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Scalar Field Models for an Accelerating Universe

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Abstract. I describe a new class of quintessence+CDM models in which late time scalar field oscillations can give rise to both quintessence and cold dark matter. Additionally, a versatile ansatz for the luminosity distance is used to reconstruct the quintessence equation of state in a *model independent* manner from observations of high redshift supernovae.

1. A new model of quintessence and cold dark matter

The supernova-based discovery that the universe may be accelerating can be explained within a general relativistic framework provided one speculates the presence of a matter component with negative pressure, the most famous example of which is the cosmological constant 'Λ' (Perlmutter et al. 1998,1999; Riess et al. 1999). Λ runs into formidable fine tuning problems since its value must be set $\sim 10^{123}$ times smaller than the energy density in the universe at the Planck time in order to ensure that Λ dominates the total energy density at precisely the present cosmological epoch. This involves a fine tuning of one part in 10^{123} at the Planck scale or one part in 10^{53} at the Electroweak scale.

One way around this difficulty is to make Λ time-dependent, perhaps by using scalar field models which successfully generate a time-dependent Λ-term during an early Inflationary epoch. In this context the exponential potential provides an interesting illustration, since the density in the ϕ-field tracks the background matter/radiation density when the latter is cosmologically dominant (Ratra & Peebles 1988, Wetterich 1988, Ferreira & Joyce 1997):

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1+w_B)}{p^2\lambda^2} \ll 1 \qquad (1)$$

($w_B = 0,\ 1/3$ respectively for dust, radiation). This behaviour allows ρ_ϕ to be fairly large initially. Based on this property we introduce a new class of cosmological models which can describe both a time-dependent Λ-term (quintessence) and cold dark matter (CDM) within the unified framework of the class of potentials (Sahni & Wang 2000)

$$V(\phi) = V_0(\cosh\lambda\phi - 1)^p. \qquad (2)$$

$V(\phi)$ has asymptotic forms:

$$V(\phi) \simeq \tilde{V}_0 e^{-p\lambda\phi} \ \text{for } |\lambda\phi| \gg 1 \ (\phi < 0), \qquad (3)$$

$$V(\phi) \simeq \tilde{V}_0 (\lambda\phi)^{2p} \ \text{for } |\lambda\phi| \ll 1 \qquad (4)$$

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where $\tilde{V}_0 = V_0/2^p$. The exponential form of $V(\phi)$ guarantees that the scalar field equation of state mimics background matter at early times so that $w_\phi \simeq w_B$. At late times oscillations of ϕ lead to a mean equation of state

$$\langle w_\phi \rangle = \left\langle \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \right\rangle = \frac{p-1}{p+1}, \tag{5}$$

resulting in cold dark matter with $\langle w_\phi \rangle \simeq 0$ if $p = 1$, or in quintessence with $\langle w_\phi \rangle \leq -1/3$ if $p \leq 1/2$. We therefore have before us the attractive possibility of describing CDM and quintessence in a common framework by the potential

$$V(\phi, \psi) = V_\phi(\cosh \lambda_\phi \phi - 1)^{p_\phi} + V_\psi(\cosh \lambda_\psi \psi - 1)^{p_\psi} \tag{6}$$

where $p_\psi = 1$ in the case of CDM and $p_\phi \leq 0.5$ in the case of quintessence. In figure 1 we show a working example of this model which agrees well with observations of high redshift supernovae and does not suffer from the fine tuning problem faced by Λ, since ρ_ϕ can be fairly large initially. We should add that most models of quintessence usually work under the assumption that the three matter fields: baryons, CDM & quintessence need not be related in any fundamental way and might even have different physical origins. If this is indeed the case then it remains somewhat of a mystery as to why Ω_ϕ, Ω_m, (and possibly Ω_b) have comparable magnitudes at the present time. By combining quintessence and CDM within a single class of potentials we make a small step in answering this question by showing that unified models of quintessence and CDM are conceivable (Sahni & Wang 2000).

An intriguing property of cold dark matter based on (6) is that it can have a large Jeans length which leads to suppression (frustration) of clustering on kiloparsec scales. *Frustrated Cold Dark Matter* (FCDM) redresses certain shortcomings of the standard CDM scenario and might provide a natural explanation for the dearth of dwarf galaxies seen in our local neighborhood (Sahni & Wang 2000).

Other quintessence potentials include $V(\phi) \propto \phi^{-\alpha}$ (Ratra & Peebles 1988), $V(\phi) \propto e^{\beta\phi^2}\phi^{-\alpha}$ (Brax & Martin 2000) and $V(\phi) \propto \sinh^{-2p}(\phi + \phi_0)$ (Sahni & Starobinsky 2000). The latter describes quintessence which maintains a constant equation of state $w = -(1+p)^{-1}$ throughout the matter dominated epoch and later, during acceleration.

