Heterogeneous Price Setting Behavior and Aggregate Dynamics: Some General Results[*]

(PRELIMINARY; COMMENTS WELCOME)

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Abstract

We analyze the effects of heterogeneity in price setting behavior in time-dependent sticky price and sticky information models characterized by quite general adjustment hazard functions. In a large class of models that includes the most commonly used price setting specifications, heterogeneity leads monetary shocks to have larger real effects than in one-sector economies with the same frequency of adjustments. Quantitatively, the effects of heterogeneity in models calibrated to match the recent empirical evidence on pricing behavior are large, even in the absence of strategic complementarity in price setting. We find that the degree of monetary non-neutrality in the calibrated heterogeneous economies can be as large as in an otherwise identical one-sector economy with roughly *three* times more nominal rigidity.

JEL classification codes: E1, E3

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1 Introduction

We analyze the effects of heterogeneity in price setting behavior in time-dependent sticky price and sticky information models characterized by quite general adjustment hazard functions. In a large class of models that includes the most commonly used price setting specifications, heterogeneity in the frequency of price changes (in sticky price economies) and in the frequency of information updating (in sticky information economies) leads monetary shocks to have larger real effects than in one-sector economies with the same frequency of adjustments. Quantitatively, the effects of heterogeneity in models calibrated to match the recent empirical evidence on pricing behavior are quite large, *even in the absence of strategic complementarity in price setting.*

The dynamics of heterogeneous economies with such general adjustment hazard functions depend on the details of the latter, and on the whole cross sectional distribution that describes how they vary across sectors. However, for commonly used specifications of monetary shocks and a sensible measure of monetary non-neutrality, we characterize analytically the effects of heterogeneity for arbitrary such hazard functions and cross-sectional distributions, in the absence of strategic complementarity or substitutability - what we refer to as *strategic neutrality* - in price setting. Besides providing analytical convenience, our focus on the case of strategic neutrality is motivated by the challenge of producing significant monetary non-neutrality with empirically plausible amounts of nominal frictions, in the absence of the strong amplification effects that complementarities in price setting are well known to generate.

Our analytical results provide conditions that determine, for a given price setting specification characterized by specific adjustment hazard functions, whether heterogeneity in price setting leads to a larger or smaller extent of monetary non-neutrality than in a one-sector economy with the same hazard function and average frequency of adjustments. We show that these conditions hold in the most commonly used sticky price and sticky information models: the extent of monetary non-neutrality in each sector is convex in the frequency of price adjustment/information updating. Jensen's inequality thus implies larger non-neutralities in the heterogeneous economies.

The intuition as to why the average frequency of adjustments can be quite misleading as an indicator of the overall degree of nominal frictions can be developed from the following limiting case: a two-sector sticky price continuous time economy with a non-negligible fraction of firms that adjust prices continuously, i.e., a flexible price sector. Then, irrespective of how low the frequency of price adjustments in the other sector is, the average frequency in the economy will be infinite. Nevertheless, monetary shocks may still have large real effects due to the sticky price sector.

The intuition of our extreme example carries through to the general case, and does not depend on continuous time. While in one-sector economies there is a direct relationship between the average frequency of price changes (information updating) and the average duration of price spells (spells

between information updating), this is not the case in heterogeneous economies: a given average frequency of adjustments is consistent with very different distributions for the average duration of spells between adjustments across sectors. As a result, in heterogeneous economies, contrary to one-sector economies, a high average frequency of adjustment need not imply small monetary non-neutralities.

We explore the quantitative effects of heterogeneity in calibrated versions of two well established models: a sticky price model with Taylor (1979) staggered price setting, and the sticky information model of Mankiw and Reis (2002). Under strategic neutrality in price setting, we find that the degree of monetary non-neutrality in the calibrated heterogeneous economies can be as large as in an otherwise identical one-sector economy with roughly *three* times more nominal rigidity. We analyze the role of complementarities and its interaction with heterogeneity in the calibrated models. In line with the results of Carvalho (2006), who analyzes heterogeneity in price stickiness in the Calvo (1983) model, and Carvalho (2008), who analyzes heterogeneity in two sticky information models, we find that strategic complementarities in price setting amplify the role of heterogeneity in generating larger monetary non-neutrality, by leading sectors that are slower to adjust prices (or update information) to have a disproportionate impact on the aggregate price level.

As a by-product of our analytical results, we uncover the features of the distribution of the duration of price spells (spells between information updating) that determine the extent of monetary non-neutrality for different types of shocks. For empirically plausible shocks, we find that the first three moments of such distribution suffice to characterize the extent of monetary non-neutrality, according to our measure. Thus, future empirical work on price setting should attempt to document these other moments of the distribution of the duration of price spells. We also uncover the somewhat surprising result that the extent of monetary non-neutrality measured as the discounted cumulative real effects of monetary surprises is the same in sticky price and sticky information economies that share the same adjustment hazard functions (as well as all other structural features).[1] It turns out to be a direct implication of optimal price setting and rational expectations.

Our case for studying heterogeneity in sticky price models builds directly on the empirical evidence. Recently, a series of important papers have documented several features of price setting behavior in both the U.S. economy and the Euro area using disaggregated price data that underlies consumer price indices (Bils and Klenow 2004, for the U.S. economy; Dhyne et al. 2006, and references cited therein for the Euro area). In particular, such papers uncover a vast amount of heterogeneity in the frequency of price changes across different sectors of these economies. As for sticky information, there is hardly any direct evidence on the frequency with which firms update their information in order to set prices. However, there is no *a priori* reason why firms in different

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[1]Of course the hazard functions play different roles in sticky price and sticky inforamtion economies. In the former they decribe the impediments to continuous price adjustment, whereas in the latter they describe the frictions that prevent continuous information updating.

sectors should behave similarly in this dimension. In fact, sticky information models are best seen as a reduced form of a microfounded model in which firms face explicit costs to acquire and process information, and optimally choose for how long to wait before updating their information again (Reis, 2006). Therefore, we should actually expect firms in different sectors to update information at different frequencies, because the optimal time period in-between such "updating dates" depends on factors such as the magnitude of those costs, the importance of sectoral shocks to demand and marginal costs and the shape of the profit function, which are all likely to vary across sectors.

