the origin of these fields. We return to this question on Sect. 6.

5. Massive stars

Evidence is accumulating that massive main sequence stars show both, the small scale fields produced by convective envelop dynamos, and the large scale stable fields just as they are observed in the intermediate mass stars.

5.1. *Small-scale fields*

Cantiello *et al.* (2009) pointed out that massive main sequence stars have convective envelops which may be capable to produce observable magnetic fields. While the convection zones occur beneath the stellar surface due to opacity peaks produced by iron and helium recombination, their distance to the surface is so small that magnetic flux tubes can buoyantly float to the surface in a short time. Their spatial extent is a significant fraction of the stellar radius (Fig. 2).

There is multiple observational evidence for the existence of these sub-surface convection zones. First, Cantiello *et al.* (2009) showed that the predicted dependence of the kinematic signature of these zones at the stellar surface on stellar mass, surface temperature and metallicity agrees with the observations of micro-turbulence as determined from spectroscopic measurements of a large number of O and early B stars in the Galaxy and the Magellanic Clouds. Secondly, stochastically excited pulsations have been measured in several massive main sequence stars (cf., Belkacem *et al.* 2009). And thirdly, the velocity field induced by the sub-surface convection may lead to a clumping of the hot star winds very near to their surface, in agreement with observations (Cantiello *et al.* 2009, Sundqvist & Owocki 2013).

Finally, and most relevant in the present context, magnetic spots at the surface of massive main sequence stars as predicted by Cantiello *et al.* (2009) may provide an explanation of the discrete absorption components (DACs) in their UV spectra lines. The DACs phenomenon appears to be best explained by assuming a disturbance of the

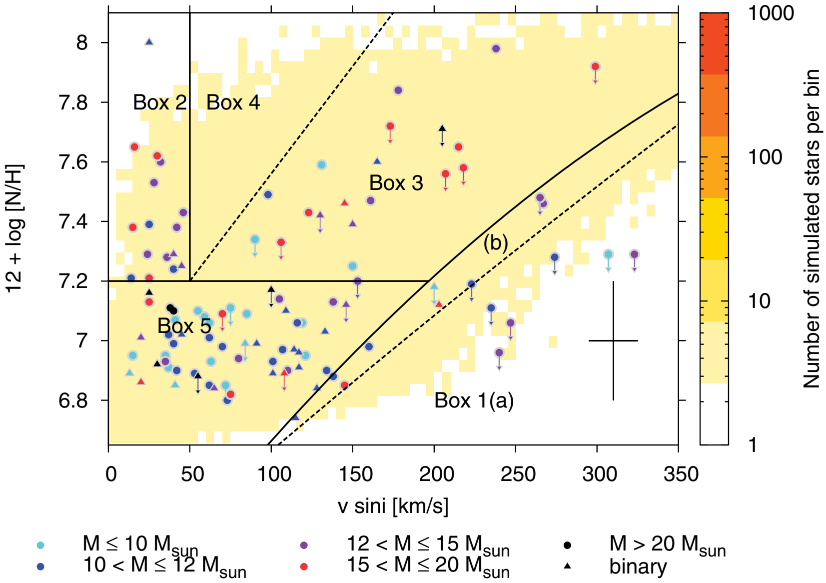

Figure 3. Hunter diagram for LMC early B type stars from the VLT-FLAMES Survey of Massive Stars (symbols), showing projected rotational velocity against their nitrogen surface abundance. Single stars are plotted as circles, radial velocity variables as triangles. A population synthesis simulation based on single star evolution models with rotational mixing (Brott *et al.* 2011a) is shown as a density plot in the background (Brott *et al.* 2011b). The color coding corresponds to the number of predicted stars per pixel. The cross in the lower right corner shows the typical error on the observations.

radiation driven wind of the star due to hot spots at its surface (Cranmer & Owocki 1996). As DACs are ubiquitous in O stars (Howarth & Prinja 1989), the tentative implication is that so are their small scale magnetic fields.

5.2. *Large-scale fields*

In recent years, the evidence has been growing that concerning large scale stable fields, the massive stars behave essentially as their intermediate mass counterparts. The magnetic fraction of OB stars has been determined to be of the order of 10% (Grunhut & Wade 2012), and the magnetic topologies are similar to those found in intermediate mass stars, with a predominance of highly inclined magnetic dipoles or low-order multipoles.