2. Reconstructing quintessence from supernova observations

Although a large class of scalar potentials can describe a time dependent Λ-term, no unique potential has so far emerged from a consideration of high energy physics theories such as supergravity or M-theory. (The situation in many respects resembles that faced by the Inflationary scenario, for a review see Sahni & Starobinsky 2000.) It is therefore meaningful to try and reconstruct $V(\phi)$ directly from observations in a model independent manner. This is easy to do if one notes that, in a flat FRW universe, the luminosity distance determines the Hubble parameter uniquely (Starobinsky 1998, Saini et al. 2000)

$$H(z) \equiv \frac{\dot{a}}{a} = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}. \tag{7}$$

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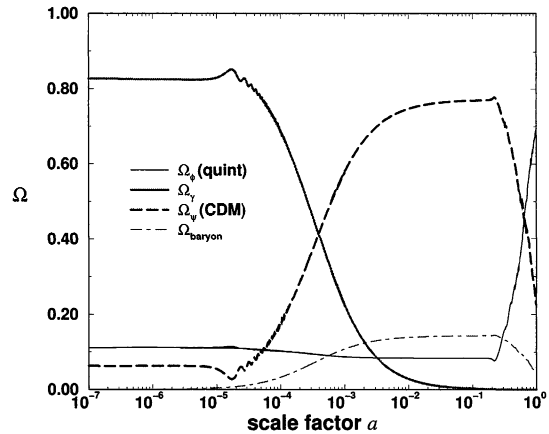

Figure 1. The evolution of the dimensionless density parameter for the CDM field Ω_ψ (dashed line) and quintessence field Ω_ϕ (thin solid line). Baryon (dash-dotted line) and radiation densities (thick solid line) are also shown. For more details see Sahni and Wang (2000).

The Einstein equations can be written in the suggestive form

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{dH^2}{dx} - \frac{1}{2}\Omega_{\rm M}\,x^3, \tag{8}$$

$$\frac{8\pi G}{3H_0^2}\left(\frac{d\phi}{dx}\right)^2 = \frac{2}{3H_0^2 x}\frac{d\ln H}{dx} - \frac{\Omega_{\rm M} x}{H^2}, \tag{9}$$

where $x \equiv 1+z$. Thus knowing D_L we can determine both $H(z)$ and $dH(z)/dz$, and hence $V(\phi)$. The cosmic equation of state can also be reconstructed from D_L since

$$w_\phi(x) \equiv \frac{p}{\rho} = \frac{(2x/3)d\ln H/dx - 1}{1-(H_0^2/H^2)\,\Omega_{\rm M}x^3}. \tag{10}$$

In order to apply our method to observations we use the following rational ansatz for the luminosity distance

$$\frac{D_L}{x} \equiv \frac{2}{H_0}\left[\frac{x-\alpha\sqrt{x}-1+\alpha}{\beta x+\gamma\sqrt{x}+2-\alpha-\beta-\gamma}\right] \tag{11}$$

where α, β and γ are fitting parameters. This function reproduces the *exact* analytical form of D_L when $\Omega_\phi = 0, \Omega_{\rm M} = 1$ and when $\Omega_\phi = 1, \Omega_{\rm M} = 0$. It also has the correct asymptotic behaviour $H(z)/H_0 \to 1$ for $z \to 0$, and $H(z)/H_0 \to$

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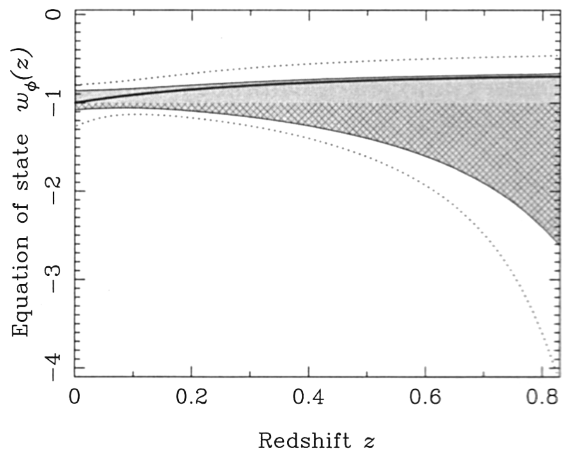

Figure 2. The equation of state parameter $w_\phi(z) = p_\phi/\rho_\phi$ as a function of redshift. The solid line corresponds to the best-fit values of the parameters. The shaded area covers the range of 68% errors, and the dotted lines the range of 90% errors. The hatched area represents the region $w_\phi \leq -1$, which is disallowed for a minimally coupled scalar field (from Saini et al. 2000).

$(1+z)^{3/2}$ for $z \gg 1$. Applying a maximum likelihood technique to D_L given by (11) and D_L^{obs} obtained from observations of high redshift supernovae, we can reconstruct $H(z)$, $V(\phi)$ and $w_\phi(z)$. Our results for w_ϕ shown in fig. 2 indicate some evidence of possible evolution in w_ϕ with $-1 \leq w_\phi \lesssim -0.80$ preferred at the present epoch, and $-1 \leq w_\phi \lesssim -0.46$ at $z = 0.83$, the farthest SN in the sample (both at 90% CL). However, a cosmological *constant* with $w = -1$ is also consistent with the data.

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References

Brax, P. & Martin, J. 2000, Phys.Rev.D, 61, 103502

Ferreira, P. G. & Joyce, M. 1997, Phys. Rev. Lett., 79, 4740

Perlmutter, S. J. et al., 1998, Nature, 391, 51

Perlmutter, S. J. et al., 1999, ApJ, 517, 565

Ratra, B. & Peebles, P. J. E. 1988, Phys.Rev.D37, 3406

Riess, A. G. et al., 1998, Astron. J., 116, 1009

Sahni, V. & Starobinsky, A. A. 2000, Int. J. Mod. Phys. D9, 373

Sahni, V. & Wang, L. 2000, Phys.Rev.D (in press); astro-ph/9910097

Saini, S., Raychaudhury, S., Sahni, V. & Starobinsky, A. A. 2000, Phys. Rev. Lett., 85, 1162

Starobinsky, A. A. 1998, JETP Lett., 68, 757

Wetterich, C. 1988, Nuclear Physics B, 302, 668

https://doi.org/10.1017/S0074180900216331 Published online by Cambridge University Press