A substantial body of research addresses issues that are related to the subjects of this paper. Following Bils and Klenow (2004), there is now a large empirical literature that documents heterogeneity in price setting behavior using micro-data that underlies price indices (Dhyne et al. 2006 list many references for the Euro area, and there are also similar papers for numerous other countries). Bils and Klenow (2002, 2004), Bils et al. (2003) and Ohanian et al. (1995) are examples of papers that allow for heterogeneity in price stickiness in the context of time-dependent models. In earlier work, Taylor (1993) extended his original model (1979, 1980) to account for wage contracts of different durations. In a different framework, with state- rather than time-dependent pricing rules, Caballero and Engel (1991, 1993) also allow for heterogeneity in the frequency of price changes. These papers do not focus, however, on isolating the role of heterogeneity in aggregate dynamics. This latter kind of analysis is undertaken by Aoki (2001) and Benigno (2001, 2004), who explore the effects of heterogeneity in price stickiness on optimal monetary policy in two-sector models.[2] Nakamura and Steinsson (2007a) perform such an analysis in a state-dependent pricing model. Dixon and Kara (2005) study the role of heterogeneity in a model with Taylor staggered wage setting. Carlstrom et al. (2006) use a two-sector model with different degrees of nominal rigidity to study how sectoral relative prices affect aggregate dynamics. Barsky et al. (2007) study a two-sector model with durable consumption goods and heterogeneity in the frequency of price changes. Sheedy (2007a) studies how heterogeneity in price stickiness affects inflation persistence. Imbs et al. (2007) study the aggregation of sectoral Phillips curves, and the statistical biases that can arise from not accounting for heterogeneity. Finally, Carvalho and Nechio (2008) show that an open economy model with heterogeneity in price stickiness can account for the sluggish dynamics of real exchange rates observed in the data, whereas a one-sector version of the model with the same average degree of price stickiness fails to do so.

The use of more general adjustment hazard functions in sticky price models is common to some recent papers. Wolman 1999, Mash 2004, Guerrieri 2006, Coenen et al. 2007, and Sheedy 2007b allow for flexible adjustment hazard functions in time-dependent models. Caballero and Engel (2007) study generalized (S,s) pricing models in which the adjustment hazard function is somewhat general. They argue that for empirically relevant parameterizations of such function, the degree

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[2]Benigno (2001) extends some of his results to a multi-region setting.

of aggregate price flexibility implied by the generalized (S,s) model is roughly three times as large as the frequency of price adjustments. None of these papers, however, considers the effects of heterogeneity in pricing behavior.

To our knowledge ours is the first paper to incorporate heterogeneity in price setting into sticky price and sticky information models with general adjustment hazard functions. Our results show that in a large class of sticky price and sticky information models that includes the ones most commonly used in the literature, heterogeneity in pricing behavior is a powerful source of amplification of monetary shocks, even in the absence of pricing complementarities. They confirm the recent findings of Carvalho (2006, 2008), whose results on heterogeneity in price setting are a particular case of our sticky price and sticky information models. Together with recent results by Nakamura and Steinsson (2007a) on the effects of heterogeneity in a calibrated menu-cost model, our work suggests that one-sector models have a strong tendency to understate the extent of monetary non-neutrality, irrespective of the nature of frictions that prevent continuous and fully informed reassessment of pricing decisions.

The rest of the paper is organized as follows. Section 2 presents the basic setup and introduces the two price setting models in separate subsections. Section 3 presents a general equivalence result between sticky price and sticky information models built on the basis of the same adjustment hazard functions. Section 4 presents the main analytical results for our measure of monetary non-neutrality. Section 5 analyzes the implications of sectoral heterogeneity for monetary non-neutrality in the most commonly used models of price setting, and provides conditions under which the results can be generalized to other models. Section 6 presents quantitative results in calibrated sticky price and sticky information models. The last section concludes.

2 Model

We start with the description of the economic environment that would obtain in the absence of any impediments to continuous price adjustments and updating of information - what we refer to as the frictionless environment. Such impediments, which characterize the sticky price and sticky information models, are introduced subsequently.

2.1 Frictionless economy

A representative consumer derives utility from a Dixit-Stiglitz composite of differentiated consumption goods and supplies a continuum of differentiated types of labor to monopolistically competitive firms, which he owns. The latter are divided into sectors, and indexed by their sector, $k \in K$, and by $j \in [0, 1]$. The distribution of firms across sectors is summarized by a density function f on K

(with cdf F).[3] Each firm hires labor of a specific type in a competitive market to produce a likewise specific variety of the consumption good. We assume a cashless economy with a risk-free nominal bond in zero net supply.

The representative consumer maximizes:

$$\int_0^\infty e^{-\rho t} \left(\frac{C(t)^{1-\sigma} - 1}{1-\sigma} - \int_{k \in K} f(k) \int_0^1 \frac{L_{kj}(t)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} dj dk \right) dt$$

$$s.t. \ \dot{B}(t) = i(t) B(t) + \int_{k \in K} f(k) \int_0^1 L_{kj}(t) W_{kj}(t) dj dk - P(t) C(t) + T(t), \ \text{for } t \geq 0,$$

and a no-Ponzi condition. Here ρ is the discount rate, $C(t)$ is consumption of the composite good, $L_{kj}(t)$ is the supplied quantity of type kj labor, $W_{kj}(t)$ is the associated nominal wage, $T(t)$ are firms' flow profits received by the consumer, $B(t)$ denotes bond holdings that accrue interest at rate $i(t)$, and $P(t)$ is a price index to be defined below. The parameters σ^{-1} and φ stand for the intertemporal elasticity of substitution in consumption and the (Frisch) elasticity of labor supply, respectively.

