CHAPTER 1.

Cosmology of Yakov Zeldovich Historical and Scientific Perspective

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Rashid Sunyaev
recalling memories and anecdotes of his supervisor
Yakov Zeldovich

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The Zeldovich Universe:
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Zeldovich's legacy in the Discovery and Understanding of the Cosmic Web

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Abstract. Modeling and understanding the structure of the universe is one the most interesting problems in modern cosmology. It is also an extremely difficult problem. Despite spectacular achievements in observations and computational capabilities at present there is no comprehensive theory of the structure formation. There are successful theoretical models however most of them rely on assumptions and various fits which are not fully physically justified. The Zeldovich Approximation is a rare exception. Suggested more than forty years ago it remains one of the most successful and profound analytic models of structure formation. Some of results stemmed from the Zeldovich Approximation will be briefly reviewed.

Keywords. large-scale structure of universe

1. Introduction

Yakov Borisovich Zeldovich – or Ya.B. as his colleagues and friends called him – celebrated his 70th birthday in 1974. The Academy of Sciences of the USSR made a present to him: it published a selection of his works in two volumes. It was translated into English almost twenty years later (Zeldovich 1993). Ya.B. concluded the second volume by "An Autobiographical Afterword". Among other things Ya.B. included a brief assessment of his contribution to astrophysics and cosmology. Here is his comment on his contribution to the theory of the large-scale structure in the universe.

"This book presents, with commentaries, my papers on astrophysics. It is not reasonable to argue with these commentaries. Today the most important individual work seems to me to be the nonlinear theory of formation of the structure of the Universe or, as it is now called for short the "pancake" theory. The structure of the Universe, its evolution and the properties of the matter which forms the hidden mass, have to this day not been fully established. A major role in this work was played by A. G. Doroshkevich, R. A. Sunyaev, S. F. Shandarin and Ya. E. Einasto. The work continues. However, the "pancake" theory is "beautiful" in and of itself: if the initial assumptions hold, then the theory gives a correct and nontrivial answer. The "pancake" theory is a contribution to synergetics. It was especially pleasant for me to learn that this work in some measure initiated mathematical investigations by V. I. Arnold and others." (Zeldovich 1993, vol. 2, p. 642).

In the last 10 - 15 years of his life Ya.B. had very close relations with V.I. Arnold who described their interactions as follows: "Usually, Yakov Borisovich telephoned me at seven in the morning. 'Doesn't it seem to you ...' he would say, and there would then follow some sort of paradox" (Zeldovich 2004, Part V, p. 195). V.I. Arnold's considered that by formulating the "pancake" theory Ya.B. opened a new topic in mathematics: "Yakov Borisovich's 'pancake theory', in essence, is equivalent to a theory for the simplest so-called Lagrange singularities in symplectic geometry – singularities of projections of

Lagrangian manifolds (on which the Poincaré invariant vanishes) from phase space onto a configurational space." (Zeldovich 2004, Part V, p. 197).

Both the great physicist and mathematician expressed very high opinions of the pancake theory. It has been substantiated by numerous studies in the following years. I begin with a brief outline of the mathematical basis of the pancake theory i.e. the Zeldovich approximation (the ZA). Then I will outline some developments that stemmed from the ZA and played important role in understanding of the origin, morphology and evolution of the web. The list of selected topics obviously reflects my personal views and does not pretend to be complete.

2. The Zeldovich approximation in a nutshell

The ZA was formulated for a collisionless fluid which is a good approximation for dark matter (DM) (Zeldovich 1970). It is particularly simple in comoving coordinates $\vec{x} = \vec{r}/a(t)$, where $a(t)$ is the scale factor and $\vec{r}$ are physical coordinates of the fluid particles. It relates the initial Lagrangian coordinates $\vec{q}$ at $t \to 0$ and Eulerian coordinates $\vec{x}$ at time t by an explicit relation

$$\vec{x}(\vec{q},t) = \vec{q} + D(t)\vec{s}(\vec{q}) \tag{2.1}$$

where $D(t)$ is the linear density growth factor fully specified by the cosmological model and the vector field $\vec{s}(\vec{q})$ is determined by the growing mode of initial density perturbations

