

# Families among the Hildas and Trojans

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**Abstract.** A search for asteroid families among the Hildas and Jupiter Trojans was performed with the use of a new set of proper elements. The proper elements were calculated by the empirical method. Besides well known families, several new probable families were found in addition.

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## 1. The Hildas and Trojans resonant regions

A search for asteroid families among the Hildas and Trojans by analytical methods is especially difficult because these asteroids move in resonance regions. The Hildas are in the 3:2 and Trojans in the 1:1 mean motion orbital resonances with Jupiter. To find asteroid families, proper elements should be calculated. A concept of proper elements has been introduced by Hirayama ([Hirayama 1918](#)). There are different methods of proper elements calculation: analytical, numerical, and empirical which uses distributions of orbital elements.

## 2. Hirayama (1918) work

What method did Hirayama use? Hirayama selected some condensations in distributions of osculating elements. For each of the selected groups he studied its distribution in the  $(\tan i \cos \Omega, \tan i \sin \Omega)$  and  $(e \cos \varpi, e \sin \varpi)$  planes. Here  $i$  is inclination,  $\Omega$  - longitude of ascending node,  $e$  - eccentricity,  $\varpi$  - longitude of perihelion. Hirayama noticed that these distributions have the form of a circle. For each of selected group he found visually the position of a center of a circle. This point corresponds to forced elements. A distance of a dot from the center is a proper element ( $i_p$  or  $e_p$ ). Also he applied the secular theory and defined that such distribution is due to secular perturbations by Jupiter. So, Hirayama used two methods: first empirical, then analytical.

## 3. The Empirical method

After Hirayama's work the empirical method was not used. The new empirical method described in ([Vinogradova \(2015\)](#)) allows us to calculate forced elements. If forced elements are known, proper ones can be calculated with the use of a coordinate transformation formula. The method is simple and not time-consuming. It allows us to use all available asteroids for a family identification. The MPC catalogue, version May. 2018, was used as a source of initial osculating elements.

## 4. Asteroid families in the Hilda-group

The number of multi-opposition Hildas now exceeds 3000. Two forced elements derived for the Hildas,  $i_f = 1.20^\circ \pm 0.05^\circ$  and  $\Omega_f = 99^\circ \pm 1^\circ$ , are in a good agreement with results of the secular theory ([Brouwer & van Woerkom \(1950\)](#)), but two other elements,  $e_f = 0.069 \pm 0.002$  and  $\varpi_f = 20^\circ \pm 2^\circ$ , differ significantly from these results (0.043 and  $12^\circ$ , respectively). One of the features of the motion in these resonant regions is the libration of the semimajor axis. It was accepted to use a value  $a_p = 3.97au + da$  as a proper semimajor

**Table 1.** Families among the Hildas and Jupiter Trojans.

Region	Name	FIN	Diam (km)	N	$d_{cut}$ $10^{-4}$	Tax	$p_v$	$\Delta a_p$ (au)	$\Delta e_p$	$\Delta i_p$ (deg)
Hilda-group	1911 Schubart	002	80+66	658	30-110	CX-7:1(36)	0.04	3.98-4.04	0.15-0.23	2.6- 3.2
	153 Hilda	001	167+100	433	90-200	CX-6:2(59)	0.06	3.98-4.03	0.11-0.25	7.8-10.4
	1212 Francette	-	83+ 21	42	110-210	P -( 1)	0.06	3.98-4.01	0.22-0.24	6.9- 7.6
	51874 2001 PZ28	-	13+ 28	52	130-250	DS-8:1( 6)	-	3.98-4.02	0.20-0.25	10.3-11.8
	2483 Guinevere	-	43+ 38	81	140-230	DC-5:2( 4)	0.07	3.98-4.03	0.20-0.27	4.7-6.3
	4757 Liselotte	-	18+ 24	17	180-230	-	-	3.99-4.02	0.14-0.16	1.1-1.8
	5661 Hildebrand	-	38+ 28	29	220-370	-	-	3.98-4.01	0.19-0.23	13.7-15.0
L4-Trojans	3548 Eurybates	005	66+ 80	317	50-120	CX-5:4( 10)	0.06	5.28-5.33	0.03-0.07	7.1- 7.8
	2148 Epeios	008	39+ 55	104	100-160	-	-	5.23-5.33	0.02-0.05	8.6-9.2
	624 Hektor	004	207+121	100	110-290	DL-9:1( 11)	0.05	5.23-5.35	0.03-0.09	17.8-19.4
	9799 1996 RJ	006	65+ 30	19	140-390	-	0.04	5.23-5.24	0.03-0.05	31.4-31.8
	9713 Oceax	-	31+ 66	122	160-210	DX-5:5( 2)	-	5.25-5.38	0.02-0.05	3.4-5.0
	2797 Teucer	-	112+113	41	160-280	D ( 2)	0.07	5.23-5.32	0.06-0.08	20.3-21.5
	1583 Antiochus	-	117+ 79	41	260-490	D ( 2)	0.09	5.23-5.33	0.01-0.08	28.3-29.5
L5-Trojans	1172 Aneas	-	137+109	55	190-300	DC-7:1( 9)	0.05	5.22-5.33	0.03-0.06	16.6-18.8
	1867 Deiphobus	009	128+ 91	139	200-270	DC-7:1( 6)	0.06	5.23-5.34	0.02-0.05	26.8-31.2
	11487 1988 RG10	-	28+ 48	37	210-270	DX-7:1( 7)	-	5.24-5.37	0.02-0.04	3.8-4.8
	37519 Amphios	010	35+ 27	14	210-270	-	-	5.22-5.23	0.04-0.05	24.4-25.1

Notes: Diam - diameter of the main asteroid + diameter of debris; N - number of members;  $d_{cut}$  - dimensionless distance; Tax - shares of taxonomical types and a number of members with known taxonomy (in brackets);  $p_v$  - albedo;  $\Delta a_p$ ,  $\Delta e_p$ ,  $\Delta i_p$  - intervals of proper elements.

axis for the Hildas ( $da$  is the libration amplitude). We adopt an approach similar to the hierarchical clustering method for identification of asteroid families. As a result, two large families were found here: (1911) Schubart and (153) Hilda. Both these families were found earlier by other authors (Schubart (1982)), (Brož & Vokrouhlický (2008)). In Asteroids IV these families were assigned FIN - family identification number (Nesvorný, Brož, Carruba (2015)). In addition, our new set of proper elements enables five probable families to be identified (see Tab.1).

### 5. Asteroid families in the Trojans

About 5000 multi-opposition asteroids are known in the Trojans now (3300 in L4 + 1700 in L5). Calculating the proper elements of the Trojans is easy, because orbital elements of Jupiter can be used as forced elements:  $i_f = 1.3^\circ$ ,  $\Omega_f = 100.5^\circ$ ,  $e_f = 0.049$ ,  $\varpi_{jup} = 14.2^\circ$ . For L4-Trojans,  $\varpi_f = \varpi_{jup} + 60^\circ = 74.2^\circ$ . For L5-Trojans,  $\varpi_f = \varpi_{jup} - 60^\circ = 314.2^\circ$ . A proper semimajor axis  $a_p = 5.20 + da$ , where  $da$  is a libration amplitude. Large discrepancies take place in the Trojans family lists obtained by different authors. The number of families found differs from one (Brož & Rozehnal 2011) to about 20 (Beaugé & Roig 2001). With our new set of proper elements seven families were found among the L4-Trojans and four among the L5-Trojans. All additional families among the Trojans were previously found by different authors, but at the same time a large number of families published earlier were not confirmed.

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