Indirect evidence for a magnetic fraction of massive stars comes from the Hunter diagram of LMC early B-stars (Fig. 3), which shows that 15% of them are nitrogen-rich slow rotators (Hunter *et al.* 2008, Brott *et al.* 2011b). While this group of stars is not reproduced from models of rotating single stars (Brott *et al.* 2011a), a magnetic field would allow to explain their slow rotation. Morel *et al.* (2008) identified a similar population in our Galaxy, and showed that a large fraction of these objects does indeed show a magnetic field. Further evidence is provided by Dufton *et al.* (2013), who showed that the velocity distribution of LMC early B-type stars is bimodal, with ∼20% of them rotating with values below 100 km/s, very reminiscent of the situation in the A-type stars (Zorec & Royer 2012).

6. The origin of stable large-scale fields

Two ideas are pursued to understand the origin of the stable, large-scale magnetic fields in intermediate and massive stars. One is that the field inside the main sequence stars is a relic of the interstellar field present in the molecular cloud at the time when it formed the stars (e.g., Mestel 2001, Moss 2001). While it may appear difficult to understand how the memory of the pristine B-field is preserved when matter is funneled through the MRI-driven accretion disc, this *fossil field* hypothesis has the basic problem to explain why it is ∼10% of the stars that obtain a large scale field in this way.

Alternatively, it has been postulated that the magnetic field as been acquired by the star in an earlier phase of its evolution, i.e., not during its formation. Confusingly, fields according to this hypothesis are also sometimes called *fossil*. Since the event with the largest appeal for field generation is strong close binary interaction, we speak here of *binary-induced fields*. The general idea that magnetic fields are generated through stellar mergers is supported by the dearth of close companions to magnetic main sequence stars at intermediate and high mass (Carrier *et al.* 2002).

Several people have suggested that strong binary interaction, in particular stellar merger, can result in the generation of strong, stable magnetic fields. Ferrario *et al.* (2009) and Tutukov & Fedorova (2010) suggested to explain the magnetic intermediate mass and massive stars through pre-main sequence mergers, while Tout *et al.* (2008) argued that the fields in magnetic white dwarfs may originate from white dwarf mergers.

However, there is evidence that a large fraction of the observed magnetic intermediate mass and massive stars are remnants of mergers between two main sequence stars. The latest determination of O star main sequence binary parameter distributions by Sana *et al.* (2012) implies that $8^{+9}_{-4}\%$ of all Galactic O stars are indeed the product of a merger between two main sequence stars (de Mink *et al.* 2014). Furthermore, Glebbeek *et al.* (2013) predict a nitrogen enrichment in main sequence merger products which is well compatible with the values found in the magnetic early B-stars analyzed by Morel *et al.* (2008; cf. Sect. 5.2).

While indeed also a significant fraction of the intermediate mass Herbig stars is found to be magnetic (Hubrig *et al.* 2004, Wade *et al.* 2007), those might also be merger products on the pre-main sequence, possibly induced by circumstellar tides as proposed by Krontreff *et al.* (2012). The ratio of nitrogen-rich to nitrogen-normal magnetic massive main sequence stars may thus give an indication of the ratio of pre-main sequence to main sequence mergers.

A stellar merging process is a most drastic event which induces strong differential rotation on a timescale close to the dynamical timescale. Binary evolution may produce such a situation also in some cases where the final merger is avoided in the end, i.e. during common envelope evolution which leads to an *almost-merger*. Perhaps Plaskett's star, which is a very massive close binary just past its rapid mass transfer phase, and of which the mass gainer has been found to host a strong magnetic field (Grunhut *et al.* 2013), and the Polars, Cataclysmic Variables with magnetic white dwarf companions (Tout *et al.* 2008), belong to this category.

It appears in any case unlikely that thermal timescale mass transfer can lead to strong magnetic fields in the mass gainer, neither accretion during star formation, which occurs on a similar time scale. The point is that the products are not observed to be magnetic. Mass transfer in massive binaries is known to lead to Be stars, which may often be seen as single stars because their companion exploded in a supernova explosion (de Mink *et al.* 2013). While perhaps not all Be stars are binary products, *none* of the many analyzed Be stars has been found to be magnetic by Grunhut & Wade (2012). Analogously, if accretion during star formation would induce strong stable fields, all stars should possess such fields, which is obviously not the case.