$$\nabla\vec{s} = -\frac{1}{D}\left(\frac{\delta\rho}{\bar{\rho}}\right)_{\rm lin,g}, \tag{2.2}$$

when $\vec{x} \to \vec{q}$. Therefore, the ZA assumes that the initial displacement field $\vec{s}(\vec{q})$ is a potential vector field,

$$\vec{s_i}(\vec{q}) = -\frac{\partial\Psi(\vec{q})}{\partial q_i}, \tag{2.3}$$

where the displacement potential Ψ is directly proportional to the linear perturbation of the gravitational field $\delta\phi_{lin}$ determined by $(\delta\rho/\bar{\rho})_{\rm lin,g}$

$$\Psi(\vec{q}) = \frac{2}{3Da^2H^2\Omega}\,\delta\phi_{lin}(\vec{q}). \tag{2.4}$$

In the above equation $H = H(t)$ is the Hubble parameter and Ω is the dimentionless mean total density capable to cluster.

The potential character of the displacement vector is a direct consequence of the potential character of the growing mode of the gravitational instability in an expanding universe. An additional important aspect of the ZA is anisotropic deformation of the mass elements. The deformation is specified by the deformation tensor

$$d_{ij}(\vec{q}) = -\frac{\partial s_i(\vec{q})}{\partial q_j}. \tag{2.5}$$

From equation (2.1) one may easily infer an explicit expression for the density as a function of Lagrangian coordinates and time. If we consider the conservation of mass in differential form (it is worth reminding that $\bar{\rho}$ does not change with time in comoving coordinates)

$$\rho(\vec{x},t)\mathrm{d}\vec{x} = \bar{\rho}\mathrm{d}\vec{q}, \tag{2.6}$$

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the density evolution directly follows from

$$\rho(\vec{x},t) = \bar{\rho}\left|J\left(\frac{\partial\vec{x}}{\partial\vec{q}}\right)\right|^{-1}, \tag{2.7}$$

where $J(\partial\vec{x}/\partial\vec{q})$ is the Jacobian determinant of the map given by equation 2.1. It is convenient to write equation 2.7 in terms of the eigen values $\lambda_i = (\alpha,\beta,\gamma)$ of the deformation tensor d_{ij}, resulting in an explicit equations for the density as a function of Lagrangian coordinates and time

$$\rho(\vec{q},t) = \left|\frac{\bar{\rho}}{[1-D(t)\alpha(\vec{q})]\,[1-D(t)\beta(\vec{q})]\,[1-D(t)\gamma(\vec{q})]}\right|, \tag{2.8}$$

where commonly used ordering of the eigen values $\alpha \geqslant \beta$ and $\beta \geqslant \gamma$ is assumed. The eigen values are nonGaussian random fields Doroshkevich (1970).

Expanding the expression for the Lagrangian density (eqn. 2.8) in a Taylor series,

$$\rho(\vec{q},t) = \bar{\rho} + \bar{\rho}D(t)\left[\alpha(\vec{q})+\beta(\vec{q})+\gamma(\vec{q})\right] + \ldots, \tag{2.9}$$

clarifies the relation between the known linear expression for the density contrast, $\delta_{\rm lin}$ and the eigen values of the deformation tensor

$$\delta_{\rm lin} = \frac{(\rho_{\rm lin}-\bar{\rho})}{\bar{\rho}} = D(t)(\alpha+\beta+\gamma). \tag{2.10}$$

Formally speaking, the ZA is valid only in the linear regime when both $|D\alpha| \ll 1$ and $|D\gamma| \ll 1$. However, Zeldovich made a bold prediction: it should be a good qualitative and arguably quantitative approximation up to the beginning of the nonlinear stage, i.e. up to the stage at which $|D\alpha| = 1$ and even a little beyond. In order to compute the density field in Eulerian space one has to apply mapping eqn. 2.1.

