Appendix: Land development and frictions to housing supply over the business cycle*

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Contents

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A Section 2

A.1 Descriptive statistics

The year-by-year Census Bureau's Survey of Construction (SOC) coverage of our data set is provided in Table A.1. We find that our SOC coverage remained relatively stable over time. Excluding 2003, the lowest and highest coverage for total housing is 39 percent and 55 percent, respectively.

A.2 Overview of the land development process

Land development is generally defined as the conversion of land from one use to another. The land development design process is lengthy and is broadly categorized into three stages: the pre-design stage, the design stage, and the post-design stage. Our data set records sites from the design stage when the preliminary site plan is submitted to and approved by the municipality. In this part, we sketch the whole land development process. The description borrows from the handbook of Dewberry (2019) where further details could be found.

Pre-design stage. At the beginning of this stage, the developer identifies a site or multiple potential sites of interest, labeled as the site selection process. Then comes the due diligence process where the site engineer performs a technical desk review of the site focusing on the regulatory aspects. Afterwards, the site engineer performs site analysis to understand the physical conditions of the site including a study on the engineering feasibility. At this phase, a particular emphasis is on the identification of environmental, cultural, and infrastructure resources.

Design stage. This stage includes both a preliminary design phase and a detailed design phase. Based on the constraints and development opportunities identified at the pre-design stage, a preliminary design is drawn to deliver the intent of the project. These preliminary design plans are submitted for an entitlement review by some municipalities. Approvals at this stage are not necessarily a guarantee of the final site plan approval, but they provide a guideline of what is to be expected during subsequent reviews. The detailed design phase builds from the approved preliminary design plan to focus on the engineering details necessary for permitting and construction. The site engineer eventually comes up with a final site plan which is submitted for a regulatory review and permit processing.

Post-design stage. After the approval of the final site plan, land development enters the post-design stage. This stage includes permits and construction. While the approval of the final site plan is typically treated as a milestone of land development, major construction activity can initiate only after permits are issued. The approval of a final site plan is a key input for permit issuance, but depending on the type and scope of the project, project bonds and other legal agreements might also be needed. Depending on the jurisdiction, a series of permits might be needed for infrastructure work such as a site permit which is often required prior to commencing any land disturbance. For the construction of structures, a building permit is typically required. Environmental permits might also be required based on site locations, natural resources present, type of construction etc. After the necessary permits are acquired and a construction contract agreement is signed, construction begins. At this stage, the general contractor coordinates with the design team to ensure compliance with the approved design documents.

A.3 TTD regression

The local controls we use in Table 3 of the main draft are listed below. Some of these are taken from Davidoff (2016).

1. Bartik: Computes the 1980 Census share of people working in each industry for each county and multiplies that with the national industry employment growth (net of the location of interest) between 1980 and the 2010-2012 American Community Survey.
2. Sand state: Dummy variable for counties in Arizona, California, Florida, and Nevada.
3. Coastal state: Dummy variable for counties adjacent to the Pacific Ocean and stops on the Acela line between Washington, D.C. and New York.
4. Immigrant: The share of adult population in 1980 that were born outside the U.S.
5. College+: The share of adult population in 1980 that had college education or more.
6. Population density: Taken from the 1980 Census.
7. County GDP: Annual county-level real GDP.

The regression coefficients of the local controls in regression (2) of Table [3] in the main text are shown in Table A.2. The regression coefficients of the local controls interacted with year fixed effects in regression (3) are omitted due to space.

A.4 Alternative TTD definitions

We first present the sensitivity of our results in Section 2 based on alternative definitions for the end date of TTD. Then, we discuss our choice of the start date of TTD.

Alternative end date of TTD. For the end date of TTD, we assume that section development is completed when 25 percent of the total units are built. Using this alternative definition, Tables 2-5 in the main text are reproduced. Table A.3 presents the section TTD statistics shown in Table 2 of the main text. As expected, the mean of total TTD decreases from 1,329 days to 1,141 days. The standard deviation as well as the IQR also decreases from 1,077 days to 1,009 days and from 1,006 days to 911 days, respectively. The relatively smaller decrease in the standard deviation and the IQR relative to the mean suggests that the heterogeneity in TTD remains robust to this definition. Note that as the end of TTD is defined as only 25 percent of completion, there are additional completed sections included in our sample.

Using this definition, Table A.4 presents the regression results in Table 3 of the main text. The regression results are quite similar.

The county-level TTD statistics using the alternative TTD definition (Table 4 in the main text) is presented in Table A.5. Despite the lower mean county-level TTD, the standard deviation and IQR remains relatively intact, suggesting that the cross-regional variation is less sensitive to the end date definition of TTD.

Table A.6 shows the county-level TTD regression results shown in Table 5 in the main text. We observe the same pattern where the Saiz elasticity and the rainfall intensity all significantly matter for county-level TTD.

Beginning date of TTD. For the beginning date of TTD, our baseline definition is the first quarter when we observe the total number of future lots to be the same as the total number of planned lots in the subdivision/section. With the help of maps submitted to the municipality, this is typically detected both by on-site drives each quarter and by acquiring satellite images from another company.

For some completed sections, our data set also includes the preliminary approval date from the municipality (typically from the planning department) as the first step in the official process of land development. Therefore, the data set with 222,868 completed sections could be classified into 4 cases:

- Case 1: Baseline missing and prelim. date missing (77,787 sections or 34.9%).
- Case 2: Baseline missing and prelim. date available (23,287 sections or 10.5%).
- Case 3: Baseline available and prelim. date missing (74,981 sections or 33.6%).
- Case 4: Baseline available and prelim. date available (46,813 sections or 21.0%).

Accordingly, we use the completed sections in cases 3 and 4 in the main text, which consists of 54.6% of the completed sections between 2003 and 2019 in the data set.

First, we show that when the start dates from our baseline definition and the official preliminary approval date are both available (case 4), our TTD definition is also consistent with an alternative definition that takes the official preliminary approval date as the start of TTD. In panel A of figure A.1, we plot the distribution of the two TTD definitions for the 46,813 completed sections in case 4. The density mostly overlaps each other. In this case, using an alternative definition does not quantitatively matter for our results.

Second, we decide to drop case 2 from our analysis even though the preliminary approval date is available. In case 2, we do not directly observe the beginning date of raw land development. We typically only observe development from an *active* stage, i.e., after the raw land development is completed. Moreover, we tend to observe the preliminary approval date to be much earlier than when the section is first recorded in the data set. In panel B of figure A.1, we plot the distribution of TTD in case 2 using the preliminary approval date as the beginning of TTD, and compare that with the same TTD distribution using sections in case 4. We find that TTD using the preliminary approval date tends to be much longer in case 2. This suggests that development in case 2 are likely to have gone through other stages not assumed in our model, such as a clearer gap between the plannings of raw land development and structures development that leads to a lengthy pause. As our goal is to understand the supply-side determinants of TTD under a comprehensive development planning at the beginning, the lengthy TTD as well as the obscure starting date of raw land development in case 2 is problematic and we decided to drop this data.

B Section 3

B.1 Derivation of Lemma 1

Recap the following six optimality conditions:

$$
\begin{aligned}
I_t &= \left(\sum_{p=0}^{P} U_{t-p|t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \\
U_{t|t+p} &= M_{t+p-P|t+p}^{1-\alpha} N_{t|t+p}^{\alpha}, \quad \text{for } p = 0, 1, \cdots, P, \\
N_t &= \sum_{p=0}^{P} N_{t|t+p}, \\
M_{t|t+P} &= \bar{M} q_t^{\gamma}, \\
\mu_{t|t+p} &= \mathbb{E}_t \left[\beta^p \left(\frac{\lambda_{t+p}}{\lambda_t} \right) q_{t+p} \left(\frac{I_{t+p}}{U_{t|t+p}} \right)^{\frac{1}{\theta}} \right] \quad \text{for } p = 0, 1, \cdots, P, \\
w_t &= \alpha \mu_{t|t+p} M_{t+p-P|t+p}^{1-\alpha} N_{t|t+p}^{\alpha-1} \quad \text{for } p = 0, 1, \cdots, P,
\end{aligned}
$$

where $\Lambda_{t|t+p} = \beta^p \lambda_{t+p} / \lambda_t$. Denoting the steady state variables of each variable with the subscript ss, the following holds at the steady state:

$$
\begin{aligned}
I_{ss} &= \left(\sum_{p=0}^{P} U_{0|p,ss}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \\
U_{0|p,ss} &= M_{ss}^{1-\alpha} N_{0|p,ss}^{\alpha}, \quad \text{for } p = 0, 1, \cdots, P, \\
N_{ss} &= \sum_{p=0}^{P} N_{0|p,ss}, \\
M_{ss} &= \bar{M} q_{ss}^{\gamma}, \\
\mu_{0|p,ss} &= \beta^p q_{ss} \left(\frac{I_{ss}}{U_{0|p,ss}} \right)^{\frac{1}{\theta}} \quad \text{for } p = 0, 1, \cdots, P, \\
w_{ss} &= \alpha \mu_{0|p,ss} M_{ss}^{1-\alpha} N_{0|p,ss}^{\alpha-1} \quad \text{for } p = 0, 1, \cdots, P.
\end{aligned}
$$

This implies the following conditions for $p = 1, \cdots, P$:

$$\frac{U_{0|p,ss}}{U_{0|p-1,ss}} = \left(\frac{N_{0|p,ss}}{N_{0|p-1,ss}}\right)^{\alpha},$$
$$\frac{\mu_{0|p,ss}}{\mu_{0|p-1,ss}} = \beta\left(\frac{U_{0|p-1,ss}}{U_{0|p,ss}}\right)^{\frac{1}{\theta}},$$
$$\frac{\mu_{0|p,ss}}{\mu_{0|p-1,ss}} = \left(\frac{N_{0|p,ss}}{N_{0|p-1,ss}}\right)^{1-\alpha}.$$

Using these, we derive the following equations:

$$N_{0|p,ss} = N_{0|p-1,ss}\tilde{\beta},$$
$$U_{0|p,ss} = U_{0|p-1,ss}\tilde{\beta}^{\alpha},$$
$$\mu_{0|p,ss} = \mu_{0|p-1,ss}\tilde{\beta}^{1-\alpha},$$

where $\tilde{\beta} = \beta^{\frac{\theta}{\theta+\alpha(1-\theta)}}$.

Based on the computed steady state values, it is straightforward to derive the five log-linearized conditions in Lemma 1. For instance, the second equation is Lemma 1 can be derived by using $\mu_{t|t+p}$ to plug the fifth optimality condition to the sixth optimality condition and then plugging in the second optimality condition using $N_{t|t+p}$.

B.2 Proof of Proposition 2

As the economy was in its steady state equilibrium before period 0, the hatted values are zero for those periods. Given a shock in period 0, the variables respond in period 0 and afterwards consistent with expectations formed in period 0. Without loss of generality, assume that $P > 2$. To derive the period-0 housing supply curve, some equations in Lemma 1 could be written as follows:

$$\hat{I}_0 = \frac{1}{B(P)}\hat{U}_{0|0},$$
$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)\hat{U}_{0|0} = \frac{1}{\theta}\hat{I}_0 + \hat{q}_0 - \hat{w}_0 + \mathbb{E}_0(\hat{\lambda}_1 - \hat{\lambda}_0).$$

Netting out $\hat{U}_{0|0}$ from the two equations, we get the period-0 housing supply curve:

$$\hat{I}_0 = \frac{1}{\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)B(P) - \frac{1}{\theta}}\hat{q}_0 - \frac{1}{\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)B(P) - \frac{1}{\theta}}\hat{w}_0,$$

which can be expressed as

$$\hat{I}_0 = \Upsilon_0(P)\hat{q}_0 - \frac{\Upsilon_0(P)}{B(0)}\hat{w}_0.$$

Similarly, the equations in Lemma 1 that are relevant to derive the period-1 housing supply curve are as follows:

$$\hat{I}_1 = \frac{1}{B(P)}\left(\hat{U}_{1|1} + \tilde{\beta}^{\alpha(\theta-1)/\theta}\hat{U}_{0|1}\right),$$
$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)\hat{U}_{1|1} = \frac{1}{\theta}\hat{I}_1 + \hat{q}_1 - \hat{w}_1,$$
$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)\hat{U}_{0|1} = \frac{1}{\theta}\hat{I}_1 + \hat{q}_1 - \hat{w}_0 + \left(\hat{\lambda}_1 - \hat{\lambda}_0\right).$$

Plugging $\hat{U}_{1|1}$ and $\hat{U}_{0|1}$ from the last two equations to the first equation, we get the period-1 housing supply curve:

$$\begin{aligned}\hat{I}_1 =& \frac{1+\tilde{\beta}^{\alpha(\theta-1)/\theta}}{\left(\frac{1-\alpha}{\alpha}+\frac{1}{\theta}\right)B(P) - \frac{1}{\theta}\left(1+\tilde{\beta}^{\alpha(\theta-1)/\theta}\right)}\hat{q}_1 \\ &- \frac{1}{\left(\frac{1-\alpha}{\alpha}+\frac{1}{\theta}\right)B(P) - \frac{1}{\theta}\left(1+\tilde{\beta}^{\alpha(\theta-1)/\theta}\right)}\left(\hat{w}_1 + \tilde{\beta}^{\alpha(\theta-1)/\theta}\hat{w}_0 + \tilde{\beta}^{\alpha(\theta-1)/\theta}(\hat{\lambda}_0 - \hat{\lambda}_1)\right),\end{aligned}$$

which can be expressed as

$$\hat{I}_1 = \Upsilon_1(P)\hat{q}_1 - \frac{\Upsilon_1(P)}{B(1)}\sum_{j=0}^{1}\left(\tilde{\beta}^{\alpha(\theta-1)/\theta}\right)^j\left(\hat{w}_{1-j} + \hat{\lambda}_{1-j} - \hat{\lambda}_1\right).$$

Similarly, the equations in Lemma 1 that are relevant to derive the period-2 housing supply elasticity are as follows:

$$\hat{I}_2 = \frac{1}{B(P)}\left(\hat{U}_{2|2} + \tilde{\beta}^{\alpha(\theta-1)/\theta}\hat{U}_{1|2} + \left(\tilde{\beta}^{\alpha(\theta-1)/\theta}\right)^2\hat{U}_{0|2}\right),$$
$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)\hat{U}_{2|2} = \frac{1}{\theta}\hat{I}_2 + \hat{q}_2 - \hat{w}_2,$$
$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)\hat{U}_{1|2} = \frac{1}{\theta}\hat{I}_2 + \hat{q}_2 - \hat{w}_1 + \left(\hat{\lambda}_2 - \hat{\lambda}_1\right),$$
$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta}\right)\hat{U}_{0|2} = \frac{1}{\theta}\hat{I}_2 + \hat{q}_2 - \hat{w}_0 + \left(\hat{\lambda}_2 - \hat{\lambda}_0\right).$$

Plugging $\hat{U}_{2|2}$, $\hat{U}_{1|2}$ and $\hat{U}_{0|2}$ from the last three equations to the first equation, we get the period-2 housing supply curve:

$$\hat{I}_2 = \Upsilon_2(P)\hat{q}_2 - \frac{\Upsilon_2(P)}{B(2)}\sum_{j=0}^{2}\left(\tilde{\beta}^{\alpha(\theta-1)/\theta}\right)^j\left(\hat{w}_{2-j} + \hat{\lambda}_{2-j} - \hat{\lambda}_2\right).$$

In general for $t < P$, the equations in Lemma 1 that are relevant to derive the period-2 housing supply elasticity are as follows:

$$\hat{I}_t = \frac{1}{B(P)} \left(\sum_{j=0}^{t} \left(\tilde{\beta}^{\alpha(\theta-1)/\theta} \right)^j \hat{U}_{t-j|t} \right),$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{t|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t - \hat{w}_t,$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{t-1|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t - \hat{w}_{t-1} + \left(\hat{\lambda}_t - \hat{\lambda}_{t-1} \right),$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{t-2|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t - \hat{w}_{t-2} + \left(\hat{\lambda}_t - \hat{\lambda}_{t-2} \right),$$

$$\vdots$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{0|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t - \hat{w}_0 + \left(\hat{\lambda}_t - \hat{\lambda}_0 \right).$$

Plugging $\{U_{t-j|t}\}_{j=0}^{t}$ from the last $t+1$ equations to the first equation, we can derive the period-t housing supply curve as in the proposition.