The composite consumption good is given by:

$$C(t) \equiv \left[\int_{k \in K} f(k)^{\eta^{-1}} C_k(t)^{\frac{\eta-1}{\eta}} dk \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

$$C_k(t) \equiv f(k) \left[\int_0^1 C_{kj}(t)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $C_k(t)$ is the subcomposite of goods produced by firms in sector k, and $C_{kj}(t)$ is consumption of the variety of the good produced by firm j from sector k (henceforth "firm kj").[4] The elasticity of substitution between varieties within a sector is $\varepsilon > 1$, and $\eta > 0$ is the elasticity of substitution across different sectors. Denoting by $P_{kj}(t)$ the price charged by firm kj at time t, the corresponding consumption price index is:

$$P(t) = \left[\int_{k \in K} f(k) P_k(t)^{1-\eta} dk \right]^{\frac{1}{1-\eta}},$$

$$P_k(t) = \left[\int_0^1 P_{kj}(t)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}},$$

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[3] In the frictionless environment described in this subsection prices are fully flexible and firms have full information about the state of the economy at all times. As a result, there are no differences between such sectors, and the notation is introduced here for convenience. In subsequent subsections we introduce sector-specific adjustment hazard functions indexed by k, which describe price setting behavior in different sectors. In addition, in this frictionless environment nominal shocks have no real effects, and so we delay the introduction of uncertainty to later sections.

[4] The normalization in the sectoral aggregators yields a symmetric equilibrium in which when firms choose the same prices they sell the same quantities. In this case the differences in total output across sectors only depends on the measure of firms in each sector.

where $P_k(t)$ is a sectoral price index.

The first order conditions for the representative consumer's optimization problem are:

$$\frac{W_{kj}(t)}{P(t)} = \frac{L_{kj}(t)^{\frac{1}{\varphi}}}{C(t)^{-\sigma}}, \quad kj \in K \times [0,1],$$

$$\frac{\dot{C}(t)}{C(t)} = -\sigma^{-1}\left[\rho - \left(i(t) - \frac{\dot{P}(t)}{P(t)}\right)\right],$$

$$C_k(t) = f(k)C(t)\left(\frac{P_k(t)}{P(t)}\right)^{-\eta}, \quad k \in K,$$

$$C_{kj}(t) = f(k)^{-1}C_k(t)\left(\frac{P_{kj}(t)}{P_k(t)}\right)^{-\varepsilon}, \quad kj \in K \times [0,1],$$

In this frictionless environment, firms solve a simple, static profit maximization problem. Firm kj hires labor of its specific type in a competitive labor market to produce its variety of the consumption good according to a linear technology:

$$Y_{kj}(t) = N_{kj}(t),$$

where $Y_{kj}(t)$ is the production of its variety and $N_{kj}(t)$ is the specific labor input.[5] The firm's problem is then given by:

$$\max_{P^*_{kj}(t)} \left[P^*_{kj}(t)Y_{kj}(t) - W_{kj}(t)N_{kj}(t)\right]$$

$$s.t. \ Y_{kj}(t) = N_{kj}(t),$$

$$Y_{kj}(t) = \left(\frac{P^*_{kj}(t)}{P_k(t)}\right)^{-\varepsilon}\left(\frac{P_k(t)}{P(t)}\right)^{-\eta}Y(t),$$

where

$$Y(t) \equiv \left[\int_{k \in K} f(k)^{\eta^{-1}}Y_k(t)^{\frac{\eta-1}{\eta}}dk\right]^{\frac{\eta}{\eta-1}},$$

$$Y_k(t) \equiv f(k)\left[\int_0^1 Y_{kj}(t)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

and where we have used the market clearing conditions in the goods markets. The optimal fric-

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[5] As in the more standard identical-firms model, the assumption of specific input markets is perfectly compatible with price-taking behavior. For a detailed discussion of this point see Woodford (2003, ch. 3).

tionless price is given by the usual mark-up rule:

$$P_{kj}^* (t) = \frac{\varepsilon}{\varepsilon - 1} W_{kj} (t) .$$

The model is closed by a monetary policy specification that ensures existence and uniqueness of a rational expectations equilibrium.

2.2 Price setting in the presence of frictions

We obtain our sticky price and sticky information models from the above frictionless environment by introducing frictions to price setting. Impediments to continuous price adjustment yield the sticky price models, whereas infrequent information updating gives rise to the sticky information models. Heterogeneity arises because the extent of such frictions is allowed to vary across sectors. In the frictionless environment of the previous section monetary shocks are immaterial, and thus were omitted for simplicity. This is no longer the case in the presence of frictions, and from now on we account for uncertainty stemming from monetary shocks.

Frictions to price setting are characterized by sector-specific "survival" functions $\widetilde{\phi}_k (t, t + s)$. The latter have different interpretations in the two types of models that we consider: in the sticky price models $\widetilde{\phi}_k (t, t + s)$ denotes the probability that a price set at time t by a firm in sector k will still be in place at time $t + s$ (and possibly thereafter); in sticky information models, $\widetilde{\phi}_k (t, t + s)$ gives the probability that a firm from sector k that updates its information set - and therefore its price plan - at time t will still be using the same price plan at time $t + s$ (and possibly thereafter).[6] At this stage, the only restriction we impose on $\widetilde{\phi}_k$ is that $\lim_{s \to \infty} \widetilde{\phi}_k (t, t + s) = 0$, which rules out no-adjustment (no-updating) on the part of firms. We emphasize that such survival functions are sector-specific in the sense that in each sector all firms follow the pricing policies that they imply, and *not* in the sense that all firms in each sector change prices (update information) at the same time.