One can easily deduce two key features of the density field at the beginning of the nonlinear stage from eqn. 2.8. Firstly the density becomes infinite as soon as $D(t)\alpha = 1$ or $D(t)\beta = 1$ or $D(t)\gamma = 1$. And secondly the collapse of a fluid particle must be anisotropic because the eigenvalues never satisfy the condition $\alpha(\vec{q}) = \beta(\vec{q}) = \gamma(\vec{q})$ for generic $\Psi(\vec{q})$. In addition, the condition

$$D(t)\lambda_i = 1 \tag{2.11}$$

determines caustics i.e. a set of surfaces where density is formally infinite.

It is easy to see that Eulerian linear perturbation theory (ELPT) is a limiting case of the ZA, assuming an additional condition $\vec{x} \approx \vec{q}$. Technically speaking, the ZA is an extrapolation of Lagrangian linear perturbation theory (LLPT) beyond the range of its formal applicability. There are two fundamental differences of LLPT from its Eulerian counterpart ELPT. As may be inferred from equations 2.8 and 2.10, the calculation of the density LLPT uses the full deformation tensor while ELPT relies only on its trace. The second difference is due to the necessity in LLPT of mapping from Lagrangian to Eulerian space in order to evaluate the density field in Eulerian space. Even at small σ_δ, where $\delta = \Delta\rho/\bar{\rho}$, the difference between LLPT and ELPT can be quite noticeable if the scale of the initial i.e. linear velocity field is considerably greater than that of density fluctuations. In this, we assume that σ_δ is evaluated in ELPT. The difference between LLPT and ELPT becomes considerable when both are extrapolated to a larger σ_δ. A particularly obvious problem occurring in ELPT is the emergence of negative densities, i.e. $\rho < 0$, in regions with a large initial density deficit. For example, if ELPT is extrapolated to $\sigma_\delta = 0.5$, the regions with negative densities occupy approximately 2.3% of the volume. This fraction increases to almost 15% at $\sigma_\delta = 1$. Evidently, for a physical model this is an unacceptable circumstance. LLPT is completely free of this problem: at all times it predicts $\rho > 0$, regardless of the magnitude of σ_δ. It is worth mentioning that each of three factors in the denominator of equation 2.8 could be negative. This is a very useful feature of the ZA which is briefly discussed bellow.

3. Further developments

3.1. *N-body simulations and accuracy of the ZA*

Cosmological N-body simulations have played a crucial role in theoretical studies and understanding of the structure in the universe. Despite of well known shortcomings they provide the most realistic data to compare with the observations. The first simulations were carried out in late 1960s. However all N-body simulations conducted in the western countries before 1983 started from Poisson distributions of particles in the simulation box. The only advantage of this initial condition consists in the easiness of its generation. But it has two major shortcomings: the only initial power spectrum of density fluctuations is flat and the smallest scales are in the nonlinear regime from the very beginning.

Doroshkevich, Ryabenkii & Shandarin (1973) were the first who generated initial conditions for a three-dimensional N-body simulation via the ZA. The goal of the study was testing the coherence of the ZA. The simulation involved two groups of particles. One of them moving according the ZA served as a source of the gravitational field. The trajectories of the other much smaller group of particles were integrated in this field. The particles that felt the gravitational field had the siblings in the first group of particles. The pairs of siblings started from identical initial conditions, this diverged with time since the ZA is not an exact solution. The accuracy of the ZA at the nonlinear stage was roughly assessed by comparing the trajectories, velocities and accelerations of each pair of siblings.