For $t \geq P$, note that $\hat{M}_{t-P|t} = \gamma \hat{q}_{t-P}$. The relevant equations in this case are

$$\hat{I}_t = \frac{1}{B(P)} \sum_{p=0}^{P} \left(\tilde{\beta}^{\alpha(\theta-1)/\theta} \right)^p \hat{U}_{t-p|t},$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{t|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t + \left(\frac{1-\alpha}{\alpha} \right) \gamma \hat{q}_{t-P} - \hat{w}_t,$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{t-1|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t + \left(\frac{1-\alpha}{\alpha} \right) \gamma \hat{q}_{t-P} - \hat{w}_{t-1} + \left(\hat{\lambda}_t - \hat{\lambda}_{t-1} \right),$$

$$\vdots$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{t-P+1|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t + \left(\frac{1-\alpha}{\alpha} \right) \gamma \hat{q}_{t-P} - \hat{w}_{t-P+1} + \left(\hat{\lambda}_t - \hat{\lambda}_{t-P+1} \right),$$

$$\left(\frac{1-\alpha}{\alpha} + \frac{1}{\theta} \right) \hat{U}_{t-P|t} = \frac{1}{\theta} \hat{I}_t + \hat{q}_t + \left(\frac{1-\alpha}{\alpha} \right) \gamma \hat{q}_{t-P} - \hat{w}_{t-P} + \left(\hat{\lambda}_t - \hat{\lambda}_{t-P} \right).$$

Substituting out $\{U_{t-j|t}\}_{j=0}^{t-P}$ in the first equation and rearranging, we get

$$\left[B(P) - \left(\frac{\frac{1}{\theta}}{\frac{1-\alpha}{\alpha} + \frac{1}{\theta}} \right) \left(\frac{\tilde{\beta}^{(\alpha(\theta-1)/\theta)(1+P)} - 1}{\tilde{\beta}^{\alpha(\theta-1)/\theta} - 1} \right) \right] \hat{I}_t =$$

$$\left(\frac{1}{\frac{1-\alpha}{\alpha} + \frac{1}{\theta}} \right) \left(\frac{\tilde{\beta}^{(\alpha(\theta-1)/\theta)(1+P)} - 1}{\tilde{\beta}^{\alpha(\theta-1)/\theta} - 1} \right) \hat{q}_t + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{\tilde{\beta}^{(\alpha(\theta-1)/\theta)(1+P)} - 1}{\tilde{\beta}^{\alpha(\theta-1)/\theta} - 1} \right) \gamma \hat{q}_{t-P} + \text{etc.}$$

Since

$$B(P) = \frac{\tilde{\beta}^{(\alpha(\theta-1)/\theta)(1+P)} - 1}{\tilde{\beta}^{\alpha(\theta-1)/\theta} - 1},$$

the proposition holds for $t \geq P$.

B.3 Proof of Corollary 3

Since $\beta < 1$, $\theta > 0$, and $\alpha \in (0,1)$,

$$\tilde{\beta} = \beta^{\frac{\theta}{\theta+\alpha(1-\theta)}} < 1.$$

To verify that $\Upsilon_t(P)$ ($t \in [0,P]$) is positive and an increasing function of t, it suffices to show that $B(t)$ is positive and an increasing function of t. First, we show that $B(t)$ is positive. Note that when $\theta > 1$, $\tilde{\beta}^{\alpha(\theta-1)/\theta} < 1$. Therefore,

$$B(t) = \frac{1 - \tilde{\beta}^{(\alpha(\theta-1)/\theta)(1+t)}}{1 - \tilde{\beta}^{\alpha(\theta-1)/\theta}} > 0.$$

Likewise, when $\theta < 1$, $\tilde{\beta}^{\alpha(\theta-1)/\theta} > 1$. which implies

$$B(t) = \frac{\tilde{\beta}^{(\alpha(\theta-1)/\theta)(1+t)} - 1}{\tilde{\beta}^{\alpha(\theta-1)/\theta} - 1} > 0.$$

Second, we show that $B(t)$ is an increasing function of t by taking its derivative with respect to t:

$$\frac{dB(t)}{dt} = \frac{\alpha\left(\frac{\theta-1}{\theta}\right)(\ln \tilde{\beta})}{\tilde{\beta}^{\alpha(\theta-1)/\theta} - 1}\tilde{\beta}^{(\alpha(\theta-1)/\theta)(1+t)} > 0.$$

The above inequality holds as $\ln \tilde{\beta} < 0$.

The second part of the corollary follows from the fact that $B(P) < B(\tilde{P})$ for $P < \tilde{P}$, as $\Upsilon_t(P)$ is a decreasing function of $B(P)$ (and P).

C Section 4

C.1 Sensitivity analysis

We set the elasticity of substitution across construction stage (θ) as 0.5 as our baseline. Appendix Figures A.2 and A.3 show the regional variations in our $T-$quarter housing supply elasticities when we set $\theta = 0.01$ and $\theta = 2.0$, respectively. As shown in these figures, the regional variation of our supply elasticity measures is less sensitive to the degree of substitution between different construction stages.

D Section 5

D.1 Details of the local general equilibrium model

The local general equilibrium model described in Section 5 closes the partial equilibrium model of housing developers and the local government in Section 3 by incorporating local households and the nondurable goods sector. Since the local economy is in a monetary union, we assume that the interest rate is exogenous. As such, the bond and nondurable goods markets do not clear, analogous to small open economy models in the international macro literature. In the next part, the local general equilibrium model is extended to a two-region New Keynesian economy with nominal rigidities for nondurable goods, national interest rates set by a standard Taylor rule by the central bank, and the bond and nondurable good markets clearing at the national level.

D.1.1 Optimality conditions

We solve the model in Section 5 with a fixed interest rate ($R_t = \bar{R} = 1/\beta$). The optimality conditions of the local general equilibrium model could be summarized as below.

1. Endogenous variables (3(P+1)+11):
   (a) TTD (3(P+1)): $U_{t|t+p}$, $N_{t|t+p}$, $\mu_{t|t+p}$ for $p = 0, 1, \cdots, P$
   (b) Quantity (8): $M_{t|t+P}$, N_t, I_t, Y_t, $N_{n,t}$, H_t, C_t, B_{t+1}
   (c) Price (3): w_t, $w_{n,t}$, q_t
2. Exogenous variable (1): φ_t
3. Predetermined values: B_0, H_{-1}.

Equations for the endogenous variables:

$$U_{t|t+p} = N_{t|t+p}^{\alpha} M_{t+p-P|t+p}^{1-\alpha} \quad \text{for } p = 0, 1, \cdots, P, \tag{D.1}$$

$$\mu_{t|t+p} = \mathbb{E}_t \left[\beta^p q_{t+p} \left(\frac{I_{t+p}}{U_{t|t+p}} \right)^{\frac{1}{\theta}} \right] \quad \text{for } p = 0, 1, \cdots, P, \tag{D.2}$$

$$w_t = \alpha \mu_{t|t+p} M_{t+p-P|t+p}^{1-\alpha} N_{t|t+p}^{\alpha-1} \quad \text{for } p = 0, 1, \cdots, P, \tag{D.3}$$

$$I_t = \left(\sum_{p=0}^{P} U_{t-p|t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \tag{D.4}$$

$$N_t = \sum_{p=0}^{P} N_{t|t+p}, \tag{D.5}$$

$$Y_t = \bar{Z} N_{n,t}, \tag{D.6}$$

$$w_{n,t} = \bar{Z}, \tag{D.7}$$

$$H_t = (1-\delta)H_{t-1} + I_t, \tag{D.8}$$

$$1 + \frac{\psi_b}{\beta} B_{t+1} = \mathbb{E}_t \left[\frac{u_c(t+1)}{u_c(t)} \right], \tag{D.9}$$

$$-u_{nn}(t) = u_c(t) w_{n,t}, \tag{D.10}$$

$$-u_n(t) = u_c(t) w_t, \tag{D.11}$$

$$u_h(t) = u_c(t) q_t - \beta(1-\delta)\mathbb{E}_t u_c(t+1) q_{t+1}, \tag{D.12}$$

$$M_{t|t+P} = q_t^{\gamma}, \tag{D.13}$$

$$C_t + \beta B_{t+1} = w_{n,t} N_{n,t} + B_t. \tag{D.14}$$

Functional forms. For household utility, we follow Guren, McKay, Nakamura and Steinsson (2020) in assuming that nondurable consumption and leisure are substitutable in household utility in the style of Greenwood, Hercowitz and Huffman (1988). The housing demand shock φ_t is modeled in a Stone-Geary fashion:

$$u(C_t, H_t, N_{n,t}, N_t; \varphi_t) = \left[\frac{1}{1-\sigma} \left(C_t - \frac{\psi_n}{1+\nu} N_{n,t}^{1+\nu} - \frac{\psi}{1+\nu} N_t^{1+\nu} \right)^{\kappa} (H_t - \varphi_t)^{1-\kappa} \right]^{1-\sigma}.$$

By defining variables $\tilde{C}_t$ and $\tilde{H}_t$ as

$$\tilde{C}_t \equiv C_t - \frac{\psi_n}{1+\nu} N_{n,t}^{1+\nu} - \frac{\psi}{1+\nu} N_t^{1+\nu},$$
$$\tilde{H}_t \equiv H_t - \varphi_t,$$

we can express the marginal utilities as follows:

$$u_c(t) = \kappa \left(\tilde{C}_t^{\kappa} \tilde{H}_t^{1-\kappa} \right)^{1-\sigma} \frac{1}{\tilde{C}_t},$$
$$u_{nn}(t) = -\kappa \psi_n \left(\tilde{C}_t^{\kappa} \tilde{H}_t^{1-\kappa} \right)^{1-\sigma} \frac{N_{n,t}^{\nu}}{\tilde{C}_t},$$
$$u_n(t) = -\kappa \psi \left(\tilde{C}_t^{\kappa} \tilde{H}_t^{1-\kappa} \right)^{1-\sigma} \frac{N_t^{\nu}}{\tilde{C}_t},$$
$$u_h(t) = (1-\kappa) \left(\tilde{C}_t^{\kappa} \tilde{H}_t^{1-\kappa} \right)^{1-\sigma} \frac{1}{\tilde{H}_t}.$$

These marginal utilities imply that the labor supply conditions of households depend solely on the real wage:

$$-u_{nn}(t) = u_c(t) w_{n,t} \qquad \Rightarrow \qquad \psi_n N_{n,t}^{\nu} = w_{n,t},$$
$$-u_n(t) = u_c(t) w_t \qquad \Rightarrow \qquad \psi N_t^{\nu} = w_t.$$

Using the above functional forms, we can rewrite the equilibrium conditions for the endogenous variables $\{U_{t|t+p}, N_{t|t+p}, \mu_{t|t+p}\}_{p=0}^{P}, M_{t|t+P}, N_t, I_t, N_{n,t}, H_t, C_t, B_{t+1}, w_t, q_t$ as follows:

$$U_{t|t+p} = N_{t|t+p}^{\alpha} M_{t+p-P|t+p}^{1-\alpha} \quad \text{for } p = 0, 1, \cdots, P, \tag{D.15}$$

$$\mu_{t|t+p} = \mathbb{E}_t\left[\beta^p q_{t+p}\left(\frac{I_{t+p}}{U_{t|t+p}}\right)^{\frac{1}{\theta}}\right] \quad \text{for } p = 0, 1, \cdots, P, \tag{D.16}$$

$$w_t = \alpha\mu_{t|t+p} M_{t+p-P|t+p}^{1-\alpha} N_{t|t+p}^{\alpha-1} \quad \text{for } p = 0, 1, \cdots, P, \tag{D.17}$$

$$I_t = \left(\sum_{p=0}^{P} U_{t-p|t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, \tag{D.18}$$

$$N_t = \sum_{p=0}^{P} N_{t|t+p}, \tag{D.19}$$

$$H_t = (1-\delta)H_{t-1} + I_t, \tag{D.20}$$

$$1 + \frac{\psi_b}{\beta} B_{t+1} = \mathbb{E}_t\left[\frac{u_c(t+1)}{u_c(t)}\right], \tag{D.21}$$

$$\psi_n N_{n,t}^{\nu} = \bar{Z}, \tag{D.22}$$

$$\psi N_t^{\nu} = w_t, \tag{D.23}$$

$$u_h(t) = u_c(t)q_t - \beta(1-\delta)\mathbb{E}_t u_c(t+1) q_{t+1}, \tag{D.24}$$

$$M_{t|t+P} = q_t^{\gamma}, \tag{D.25}$$

$$C_t + \beta B_{t+1} = \bar{Z} N_{n,t} + B_t. \tag{D.26}$$

D.1.2 Steady state

The relevant steady states (with zero net bond holdings) are expressed as follows:

$$U_{0|p} = N_{0|p}^{\alpha} M_{0|P}^{1-\alpha} \quad \text{for } p = 0, 1, \cdots, P,$$

$$\mu_{0|p} = \beta^p q_0 \left(\frac{I_0}{U_{0|p}}\right)^{\frac{1}{\theta}} \quad \text{for } p = 0, 1, \cdots, P,$$

$$w_0 = \alpha\mu_{0|p} M_{0|P}^{1-\alpha} N_{0|p}^{\alpha-1} \quad \text{for } p = 0, 1, \cdots, P,$$

$$I_0 = \left(\sum_{p=0}^{P} U_{0|p}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$

$$N_0 = \sum_{p=0}^{P} N_{0|p},$$

$$\delta H_0 = I_0,$$

$$\psi_n = \frac{\bar{Z}}{N_{n,0}^{\nu}},$$

$$\psi = \frac{w_0}{N_0^{\nu}},$$

$$\left(\frac{1-\kappa}{\kappa}\right)\frac{\tilde{C}_0}{\tilde{H}_0} = (1-\beta(1-\delta))q_0,$$

$$M_{0|P} = q_0^{\gamma},$$
$$C_0 = \bar{Z} N_{n,0}.$$

As such, the steady state ratios of TTD variables for each $p = 1, \cdots, P$ are as follows:

$$\frac{U_{0|p}}{U_{0|p-1}} = \left(\frac{N_{0|p}}{N_{0|p-1}}\right)^{\alpha},$$
$$\frac{\mu_{0|p}}{\mu_{0|p-1}} = \beta \left(\frac{U_{0|p-1}}{U_{0|p}}\right)^{\frac{1}{\theta}},$$
$$\frac{\mu_{0|p}}{\mu_{0|p-1}} = \left(\frac{N_{0|p}}{N_{0|p-1}}\right)^{1-\alpha}.$$

This implies the TTD ratios as follows:

$$\frac{N_{0|p}}{N_{0|p-1}} = \beta^{\frac{\theta}{\alpha+\theta(1-\alpha)}} \equiv \bar{\beta}_n,$$
$$\frac{U_{0|p}}{U_{0|p-1}} = \beta^{\frac{\theta\alpha}{\alpha+\theta(1-\alpha)}} \equiv \bar{\beta}_u,$$
$$\frac{\mu_{0|p}}{\mu_{0|p-1}} = \beta^{\frac{\theta(1-\alpha)}{\alpha+\theta(1-\alpha)}} \equiv \bar{\beta}_\mu.$$

Using these ratios, we can reduce the steady state expressions as follows:

$$U_{0|0} = N_{0|0}^{\alpha} M_{0|P}^{1-\alpha},$$
$$\mu_{0|0} = q_0 \left(\frac{I_0}{U_{0|0}}\right)^{\frac{1}{\theta}},$$
$$w_0 = \alpha \mu_{0|0} M_{0|P}^{1-\alpha} N_{0|0}^{\alpha-1},$$
$$I_0 = U_{0|0} \left[\sum_{p=0}^{P} \left(\bar{\beta}_u^{\frac{\theta-1}{\theta}}\right)^p\right]^{\frac{\theta}{\theta-1}} = U_{0|0} \left[\frac{1-\left(\bar{\beta}_u^{\frac{\theta-1}{\theta}}\right)^{P+1}}{1-\bar{\beta}_u^{\frac{\theta-1}{\theta}}}\right]^{\frac{\theta}{\theta-1}},$$
$$N_0 = N_{0|0}\left(\sum_{p=0}^{P} \bar{\beta}_n^p\right) = N_{0|0}\left[\frac{1-\bar{\beta}_n^{P+1}}{1-\bar{\beta}_n}\right],$$
$$\delta H_0 = I_0,$$
$$\psi_n = \frac{\bar{Z}}{N_{n,0}^{\nu}},$$
$$\psi = \frac{w_0}{N_0^{\nu}},$$
$$\left(\frac{1-\kappa}{\kappa}\right)\frac{\tilde{C}_0}{\tilde{H}_0} = (1-\beta(1-\delta))q_0,$$

$$M_{0|P} = q_0^{\gamma},$$
$$C_0 = \bar{Z} N_{n,0}.$$

We calibrate the utility parameters ψ_n, ψ, and κ to target the given steady state values of N_n, N, and C/H. The following steady state values are similarly derived as above: $\boxed{B_0 = 0}$, $\boxed{\psi_n = \frac{\bar{Z}}{N_{n,0}^{\nu}}}$, $\boxed{C_0 = \bar{Z} N_{n,0}}$, $\boxed{H_0 = C_0/(C_0/H_0)}$, $\boxed{I_0 = \delta H_0}$, $\boxed{U_{0|0} = I_0 \left[\sum_{p=0}^{P} \left(\bar{\beta}_u^{\frac{\theta-1}{\theta}}\right)^p\right]^{-\frac{\theta}{\theta-1}}}$, $\boxed{N_{0|0} = N_0 \left(\sum_{p=0}^{P} \bar{\beta}_n^p\right)^{-1}}$, $\boxed{M_{0|P} = \left(\frac{U_{0|0}}{N_{0|0}^{\alpha}}\right)^{\frac{1}{1-\alpha}}}$, $\boxed{q_0 = M_{0|P}^{\frac{1}{\gamma}}}$, $\boxed{\mu_{0|0} = q_0 \left(\frac{I_0}{U_{0|0}}\right)^{\frac{1}{\theta}}}$, and $\boxed{w_0 = \alpha \mu_{0|0} M_{0|P}^{1-\alpha} N_{0|0}^{\alpha-1}}$. In turn, we obtain $\boxed{\psi = \frac{w_0}{N_0^{\nu}}}$. Plugging these into the following variables

$$\tilde{C}_0 = C_0 - \frac{\psi_n}{1+\nu} N_{n,0}^{1+\nu} - \frac{\psi}{1+\nu} N_0^{1+\nu}, \qquad \tilde{H}_0 = H_0 - \varphi_0,$$

we obtain κ by the equation $\boxed{\frac{1-\kappa}{\kappa} = (1-\beta(1-\delta)) q_0 \frac{\tilde{H}_0}{\tilde{C}_0}}$. Finally, the remaining TTD variables for each $p = 1, \cdots, P$ are computed as $\boxed{N_{0|p} = N_{0|p-1} \bar{\beta}_n}$, $\boxed{U_{0|p} = U_{0|p-1} \bar{\beta}_u}$, $\boxed{\mu_{0|p} = \mu_{0|p-1} \bar{\beta}_{\mu}}$.