2.2.1 Sticky prices

In the presence of the aforementioned impediments to continuous price adjustments, when setting its price $X_{kj}^{sp} (t)$ at time t firm kj solves:

$$\max \; E_t \int_0^\infty Q^{sp} (t, t + s) \, \widetilde{\phi}_k (t, t + s) \left[X_{kj}^{sp} (t) Y_{kj}^{sp} (t + s) - W_{kj}^{sp} (t + s) N_{kj}^{sp} (t + s) \right] ds$$

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[6]Without loss of generality we assume that $\widetilde{\phi}_k (t, t + s)$ is defined for $s \in [0, \infty)$. Models in which the duration of price spells (or of price plans) is finite with probability one are naturally included.

$$s.t. \ Y_{kj}^{sp}\left(t+s\right)=N_{kj}^{sp}\left(t+s\right),$$

$$Y_{kj}^{sp}\left(t+s\right)=\left(\frac{X_{kj}^{sp}\left(t\right)}{P_{k}^{sp}\left(t+s\right)}\right)^{-\varepsilon}\left(\frac{P_{k}^{sp}\left(t+s\right)}{P^{sp}\left(t+s\right)}\right)^{-\eta}Y^{sp}\left(t+s\right),$$

where the "*sp*" superscript stands for sticky price, and the nominal discount factor between t and $t+s$ used to price firms' profits, $Q^{sp}\left(t,t+s\right)$, is given by:[7]

$$Q^{sp}\left(t,t+s\right)=e^{-\rho s}\frac{P^{sp}\left(t\right)}{P^{sp}\left(t+s\right)}.$$

The first order condition yields:

$$X_{kj}^{sp}\left(t\right)=\frac{\varepsilon}{\varepsilon-1}\frac{E_{t}\int_{t}^{\infty}Q^{sp}\left(t,s\right)\widetilde{\phi}_{k}\left(t,s\right)P_{k}^{sp}\left(s\right)^{\varepsilon-\eta}P^{sp}\left(s\right)^{\eta}Y^{sp}\left(s\right)W_{kj}^{sp}\left(s\right)ds}{E_{t}\int_{t}^{\infty}Q^{sp}\left(t,s\right)\widetilde{\phi}_{k}\left(t,s\right)P_{k}^{sp}\left(s\right)^{\varepsilon-\eta}P^{sp}\left(s\right)^{\eta}Y^{sp}\left(s\right)ds}.$$

We focus on the symmetric equilibrium in which, conditional on time t information, the joint distribution of future variables that matter for price setting is the same for all firms in sector k that change prices in period t, and therefore they choose the same nominal price. Thus, the sectoral price indices are given by:

$$P_{k}^{sp}\left(t\right)=\left[\int_{-\infty}^{t}\Lambda_{k}\left(s\right)\widetilde{\phi}_{k}\left(s,t\right)X_{k}^{sp}\left(s\right)^{1-\varepsilon}ds\right]^{\frac{1}{1-\varepsilon}}, \tag{3}$$

where $\Lambda_{k}\left(s\right)$ denotes the fraction of firms from sector k that change prices at s.

Given an arbitrary heterogeneous sticky price economy characterized by $\left\{f,\widetilde{\phi}_{k},\Lambda_{k}\right\}_{k\in K}$, let its *corresponding sticky information economy* be the sticky information economy characterized by the same primitives, including the functions $\left\{f,\widetilde{\phi}_{k},\Lambda_{k}\right\}_{k\in K}$, where the interpretation of $\widetilde{\phi}_{k}$ is, of course, different. We derive and aggregate firms' optimal pricing policies in the corresponding sticky information economy in the next subsection. For notational convenience, henceforth we use $\left\{S_{k}\right\}_{k\in K}$ to denote the $\left\{f,\widetilde{\phi}_{k},\Lambda_{k}\right\}_{k\in K}$ structure.

2.2.2 Sticky information

In line with the sticky information model proposed by Mankiw and Reis (2002), firms only update their information sets sporadically. There are no impediments to price adjustment, so that firms set prices at each instant to maximize current expected profits, conditional on their current (possibly outdated) information. At time t a firm kj that last updated its information set at time $t-s$

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[7]Alternatively, under complete markets the nominal stochastic discount factor would take on the usual form $Q^{sp}\left(t,t+s\right)=e^{-\rho s}\left(\frac{C^{sp}\left(t+s\right)}{C^{sp}\left(t\right)}\right)^{-\sigma}\frac{P^{sp}\left(t\right)}{P^{sp}\left(t+s\right)}$. This difference is immaterial in the log-linear approximate model analyzed subsequently.

solves:[8]

$$\max_{X_{kj}^{si}(t-s,t)} \; E_{t-s}\left[\left(X_{kj}^{si}(t-s,t) - W_{kj}^{si}(t)\right)\left(\frac{X_{kj}^{si}(t-s,t)}{P_k^{si}(t)}\right)^{-\varepsilon}\left(\frac{P_k^{si}(t)}{P^{si}(t)}\right)^{-\eta}Y^{si}(t)\right],$$

where the "*si*" superscript stands for sticky information. Thus, it sets:

$$X_{kj}^{si}(t-s,t) = \frac{\varepsilon}{\varepsilon-1}\frac{E_{t-s}\left[P_k^{si}(t)^{\varepsilon-\eta}P^{si}(t)^{\eta}Y^{si}(t)W_{kj}^{si}(t)\right]}{E_{t-s}\left[P_k^{si}(t)^{\varepsilon-\eta}P^{si}(t)^{\eta}Y^{si}(t)\right]}.$$

Firms from sector k that share the same information set (from $s \leq t$, say) charge the same price, denoted $X_k^{si}(s,t)$. Thus, sectoral price indices can be written as:

$$P_k^{si}(t) = \left[\int_{-\infty}^{t}\Lambda_k(s)\,\widetilde{\phi}_k(s,t)\,X_k^{si}(s,t)^{1-\varepsilon}\,ds\right]^{\frac{1}{1-\varepsilon}}. \tag{4}$$

2.2.3 A short digression about the $\{S_k\}_{k\in K}$ structure

At this stage our framework is quite general. In particular, the $\widetilde{\phi}_k$ functions that describe the distribution of adjustment times for firms in sector k are essentially unrestricted, and so are the Λ_k functions. The two are, of course, related, since the integrals in (3) (and 4) must account for all firms in sector k. Thus:

$$\forall\,(t,k)\;\int_0^\infty \Lambda_k(t-s)\,\widetilde{\phi}_k(t-s,t)\,ds \equiv 1. \tag{5}$$

An equivalent way to see how $\widetilde{\phi}_k$ and Λ_k have to be related proves to be more useful for later use. Define $g_k(t,t')$ to be the likelihood that a firm from sector k that adjusts its price (updates its information) at time t will adjust again *exactly* at time t'. Then:

$$\widetilde{\phi}_k(t,t+s) = \int_{t+s}^{\infty} g_k(t,r)\,dr,$$

$$\Lambda_k(t) = \int_0^\infty \Lambda_k(t-s)\,g_k(t-s,t)\,ds.$$

3   An equivalence result

In this section we present an equivalence result that simplifies the analysis of the role of heterogeneity in our sticky price and sticky information models. Briefly, it states that under strategic neutrality in price setting, the present discounted value of the real effects of monetary shocks is the

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[8]As in Mankiw and Reis (2006, 2007), firms with outdated information do not make any inference about the state of the economy in non-information-updating times.

same in sticky price and sticky information economies that share the same $\{S_k\}_{k \in K}$ structure.