7. Evolution

While in the sections above, we were mostly assessing main sequence star, one may wonder how the stable magnetic fields survive during the post main sequence evolution of intermediate mass and massive stars. Theoretical ideas for this are scarce, since after the main sequence, ever changing parts of the post main sequence stars become convective. In particular, most of these stars may evolve into red giants and supergiants, which have convective cores and deep convective envelopes and thus leave little room for a stable magnetic field. On the other hand, there is no stage where the whole star would become convective, and it may thus be possible that the stable field of the main sequence stars is preserved throughout their post-main sequence evolution deep inside the star.

It has indeed been postulated that the magnetic white dwarfs — again: about 10% of all white dwarfs — are the remnants of magnetic intermediate mass main sequence stars, as the magnetic fluxes of both are quite comparable. In the light of the previous section, it may be plausible to assume that the magnetic white dwarfs have perhaps two components, one evolving from magnetic main sequence stars, and the other from white dwarf mergers. The latter channel may lead to an on average larger mass of magnetic white dwarfs, while the former might stand out by a slower-than-average spin. If magnetic white dwarfs are really merger products, it may be interesting to note that they will only play a minor role in Type Ia supernovae, if any at all. However, magnetic fields could form in double degenerate mergers, which are thought to provide one channel towards Type Ia explosions.

While the fraction of neutron stars which have extreme fields, the magnetars, is not well established, it appears again compatible with an order of magnitude of $\sim 10\%$. Also the flux freezing argument could apply. If massive main sequence stars had a B-field of $\sim 10^4$ G in their core — which appears plausible as they have surface fields of up to $\sim 10^3$ G —, the resulting B-field in the neutron stars would be of the order of 10^{14} G, which is two orders of magnitude larger than typical neutron star magnetic field strengths (Ferrario & Wickramasinghe 2006). In contrast to the scenario by Duncan & Thompson (1992), where the magnetar field forms from an extremely rapidly rotating collapsing iron core, magnetars as successors of magnetic main sequence stars would form slowly rotating neutron stars. This appears not only to be more compatible with the young supernova remnants surrounding some magnetars (Vink 2008), but also with the dearth of progenitors which can produce rapidly rotating iron cores in a high metallicity environment as our Galaxy (Yoon *et al.* 2006), and would argue against magnetar-powered supernovae (Woosley 2010). As also proposed by Duncan & Thompson (1992), the proto-neutron stars with ordinary spin rates (corresponding to $j \simeq 10^{14}\,\mathrm{cm}^2/\mathrm{s}$; see Fig. 1) may well produce the 10^{12} G fields found in most neutron stars.

8. Outlook

While all low mass stars appear to have small scale, dynamo-produced surface fields, the stronger the faster they rotate, only a fraction of $\sim 10\%$ of the intermediate mass stars possess strong B-field, which are large scale and occur mostly in slowly rotating single stars. Both types of fields may be combined in the massive stars, the small scale ones in all of the, the large scale one again in a fraction of about 10%. In addition, all stars may contain internal toroidal magnetic fields induced by differential rotation, which couples their core and envelope spins.

The hypothesis for the formation of the large scale fields which is consistent with all currently known constraints is that of strong binary interaction, preferentially via stellar mergers. In contrast to the fossil field hypothesis, it makes several clear predictions. Due to the large observational efforts currently underway, we can expect that this topic will be settled within the next years.

The question of the influence and survival of the large scale fields during the post-main sequence evolution of intermediate mass and massive stars appears more difficult to answer. Since red supergiants will likely not allow to assess this question observationally due to their deep convective envelopes — which may produce its own field through a dynamo process —, it may be interesting to focus on blue supergiants and Wolf-Rayet stars. If descendants of magnetic main sequence stars evolve into long-lived blue supergiants, a fair fraction of them might show surface magnetic fields, although considerably weaker ones if the magnetic flux is conserved. Also some Wolf-Rayet stars may have magnetic main sequence stars as precursors, but a field detection in these objects appears difficult due to their strong winds (de la Chevrotière *et al.* 2013).

Whether the large scale fields survive even until the formation of the compact remnant remains an open question, although there may be more arguments in favor of this idea than against it (cf. Sect. 7). However, it remains a challenge to produce solid theoretical predictions about the survival of the field during the post-main sequence evolution, as well as to identify observational strategies which would allow to settle the case.

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