The authors also suggested an analytical estimate of the coherence of the ZA by computing the relative difference of two densities. One of which ρ is given by eq. 2.8 and the other $\tilde{\rho}$ is determined by the divergence of the acceleration field obtained from the ZA (eq. 2.1). If the ZA was exact solution then $\rho \equiv \tilde{\rho}$. It was shown that (see also Shandarin & Zeldovich (1989))

$$\frac{\tilde{\rho} - \rho}{\rho} \equiv D^2(-J_2 + 2DJ_3), \tag{3.1}$$

where $J_2 = \alpha\beta + \alpha\gamma + \beta\gamma$ and $J_3 = \alpha\beta\gamma$. It is interesting that even at the time of the emergence of the pancake at $D = 1/\alpha$ when $\rho \to \infty$ the relative difference between two densities remain finite $(\tilde{\rho} - \rho)/\rho = -\beta/\alpha - \gamma/\alpha + \beta\gamma/\alpha^2$. Equation 3.1 also shows that the ZA is exact in one-dimensional case until shell crossing since $J_2 = J_3 = 0$.

Numerous studies elaborated on the quantitative accuracy and limitations of the approximation, see e.g. Coles, Melott & Shandarin (1993); Melott, Shandarin & Weinberg (1994); Yoshisato, Matsubara & Morikawa (1998); Yoshisato, *et al.* (2006) and references therein.

The first cosmological N-body simulations with Gaussian initial conditions generated via the ZA were carried out by Doroshkevich *et al.* (1980) and (Klypin & Shandarin 1983; Shandarin & Klypin 1984) in two- and three-dimensional cases respectively. This method of the initiation of cosmological simulations has become universal.

3.2. *Relation to Catastrophe Theory*

Ya.B published his paper on pancakes almost ten years prior to first observational evidences for existing highly anisotropic superclusters of galaxies Gregory & Thompson

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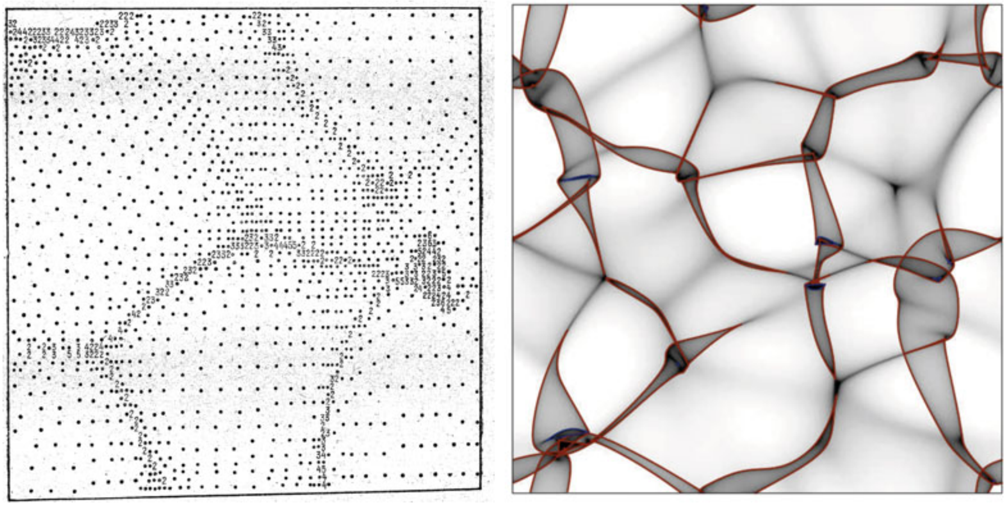

Figure 1. Left Panel: The first theoretical hint of the web structure (Doroshkevich, Zeldovich & Sunyaev 1976). The Russian caption in the original publication says: ' A typical distribution of mass particles obtained by S.F. Shandarin in two-dimensional simulation (see text). Symbols show the number of particles in the mesh cells. Initially particles were placed on a regular grid and slightly perturbed by smooth potential random vector field.' Right Panel: Modern simulation of the structure in 2D using the ZA. (Hidding, Shandarin & van de Weygaert 2014). Gray shades show the density while red and blue lines show the caustics corresponding to two eigen values.

(1978); Chincarini & Rood (1979); for a review see Oort (1983). However most of the cosmologists regarded the prediction of anisotropic structures with conspicuous skepticism for almost twenty years after it was published as described by Shandarin & Sunyaev (2009).