In general, the dynamics of the heterogeneous economies encompassed by our framework depend on the whole cross sectional distribution of the functions $\widetilde{\phi}_k$ and Λ_k, and analytical results are difficult to obtain. To make the problem more tractable, we work with log-linear approximate versions of the models, and with a sensible scalar measure of monetary non-neutrality for which we can derive analytical results. We also assume that monetary shocks take the form of innovations to an exogenous nominal aggregate demand process.[9]

We start by presenting log-linear versions of the price setting models, approximated around a (common) deterministic zero inflation steady state. Then we present our equivalence result.

3.1 Log-linear sticky price models

Firms from sector k that change prices at t set:[10]

$$x_k^{sp}(t) = \frac{\int_t^\infty e^{-\rho(s-t)} \widetilde{\phi}_k(t,s) E_t \left[\frac{1+\varphi^{-1}\eta}{1+\varphi^{-1}\varepsilon} p^{sp}(s) + \frac{\varphi^{-1}(\varepsilon-\eta)}{1+\varphi^{-1}\varepsilon} p_k^{sp}(s) + \frac{\varphi^{-1}+\sigma}{1+\varphi^{-1}\varepsilon} y^{sp}(s) \right] ds}{\int_t^\infty e^{-\rho(s-t)} \widetilde{\phi}_k(t,s) \, ds}.$$

When $\frac{\varphi^{-1}(\varepsilon-\eta)}{1+\varphi^{-1}\varepsilon} = 0$, $\theta \equiv \frac{\varphi^{-1}+\sigma}{1+\varphi^{-1}\varepsilon}$ corresponds to the Ball and Romer (1990) index of real rigidities. In that case, prices are strategic complements (substitutes) if $\theta < 1 \ (> 1)$. The dividing case implies neither complementarity nor substitutability, and we refer to it as *neutrality in price setting*. It arises, for instance, under logarithmic utility and linear disutility of labor.

The aggregate price level is given by:

$$p^{sp}(t) = \int_{k \in K} f(k) \, p_k^{sp}(t) \, dk,$$

where the sectoral price indices evolve according to:

$$p_k^{sp}(t) = \int_{-\infty}^t \Lambda_k(s) \, \widetilde{\phi}_k(s,t) \, x_k^{sp}(s) \, ds.$$

3.2 Log-linear sticky information models

Firms from sector k that last updated information at $t - s$ set:

$$x_k^{si}(t - s, t) = E_{t-s}\left[\frac{1 + \varphi^{-1}\eta}{1 + \varphi^{-1}\varepsilon}p^{si}(t) + \frac{\varphi^{-1}(\varepsilon - \eta)}{1 + \varphi^{-1}\varepsilon}p_k^{si}(t) + \frac{\varphi^{-1} + \sigma}{1 + \varphi^{-1}\varepsilon}y^{si}(t)\right].$$

The aggregate price level is given by:

$$p^{si}(t) = \int_{k \in K} f(k)\, p_k^{si}(t)\, dk,$$

with sectoral price indices evolving according to:

$$p_k^{si}(t) = \int_{-\infty}^{t} \Lambda_k(s)\, \widetilde{\phi}_k(s, t)\, x_k^{si}(s, t)\, ds.$$

3.3 The equivalence result

For any given realization of the nominal aggregate demand process $m(t)$, define the present discounted value of deviations of the output gap from its expected path as of time t:

$$\Gamma^j(t) \equiv \int_{t}^{\infty} e^{-\rho(s-t)}\left(y^j(s) - E_t y^j(s)\right) ds,$$

where $y^j(t)$ is the output gap,[11] and $j = $ "sp", "si". Note that $\Gamma^j(t)$ is itself a stochastic process.

We prove the following result:[12]

Proposition 1 *Under strategic neutrality in price setting, given an arbitrary heterogeneous sticky price economy characterized by $\left\{f, \widetilde{\phi}_k, \Lambda_k\right\}_{k \in K}$ and its corresponding sticky information economy:*

$$\forall t,\ \Gamma^{sp}(t) = \Gamma^{si}(t).$$

This is a striking result given the level of generality of our $\{S_k\}_{k \in K}$ structure, and the fact that it applies for any realization of the nominal aggregate demand process. It follows directly from optimal price setting behavior and rational expectations. The reason is that when adjusting its price at a given time t, a sticky price firm sets a price (deviation from steady state) that is equal to a weighted average of the path for price deviations that would be chosen by a sticky information firm in the corresponding information economy, if the sticky information firm were to update its

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[11] Due to the absence of shocks that can change the natural rate of output in the models, it is also equal to the (log deviation from the steady state of the) output level.

[12] All proofs are in the Appendix.

information set at the same given time t. The weights are given by the discount factor and by the likelihood that the price will still be in effect until the various dates.

Proposition 1 should not be taken to imply that the dynamic responses of a heterogeneous sticky price economy and of its corresponding sticky information economy to monetary shocks are the same. In fact, we know that in general they differ. Nevertheless, this result turns out to be extremely useful. The reason is that it implies that the extent of monetary non-neutrality in sticky price and sticky information economies that share the same $\{S_k\}_{k \in K}$ structure is the same, according to the scalar measure of non-neutrality that we use to study the effects of heterogeneity in price setting. This allows us to focus on just one class of models, and to obtain sharp analytical results, to which we turn next.