D.1.3 Log-linearized conditions

Define $B_{t+1}^* \equiv e^{B_{t+1}}$. Log-linearizing the variables at their respective steady state values, we get the following $3(P+1) + 11$ equations:

$$\hat{u}_{t|t+p} = \alpha \hat{n}_{t|t+p} + (1-\alpha)\hat{m}_{t+p-P|t+p} \quad \text{for } p = 0, 1, \cdots, P,$$
$$\hat{\mu}_{t|t+p} = \mathbb{E}_t \hat{q}_{t+p} + \frac{1}{\theta}\mathbb{E}_t \hat{i}_{t+p} - \frac{1}{\theta}\hat{u}_{t|t+p} \quad \text{for } p = 0, 1, \cdots, P,$$
$$\hat{w}_t = \hat{\mu}_{t|t+p} + (1-\alpha)\hat{m}_{t+p-P|t+p} + (\alpha-1)\hat{n}_{t|t+p} \quad \text{for } p = 0, 1, \cdots, P,$$
$$\left[\frac{1 - \left(\bar{\beta}_u^{\frac{\theta-1}{\theta}}\right)^{P+1}}{1 - \bar{\beta}_u^{\frac{\theta-1}{\theta}}}\right] \hat{i}_t = \sum_{p=0}^{P} \left(\bar{\beta}_u^{\frac{\theta-1}{\theta}}\right)^p \hat{u}_{t-p|t},$$
$$\left(\frac{1-\bar{\beta}_n^{P+1}}{1-\bar{\beta}_n}\right)\hat{n}_t = \sum_{p=0}^{P} (\bar{\beta}_n)^p \hat{n}_{t|t+p},$$
$$\hat{h}_t = (1-\delta)\hat{h}_{t-1} + \delta \hat{i}_t,$$
$$\frac{\psi_b}{\beta}\hat{b}_{t+1}^* = [\kappa(1-\sigma)-1](\mathbb{E}_t \hat{\tilde{c}}_{t+1} - \hat{\tilde{c}}_t) + (1-\kappa)(1-\sigma)(\mathbb{E}_t \hat{\tilde{h}}_{t+1} - \hat{\tilde{h}}_t),$$
$$\hat{n}_{n,t} = 0,$$
$$\nu \hat{n}_t = \hat{w}_t,$$
$$[1-(\kappa-\kappa\sigma)\beta(1-\delta)]\hat{\tilde{c}}_t - [1-(\kappa-\kappa\sigma)]\beta(1-\delta)\mathbb{E}_t \hat{\tilde{c}}_{t+1} =$$

$$[1-(\kappa+\sigma-\kappa\sigma)\beta(1-\delta)]\hat{\tilde{h}}_t-(1-(\kappa+\sigma-\kappa\sigma))\beta(1-\delta)\mathbb{E}_t\hat{\tilde{h}}_{t+1}+\hat{q}_t-\beta(1-\delta)\mathbb{E}_t\hat{q}_{t+1},$$

$$\hat{m}_{t|t+P}=\gamma\hat{q}_t,$$

$$\bar{C}\hat{c}_t+\beta\hat{b}^*_{t+1}=\bar{Z}\bar{N}_n\hat{n}_{n,t}+\hat{b}^*_t,$$

$$\bar{\tilde{C}}\hat{\tilde{c}}_t=\bar{C}\hat{c}_t-\psi_n\bar{N}_n^{1+\nu}\hat{n}_{n,t}-\psi\bar{N}^{1+\nu}\hat{n}_t,$$

$$\bar{\tilde{H}}\hat{\tilde{h}}_t=\bar{H}\hat{h}_t-\bar{\varphi}\hat{\varphi}_t.$$

The $3(P+1)+11$ endogenous variables are $\{\hat{u}_{t|t+p},\hat{n}_{t|t+p},\hat{\mu}_{t|t+p}\}_{p=0}^P$, $\hat{m}_{t|t+P}$, $\hat{q}_t$, $\hat{i}_t$, $\hat{w}_t$, $\hat{n}_t$, $\hat{h}_t$, $\hat{b}^*_{t+1}$, $\hat{\tilde{c}}_t$, $\hat{\tilde{h}}_t$, $\hat{n}_{n,t}$, $\hat{c}_t$.

D.1.4 Calibration

Table A.7 presents our calibration for the local GE model. The model is calibrated at a quarterly frequency with a time discount factor of $\beta=0.98^{\frac{1}{4}}$. We set the inverse of the Frisch elasticity (ν) to be 1.0 and the inverse of the elasticity of intertemporal substitution (σ) to be 2.0, following Guren et al. (2020). The elasticity of substitution across construction stages (θ) is set at 0.5 as our baseline.

The construction labor share (α) is set at 0.385 which implies that a county with the smallest Saiz's supply elasticity has a permit elasticity at its lower bound of zero. This value of is consistent with our estimate of the construction labor income of 37 percent in the KLEMS account when we assume that overhead labor costs are about 10 percent of the total labor cost. We set the preference weight on effective consumption (κ) as 0.75 to target a 25% expenditure share on housing, which is the average housing expenditure in the Consumer Expenditure Survey (CEX).

We set the depreciation rate on housing (δ) to 3% annually and the scale of the portfolio holding cost (ψ_b) to 0.001 as in Guren et al. (2020). The housing demand shock (φ_t) follows an AR(1) with quarterly persistence of 0.95.

D.2 Empirical regularities of the housing market

Cross-county price-quantity correlation. The cross-county rank correlation coefficients between one-year house price growth and one-year housing unit growth from 2001 through 2019 are shown in Figure A.4. The average cross-county rank correlation is 0.13, and the correlation is positive in the sample except 2008-11. Excluding the Great Recession period (2007-09), the average correlation is 0.17.

Regression of house price dynamics with short- and long-run elasticities. Section 5.3 in the main text presents the following regression:

$$\Delta_{\tau_1}^{\tau_2}\log\left(P_{i,t}/P_{N,t}\right)=\kappa\mathcal{E}^i_\infty+\eta\mathcal{E}^i_5+\Omega\mathbf{X_i}+u_i,$$

where $\mathcal{E}^i_\infty$ and $\mathcal{E}^i_5$ are county i's respective long-run and short-run (5-year) housing supply elasticities. To verify the intuition provided in the main text, we simulate our model with a common housing demand shock (with persistence $\rho_\varphi = 0.95$) for each county and run the above regression. The results are plotted in Figure A.5. For a relatively persistent shock, both the long-run elasticity and our short-run elasticity correctly predicts a negative sign in the short horizons. Also consistent with our intuition is that only the regression coefficient of the long-run elasticity is significantly negative in the longer horizons.

In Figure A.6, we run the regression without the long-run elasticity. In this case, the coefficients are negative throughout the horizon as our four-year elasticity is a weighted sum of the short-run elasticity driven by TTD and the long-run elasticity.

D.3 Extension: Two-region general equilibrium model

In this part, we develop a two-region general equilibrium model to study the general equilibrium forces that govern new construction and house price responses when house price elasticities are different across regions. The model consists of two regions with local governments that belong to a monetary union. We refer to the regions as "home" and "foreign". The population of the entire economy is normalized to one and the population of the home region is denoted by n.

D.3.1 Household

Utility. Home households maximize their expected lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, H_t, N_t, N_{c,t}; \varphi_t\right), \tag{D.27}$$

where C_t is the household consumption of a composite consumption good, H_t is the service flow of housing, N_t is the labor supply for the (non-construction) output sector, $N_{c,t}$ is the labor supply for the construction sector, and φ_t is an exogenous process for housing demand. The parameter β is the household subjective discount factor. Foreign households maximize the same utility and we use the asterisk (*) to denote foreign variables.

Consumption good. The composite consumption good of the home region, C_t, is a constant elasticity of substitution (CES) aggregator of final goods produced in both home and foreign regions:

$$C_t = \left[\phi^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + (1-\phi)^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where $C_{H,t}$ is home consumption of goods produced in the home region and $C_{F,t}$ is home consumption of goods produced in the foreign region. The parameter ϕ captures the degree of home bias in the demand for goods at the home region and η is the elasticity of substitution between home and foreign goods. Similarly, the composite consumption good of the foreign region, C_t^*, is

$$C_t^* = \left[(\phi^*)^{\frac{1}{\eta}} \left(C_{F,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\phi^*)^{\frac{1}{\eta}} \left(C_{H,t}^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where $C^*_{F,t}$ is foreign consumption of goods produced in the foreign region, $C^*_{H,t}$ is foreign consumption of goods produced in the home region, and ϕ^* captures the foreign region's degree of home bias.

The regional final goods, $C_{H,t}$ and $C_{F,t}$, are given by

$$C_{H,t} = \left(\int_0^1 \left(C_{H,t}\left(j\right) \right)^{\frac{\theta_c - 1}{\theta_c}} dj \right)^{\frac{\theta_c}{\theta_c - 1}} \text{ and } C_{F,t} = \left(\int_0^1 \left(C_{F,t}\left(j\right) \right)^{\frac{\theta_c - 1}{\theta_c}} dj \right)^{\frac{\theta_c}{\theta_c - 1}},$$

where $C_{H,t}\left(j\right)$ and $C_{F,t}\left(j\right)$ are the home consumption of variety $j \in [0,1]$ of home- and foreign-produced goods, respectively. We assume that goods markets are competitive and integrated across regions. Thus, home and foreign households face the same prices for each variety j of goods produced in the economy, denoted by $P_{H,t}\left(j\right)$ and $P_{F,t}\left(j\right)$.

Solving the cost minimization problem of home households, we obtain the following home region's demand for home- and foreign-produced goods:

$$C_{H,t}\left(j\right) = \left(\frac{P_{H,t}\left(j\right)}{P_{H,t}} \right)^{-\theta_c} C_{H,t}, \quad C_{F,t}\left(j\right) = \left(\frac{P_{F,t}\left(j\right)}{P_{F,t}} \right)^{-\theta_c} C_{F,t},$$

$$C_{H,t} = \phi \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad \text{and} \quad C_{F,t} = \left(1 - \phi\right) \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t,$$

where

$$P_{H,t} = \left(\int_0^1 \left(P_{H,t}\left(j\right) \right)^{1-\theta_c} dj \right)^{\frac{1}{1-\theta_c}}, \quad P_{F,t} = \left(\int_0^1 \left(P_{F,t}\left(j\right) \right)^{1-\theta_c} dj \right)^{\frac{1}{1-\theta_c}},$$

and the home region's composite price level, P_t, is given by

$$P_t = \left[\phi P_{H,t}^{1-\eta} + \left(1 - \phi\right) P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

Household's total consumption spending can be expressed as follows:

$$\int_0^1 \left[P_{H,t}(j) C_{H,t}(j) + P_{F,t}(j) C_{F,t}(j) \right] dj = P_t C_t.$$

Similarly, foreign region's demand for foreign- and home-produced goods are

$$C^*_{F,t}\left(j\right) = \left(\frac{P_{F,t}\left(j\right)}{P_{F,t}} \right)^{-\theta_c} C^*_{F,t}, \quad C^*_{H,t}\left(j\right) = \left(\frac{P_{H,t}\left(j\right)}{P_{H,t}} \right)^{-\theta_c} C^*_{H,t},$$

$$C^*_{F,t} = \phi^* \left(\frac{P_{F,t}}{P^*_t} \right)^{-\eta} C^*_t, \quad \text{and} \quad C^*_{H,t} = \left(1 - \phi^*\right) \left(\frac{P_{H,t}}{P^*_t} \right)^{-\eta} C^*_t,$$

where the foreign region's composite price level, P^*_t, is given by

$$P^*_t = \left[\phi^* P_{F,t}^{1-\eta} + \left(1 - \phi^*\right) P_{H,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

Housing stock and flow. Households' service flow of housing is proportional to their housing stock. Home region's housing stock evolves over time by

$$H_t = (1-\delta) H_{t-1} + I_t, \tag{D.28}$$

where I_t is the home region's new housing investment and δ is the depreciation rate of the housing stock. We assume separate housing markets in each region. The foreign region's housing stock evolves over time in a similar fashion.

Labor supply. To introduce wage stickiness in the output sector, we follow Schmitt-Grohe and Uribe (2007) in assuming that labor decisions are made by a central authority within the home household which supplies labor monopolistically to a continuum of labor markets indexed by $k \in [0,1]$. In each labor market k, the central authority faces a demand for labor, N_t^k, given by

$$N_t^k = \left(\frac{W_t^k}{W_t}\right)^{-\tilde{\eta}} N_t^d,$$

where W_t^k denotes the nominal wage charged changed by the central authority in labor market k at period t, W_t is the home region's nominal wage index in the output sector, and N_t^d is the population-adjusted aggregate labor demand by firms. This labor variety demand function is later derived from the firm's problem. The central authority takes W_t and N_t^d as exogenous and sets W_t^k to satisfy labor demand. The sum of labor supply to each labor market must be equal to the household's total labor supply

$$N_t = \int_0^1 N_t^k dk.$$

Combined with the labor variety demand function, we get

$$N_t = N_t^d \int_0^1 \left(\frac{w_t^k}{w_t}\right)^{-\tilde{\eta}} dk, \tag{D.29}$$

where we alternatively use real wage variables: $w_t^k = W_t^k/P_t$ and $w_t = W_t/P_t$. In the construction sector, we assume that labor markets are perfectly competitive. The foreign region's labor decision is made in a similar fashion.

Budget constraint. We assume incomplete financial markets across regions in the sense that households only have access to risk-free nominal bonds. The real flow budget constraint of the home region household is given by

$$C_t + q_t I_t + \frac{B_{t+1}}{R_t} + \frac{\psi_b}{2} B_{t+1}^2 = \int_0^1 w_t^k \left(\frac{w_t^k}{w_t}\right)^{-\tilde{\eta}} N_t^d dk + w_{c,t} N_{c,t} + \frac{B_t}{\pi_t} + \frac{1}{n} T_t + \frac{1}{n}\Phi_t, \tag{D.30}$$

where q_t is the real price of a housing unit (Q_t/P_t), B_{t+1} is real bond holdings, R_t is the risk-free nominal interest rate between periods t and $t+1$, $\pi_t = P_t/P_{t-1}$ is the price inflation rate, $w_{c,t} = W_{c,t}/P_t$ is the real wage in the construction sector, T_t is the real transfer from the local government, and $\Phi_t = \int_0^1 \Phi_t(j)dj$ is the aggregate of home firms' real profits. Both the transfer and the firms' profits are distributed equally to the households based on the home population. For real bond holding B_{t+1}, we impose a convex portfolio holding cost of $\psi_b B_{t+1}^2/2$ that the local government rebates equally to the households.

We introduce wage stickiness by assuming that the central authority in the household cannot set the nominal wage optimally with probability $\tilde{\omega} \in [0,1]$ of a random labor market in each period. When the household cannot set the nominal wage optimally in market k, we assume $W_t^k = W_{t-1}^k$.

The foreign region household's budget constraint is also written analogously with the respective foreign variables with the stochastic discount factor that is common across regions. Moreover, transfers and the firms' profits are distributed equally to the foreign households based on the foreign population share $1-n$.