Ya.B. described just one generic type of structure formed by caustics - pancakes which emerged as the first structures at the shell crossing stage. The collapse of a uniform isolated ellipsoid was studied by Lin, Mestel & Shu (1965) who showed that the initially oblate ellipsoid tends toward a disk remaining uniform at all stages. The Zeldovich pancakes formed as a three-stream flow regions bounded by caustic surfaces. The shape of pancakes formed from generic Gaussian initial conditions is very different from ellipses. The first simulation of the ZA in two-dimensional case showed that the structure begins to emerge as a set of isolated pancakes however they weave in essentially single structure very quickly (see left hand side panel Fig. 1).

The explanation of this phenomenon required a dipper understanding of the geometry and topology of the mapping generated by eq. 2.1. Zeldovich turned to Arnold who enthusiastically began working on this problem. Soon the normal form for all generic singularities have been found (Arnold 1982) and the results was applied to cosmology (Arnold, Shandarin & Zeldovich 1982). Arnold, Shandarin & Zeldovich (1982) were able to crudely outline the richness of the web in 2D. A far more detailed study of the two-dimensional web was performed by Hidding, Shandarin & van de Weygaert (2014). Figure 1 (the right hand side panel) illustrates the improvements in both computation and visualization. The visualization of the web in 3D in its full complexity was a serious challenge in 1980s and remain a difficult problem even nowadays. The comprehensive analysis of the ZA in 3D is challenging computationally as well because it requires to study geometry and topology of highly nonGausssian fields which are $\lambda_i(\vec{q})$. Mapping to Eulerian space further complicates the analysis. In order to understand the dark matter

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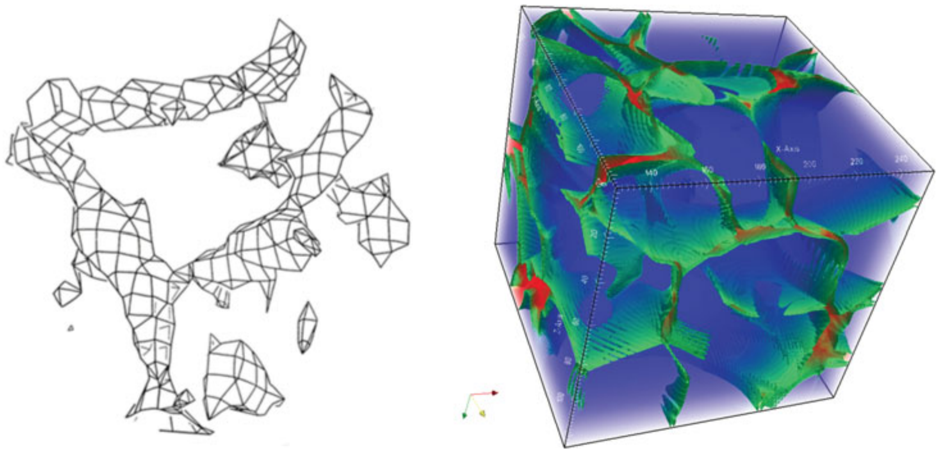

Figure 2. Left Panel: The density contour $\rho = 2.5\bar{\rho}$ obtained in the N-body simulation of the hot dark matter model (Shandarin & Klypin 1984). Right Panel: Prediction of the structure in 3D by the ZA. (Hidding, Shandarin & van de Weygaert 2014). The green surfaces are the α–caustics corresponding to the condition $D(t)\alpha = 1$ and red surfaces are the β-caustics seen only in the openings at the boundaries of the box; γ-caustics are completely obscured by the α- and β-caustics.

structure in the universe we have to understand much simpler structure described by the ZA (see the right hand panel of Fig. 2). It is very likely that the types of singularities in gravitating cold collisionless fluid evolved from potential initial condition will be similar to that in the ZA since the flow in individual streams remains potential as clearly demonstrated by Hahn, Angulo & Abel (2014).