4   Measuring the extent of monetary non-neutrality

The result of the previous section is quite general. However, it does not allow us to quantify the extent of monetary non-neutrality. In particular, the fact that the $\{S_k\}_{k \in K}$ structure potentially varies over time makes the problem non-stationary, in the sense that the dates on which the shocks hit matter through the distribution of price adjustment and price plan revision times implied by $\{S_k\}_{k \in K}$.

To arrive at a useful measure of the extent of monetary non-neutrality we impose restrictions on $\{S_k\}_{k \in K}$. to obtain stationarity. This allows us to derive a general expression for our measure of monetary non-neutrality. We then specialize to two types of monetary shocks for which we can derive sharp analytical results: shocks to the level of nominal aggregate demand ("level shocks"), and shocks to the growth rate of nominal aggregate demand ("growth rate shocks").

We use these results in Section (5) to analyze the effects of heterogeneity on the extent of monetary non-neutrality in the most commonly used sticky price and sticky information models. We also provide general conditions under which heterogeneity in price setting behavior increases the extent of monetary non-neutrality.

4.1   Restrictions on $\widetilde{\phi}_k$ and Λ_k, and auxiliary results

From the definition of the $\widetilde{\phi}_k, \Lambda_k$ and g_k functions:

$$
\begin{aligned}
\widetilde{\phi}_k\left(t, t'\right) &= \int_{t'}^{\infty} g_k\left(t, s\right) ds, \\
\Lambda_k\left(t\right) &= \int_{0}^{\infty} \Lambda_k\left(t - s\right) g_k\left(t - s, t\right) ds.
\end{aligned}
$$

For $\widetilde{\phi}_k$ to be stationary (i.e., to depend only on $t' - t$; not on t) we need $g_k(t, s)$ to depend only on $s - t$. In that case, we can write:

$$\widetilde{\phi}_k(\tau) = \int_\tau^\infty g_k(s) \, ds,$$

where now $\widetilde{\phi}_k(\tau)$ denotes the probability that a price (price plan) of a firm from sector k will last for an interval of length τ or more. Accordingly, $g_k(s)$ denotes the likelihood that such a price (price plan) will last exactly for an interval of length s.

In addition, for the extent of monetary non-neutrality to be independent of the date on which the shock hits, we also need $\Lambda_k(t)$ to be time-invariant: $\forall t, \Lambda_k(t) \equiv \Lambda_k$. This amounts to requiring that the measure of firms adjusting prices (updating information) at any given point in time be constant, and corresponds to the assumption of uniform staggering, common in this class of models.

It follows from (5) that:

$$\forall k, \quad \int_0^\infty \Lambda_k \widetilde{\phi}_k(\tau) \, d\tau = 1,$$

so that $\Lambda_k = \left(\int_0^\infty \widetilde{\phi}_k(\tau) \, d\tau \right)^{-1}$.

We can now define, for each sector, $\phi_k(\tau) \equiv \Lambda_k \widetilde{\phi}_k(\tau)$, and prove the following results:

Lemma 1 *At any given time t, $\phi_k(\tau)$ gives the density of firms in sector k that will adjust prices (revise price plans) again exactly at $t + \tau$.*

As a result of **Lemma 1**, we shall refer to $\phi_k(\tau)$ as the density associated with the distribution of *remaining durations* of prices (price plans) already in place at any given point in time.

Lemma 2 *Let $\tau_k \equiv E_{g_k}[\tau] \equiv \int_0^\infty g_k(\tau) \tau d\tau$ denote the expected duration of price spells (price plans) for newly set prices (price plans) in sector k. Then:*

$$\tau_k = \frac{1}{\Lambda_k}.$$

As a result of **Lemma 2**, we shall refer to Λ_k as the *average frequency of price changes (information updating) in sector k.*

Note that in general the expected duration of price spells (price plans) for newly set prices (price plans) by firms in sector k, τ_k, differs from the expected remaining duration of prices (price plans) already in place at any point in time in that sector, $E_{\phi_k}[\tau] \equiv \int_0^\infty \phi_k(\tau) \tau d\tau$. However, if the time elapsed since the price (price plan) was set does not affect the distribution of the time interval until the next adjustment (plan revision), then the two expected durations should be equal. This is exactly the case for models with constant *adjustment hazards*. The *adjustment hazard function* gives the likelihood of adjustment (information updating) taking place at a given point in time in the life of the price spell (price plan), conditional on it not having taken place until then. In terms

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of our notation, the adjustment hazard function for sector k is given by:

$$h_k(\tau) = \frac{g_k(\tau)}{1 - G_k(\tau)},$$

where:

$$G_k(\tau) = \int_0^\tau g_k(s)\,ds.$$

The irrelevance of the time elapsed since the last price adjustment (price plan revision) in constant hazard models is formalized in the following:

Lemma 3 *For models with constant adjustment hazards:*

$$g_k(\tau) = \phi_k(\tau).$$

Finally, in the following sections we derive results that relate the extent of monetary non-neutrality to moments of the distributions that characterize price setting behavior in our economies. Thus, from now on we assume the technical condition $\forall k$, $\lim_{\tau \to \infty}\left[\tau^3\left(1 - G_k(\tau)\right)\right] = 0$, which guarantees that the relevant moments exist.

4.2   A measure of monetary non-neutrality

A one-time monetary shock hits the economy at time zero, yielding thereafter a (perfect foresight) path for nominal aggregate demand denoted by $m^*(t)$.[13] Prior to the shock the latter is assumed to have been evolving according to $m(t)$. We measure the degree of monetary non-neutrality by the discounted cumulative effect of the shock on the output gap. More specifically, our measure of non-neutrality is given by $\Gamma = \int_0^\infty e^{-\rho s}y^j(s)\,ds$, which from **Proposition 1** is the same in sticky price and sticky information economies that share the same $\{f, \phi_k, \Lambda_k\}_{k \in K}$ structure, under strategic neutrality in price setting. For ease of exposition, from now on we assume those conditions and focus on heterogeneous sticky information economies.