Household choice. The home household chooses C_t, I_t, H_t, B_{t+1}, w_t^k, N_t, and $N_{c,t}$ so as to maximize the utility function (D.27) subject to (D.29), (D.30), the wage stickiness assumption, and a no-Ponzi constraint, taking as given the processes q_t, w_t, R_t, π_t, N_t^d, T_t, Φ_t, and the initial conditions B_0 and H_{-1}. The Lagrangian associated with the home household problem is

$$
\begin{aligned}
\mathcal{L} =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ U(C_t, H_t, N_t, N_{c,t}; \varphi_t) \\
&+ \xi_t \left[\int_0^1 w_t^k \left(\frac{w_t^k}{w_t} \right)^{-\tilde{\eta}} N_t^d dk + w_{c,t} N_{c,t} + \frac{B_t}{\pi_t} + \frac{1}{n} T_t + \frac{1}{n} \Phi_t \right. \\
&\left. - C_t - q_t I_t - \frac{B_{t+1}}{R_t} - \frac{\psi_b}{2} B_{t+1}^2 \right] \\
&+ \frac{\xi_t w_t}{\tilde{\mu}_t} \left[N_t - N_t^d \int_0^1 \left(\frac{w_t^k}{w_t} \right)^{-\tilde{\eta}} dk \right] \\
&+ \xi_t \nu_t \left[(1-\delta) H_{t-1} + I_t - H_t \right] \} .
\end{aligned}
$$

The optimal first-order conditions with respect to C_t, B_{t+1}, N_t, $N_{c,t}$, H_t, I_t, and w_t^k, in that order, are given by

$$U_C(t) = \xi_t, \tag{D.31}$$

$$\xi_t (R_t^{-1} + \psi_b B_{t+1}) = \beta \mathbb{E}_t \frac{\xi_{t+1}}{\pi_{t+1}}, \tag{D.32}$$

$$-U_N(t) = \frac{\xi_t w_t}{\tilde{\mu}_t}, \tag{D.33}$$

$$-U_{Nc}(t) = \xi_t w_{c,t}, \tag{D.34}$$

$$U_H(t) = \xi_t \nu_t - \beta(1-\delta) \mathbb{E}_t \xi_{t+1} \nu_{t+1}, \tag{D.35}$$

$$q_t = \nu_t, \tag{D.36}$$

$$w_t^k = \begin{cases} \tilde{w}_t, & \text{if wages are set optimally in } t \\ w_{t-1}^k/\pi_t, & \text{otherwise} \end{cases}, \tag{D.37}$$

where $\tilde{w}_t$ is the real wage in the $1-\tilde{\omega}$ labor markets where the central authority can set wages optimally in period t. In equilibrium, the real wage and labor supply are identical across varieties that are allowed to update, which we denote as $\tilde{w}_t$ and $\tilde{N}_t$ respectively. Plugging this into the labor demand curve, we get $\tilde{w}_t^{\tilde{\eta}}\tilde{N}_t = w_t^{\tilde{\eta}}N_t^d$. For labor variety k with s periods after the last optimization, its real wage w_{t+s}^k becomes

$$w_{t+s}^k = \tilde{w}_t \prod_{\tau=1}^{s} \pi_{t+\tau}^{-1}.$$

Separating out the Lagrangian associated with setting wages for optimizing central authorities,

$$\mathcal{L}^w = \mathbb{E}_t \sum_{s=0}^{\infty} (\tilde{\omega}\beta)^s \xi_{t+s} N_{t+s}^d w_{t+s}^{\tilde{\eta}} \prod_{\tau=1}^{s} \pi_{t+\tau}^{\tilde{\eta}} \left[\tilde{w}_t^{1-\tilde{\eta}} \prod_{\tau=1}^{s} \pi_{t+\tau}^{-1} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}} \tilde{w}_t^{-\tilde{\eta}} \right].$$

The first-order condition with respect to $\tilde{w}_t$ is

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\tilde{\omega}\beta)^s \xi_{t+s} N_{t+s}^d w_{t+s}^{\tilde{\eta}} \prod_{\tau=1}^{s} \pi_{t+\tau}^{\tilde{\eta}} \left[\left(\frac{\tilde{\eta}-1}{\tilde{\eta}} \right) \tilde{w}_t \prod_{\tau=1}^{s} \pi_{t+\tau}^{-1} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right].$$

Then, the optimal $\tilde{w}_t$ is given by

$$\tilde{w}_t = \frac{\tilde{\eta}}{\tilde{\eta}-1} \frac{f_{H,t}^a}{f_{H,t}^b}, \tag{D.38}$$

where

$$f_{H,t}^a = \mathbb{E}_t \sum_{s=0}^{\infty} (\tilde{\omega}\beta)^s \xi_{t+s} N_{t+s}^d w_{t+s}^{\tilde{\eta}} \left(\prod_{\tau=1}^{s} \pi_{t+\tau}^{\tilde{\eta}} \right) \frac{w_{t+s}}{\tilde{\mu}_{t+s}},$$
$$f_{H,t}^b = \mathbb{E}_t \sum_{s=0}^{\infty} (\tilde{\omega}\beta)^s \xi_{t+s} N_{t+s}^d w_{t+s}^{\tilde{\eta}} \left(\prod_{\tau=1}^{s} \pi_{t+\tau}^{\tilde{\eta}-1} \right).$$

Note that we can write $f_{H,t}^a$ and $f_{H,t}^b$ in the following recursive forms:

$$f_{H,t}^a = \xi_t N_t^d w_t^{\tilde{\eta}} \frac{w_t}{\tilde{\mu}_t} + \tilde{\omega}\beta \mathbb{E}_t \pi_{t+1}^{\tilde{\eta}} f_{H,t+1}^a, \tag{D.39}$$
$$f_{H,t}^b = \xi_t N_t^d w_t^{\tilde{\eta}} + \tilde{\omega}\beta \mathbb{E}_t \pi_{t+1}^{\tilde{\eta}-1} f_{H,t+1}^b. \tag{D.40}$$

D.3.2 Developer

The home region's representative developer produces new housing units, I_t, using construction inputs produced in current and previous periods, $\left\{U_{t-p|t}\right\}_{p=0,1,\cdots,P}$, where the subscript $t-p|t$

refers to stage-p construction input produced in period $t-p$ for new housing in period t. The production function is:

$$I_t = \left(\sum_{p=0}^{P} U_{t-p|t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \ \theta > 0,$$

where the parameter θ governs the substitutability of the different stages of construction.

Stage-by-stage building of the lot takes place using the following construction technology:

$$U_{t|t+p} = Z_{c,t} \left(N_{c,t|t+p} \right)^{\alpha} \left(M_{t+p-P|t+p} \right)^{1-\alpha},$$

where $Z_{c,t}$ is an exogenous shock to construction productivity, $N_{c,t|t+p}$ is time-t construction labor input for new housing to be completed in period $t+p$, and $M_{t+p-P|t+p}$ is the housing permit approved in period $t+p-P$ for new housing that is expected to be completed in period $t+p$.

Taking the real house price $q_t (= Q_t/P_t)$ as well as the real input prices $q_{M,t}$ and $w_{c,t} (= W_{c,t}/P_t)$ as given, the representative developer solves the following profit-maximization problem:

$$\begin{aligned}
\max_{\{I_t, N_{c,t}, M_{t|t+P}, \{U_{t|t+p}, N_{t|t+p}\}_{p=0}^{P}\}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0|t} \left(q_t I_t - q_{M,t} M_{t|t+P} - w_{c,t} N_{c,t} \right), \\
\text{subject to} \quad & U_{t|t+p} = Z_{c,t} N_{c,t|t+p}^{\alpha} \left(M_{t+p-P|t+p} \right)^{1-\alpha} \quad \text{for } p = 0, 1, \cdots, P, \\
& N_{c,t} = \sum_{p=0}^{P} N_{c,t|t+p}, \\
& I_t = \left(\sum_{p=0}^{P} U_{t-p|p}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}.
\end{aligned}$$

The foreign region's representative developer solves an analogous problem.

D.3.3 Goods producer

Home region. A monopolistically competitive firm in the home region produces the tradable j-variety output $Y_{H,t}(j)$ using the following production technology:

$$Y_{H,t}(j) = Z_{H,t} N_{H,t}(j),$$

where $Z_{H,t}$ is the common total factor productivity across varieties and $N_{H,t}(j)$ is the labor input. The demand for the variety that the firm is required to satisfy, $Y_{H,t}^d(j)$, is as follows:

$$Y_{H,t}^d(j) = nC_{H,t}(j) + (1-n)C_{H,t}^*(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta_c} \left(nC_{H,t} + (1-n) C_{H,t}^* \right),$$

where the second equality is derived from the consumption variety demand functions. The period-t real profit is given by

$$\Phi_{H,t}(j) = \frac{P_{H,t}(j)}{P_t} Y_{H,t}(j) - \frac{W_t N_{H,t}(j)}{P_t}.$$

Prices are sticky in the sense that firms can adjust its price only with probability $1 - \omega$ in each period. As such, the profit maximization problem of the firm that is allowed to adjust its price is given by

$$\max_{P^{\diamond}_{H,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t|t+s} \left[\frac{P^{\diamond}_{H,t}}{P_{t+s}} Y_{H,t+s}(j) - \frac{W_{t+s} N_{H,t+s}(j)}{P_{t+s}} \right],$$

subject to

$$\begin{aligned} Y_{H,t+s}(j) &= Z_{H,t+s} N_{H,t+s}(j), \\ Y_{H,t+s}(j) &\geq \left(\frac{P^{\diamond}_{H,t}}{P_{H,t+s}} \right)^{-\theta_c} \left(n C_{H,t+s} + (1-n) C^*_{H,t+s} \right). \end{aligned}$$

The labor input used by the j-variety producing firm is assumed to be a CES aggregate of a continuum of labor services, $N^k_{H,t}(j)$ for $k \in [0,1]$, in the following manner:

$$N_{H,t}(j) = \left(\int_0^1 (N^k_{H,t}(j))^{1-1/\tilde{\eta}} dk \right)^{1/(1-1/\tilde{\eta})},$$

where $\tilde{\eta} > 1$ is the elasticity of substitution across labor services. In each period, the demand for each labor variety is derived by minimizing the total labor cost, $\int_0^1 W^k_t N^k_{H,t}(j) dk$, while satisfying the above CES aggregation, where W^k_t is the nominal wage to labor variety k in period t. This implies the following labor variety demand:

$$N^k_{H,t}(j) = \left(\frac{W^k_t}{W_t} \right)^{-\tilde{\eta}} N_{H,t}(j),$$

where

$$W_t = \left(\int_0^1 (W^k_t)^{1-\tilde{\eta}} dk \right)^{\frac{1}{1-\tilde{\eta}}}.$$

It follows that $W_t N_{H,t}(j) = \int_0^1 W^k_t N^k_{H,t}(j) dk$.

Foreign region. A monopolistically competitive firm in the foreign region has a similar problem. We summarize the profit maximization problem of the j-variety firm that is allowed to adjust its price as follows:

$$\max_{P^{\diamond}_{F,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \Lambda^*_{t|t+s} \left[\frac{P^{\diamond}_{F,t}}{P^*_{t+s}} Y_{F,t+s}(j) - \frac{W^*_{t+s} N_{F,t+s}(j)}{P^*_{t+s}} \right],$$

subject to

$$Y_{F,t+s}(j) = Z_{F,t+s} N_{F,t+s}(j),$$
$$Y_{F,t+s}(j) \geq \left(\frac{P^{\diamond}_{F,t}}{P_{F,t+s}}\right)^{-\theta_c} \left(nC_{F,t+s} + (1-n) C^*_{F,t+s}\right).$$

The labor input used by the firm in the foreign region is assumed to be a CES aggregate analogous to firms in the home region.

D.3.4 Fiscal and monetary policy

The supply of housing permits in each region is determined by its local government, which in turn is elastic to the region's equilibrium house price. In detail, the home and foreign local governments issue their respective housing permits, M_t and M_t^*, according to

$$M_{t|t+P} = q_t^{\gamma} \quad \text{and} \quad M^*_{t|t+P^*} = (q_t^*)^{\gamma^*}.$$

The respective real cost of a housing permit is $q_{M,t}$ and $q^*_{M,t}$. Local governments also levy portfolio holding costs to households. Local governments follow a balanced budget in each period:

$$T_t = q_{M,t} M_{t|t+P} + n\frac{\psi_b}{2} B^2_{t+1} \quad \text{and} \quad T^*_t = q^*_{M,t} M^*_{t|t+P^*} + (1-n)\frac{\psi_b}{2} B^{*2}_{t+1},$$

where T_t and T_t^* are real transfers to households of home and foreign local governments.

Both regions are in a monetary union. Monetary policy follows a standard Taylor rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{\tilde{\pi}_t}{\bar{\pi}}\right)^{\phi_\pi},$$

where $\tilde{\pi}_t = (\pi_t)^n (\pi^*_t)^{1-n}$ is the population-weighted aggregate inflation, $\bar{\pi}$ is the aggregate inflation target, and $\bar{R}$ is the subsequent nominal interest rate target. The parameter ϕ_π is the Taylor coefficient on the deviation of inflation from target.

D.3.5 Market clearing

Labor market. Taking into account that the output sector wage adjustments are identical at all labor markets when allowed to change optimally, the home household's aggregate labor demand (D.29) could be expressed as

$$\begin{aligned} N_t &= N_t^d \int_0^1 \left(\frac{W_t^k}{W_t}\right)^{-\tilde{\eta}} dk \\ &= N_t^d \left[(1-\tilde{\omega})\left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}} + (1-\tilde{\omega})\tilde{\omega}\left(\frac{\tilde{W}_{t-1}}{W_t}\right)^{-\tilde{\eta}} + (1-\tilde{\omega})\tilde{\omega}^2\left(\frac{\tilde{W}_{t-2}}{W_t}\right)^{-\tilde{\eta}} + \cdots\right] \end{aligned}$$

$$= (1-\tilde{\omega})N_t^d \sum_{s=0}^{\infty}\left[\tilde{\omega}^s\left(\frac{\tilde{W}_{t-s}}{W_t}\right)^{-\tilde{\eta}}\right].$$

This expression could be written as

$$N_t = \tilde{\Xi}_t N_t^d, \tag{D.41}$$

where

$$\tilde{\Xi}_t = (1-\tilde{\omega})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\omega}\pi_t^{\tilde{\eta}}\left(\frac{\tilde{w}_{t-1}}{w_t}\right)^{-\tilde{\eta}}\tilde{\Xi}_{t-1}. \tag{D.42}$$

Note that $\tilde{\Xi}_t$ is the wage dispersion term that could be shown as bounded below by one. The aggregate wage index is also written as

$$\begin{aligned} W_t^{1-\tilde{\eta}} &= \int_0^1 (W_t^k)^{1-\tilde{\eta}} dk \\ &= (1-\tilde{\omega})\tilde{W}_t^{1-\tilde{\eta}} + (1-\tilde{\omega})\tilde{\omega}\tilde{W}_{t-1}^{1-\tilde{\eta}} + (1-\tilde{\omega})\tilde{\omega}^2\tilde{W}_{t-2}^{1-\tilde{\eta}} + \cdots \\ &= (1-\tilde{\omega})\tilde{W}_t^{1-\tilde{\eta}} + \tilde{\omega}W_{t-1}^{1-\tilde{\eta}}, \end{aligned}$$

which implies the following recursive form:

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\omega})\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\omega}\pi_t^{\tilde{\eta}-1}w_{t-1}^{1-\tilde{\eta}}. \tag{D.43}$$

Labor market clearing in the output sector implies that the aggregation of all labor demand across firms adjusted by the population should be equal to the per household aggregate labor demand:

$$N_t^d = \frac{1}{n}\int_0^1 N_{H,t}(j)dj. \tag{D.44}$$

Labor market in the construction sector also clears. Moreover, the foreign labor market in both the output and construction sectors clear in a similar fashion.