3.3. *Morphology of the web*

Equation (2.1) encapsulates the mapping by the LLPT from Lagrangian space to Eulerian space. Its ramifications can be explored analytically before an overwhelming fraction of mass elements starts to experience shell crossing. The mathematical complications increase rapidly as the number and extent of the multistream flow regions proliferates. In particular interesting are those mass elements which are separated by finite distances in Lagrangian space and end up at the same places in Eulerian space. They belong to different streams and are key manifestations of the dynamically evolving mass distribution and mark the emerging cosmic web. To understand and assess this aspect of the Zeldovich approximation, we require numerical modeling, in particular for the cosmologically relevant situation of random initial conditions. The surfaces separated the regions with different number of streams in Eulerian space are caustics. Their progenitors in Lagrangian space isolate the fluid elements that experienced different number of turns inside out or flip-flops.

Shandarin & Klypin (1984) suggested that the structure emerges in the sequence: pancakes, filaments and finally haloes. The pancakes begin to emerge in the vicinity of $\alpha(\vec{q}) = max$ due to the collapse along the corresponding eigen vector $\vec{n}_\alpha$. Then the following collapse along the eigen vector $\vec{n}_\beta$ results in the emergence of filaments. Finally the collapse along the eigen vector $\vec{n}_\gamma$ leads to the origin of halos. The model obviously represents a substantial simplification of the dynamics in the dark matter component (Hidding, Shandarin & van de Weygaert 2014).

However recently Falck, Neyrinck & Szalay (2012) exploited a very similar idea for developing the ORIGAMI method of identifying structures in cosmological N-body

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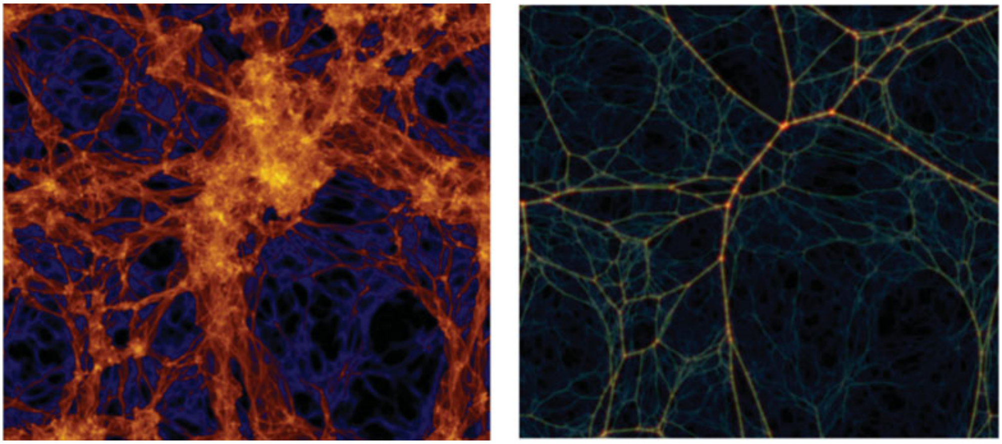

Figure 3. The ZA (left) v.s. Adhesion approximation (right). Courtesy of J. Hidding.

simulations. Their analysis shows a good agreement of the model with cosmological N-body simulations. Another successful numerical algorithm NEXUS for identifying the basic elements of the web (halos, filaments, walls aka pancakes, and voids) that incorporates some essential concepts of the ZA has been recently suggested by Cautun *et al.* (2014).

Klypin and Shandarin (1983, 1984) were puzzled why only the filamentary structure was observed in the simulation of the universe dominated by hot dark matter (see the left hand side panel of Fig. 2) but not Zeldovich's pancakes. They correctly conjectured that the pancakes have too low density contrast to be seen in their simulations. The right hand panel of Fig. 2 demonstrates that in order to see pancakes clearly one has to compute and plot the caustic surfaces or achieve the mass resolution significantly higher than in common cosmological N-body simulations.