The price level in our heterogeneous sticky information economy at a time $t \geq 0$ is given by:

$$p(t) = \int_{k \in K}\left(\int_{-\infty}^0 \phi_k(t-s)\,m(t)\,ds + \int_0^t \phi_k(t-s)\,m^*(t)\,ds\right)dF(k). \tag{6}$$

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[13] In looking at the perfect foresight response to a one-time shock we make use of the fact that certainty equivalence holds in our (loglinear) sticky price and sticky information models.

Using (6), our measure of monetary non-neutrality becomes:

$$\Gamma \;=\; \int_0^\infty e^{-\rho t}\left[m^*(t) - p(t)\right] dt \tag{7}$$

$$\;=\; \int_{k\in K}\left(\int_0^\infty e^{-\rho t}\left[m^*(t) - m(t)\right]\left[1 - \Phi_k(t)\right] dt\right) dF(k),$$

where $\Phi_k(t) = \int_0^t \phi_k(s)ds = 1 - \int_t^\infty \phi_k(s)ds$. Given the restrictions imposed in the previous subsection, our measure of non-neutrality amounts to the average discounted surprises in nominal aggregate demand weighted by the total fraction of firms at each point in time which last updated their information sets before the shock hit.

4.3 Level shocks to nominal aggregate demand

A shock to the level of nominal aggregate demand that dies out exponentially at rate γ can be modeled as:

$$m^*(t) - m(t) = \begin{cases} 0, t < 0 \\ e^{-\gamma t}, t \geq 0, \end{cases}$$

where without loss of generality the size of the shock has been normalized to unity.

In this context, we prove the following:

Proposition 2 *For a small discount rate* $(\rho \to 0)$, *the expected discounted cumulative effect on the output gap of a persistent shock* $(\gamma \to 0)$ *to the level of nominal aggregate demand equals the economy-wide expected remaining duration of price plans put in place before the shock hit:*

$$\lim_{\rho,\gamma\to 0}\Gamma = \int_{k\in K} E_{\phi_k}[\tau]dF(k),$$

where, as defined previously, $E_{\phi_k}[\tau] \equiv \int_0^\infty \phi_k(\tau)\tau dt.$

Proposition 2 relates the extent of monetary non-neutrality with a moment of the economy-wide distribution of times until the next price plan revisions, $\{\phi_k\}_{k\in K}$. As explained in subsection (4.1), that distribution refers to price plans that were in place when the shock hit, and in general differs from the distribution of durations of newly set price plans. Our next result relates the extent of non-neutrality directly to moments of the cross-sectional distribution of durations of newly set price plans.

Proposition 3 *For a small discount rate* $(\rho \to 0)$, *the expected discounted cumulative effect on the*

output gap of a persistent shock $(\gamma \rightarrow 0)$ *to the level of nominal aggregate demand is given by:*

$$\lim_{\rho,\gamma \rightarrow 0} \Gamma = \frac{1}{2} \int_{k \in K} \frac{\tau_k^2 + \sigma_k^2}{\tau_k} dF(k),$$

where as defined previously $\tau_k \equiv E_{g_k}[\tau] \equiv \int_0^\infty g_k(\tau)\tau dt$ *is the expected duration of a newly set price plan by a firm in sector* k, *and* $\sigma_k^2 \equiv Var_{g_k}[\tau] \equiv \int_0^\infty g_k(\tau)(\tau - \tau_k)^2 d\tau$ *is the variance of the duration of such a plan.*

4.4 Growth rate shocks to nominal aggregate demand

A shock to the growth rate of nominal aggregate demand that dies out exponentially at rate λ can be modeled as:

$$m^*(t) - m(t) = \begin{cases} 0, t < 0 \\ \frac{1-e^{-\lambda t}}{\lambda}, t \geq 0, \end{cases}$$

where without loss of generality the size of the shock has been normalized to unity.

In this context, we prove the following:

Proposition 4 *For a small discount rate* $(\rho \rightarrow 0)$, *the expected discounted cumulative effect on the output gap of a persistent shock* $(\lambda \rightarrow 0)$ *to the growth rate of nominal aggregate demand equals (half) the cross-sectional average of the second moment of the distribution of remaining durations of price plans put in place before the shock hit:*

$$\lim_{\rho,\lambda \rightarrow 0} \Gamma = \frac{1}{2} \int_{k \in K} E_{\phi_k}[\tau^2] dF(k),$$

where $E_{\phi_k}[\tau^2] \equiv \int_0^\infty \phi_k(\tau)\tau^2 d\tau.$

In analogy with **Proposition 3**, our next result relates the extent of non-neutrality generated by growth rate shocks directly with moments of the cross-sectional distribution of durations of newly set price plans (prices).

Proposition 5 *For a small discount rate* $(\rho \rightarrow 0)$, *the expected discounted cumulative effect on the output gap of a persistent shock* $(\lambda \rightarrow 0)$ *to the growth rate of nominal aggregate demand equals*

$$\lim_{\rho,\lambda \rightarrow 0} \Gamma = \frac{1}{6} \int_{k \in K} \tau_k^2 + 3\sigma_k^2 + \frac{Skew_{g_k}[\tau]}{\tau_k} dF(k),$$

where $Skew_{g_k}[\tau] = \int_0^\infty g_k(\tau)(\tau - \tau_k)^3 d\tau$ *is the skewness of the distribution of the duration of a newly set price plan (price) by a firm in sector* k.

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5 The effects of sectoral heterogeneity

In this section we assess the implications of heterogeneity in price setting for the extent of monetary non-neutrality. To isolate the role of heterogeneity, we make comparisons with otherwise identical one-sector economies with the same average frequency of price changes (information updating) as the heterogeneous economy. We start by looking at the most commonly used sticky price and sticky information specifications, and then draw lessons for a wider class of models.