Goods market. Defining per household aggregate goods production in each region as $Y_{H,t}$ and $Y_{F,t}$, we get

$$Y_{H,t} \equiv \frac{1}{n}\int_0^1 Y_{H,t}(j)\,dj = Z_{H,t}N_t^d, \text{ and } Y_{F,t} \equiv \frac{1}{1-n}\int_0^1 Y_{F,t}(j)\,dj = Z_{F,t}N_t^{d*}.$$

Then, the aggregated goods market clearing conditions are

$$Y_{H,t} = \frac{1}{n}\int_0^1 Y_{H,t}^d(j)\,dj = \frac{1}{n}\tilde{Y}_{H,t}\Xi_{H,t}, \text{ and } Y_{F,t} = \frac{1}{1-n}\int_0^1 Y_{F,t}^d(j)\,dj = \frac{1}{1-n}\tilde{Y}_{F,t}\Xi_{F,t},$$

where

$$\tilde{Y}_{H,t} \equiv nC_{H,t} + (1-n)\,C_{H,t}^* \quad \text{and} \quad \tilde{Y}_{F,t} \equiv nC_{F,t} + (1-n)\,C_{F,t}^*,$$

and $\Xi_{H,t}$ and $\Xi_{F,t}$ are the respective price dispersion terms:

$$\Xi_{H,t} = \int_0^1\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\theta_c} dj \quad \text{and} \quad \Xi_{F,t} = \int_0^1\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\theta_c} dj.$$

Bond market. The nominal bond market clearing condition is

$$nP_tB_{t+1} + (1-n)P_t^*B_{t+1}^{*2} = 0.$$

Resource constraint. Under a monetary union with a common nominal interest rate, we can also derive the aggregated resource constraint by combining households' budget constraints:

$$\begin{aligned} nP_tC_t + (1-n)\,P_t^*C_t^* =& nW_tN_t^d + \int_0^1 \left(P_{H,t}(j)\,Y_{H,t}(j) - W_tN_{H,t}(j)\right)dj \\ &+ (1-n)\,W_t^*N_t^{d*} + \int_0^1 \left(P_{F,t}(j)\,Y_{F,t}(j) - W_t^*N_{F,t}(j)\right)dj, \end{aligned}$$

which implies

$$nC_t + (1-n)\,C_t^* = \tilde{Y}_{H,t}\frac{P_{H,t}}{P_t} + \tilde{Y}_{F,t}\frac{P_{F,t}}{P_t^*}.$$

Lastly, we define (population-weighted) aggregate output as follows:

$$Y_t = nY_{H,t} + (1-n)\,Y_{F,t}.$$

D.3.6 Functional forms and calibration

Following Guren et al. (2020), we assume that consumption and leisure are substitutable in the style of Greenwood et al. (1988), which eliminates the wealth effects of labor supply. We also model the housing demand shock using a Stone-Geary formulation:

$$U(C_t, H_t, N_t, N_{c,t}; \varphi_t) = \left(\frac{\left(C_t - \frac{\psi}{1+\nu}N_t^{1+\nu} - \frac{\psi_c}{1+\nu}N_{c,t}^{1+\nu}\right)^{\kappa}(H_t - \varphi_t)^{1-\kappa}}{1-\sigma}\right)^{1-\sigma}$$

Under this assumption, the households' marginal utilities are defined as follows:

$$\begin{aligned} U_{c,t} &= \kappa\left(\left(\tilde{C}_t\right)^{\kappa}\left(\tilde{H}_t\right)^{1-\kappa}\right)^{1-\sigma}\frac{1}{\tilde{C}_t} \\ U_{n,t} &= -\kappa\psi\left(\left(\tilde{C}_t\right)^{\kappa}\left(\tilde{H}_t\right)^{1-\kappa}\right)^{1-\sigma}\frac{1}{\tilde{C}_t}N_t^{\nu} \\ U_{nc,t} &= -\kappa\psi_c\left(\left(\tilde{C}_t\right)^{\kappa}\left(\tilde{H}_t\right)^{1-\kappa}\right)^{1-\sigma}\frac{1}{\tilde{C}_t}N_{c,t}^{\nu} \\ U_{h,t} &= (1-\kappa)\left(\left(\tilde{C}_t\right)^{\kappa}\left(\tilde{H}_t\right)^{1-\kappa}\right)^{1-\sigma}\frac{1}{\tilde{H}_t} \end{aligned}$$

where

$$\tilde{C}_t = C_t - \frac{\psi}{1+\eta}N_t^{1+\nu} - \frac{\psi_c}{1+\eta}N_{c,t}^{1+\nu} \text{ and } \tilde{H}_t = H_t - \varphi_t$$

We pick parameter values based on long-run averages or from the literature. Table A.8 presents our calibration.

D.3.7 Model responses to shocks

We present the impulse response functions for house prices and housing investment with regards to a common housing demand shock in our two-region general equilibrium model. These examples show that the results for our local general equilibrium model extends to the two-region general equilibrium model. In all cases, we assume that the persistence parameter of the housing demand shock is 0.95.

In Figure A.7, we present the result when the home and foreign regions are only different in terms of the long-run elasticity parameters $\gamma > \gamma^*$. In the left panel, we observe that the partial equilibrium housing supply elasticity is the same in horizons lower than 12 quarters. Afterwards, housing supply elasticity is higher in the home region, consistent with its higher long-run elasticity. The second panel shows that house prices in the foreign region respond higher than in the home region, consistent with the result of the first panel. Note that even though the short-run elasticities are common across the two regions, house price responses in this case are likely to take into account supply elasticities beyond 12 quarters. The third panel shows the housing investment response in the two regions. Housing investment at an after 12 quarters is much higher in the home region compared to the foreign region. In this example, we observe that the long-run elasticity difference drives the difference in house prices both in business-cycle frequency and in the long run.

In Figure A.8, we conduct the same exercise under a different calibration. In this case, the home and foreign regions are only different in terms of TTD ($P < P^*$). In the left panel, we observe that the partial equilibrium housing supply elasticity is different at shorter horizons, but begins converging after 18 quarters. Note that the long-run housing supply elasticities are the same in both regions. The second panel shows that housing prices in the foreign region respond higher than in the home region, again consistent with the result of the first panel. Note that even though the long-run elasticities are common across the two regions, house price responses in this case are consistent with the supply elasticities in the short to medium run. The third panel shows the housing investment response in the two regions consistent with the difference in TTD. In this example, we observe that TTD alone can drive a sizable difference in the house price response, consistent with the implied gap in the short- to medium-run housing supply elasticities.

Figures A.9 and A.10 presents the responses when both the long-run elasticities and TTD are different in the two regions. Note that both the long-run elasticities and TTD are within the range of our sample counties, suggesting that our calibration exercise in these examples are sensible. A key takeaway is that TTD can work as both widening or reversing the price response difference between two regions. In the second case with reversal, housing supply elasticity is lower in the home region despite the higher long-run supply elasticity, as TTD is much higher at home. The model impulse responses in this case shows that house price responses become higher in the home region for about the first five quarters, until house prices in the foreign region becomes higher consistent with its lower long-run elasticity. These examples suggest the need to take into account both the short- to long-run housing supply elasticities in accounting for house price responses to a common housing demand shock.

D.3.8 Details: Equilibrium conditions

We first define variables for relative prices and inflation:

$$\pi_t = \frac{P_{t+1}}{P_t},\ \pi_t^* = \frac{P_{t+1}^*}{P_t^*},\ X_t = \frac{P_t^*}{P_t},\ X_{H,t} = \frac{P_{H,t}}{P_t},\ X_{F,t} = \frac{P_{F,t}}{P_t^*},$$

$$\pi_{H,t} = \frac{X_{H,t}}{X_{H,t-1}}\pi_t,\ \pi_{F,t} = \frac{X_{F,t}}{X_{F,t-1}}\pi_t^*,\ p_{H,t}^{\diamond}=\frac{P_{H,t}^{\diamond}}{P_{H,t}}, p_{F,t}^{\diamond}=\frac{P_{F,t}^{\diamond}}{P_{H,t}}$$

Variables

- (Home) Non-construction (7 variables): $\left\{C_{H,t}, C_{F,t}, C_t, \tilde{\mu}_t, N_t, N_t^d, \tilde{Y}_{H,t}\right\}$
- (Home) Construction ($7 + 3P$ variables): $\left\{N_{c,t}, H_t, I_t, \{U_{t-p|t}\}_{p=0}^P, \{N_{c,t|t+p}\}_{p=0}^P, \{\mu_{t|t+p}\}_{p=0}^P, q_t, q_{M,t}, w_{c,t}, M_t\right\}$
- (Foreign) Non-construction (7 variables): $\left\{C_{H,t}^*, C_{F,t}^*, C_t^*, \tilde{\mu}_t^*, N_t^*, N_t^{d*}, \tilde{Y}_{F,t}\right\}$
- (Foreign) Construction ($7 + 3P^*$ variables): $\left\{N_{c,t}^*, H_t^*, I_t^*, \{U_{t-p|t}^*\}_{p=0}^{P^*}, \{N_{c,t|t+p}^*\}_{p=0}^{P^*}, \{\mu_{t|t+p}^*\}_{p=0}^{P^*}, q_t^*, q_{M,t}^*, w_{c,t}^*, M_t^*\right\}$
- Nominal variables (17 variables): $\left\{\pi_t, \pi_t^*, \pi_{H,t}, \pi_{F,t}, mc_{H,t}, mc_{F,t}, X_t, X_{H,t}, X_{F,t}, \Xi_{H,t}, \Xi_{F,t}, p_{H,t}^{\diamond}, p_{F,t}^{\diamond}, \tilde{Z}_{H,t}^a, \tilde{Z}_{H,t}^b, \tilde{Z}_{F,t}^a, \tilde{Z}_{H,t}^b\right\}$
- Wages (10 variables): $\left\{w_t, w_t^*, \tilde{w}_t, \tilde{w}_t^*, f_{H,t}^a, f_{H,t}^b, f_{F,t}^a, f_{F,t}^b, \tilde{\Xi}_t, \tilde{\Xi}_t^*\right\}$
- Aggregate variables (3 variables): $\{R_t, \tilde{\pi}_t, Y_t\}$
- Marginal utilities (8 variables): $\{U_{c,t}, U_{n,t}, U_{nc,t}, U_{h,t}, U_{c^*,t}, U_{n^*,t}, U_{nc^*,t}, U_{h^*,t}\}$

Notices that the number of variables excluding marginal utility variables is $58+3P+3P^*$, which is the same with the number of equations shown below (C.1)-(C.64).

- (Home) Home demand for home- and foreign-produced goods

$$C_{H,t} = \phi\left(X_{H,t}\right)^{-\eta} C_t \tag{D.45}$$
$$C_{F,t} = (1-\phi)\left(X_{F,t}X_t\right)^{-\eta} C_t \tag{D.46}$$

- (Home) Supply and demand for non-construction labor

$$-U_{n,t} = w_t \frac{U_{c,t}}{\tilde{\mu}_t} \tag{D.47}$$
$$N_t = N_t^d \tilde{\Xi}_t \tag{D.48}$$

- (Home) Wage dispersion

$$\tilde{\Xi}_t = (1-\tilde{\omega})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\omega}\pi_t^{\tilde{\eta}}\left(\frac{\tilde{w}_{t-1}}{w_t}\right)^{-\tilde{\eta}}\tilde{\Xi}_{t-1} \tag{D.49}$$

- (Home) Wage index

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\omega})\,\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\omega}\left(\frac{w_{t-1}}{\pi_t}\right)^{1-\tilde{\eta}} \tag{D.50}$$

- (Home) Wage Phillips curve

$$\tilde{w}_t = \frac{\tilde{\eta}}{\tilde{\eta}-1}\frac{f_{H,t}^a}{f_{H,t}^b}, \tag{D.51}$$

$$f_{H,t}^a = N_t^d w_t^{\tilde{\eta}}\frac{w_t}{\tilde{\mu}_t} + \tilde{\omega}\beta\mathbb{E}_t\left[\frac{U_{c,t+1}}{U_{c,t}}\pi_{t+1}^{\tilde{\eta}} f_{H,t+1}^a\right] \tag{D.52}$$

$$f_{H,t}^b = N_t^d w_t^{\tilde{\eta}} + \tilde{\omega}\beta\mathbb{E}_t\left[\frac{U_{c,t+1}}{U_{c,t}}\pi_{t+1}^{\tilde{\eta}-1} f_{H,t+1}^b\right] \tag{D.53}$$

- (Home) Housing demand

$$\frac{U_{h,t}}{U_{c,t}} = q_t - \beta\,(1-\delta)\,\mathbb{E}_t\left[\frac{U_{c,t+1}}{U_{c,t}}q_{t+1}\right] \tag{D.54}$$

- (Home) Supply of construction labor

$$-U_{nc,t} = w_{c,t}U_{c,t} \tag{D.55}$$

- (Home) Housing accumulation

$$H_t = (1-\delta)\,H_{t-1} + I_t \tag{D.56}$$

- (Home) Housing construction function

$$I_t = \left(\sum_{p=0}^{P} U_{t-p|t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{D.57}$$

- (Home) Demand for p-period ahead construction labor

$$w_{c,t} = \alpha\mu_{t|t+p}\frac{U_{t|t+p}}{N_{c,t|t+p}} \text{ for } p = 0,1,\cdots,P \tag{D.58}$$

- (Home) Demand for p-period ahead construction input

$$\mu_{t|t+p} = \mathbb{E}_t \left[\beta^p \frac{U_{c,t+p}}{U_{c,t}} q_{t+p} \left(\frac{I_{t+p}}{U_{t|t+p}} \right)^{\frac{1}{\theta}} \right] \quad \text{for } p = 0, 1, \cdots, P \tag{D.59}$$

- (Home) Demand for housing permit

$$q_{M,t} = (1-\alpha)\, \mathbb{E}_t \left[\beta^P \frac{U_{c,t+P}}{U_{c,t}} q_{t+P} \sum_{p=0}^{P} \left(\frac{I_{t+P}}{U_{t+p|t+P}} \right)^{\frac{1}{\theta}} \frac{U_{t+p|t+P}}{M_{t|t+P}} \right] \tag{D.60}$$

- (Home) Production of p-period ahead construction input

$$U_{t|t+p} = Z_{c,t} N_{c,t|t+p}^{\alpha} \left(M_{t+p-P|t+p} \right)^{1-\alpha} \quad \text{for } p = 0, 1, \cdots, P \tag{D.61}$$

- (Home) Demand for total construction labor

$$N_{c,t} = \sum_{p=0}^{P} N_{c,t|t+p} \tag{D.62}$$

- (Home) Housing permit supply

$$M_{t|t+P} = (q_t)^{\gamma} \tag{D.63}$$

- (Home) Supply and demand for non-construction output

$$\tilde{Y}_{H,t} = nC_{H,t} + (1-n)\, C_{H,t}^{*} \tag{D.64}$$

$$\tilde{Y}_{H,t} \Xi_{H,t} = n Z_{H,t} N_t^d \tag{D.65}$$

- (Home) Price dispersion

$$\Xi_{H,t} = (1-\omega) \left(p_{H,t}^{\diamond} \right)^{-\theta_c} + \omega \left(\pi_{H,t} \right)^{\theta_c} \Xi_{H,t-1} \tag{D.66}$$

- (Home) Aggregate price index

$$\left(\pi_{H,t} \right)^{1-\theta_c} = (1-\omega) \left(p_{H,t}^{\diamond} \right)^{1-\theta_c} + \omega \tag{D.67}$$

- (Home) Phillips curve

$$p_{H,t}^{\diamond} = \frac{\theta_c}{\theta_c - 1} \frac{\tilde{Z}_{H,t}^{a}}{\tilde{Z}_{H,t}^{b}} \tag{D.68}$$

$$\tilde{Z}_{H,t}^{a} = mc_{H,t} \tilde{Y}_{H,t} + \beta\omega \mathbb{E}_t \left[\frac{U_{c,t+1}}{U_{c,t}} \left(\pi_{H,t+1} \right)^{\theta_c} \tilde{Z}_{H,t+1}^{a} \right] \tag{D.69}$$

$$\tilde{Z}_{H,t}^{b} = X_{H,t} \tilde{Y}_{H,t} + \beta\omega \mathbb{E}_t \left[\frac{U_{c,t+1}}{U_{c,t}} \left(\pi_{H,t+1} \right)^{\theta_c - 1} \tilde{Z}_{H,t+1}^{b} \right] \tag{D.70}$$

- (Home) Marginal cost

$$mc_{H,t} = \frac{w_t}{Z_{H,t}} \tag{D.71}$$

- (Foreign) Foreign demand for home- and foreign-produced good

$$C^*_{F,t} = \phi^* \left(X_{F,t}\right)^{-\eta} C^*_t \tag{D.72}$$

$$C^*_{H,t} = \left(1 - \phi^*\right) \left(\frac{X_{H,t}}{X_t}\right)^{-\eta} C^*_t \tag{D.73}$$

- (Foreign) Supply and demand for non-construction labor

$$-U_{n^*,t} = w^*_t \frac{U_{c^*,t}}{\tilde{\mu}^*_t} \tag{D.74}$$

$$N^*_t = N^{d*}_t \tilde{\Xi}^*_t \tag{D.75}$$

- (Foreign) Wage dispersion

$$\tilde{\Xi}^*_t = \left(1 - \tilde{\omega}\right) \left(\frac{\tilde{w}^*_t}{w^*_t}\right)^{-\tilde{\eta}} + \tilde{\omega} \pi^{\tilde{\eta}}_t \left(\frac{\tilde{w}^*_{t-1}}{w^*_t}\right)^{-\tilde{\eta}} \tilde{\Xi}^*_{t-1} \tag{D.76}$$

- (Foreign) Wage index

$$\left(w^*_t\right)^{1-\tilde{\eta}} = \left(1 - \tilde{\omega}\right) \left(\tilde{w}^*_t\right)^{1-\tilde{\eta}} + \tilde{\omega} \left(\frac{w^*_{t-1}}{\pi^*_t}\right)^{1-\tilde{\eta}} \tag{D.77}$$

- (Foreign) Wage Phillips curve

$$\tilde{w}^*_t = \frac{\tilde{\eta}^*}{\tilde{\eta}^* - 1} \frac{f^a_{F,t}}{f^b_{F,t}}, \tag{D.78}$$

$$f^a_{F,t} = N^{d*}_t \left(w^*_t\right)^{\tilde{\eta}} \frac{w^*_t}{\tilde{\mu}^*_t} + \tilde{\omega} \beta \mathbb{E}_t \left[\frac{U^*_{c,t+1}}{U_{c,t}} \left(\pi^*_{t+1}\right)^{\tilde{\eta}} f^a_{F,t+1}\right] \tag{D.79}$$

$$f^b_{F,t} = N^{d*}_t \left(w^*_t\right)^{\tilde{\eta}} + \tilde{\omega} \beta \mathbb{E}_t \left[\frac{U^*_{c,t+1}}{U_{c,t}} \left(\pi^*_{t+1}\right)^{\tilde{\eta}-1} f^b_{F,t+1}\right] \tag{D.80}$$