3.4. *Topology of the large-scale structure*

The both two and three–dimensional N-body simulations (Doroshkevich *et al.* 1980; Klypin & Shandarin 1983; Shandarin & Klypin 1984) as well as two– and three–dimensional models based on the ZA (Fig. 1 and 2) showed that the filamentary structure spans throughout the entire simulation box. This observation raised the question of the topology of the large-scale structure. Ya.B. initiated the study of the topology of the structure resulted in a series of papers based on percolation theory (Zeldovich 1982; Zeldovich Einasto & Shandarin 1982; Shandarin 1983; Shandarin & Zeldovich 1983, 1984; Klypin 1987; Klypin & Shandarin 1993). Later Gott, Dickinson & Melott (1986) suggested to use the genus statistics for quantifying the topology of the structure. Sahni, Sathyaprakash & Shandarin (1997) compared two techniques and concluded that the percolation method along with ability to distinguish nonGaussian from Gaussian fields also reveals the structure difference in Gaussian fields with different power spectra. The latter property allowed to relate the effects of nature (initial field) and nurture (nonlinear evolution) on the topology of the web (Shandarin, Habib & Heitmann 2010).

At present the term "the large-scale structure" is used less often than "the Cosmic Web" coined by Bond, Kofman & Pogosyan (1996). The difference in the meaning of these two terms is substantial. "The large-scale structure" does not invoke any particular geometry or pattern while "the Cosmic Web" almost inevitably invokes an image of a network made of filaments like a spider-web. Shandarin & Klypin (1984) discussed this issue and and came to the following conclusion: "The regions of high density seem to form

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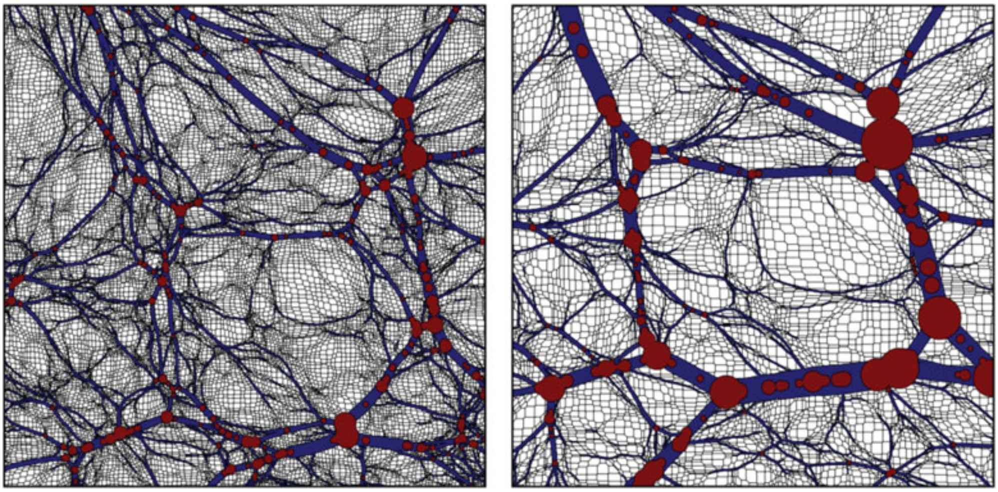

Figure 4. A modern version of the Adhesion approximation. A two-dimensional model shows two stages of the evolution of the web in coming coordinates. One can see the multi scale nature of the web and multiple nature of the halo mergers when small voids are squeezed by expanding large voids. The red circles symbolize the halos which sizes indicate the masses. From Hidding *et al.* (2012).

a single three-dimensional web structure. However, it is not clear from our simulations whether honeycomb structure arises or not." At present the answer is unambiguous: in both the ZA (Fig. 2) and N-body simulations (Ramachandra & Shandarin 2014) the three-stream flow regions form irregular cellular structure made of very thin quasi two-dimensional regions most of which are interconnected in a single percolating structure.

