A Note on the Interpretation of Cross-Sectional Evidence Against the Beta-Expected Return Relationship

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Introduction

Recent empirical evidence suggests the absence of any significant relation between average stock returns and market βs, contrary to the Capital Asset Pricing Model of Sharpe (1964), Lintner (1965) and Black (1972). Employing the cross-sectional regression approach of Fama and MacBeth (1973), Fama and French (1992) suggest that the average premium for β is economically unimportant, calling into question the widely assumed relation between risk, as measured by β, and expected return. This paper demonstrates that using monthly returns data, the cross-sectional regression approach will accept the null hypothesis of no relation between β and stock returns even when the underlying model is true. The case against β is therefore much weaker than this empirical evidence suggests.

A brief review of the SLB model

The central prediction of the Sharpe-Lintner-Black (SLB) model is that the expected return R_i on a security i satisfies

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f\right], \tag{1}$$

where R_m is the return on an efficient market portfolio, R_f is the risk-free rate of return, β_i is the slope in the regression of R_i on R_m, and $E(\,)$ denotes the expectations operator. Note that the expected excess market return is assumed to be nonnegative [1]. Thus, equation (1) describes a positive relation between expected return and risk as measured by β.

The proper way to eliminate the expectations operator, consistent with the definition of β_i, is to introduce a white noise error term ϵ_i which is uncorrelated with R_m. Equation (1) may then be rewritten as [2]

$$R_i = R_f + \beta_i \left[R_m - R_f\right] + \epsilon_i. \tag{2}$$

[1] If expected excess market returns were negative, the market portfolio would immediately decline to the point where nonnegative expected excess returns were restored.

[2] An incorrect alternative would be to introduce an additional term $-\beta_i\epsilon_m$, where ϵ_m is the expectational error in forecasting market returns. By the nonnegativity of expected market returns, this term would be negatively correlated with R_m, implying that β_i is always estimated inconsistently. Moreover, such a term implies that a stock should advance in a declining market simply because it has a positive β.

Cross-sectional regressions

The cross-sectional regression approach of Fama and MacBeth (1973) is implemented as follows. Each month, the cross-section of monthly returns is regressed on a given set of explanatory factors. The time series means of these monthly regression slopes, along with their time series standard errors, are used to construct standard t-statistics which indicate whether the various factors are related to returns in a statistically significant manner.

Consider the application of this technique, regressing the column vector of stock returns $\tilde{R}$ for a given month on a column vector of ones $\tilde{\ell}$, and the column vector $\tilde{\beta}$ of individual β_is. The regression equation is given by

$$\tilde{R} = \tilde{\ell}c + \tilde{\beta}\gamma + \tilde{\varepsilon}. \tag{3}$$

Under SLB model, equation (2) should hold for each row of this regression. Comparing equation (3) to (2) yields a simple interpretation of the cross-sectional regression slope : the cross-sectional regression slope γ is simply the monthly excess market return $[R_m - R_f]$. In months when the market return is highly positive, the cross-sectional regression slope γ will also be highly positive. In months such as October 1987 when the market return is sharply negative, γ will be sharply negative as well.

Consider now the construction of the relevant t-statistic. Under the SLB model, the time series average of the cross-sectional slopes is simply the average monthly excess market return over the T periods being examined, and the standard error is the standard deviation of monthly excess returns, divided by $\sqrt{T}$.

Empirically, the average of monthly excess market returns is quite small relative to the standard deviation of those returns, except in data sets confined to strongly advancing stock prices. Thus, even when the SLB model is true, and the market portfolio is efficient[3], and the βs are measured without error, the cross-sectional regression approach will tend to incorrectly accept the hypothesis of no relation between β and return.

[3] Roll (1992) notes that when the market index against which βs are computed is not on the efficient frontier, the *ex ante* relation in equation (1) does not necessarily hold. Moreover, other variables may enter the cross-sectional regressions with spurious explanatory power, violating the exclusion restrictions implied by (1).

The tendency of the cross-sectional technique to reject the SLB model - even when the SLB model is true - cannot be remedied by increasing the number of months over which returns are calculated. Consider the case in which monthly excess market returns are randomly distributed, with mean μ and variance σ_m^2

$$R_m - R_f = \mu + \epsilon_m.$$

Let N be the number of months over which returns are calculated. In a data set of T months, the number of individual observations is then T/N. If the SLB model is true, the average regression slope γ will be equal to $N\mu$, with a time series standard error of $\sqrt{N}\sigma_m/\sqrt{T/N}$. The t-statistic of the regression slope, using single period returns, reduces to

$$t = \frac{\mu\sqrt{T}}{\sigma_m}.$$

This is identical to the t-statistic obtained using individual monthly regressions. Under the SLB model, increasing the number of periods N over which returns are measured has no impact on the the estimated t-statistic from the cross-sectional regression technique in a finite data set.

Excess returns

Using the CRSP value weighted return (including dividends) on all NYSE and ASE stocks, and the monthly total return index for Treasury bills reported by Ibbotson and Associates (1991), the average monthly excess market return for the period from July 1963 to December 1990 is calculated to be 0.35%. The time series standard error of this excess return is 0.25. For the Standard & Poor's Composite, the corresponding mean excess return is 0.34% with a time series standard error of 0.24. If the SLB model is strictly true and betas are measured without error, the t-statistic from the cross-sectional regression will therefore have an "insignificant" value of 1.4. To conclude that β is economically unimportant on this basis would be equivalent to saying that the excess market return provided by equities is also unimportant.

Fama and French (1992) report that the most damaging evidence against the SLB model is provided by a set of univariate regressions, each yielding an average slope estimate for β which is statistically insignificant relative to its standard error. For the period from July 1963 to December 1990, this average slope is 0.15% monthly for portfolios of NYSE, ASE and NASDAQ stocks, with a standard error of approximately 0.33. For the period from 1941 to 1990, the slope estimate is 0.24% for individual NYSE stocks, with a standard error of 0.23. These sample means are well within one standard error of the mean excess market returns for the CRSP value weighted index and the S&P Composite, and the standard errors of the mean slope estimates are similar in magnitude to the standard errors of actual excess market returns. Despite assertions to the contrary, the cross sectional evidence of Fama and French (1992) cannot be interpreted as meaningfully unfavorable to the SLB model.

Concluding remarks

The foregoing results suggest that the cross-sectional regression approach has virtually no power to reject the null hypothesis that β and average return are unrelated. Indeed, the cross-sectional regression approach will reject this null hypothesis only if

1) the average excess market return is sufficiently high during the sample period, relative to its standard error, to generate a "significant" t-statistic, or
2) β commands an additional premium not predicted by theory, sufficiently large to dominate the standard error of average market returns in the construction of the relevant t-statistic.

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