- (Foreign) Housing demand

$$\frac{U_{h^*,t}}{U_{c^*,t}} = q^*_t - \beta \left(1 - \delta\right) \mathbb{E}_t \left[\frac{U_{c^*,t+1}}{U_{c^*,t}} q^*_{t+1}\right] \tag{D.81}$$

- (Foreign) Supply of construction labor

$$- U_{nc^*,t} = w^*_{c,t} U_{c^*,t} \tag{D.82}$$

- (Foreign) Housing accumulation

$$H_t^* = (1-\delta) H_{t-1}^* + I_t^* \tag{D.83}$$

- (Foreign) Housing construction function

$$I_t^* = \left(\sum_{p=0}^{P^*} \left(U_{t-p|t}^* \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{D.84}$$

- (Foreign) Demand for p-period ahead construction labor

$$w_{c,t}^* = \alpha \mu_{t|t+p}^* \frac{U_{t|t+p}^*}{N_{c,t|t+p}^*} \text{ for } p = 0, 1, \cdots, P^* \tag{D.85}$$

- (Foreign) Demand for p-period ahead construction input

$$\mu_{t|t+p}^* = \mathbb{E}_t \left[\beta^p \frac{U_{c^*,t+p}}{U_{c^*,t}} q_{t+p}^* \left(\frac{I_{t+p}^*}{U_{t|t+p}^*} \right)^{\frac{1}{\theta}} \right] \text{ for } p = 0, 1, \cdots, P^* \tag{D.86}$$

- (Foreign) Demand for housing permit

$$q_{M,t}^* = (1-\alpha) \mathbb{E}_t \left[\beta^{P^*} \frac{U_{c^*,t+P^*}}{U_{c^*,t}} q_{t+P^*}^* \sum_{p=0}^{P^*} \left(\frac{I_{t+P^*}}{U_{t+p|t+P^*}^*} \right)^{\frac{1}{\theta}} \frac{U_{t+p|t+P^*}^*}{M_{t|t+P^*}^*} \right] \tag{D.87}$$

- (Foreign) Production of p-period ahead construction input

$$U_{t|t+p}^* = Z_{c,t}^* \left(N_{c,t|t+p}^* \right)^{\alpha} \left(M_{t+p-P^*|t+p}^* \right)^{1-\alpha} \text{ for } p = 0, 1, \cdots, P^* \tag{D.88}$$

- (Foreign) Demand of total construction labor

$$N_{c,t}^* = \sum_{p=0}^{P} N_{c,t|t+p}^* \tag{D.89}$$

- (Foreign) Housing permit supply

$$M_{t|t+P^*}^* = (q_t^*)^{\gamma^*} \tag{D.90}$$

- (Foreign) Supply and demand for non-construction output

$$\tilde{Y}_{F,t} = nC_{F,t} + (1-n) C_{F,t}^* \tag{D.91}$$

$$\tilde{Y}_{F,t} \Xi_{F,t} = (1-n) Z_{F,t} N_t^{d*} \tag{D.92}$$

- (Foreign) Price dispersion

$$\Xi_{F,t} = (1-\omega)\left(p^{\diamond}_{F,t}\right)^{-\theta_c} + \omega\left(\pi_{F,t}\right)^{\theta_c}\Xi_{F,t-1} \tag{D.93}$$

- (Foreign) Aggregate price index

$$\left(\pi_{F,t}\right)^{1-\theta_c} = (1-\omega)\left(p^{\diamond}_{F,t}\right)^{1-\theta_c} + \omega \tag{D.94}$$

- (Foreign) Phillips curve

$$p^{\diamond}_{F,t} = \frac{\theta_c}{\theta_c - 1}\frac{\tilde{Z}^a_{F,t}}{\tilde{Z}^b_{F,t}} \tag{D.95}$$

$$\tilde{Z}^a_{F,t} = mc_{F,t}\tilde{Y}_{F,t} + \beta\omega\mathbb{E}_t\left[\frac{U_{c^*,t+1}}{U_{c^*,t}}\left(\pi_{F,t+1}\right)^{\theta_c}\tilde{Z}^a_{F,t+1}\right] \tag{D.96}$$

$$\tilde{Z}^b_{F,t} = X_{F,t}\tilde{Y}_{F,t} + \beta\omega\mathbb{E}_t\left[\frac{U_{c^*,t+1}}{U_{c^*,t}}\left(\pi_{F,t+1}\right)^{\theta_c-1}\tilde{Z}^b_{F,t+1}\right] \tag{D.97}$$

- (Foreign) Marginal cost

$$mc_{F,t} = \frac{w^*_t}{Z_{F,t}} \tag{D.98}$$

- Common stochastic discount factor (“Backus-Smith”)

$$\frac{U_{c^*,t}}{U_{c,t}} = X_t \tag{D.99}$$

- Euler equation and nominal interest rate

$$1 = \beta R_t \mathbb{E}_t\left[\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right] \tag{D.100}$$

- Aggregate output

$$Y_t = nZ_{H,t}N^d_t + (1-n)\,Z_{F,t}N^{d*}_t \tag{D.101}$$

- Monetary policy

$$\frac{R_t}{\bar{R}} = \left(\frac{\tilde{\pi}_t}{\bar{\pi}}\right)^{\phi} \tag{D.102}$$

- Aggregate inflation

$$\tilde{\pi}_t = \left(\pi_t\right)^n\left(\pi^*_t\right)^{1-n} \tag{D.103}$$

- Resource constraint

$$nC_t + (1-n)\,C^*_t = \tilde{Y}_{H,t}X_{H,t} + \tilde{Y}_{F,t}X_{F,t} \tag{D.104}$$

- Relative price relationship

$$1 = \phi \left(X_{H,t}\right)^{1-\eta} + (1-\phi)\left(X_{F,t}X_t\right)^{1-\eta} \tag{D.105}$$

- Inflation relationship

$$\pi_t^* = \frac{X_t}{X_{t-1}}\pi_t \tag{D.106}$$

$$\pi_{H,t} = \frac{X_{H,t}}{X_{H,t-1}}\pi_t \tag{D.107}$$

$$\pi_{F,t} = \frac{X_{F,t}}{X_{F,t-1}}\pi_t^* \tag{D.108}$$

D.3.9 Details: Steady state

Baseline GHH preference for household utility.

$$\begin{aligned}
\bar{U}_c &= \kappa \left(\left(\bar{\tilde{C}}\right)^{\kappa} \left(\bar{\tilde{H}}\right)^{1-\kappa} \right)^{1-\sigma} \frac{1}{\bar{\tilde{C}}}, \\
\bar{U}_n &= -\kappa\psi \left(\left(\bar{\tilde{C}}\right)^{\kappa} \left(\bar{\tilde{H}}\right)^{1-\kappa} \right)^{1-\sigma} \frac{1}{\bar{\tilde{C}}}\bar{N}^{\nu}, \\
\bar{U}_{nc} &= -\kappa\psi_c \left(\left(\bar{\tilde{C}}\right)^{\kappa} \left(\bar{\tilde{H}}\right)^{1-\kappa} \right)^{1-\sigma} \frac{1}{\bar{\tilde{C}}}\bar{N}_c^{\nu}, \\
\bar{U}_h &= (1-\kappa) \left(\left(\bar{\tilde{C}}\right)^{\kappa} \left(\bar{\tilde{H}}\right)^{1-\kappa} \right)^{1-\sigma} \frac{1}{\bar{\tilde{H}}},
\end{aligned}$$

where

$$\begin{aligned}
\bar{\tilde{C}} &= \bar{C} - \frac{\psi}{1+\nu}\bar{N}^{1+\nu} - \frac{\psi_c}{1+\nu}\bar{N}_c^{1+\nu}, \\
\bar{\tilde{H}} &= \bar{H}_t - \bar{\varphi}
\end{aligned}$$

- (Home) Home demand for home- and foreign-produced goods

$$\begin{aligned}
\bar{C}_H &= \phi \left(\bar{X}_H\right)^{-\eta} \bar{C} \\
\bar{C}_F &= (1-\phi)\left(\bar{X}_F\bar{X}\right)^{-\eta} \bar{C}
\end{aligned}$$

- (Home) Supply and demand for non-construction labor

$$\bar{\tilde{\mu}} = \frac{\tilde{\eta}}{\tilde{\eta}-1}$$

$$\begin{aligned}
-\bar{U}_n = \bar{w}\frac{\bar{U}_c}{\bar{\tilde{\mu}}} &\Leftrightarrow \psi = \frac{1}{\bar{\tilde{\mu}}}\frac{\bar{w}}{\bar{N}^{\nu}} \\
\bar{N} &= \bar{N}^d
\end{aligned}$$

- (Home) Wage dispersion

$$\bar{\tilde{\Xi}} = 1$$

- (Home) Wage index

$$\bar{w} = \bar{\tilde{w}}$$

- (Home) Wage Phillips curve

$$\bar{\tilde{w}} = \frac{\tilde{\eta}}{\tilde{\eta}-1}\frac{\bar{f}_H^a}{\bar{f}_H^b},$$
$$\bar{f}_H^a = \frac{1}{1-\tilde{\omega}\beta}\bar{N}^d\bar{w}^{1+\tilde{\eta}}\frac{1}{\bar{\tilde{\mu}}}$$
$$\bar{f}_H^b = \frac{1}{1-\tilde{\omega}\beta}\bar{N}^d\bar{w}^{\tilde{\eta}}$$

- (Home) Housing demand

$$\left(\frac{1-\kappa}{\kappa}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{H}}} = \left(1-\left(1-\delta\right)\beta\right)\bar{q}$$

- (Home) Supply of construction labor

$$-\bar{U}_{nc} = \bar{w}_c\bar{U}_c$$

- (Home) Housing accumulation

$$\delta\bar{H} = \bar{I}$$

- (Home) Housing construction function

$$\bar{I} = \left(\sum_{p=0}^{P}\bar{U}_p^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$

- (Home) Demand for p-period ahead construction labor

$$\bar{w}_c = \alpha\bar{\mu}_p\frac{\bar{U}_p}{\bar{N}_{c,p}} \text{ for } p = 0, 1, \cdots, P$$

- (Home) Demand for p-period ahead construction input

$$\bar{\mu}_p = \beta^p\bar{q}\left(\frac{\bar{I}}{\bar{U}_p}\right)^{\frac{1}{\theta}} \quad \text{for } p = 0, 1, \cdots, P$$

- (Home) Demand for housing permit

$$\begin{aligned}\bar{q}_M &= (1-\alpha)\left[\sum_{p=0}^{P} \beta^p \bar{\mu}_{P-p} \frac{\bar{U}_{P-p}}{\bar{M}}\right] \\ &= (1-\alpha)\,\beta^P \bar{q} \frac{\bar{I}}{\bar{M}}\end{aligned}$$

- (Home) Production of p-period ahead construction input

$$\bar{U}_p = \bar{Z}_c \left(\bar{N}_{c,p}\right)^{\alpha} \left(\bar{M}\right)^{1-\alpha} \quad \text{for } p = 0, 1, \cdots, P$$

- (Home) Demand for total construction labor

$$\bar{N}_c = \sum_{p=0}^{P} \bar{N}_{c,p}$$

- (Home) Supply of housing permit

$$\bar{M} = (\bar{q})^{\gamma}$$

- (Home) Supply and demand for non-construction output

$$\begin{aligned}\bar{\tilde{Y}}_H &= n\bar{C}_H + (1-n)\,\bar{C}^*_H \\ \bar{\tilde{Y}}_H &= n\bar{Z}_H \bar{N}^d\end{aligned}$$

- (Home) Price dispersion of home-produced good

$$\bar{\Xi}_H = 1$$

- (Home) Aggregate price index

$$\overline{p^{\diamond}_H} = 1$$

- (Home) Phillips curve

$$\begin{aligned}\overline{p^{\diamond}_{H,t}} &= \frac{\theta_c}{\theta_c - 1} \frac{\bar{\tilde{Z}}^a_H}{\bar{\tilde{Z}}^b_H} \\ \bar{\tilde{Z}}^a_H &= \frac{1}{1-\omega\beta} \bar{mc}_H \bar{\tilde{Y}}_H \\ \bar{\tilde{Z}}^b_H &= \frac{1}{1-\omega\beta} \bar{X}_H \bar{\tilde{Y}}_H\end{aligned}$$

- (Home) Marginal cost

$$\bar{mc}_H = \frac{\bar{w}}{\bar{Z}_H}$$

- (Foreign) Foreign demand for home- and foreign-produced good

$$\bar{C}_F^* = \phi^* \left(\bar{X}_F\right)^{-\eta} \bar{C}^*$$
$$\bar{C}_H^* = (1-\phi^*) \left(\frac{\bar{X}_H}{\bar{X}}\right)^{-\eta} \bar{C}^*$$

- (Foreign) Supply and demand for non-construction labor

$$-\bar{U}_{n^*} = \bar{w}^* \frac{\bar{U}_{c^*}}{\bar{\tilde{\mu}}^*}$$
$$\bar{N}^* = \bar{N}^{d*}$$

- (Foreign) Wage dispersion

$$\bar{\tilde{\Xi}}^* = 1$$

- (Foreign) Wage index

$$\bar{w}^* = \bar{\tilde{w}}^*$$

- (Foreign) Wage Phillips curve

$$\bar{\tilde{w}}^* = \frac{\tilde{\eta}^*}{\tilde{\eta}^* - 1} \frac{\bar{f}_F^a}{\bar{f}_F^b},$$
$$\bar{f}_F^a = \frac{1}{1-\tilde{\omega}\beta} \bar{N}_t^{d*} \left(\bar{w}^*\right)^{1+\tilde{\eta}} \frac{1}{\bar{\tilde{\mu}}^*}$$
$$\bar{f}_F^b = \frac{1}{1-\tilde{\omega}\beta} \bar{N}^{d*} \left(\bar{w}^*\right)^{\tilde{\eta}}$$

$$\bar{\tilde{\mu}}^* = \frac{\tilde{\eta}^*}{\tilde{\eta}^* - 1}$$

- (Foreign) Housing demand

$$\left(\frac{1-\kappa}{\kappa}\right) \frac{\bar{\tilde{C}}^*}{\bar{\tilde{H}}^*} = \left(1 - (1-\delta)\beta\right) \bar{q}^*$$

- (Foreign) Supply of construction labor

$$-\bar{U}_{nc^*} = \bar{w}_c^* \bar{U}_{c^*}$$

- (Foreign) Housing accumulation

$$\delta \bar{H}^* = \bar{I}^*$$

- (Foreign) Housing construction function

$$\bar{I}^* = \left(\sum_{p=0}^{P^*} \left(\bar{U}_p^* \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

- (Foreign) Demand for p-period ahead construction labor

$$\bar{w}_c^* = \alpha \bar{\mu}_p^* \frac{\bar{U}_p^*}{\bar{N}_{c,p}^*} \text{ for } p = 0, 1, \cdots, P^*$$

- (Foreign) Demand for p-period ahead construction input

$$\bar{\mu}_p^* = \beta^p \bar{q}^* \left(\frac{\bar{I}^*}{\bar{U}_p^*} \right)^{\frac{1}{\theta}} \quad \text{for } p = 0, 1, \cdots, P^*$$

- (Foreign) Demand for housing permit

$$\begin{aligned} \bar{q}_M^* &= (1-\alpha) \left[\sum_{p=0}^{P^*} \beta^p \bar{\mu}_{P^*-p}^* \frac{\bar{U}_{P^*-p}}{\bar{M}^*} \right] \\ &= (1-\alpha) \beta^{P^*} \bar{q}^* \frac{\bar{I}^*}{\bar{M}^*} \end{aligned}$$

- (Foreign) Production of p-period ahead construction input

$$\bar{U}_p^* = \bar{Z}_c^* \left(\bar{N}_{c,p}^* \right)^{\alpha} \left(\bar{M}^* \right)^{1-\alpha} \quad \text{for } p = 0, 1, \cdots, P^*$$

- (Foreign) Demand for total construction labor

$$\bar{N}_c^* = \sum_{p=0}^{P^*} \bar{N}_{c,p}^*$$

- (Foreign) Supply of housing permit

$$\bar{M}^* = (\bar{q}^*)^{\gamma^*}$$

- (Foreign) Supply and demand for non-construction output

$$\begin{aligned} \bar{\tilde{Y}}_F &= n\bar{C}_F + (1-n) \bar{C}_F^* \\ \bar{\tilde{Y}}_F &= (1-n) \bar{Z}_F \bar{N}^* \end{aligned}$$