3.5. *The Adhesion model*

Not being accurate the ZA remains qualitatively correct until massive shell crossing happens. The streams continue moving through each other without bounds. This results in unlimited growth of the thickness of the structure as illustrated by the left hand side panel in Fig. 3. The numerical simulations showed that the correctly computed gravitational forces keep the structure very thin. In order to suppress the growth of the thickness of the pancakes Gurbatov, Saichev & Shandarin (1985, 1989) (see also Gurbatov, Saichev & Shandarin (2012) for a recent review) suggested to introduce artificial viscosity into the dynamical equations describing the ZA. Since this modification cannot yield the correct result the form of the viscosity was chosen in the form that results in the Burgers equation well known in theory of turbulence. An important advantage of the Burgers model consists in that it allows an analytic solution. In the limit of the infinitesimal viscosity the structure becomes a two-dimensional surface of a very complicated shape in 3D or a one-dimensional line in 2D as shown in the right hand panel of Fig. 3. The web predicted by the adhesion approximation can be called a skeleton of the web. The accuracy and limitations of the adhesion approximation have been carefully studied (Kofman, Pogosyan & Shandarin 1990; Nusser & Dekel 1990; Kofman *et al.* 1992; Melott, Shandarin & Weinberg 1994). It has also been used for the simulation of deep redshift surveys (Weinberg & Gunn 1990a,b).

A recent very interesting new development of the Adhesion model revealed a deep connection of the relation between Eulerian and Lagrangian space with that between Voronoi and Delaunay tessellations (Hidding *et al.* 2012). It has been found that the walls aka pancakes of the web as edge-like objects in Lagrangian space, whereas filaments have

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a flattened signature in Lagrangian space. Halos, being the most massive concentrations of mass are therefore most extended progenitors in Lagrangian space. The Adhesion model can be naturally interpreted as a skeleton of the web as illustrated by Fig. 4.

The Adhesion model naturally incorporates the multi scale character of the web, anisotropic accretion of mass on halos from the filaments and to lesser extent from walls, extinguishing of small voids when they pushed to the walls by large voids, simultaneous merger of several halos when a void collapses.

4. Other recent developments

Illustrating the idea of a pancake Ya.B. had a plot (Fig.2 in Zeldovich (1970)) Eulerian coordinate as a function of Lagrangian coordinate at three times: before, after and exactly at the time of birth of a pancake. This is one of the Lagrangian sub-manifolds playing fundamental role in classical mechanics. The distribution of dark matter in $(\vec{q}, \vec{x})$–space, where $\vec{q}$ and $\vec{x}$ are initial and actual comoving coordinates of particles in N-body simulation at time t, is highly degenerate. It occupies a very thin sheet around three-dimensional submanifold due to very low temperature of dark matter.

Recently a very promising technique has emerged based on a triangulation of a three-dimensional submanifold $\vec{x} = \vec{x}(\vec{q}, t)$ in six-dimensional space $(\vec{q}, \vec{x})$ (Shandarin, Habib & Heitmann 2012; Able, Hahn & Kaehler 2012). Based on the triangulation one can compute density, velocity, number of streams, and other fields with considerably higher spatial resolution than other techniques can achieve (Angulo, Hahn & Abel 2013; Hahn, Angulo & Abel 2014; Ramachandra & Shandarin 2014; Shandarin & Medvedev 2014). Therefore this technique allows to visualize and study the web with unprecedented accuracy.

Another extremely interesting example of studies stemmed from the ZA. The ZA was used for computation of the two-point function of the matter and biased tracers (White 2014). The comparison with the N-body simulations and other Lagrangian perturbation theories showed the ZA provides a good fit the N-body results. The ZA was also used to compute the ingredients of the Gaussian streaming model of Reid & White (2011). It was found that this hybrid model, refered to as the Zeldovich streaming model and which involves only simple integrals of the linear theory power spectrum, provides a good match to the N-body measurements down to tens of Mpc.

5. Summary

The ZA continues to inspire cosmologists to create new approaches to the study and understanding of the web: its origin, evolution and morphology. Currently the ZA is 'one of the most successful and insightful analytic models of structure formation' (White 2014). It provides a very sophisticated framework for studies and understanding the complexity of the large-scale structure in the universe. It incoporates a number of necessary concepts and therefore provides language for accounting of the highly nontrivial morphology of the web. It is also a very good approximation for statistical calculations.

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