- (Foreign) Price dispersion of foreign-produced good

$$\bar{\Xi}_F = 1$$

- (Foreign) Aggregate price index

$$\bar{\hat{p}}_F = 1$$

- (Foreign) Phillips curve

$$\bar{\hat{p}}_F = \frac{\theta_c}{\theta_c - 1}\frac{\bar{\tilde{Z}}_F^a}{\bar{\tilde{Z}}_F^b}$$
$$\bar{\tilde{Z}}_F^a = \frac{1}{1-\omega\beta}\bar{mc}_F\bar{\tilde{Y}}_F$$
$$\bar{\tilde{Z}}_F^b = \frac{1}{1-\omega\beta}\bar{X}_F\bar{\tilde{Y}}_F$$

- (Foreign) Marginal cost

$$\bar{mc}_F = \frac{\bar{w}^*}{\bar{Z}_F}$$

- "Backus-Smith"

$$\frac{\bar{U}_{c^*}}{\bar{U}_c} = \bar{X}$$

- Euler equation and nominal interest rate

$$\bar{R} = \frac{1}{\beta}$$

- Aggregate output

$$\bar{Y} = n\bar{Z}_H\bar{N}^d + (1-n)\,\bar{Z}_F\bar{N}^{d*}$$

- Resource constraint

$$n\bar{C} + (1-n)\,\bar{C}^* = \bar{\tilde{Y}}_H\bar{X}_H + \bar{\tilde{Y}}_F\bar{X}_F$$

- Relative price relationship

$$1 = \phi\left(\bar{X}_H\right)^{1-\eta} + (1-\phi)\left(\bar{X}_F\bar{X}\right)^{1-\eta}$$

- Inflation relationship

$$\bar{\pi}^* = \bar{\pi} = \bar{\pi}_H = \bar{\pi}_F$$

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List of Figures

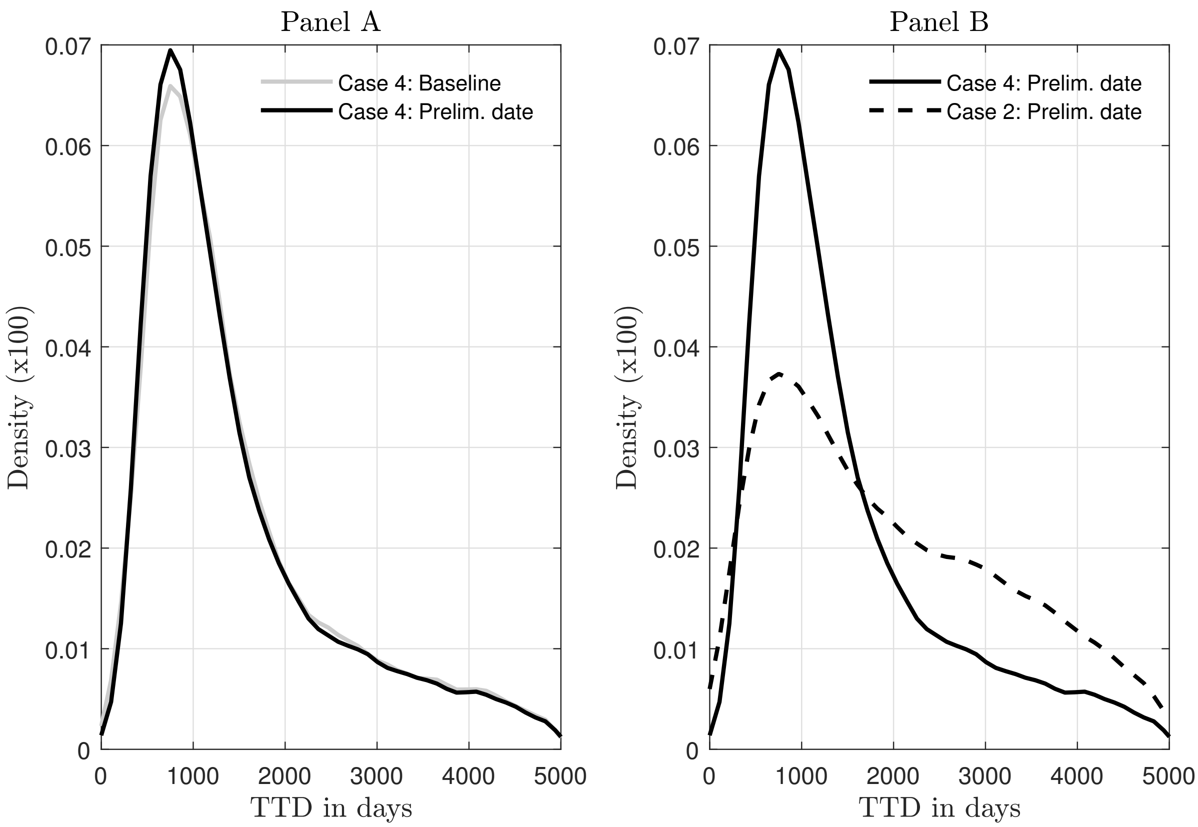

Figure A.1: TTD based on preliminary approval date

Note: The kernel density is plotted for a range of TTD data from 0 days to 5,000 days.

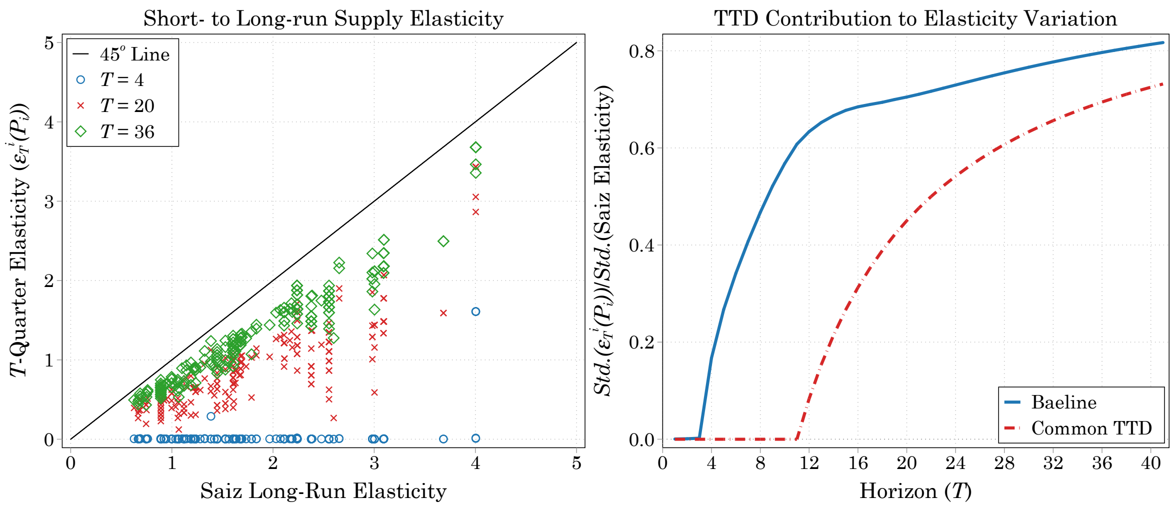

Figure A.2: Supply elasticities in each horizon ($\theta = 0.01$)

Note: The left panel shows the scatter plot comparing the Saiz supply elasticity and the $T-$horizon supply elasticities with $T = 4, 16, 32$ quarters. The middle panel of this figure shows the Spearman's rank correlation of our horizon-specific elasticities with the Saiz elasticity. The right panel shows the cross-county variation of the $T-$horizon supply elasticities when we use (i) the TTD measure constructed by using regression (3) in Table 3 (blue solid line) and (ii) the median county TTD measure of 11 quarters (red dashed line). We set the elasticity of substitution across construction stages (θ) as 0.01.

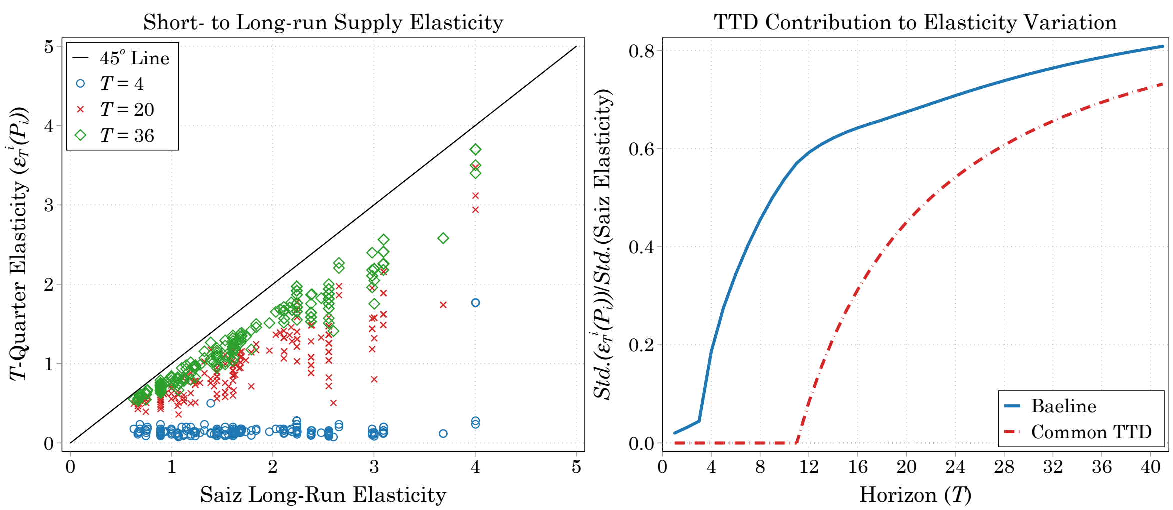

Figure A.3: Supply elasticities in each horizon ($\theta = 2.0$)

Note: The left panel shows the scatter plot comparing the Saiz supply elasticity and the $T-$horizon supply elasticities with $T = 4, 16, 32$ quarters. The middle panel of this figure shows the Spearman's rank correlation of our horizon-specific elasticities with the Saiz elasticity. The right panel shows the cross-county variation of the $T-$horizon supply elasticities when we use (i) the TTD measure constructed by using regression (3) in Table A.2 (blue solid line) and (ii) the median county TTD measure of 11 quarters (red dashed line). We set the elasticity of substitution across construction stages (θ) as 2.0.

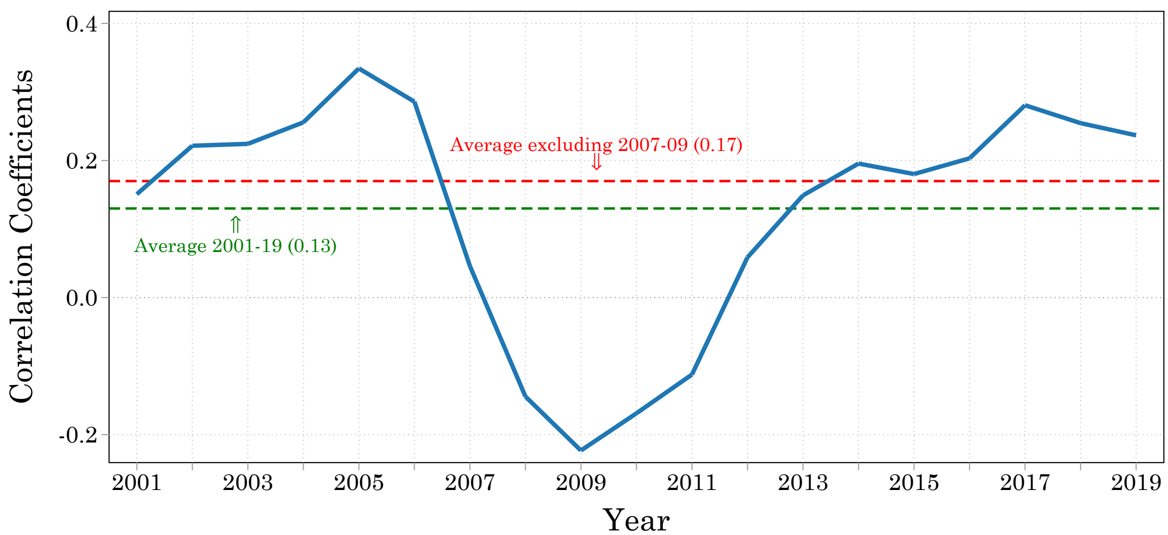

Figure A.4: Cross-sectional correlation between annual HPI growth and housing unit growth

Note: This figure shows the cross-county rank correlation coefficients between one-year HPI growth and one-year housing unit growth from 2001 through 2019. County-level housing units are from the Census Bureau's Annual Estimates of County Housing Units and county-level house price data are from Federal Housing Finance Agency. Green line shows average of correlation coefficients from 2001 to 2019 and red line shows the average correlation excluding the Great Recession period (2007-09).

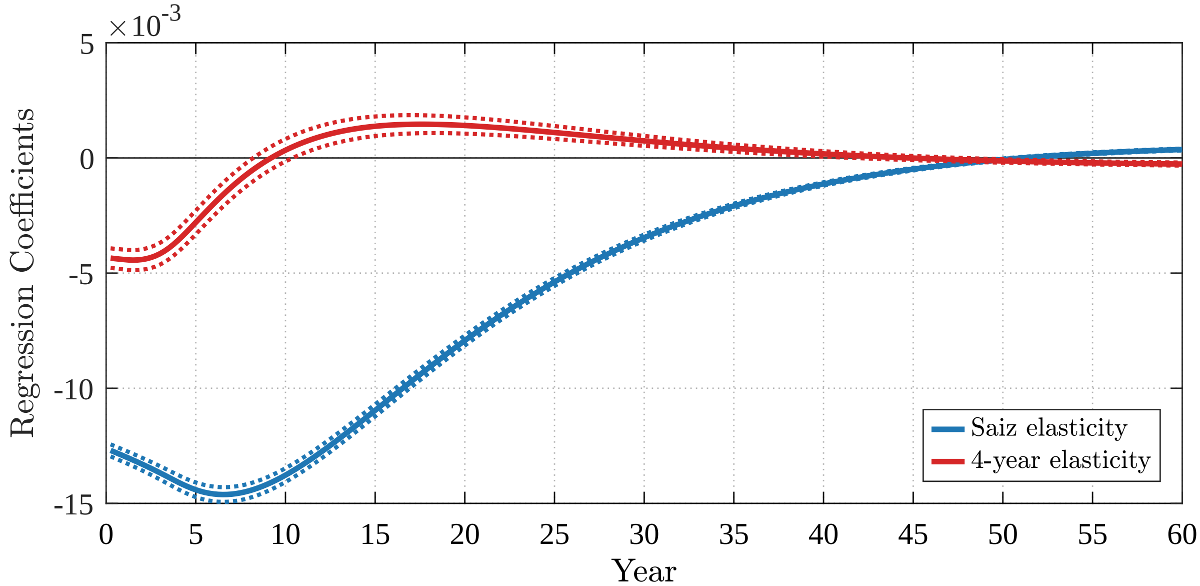

Figure A.5: House price growth regression of short- and long-run elasticities

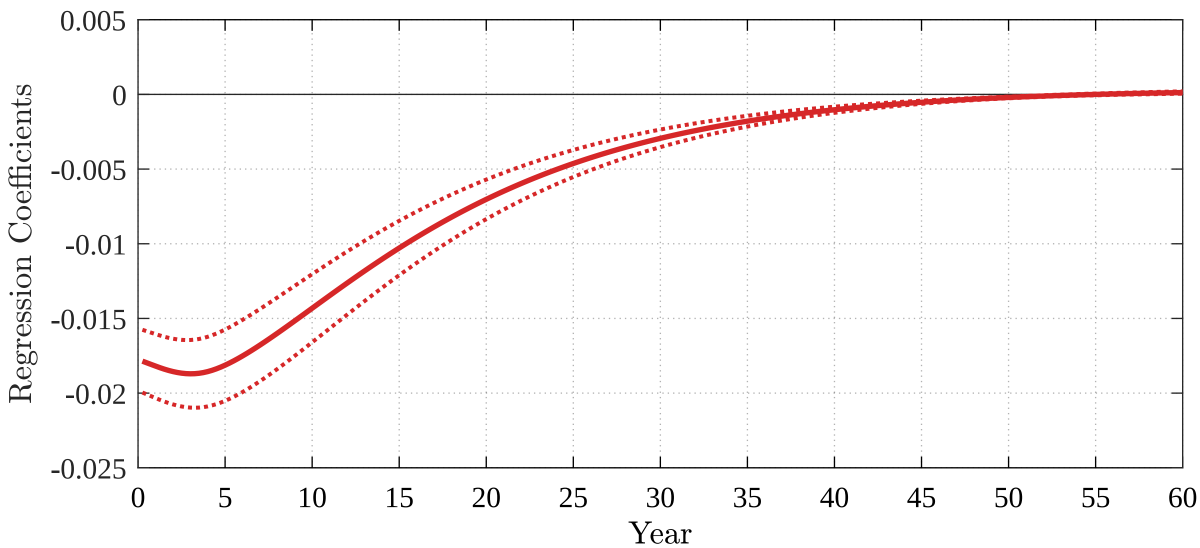

Figure A.6: House price growth regression of short-run elasticities

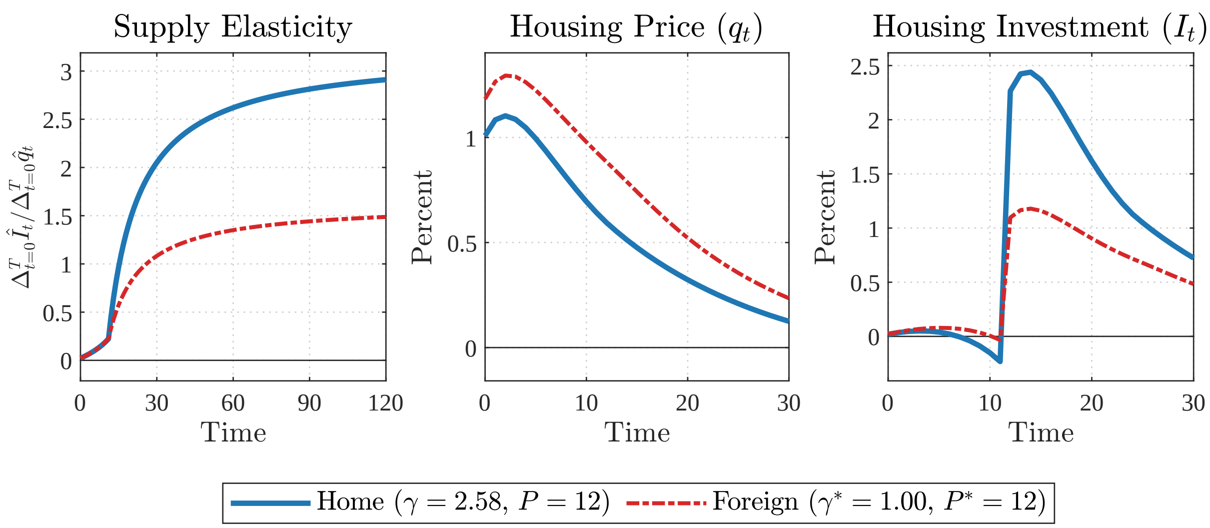

Figure A.7: Model Responses to Housing Demand Shocks in Each Region (Different γ)

Notes:

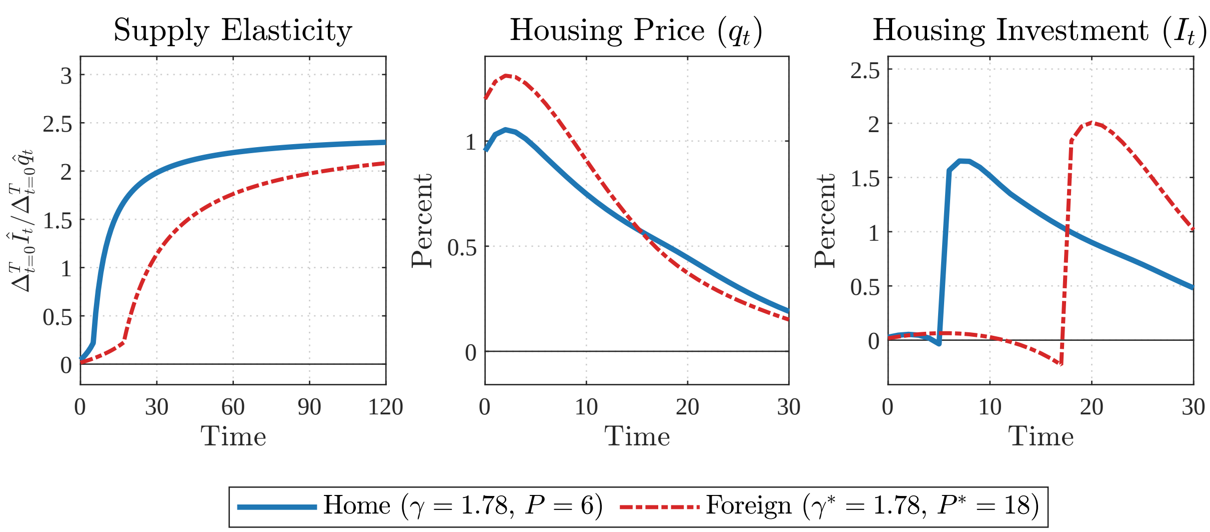

Figure A.8: Model Responses to Housing Demand Shocks in Each Region (Different P)

Notes:

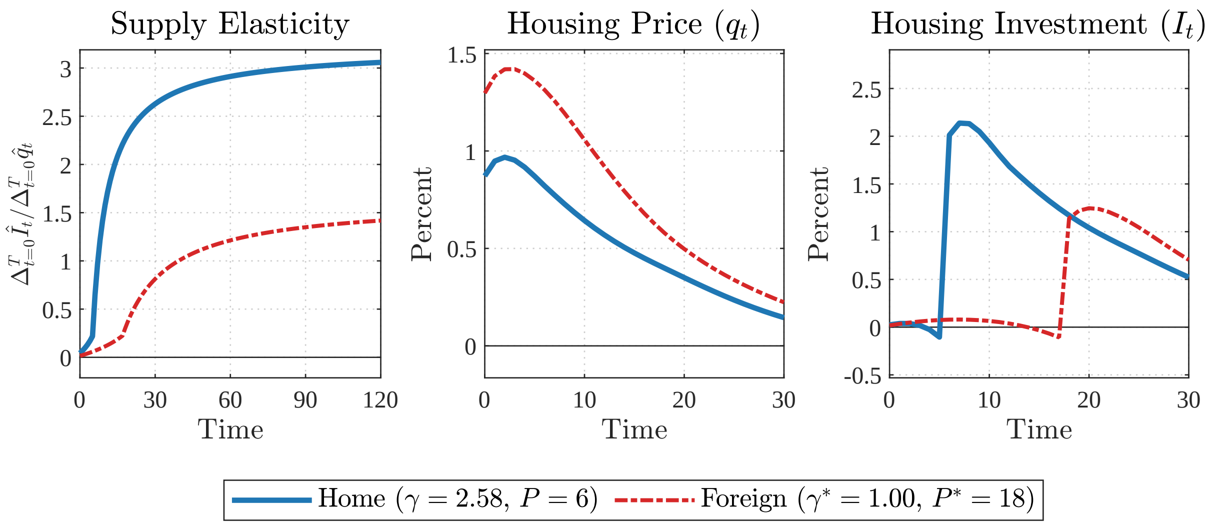

Figure A.9: Model Responses to Housing Demand Shocks in Each Region (Different γ and P 1)

Notes:

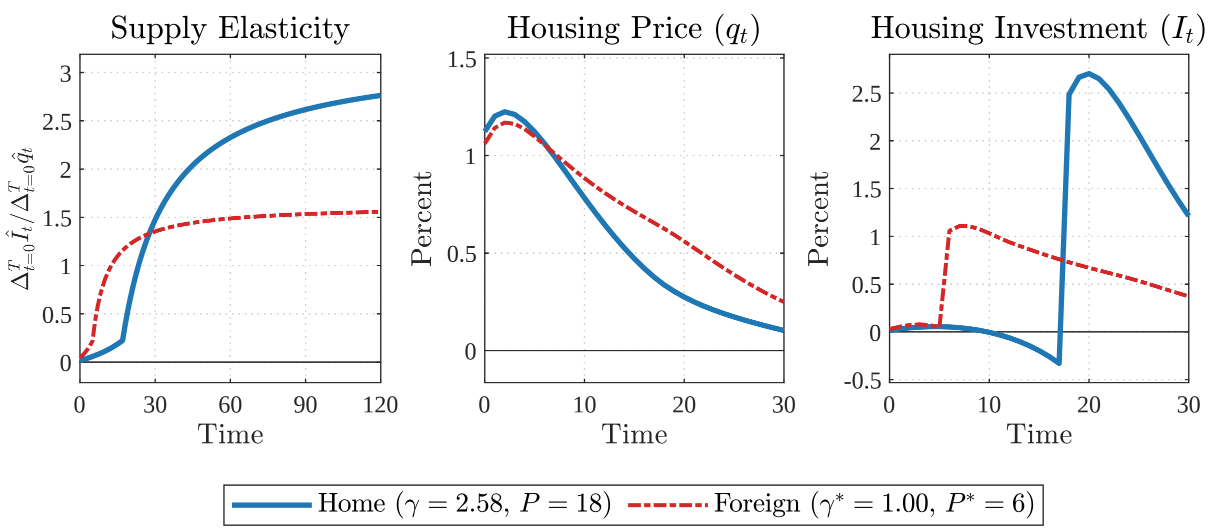

Figure A.10: Model Responses to Housing Demand Shocks in Each Region (Different γ and P 2)

Notes:

List of Tables

Table A.1: New housing completions by each year between 2003 and 2019

(unit: $1,000$ housing)	Zonda	Census	Coverage
Total housing			
2003	555	$1,677$	33%
2004	820	$1,835$	45%
2005	835	$1,929$	43%
2006	902	$1,989$	45%
2007	754	$1,514$	50%
2008	495	$1,127$	44%
2009	348	796	44%
2010	319	654	49%
2011	264	585	45%
2012	301	641	47%
2013	419	763	55%
2014	344	883	39%
2015	372	965	39%
2016	417	$1,061$	39%
2017	465	$1,152$	40%
2018	496	$1,190$	42%
2019	504	$1,260$	40%
Single family housing			
2003	458	$1,381$	33%
2004	672	$1,528$	44%
2005	612	$1,634$	37%
2006	640	$1,662$	38%
2007	537	$1,228$	44%
2008	361	826	44%
2009	247	522	47%
2010	234	495	47%
2011	202	446	45%
2012	237	478	50%
2013	316	570	55%
2014	282	619	46%
2015	302	647	47%
2016	342	737	46%
2017	381	795	48%
2018	405	842	48%
2019	405	904	45%

Note: "Zonda" indicates total new housing completions for each year in our data set. "Census" indicates total new housing completions for each year in the Census Bureau's SOC. "Coverage" is the ratio between "Zonda" and "Census" in percentage.

Table A.2: Section TTD regression results: Local controls

Variables	(1)	(2)	(3)
Bartik		0.277***	
		(0.0355)	
Sand state		−0.101***	
		(0.00866)	
Coastal state		0.0132**	
		(0.00599)	
Immigrant		0.400***	
		(0.0850)	
College+		0.0841	
		(0.0654)	
Log(population density)		0.0335***	
		(0.00322)	
Log(county gdp)		−0.100***	
		(0.0107)	
Builder fixed effect	✓	✓	✓
Year fixed effect	✓	✓	✓
Local controls		✓	
Local controls × Year			✓
Constant	4.505***	4.432***	5.284***
	(0.0505)	(0.0883)	(0.184)
Observations	104,196	104,196	104,196
R-squared	0.282	0.289	0.330

Note: Same as Table [3] in the main text. Regression on log(TTD). *** p<0.01, ** p<0.05, * p<0.1. Robust standard errors in parentheses.

Table A.3: Section TTD statistics (25 percent completion)

(unit: days)	Land TTD	Building TTD	Total TTD
Mean	573	568	1,141
Std. dev.	767	667	1,009
IQR	458	454	911
P10	91	92	365
P25	181	184	458
P50	275	365	821
P75	639	638	1,369
P90	1,278	1,188	2,556
Observations	104,936	104,936	104,936

Note: Each observation is a subdivision or a section of a subdivision when there are multiple sections in a subdivision. IQR stands for the interquartile range (P75−P25). Five different percentiles of each TTD distribution are shown, e.g. P50 referring to the median (50th percentile) of the distribution.

Table A.4: Section TTD regression results (25 percent completion)

Variables	(1)	(2)	(3)
Log(number of units)	0.124***	0.131***	0.128***
	(0.00442)	(0.00453)	(0.00437)
Log(lot size)	0.126***	0.135***	0.135***
	(0.00479)	(0.00492)	(0.00478)
Single family	–	–	–
Townhouse	0.180***	0.174***	0.164***
	(0.0115)	(0.0118)	(0.0116)
Condo	0.176***	0.218***	0.244***
	(0.0413)	(0.0414)	(0.0392)
Duplex	−0.0197	−0.0255	−0.0157
	(0.0307)	(0.0308)	(0.0297)
Etc.	0.0296	0.0473*	0.0426*
	(0.0253)	(0.0251)	(0.0235)
Builder fixed effect	✓	✓	✓
Year fixed effect	✓	✓	✓
Local controls		✓	
Local controls × Year			✓
Constant	4.524***	4.252***	4.901***
	(0.0513)	(0.0894)	(0.177)
Observations	104,936	104,936	104,936
R-squared	0.231	0.240	0.277

Note: Regression with log(TTD) as the dependent variable. Local control variables include Bartik-type predicted industry employment growth, indicators for sand state and coastal state, population share of immigrants, population share of college educated, population density, and county real GDP. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table A.5: County-level TTD statistics (25 percent completion)

(unit: days)	Raw TTD	Reg. (1)	Reg. (2)	Reg. (3)
Mean	848	797	790	798
Std. dev.	452	330	324	326
IQR	364	358	346	312
P10	458	472	442	470
P25	639	611	603	628
P50	821	760	752	764
P75	1,003	970	949	939
P90	1,096	1,172	1,126	1,119
Observations	298	298	298	298

Note: Each observation is a county's median TTD. We use counties with at least 10 completed sections observed. IQR stands for the interquartile range (P75−P25). Five different percentiles of each TTD distribution are shown, e.g. P50 referring to the median (50th percentile) of the distribution.

Table A.6: County-level TTD regression results (25 percent completion)

Variables	Reg. (1)	Reg. (2)	Reg. (3)
Saiz elasticity	−0.176***	−0.150***	−0.132***
	(0.041)	(0.042)	(0.037)
Rainfall intensity	0.108***	0.082***	0.076***
	(0.023)	(0.023)	(0.022)
Heat	0.042*	0.060***	0.053**
	(0.021)	(0.022)	(0.021)
Observations	223	223	223
R-squared	0.216	0.190	0.182

Note: We use counties with at least 10 completed sections observed. "Rainfall intensity" measures the rainfall inches per hour on a storm of one-hour duration and a 100-year return period (Data source: National Oceanic and Atmospheric Administration's Atlas 14 precipitation frequency estimates). "Heat", i.e. cooling degree days, is a measure of the year's temperature hotness, calculated as the difference between the daily temperature mean (the sum of the high and low temperatures divided by two) and 65 degrees Fahrenheit, multiplied by the number of days with a positive value of this difference in a given year (Data source: National Centers for Environmental Information's Annual Climatological Data). Robust standard errors are reported in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table A.7: Local GE Model: Calibration

	Value	Description	Source/Target
β	$0.98^{\frac{1}{4}}$	Time preference	Quarterly frequency
σ	2.0	Inverse of EIS	Guren et al. (2020)
ν	1.0	Inverse of Frisch elasticity	Guren et al. (2020)
ψ_n	1.0	Labor supply disutility parameter	Steady-state labor ($\bar{N}_n = 1$)
ψ	0.05	Construction labor supply disutility parameter	Steady-state construction labor ($\bar{N} = 1$)
θ	0.5	ES between different stages of housing production	Assigned
α	0.385	Construction elasticity of labor	Construction labor income share (KLEMS)
κ	0.75	Preference weight on effective consumption	Expenditure share on housing (CEX)
δ	0.0075	Housing depreciation rate	Guren et al. (2020)
ψ_b	0.001	Scale of the portfolio holding cost	Guren et al. (2020)
ρ_φ	0.95	Persistence of housing demand shock	Guren et al. (2020)

Notes: This table shows model parameter values used for our local GE model simulation. See Section D.1.4 for details.

Table A.8: Calibration

	Value	Description	Source/Target
β	$0.98^{\frac{1}{4}}$	Time preference	Quarterly frequency
κ	0.58	Preference weight on effective consumption	Guren et al. (2020)
σ	2.0	Inverse of EIS	Guren et al. (2020)
ν	1.0	Inverse of Frisch elasticity	Guren et al. (2020)
ϕ	0.4	Degree of home bias in the home region	*Guren et al. (2020)*
ϕ^*	0.4	Degree of home bias in the foreign region	*Guren et al. (2020)*
η	2.0	ES between goods produced in home and foreign region	Assigned
θ	0.5	ES between different stages of housing production	Assigned
θ_c	5.0	ES across differentiated goods in each region	Iacoviello and Neri (2010)
ω	0.84	Degree of price stickiness	Iacoviello and Neri (2010)
ω	0.91	Degree of wage stickiness	Iacoviello and Neri (2010)
$\tilde{\eta}$	5.0	ES across differentiated goods in each region	Iacoviello and Neri (2010)
δ_h	0.01	Housing depreciation rate	Iacoviello and Neri (2010)
α	0.385	Construction elasticity of labor	Guren et al. (2020)
ϕ_π	1.5	Inflation feedback in Taylor rule	Standard

Notes: This table shows model parameter values used for our baseline simulation. See Section D.3.6 for details.