5.1 Constant hazard models

The most common sticky price models - built on the Calvo (1983) model - and the seminal sticky information model of Mankiw and Reis (2002) assume constant adjustment hazards. In that context the distribution of duration of price spells (price plans) in sector k is given by:

$$g_k(\tau) = \Lambda_k e^{-\Lambda_k \tau}.$$

By definition, $\tau_k = \Lambda_k^{-1}$, and direct calculations yield:

$$\sigma_k^2 = \tau_k^2,$$
$$Skew_{g_k}[\tau] = 2\tau_k^3.$$

The effects of persistent level and growth rate shocks to nominal aggregate demand are, respectively:

$$\text{Level:} \qquad \lim_{\rho,\gamma \to 0} \Gamma = \frac{1}{2} \int_{k \in K} \frac{\tau_k^2 + \sigma_k^2}{\tau_k} dF(k) = \int_{k \in K} \tau_k dF(k),$$

$$\text{Growth rate:} \qquad \lim_{\rho,\lambda \to 0} \Gamma = \frac{1}{6} \int_{k \in K} \tau_k^2 + 3\sigma_k^2 + \frac{Skew_{g_k}[\tau]}{\tau_k} dF(k) = \int_{k \in K} \tau_k^2 dF(k).$$

An otherwise identical one-sector economy with the same average frequency of price changes (information updating) has constant adjustment hazard equal to $\Lambda = \int_{k \in K} \Lambda_k dF(k)$, with implied duration of price rigidity equal to Λ^{-1}. Thus, because $\tau_k = \Lambda_k^{-1}$, Jensen's inequality directly implies that both for persistent level and growth rate shocks to nominal aggregate demand, the extent of monetary non-neutrality is larger in heterogeneous economies:

$$\text{Level:} \qquad \int_{k \in K} \tau_k dF(k) = \int_{k \in K} \Lambda_k^{-1} dF(k) > \left(\int_{k \in K} \Lambda_k dF(k) \right)^{-1} = \Lambda^{-1},$$

$$\text{Growth rate:} \qquad \int_{k \in K} \tau_k^2 dF(k) = \int_{k \in K} \Lambda_k^{-2} dF(k) > \left(\int_{k \in K} \Lambda_k dF(k) \right)^{-2} = \Lambda^{-2}.$$

This simply reflects the fact that what matters for the extent of non-neutrality is the distribution of times until prices (price plans) are adjusted after the shock hits. To further sharpen the intuition as to why the average frequency of adjustments can be quite misleading as an indicator of the overall degree of nominal frictions, consider the following limiting case: a two-sector sticky price economy with a non-negligible fraction of firms that adjust prices continuously, i.e., a flexible price sector. Then, irrespective of how low the frequency of price adjustments in the other sector is, the average frequency in the economy will be infinite. Nevertheless, monetary shocks may still have large real effects due to the sticky price sector.

The intuition of our extreme example carries through to the general case. While in one-sector economies there is a direct relationship between the average frequency of price changes (information updating) and the average duration of spells between adjustments, this is not the case in heterogeneous economies: a given average frequency of adjustments is consistent with very different distributions for the average duration of spells between adjustments across sectors. As a result, in heterogeneous economies, contrary to one-sector economies, a high average frequency of adjustment need not imply small monetary non-neutralities.

5.2 Constant duration models

The sticky price models inspired by the seminal work of Taylor (1979) assume that the duration of price spells is constant. In turn, Dupor and Tsuruga (2005) analyze a sticky information model with constant time intervals between information updatings. In those cases the distribution of duration of price spells (price plans) is degenerate: every newly set price (price plan) by firms in sector k lasts τ_k with probability one.[14] Thus, for constant duration models, $\sigma_k^2 = Skew_{g_k}[\tau] = 0$.

The effects of persistent level and growth rate shocks to nominal aggregate demand in those models are, respectively:

$$\text{Level:} \qquad \lim_{\rho,\gamma \to 0} \Gamma = \frac{1}{2} \int_{k \in K} \frac{\tau_k^2 + \sigma_k^2}{\tau_k} dF(k) = \int_{k \in K} \frac{1}{2} \tau_k dF(k),$$

$$\text{Growth rate:} \qquad \lim_{\rho,\lambda \to 0} \Gamma = \frac{1}{6} \int_{k \in K} \tau_k^2 + 3\sigma_k^2 + \frac{Skew_{g_k}[\tau]}{\tau_k} dF(k) = \int_{k \in K} \frac{1}{6} \tau_k^2 dF(k).$$

An otherwise identical one-sector economy with the same average frequency of price changes (information updating) has a duration of price rigidity equal to Λ^{-1}, where $\Lambda = \int_{k \in K} \Lambda_k dF(k)$. Thus, arguments analogous to those in the previous subsection imply that both for persistent level and growth rate shocks to nominal aggregate demand, the extent of monetary non-neutrality is larger in heterogeneous economies.

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[14]Formally, in this case there is no underlying density $g_k(\tau)$, and the problem should be written with the *Dirac* measure.

5.3 Other models

The two previous subsections show that in the most common sticky price and sticky information models, heterogeneity in price setting increases the extent of monetary non-neutrality. Beyond that, however, the generality of our framework comes at the cost of making it hard to make precise statements about models with adjustment hazard functions that are essentially unrestricted. To illustrate the difficulty, consider a two-sector sticky price economy in which one sector features Calvo, and the other Taylor pricing (yes, the generality of our framework allows for that). One can obviously compute moments of the distribution of the frequency of price changes in the heterogeneous economy, but how should one construct a one-sector economy with which to make comparisons? While potentially an interesting research question, it is beyond the scope of this paper. We avoid these complications by restricting the nature of heterogeneity so that the adjustment hazard functions in all sectors come from a single parametric family with a finite number of parameters, which in turn are allowed to differ across sectors.

Our previous analytical results suggest that for the types of shocks that we analyze, the first three moments of the distribution of duration of price spells (spells between information updatings) suffice to determine the extent of monetary non-neutrality.[15] Thus, it seems natural to restrict attention to three-parameter parametric families for the adjustment hazard functions. However, the empirical literature on price setting does not provide us with enough detailed information to go beyond the frequency of price changes.[16] As a result, we illustrate one application of the analytical results in our general framework with a one-parameter family. To make the narrative simpler we describe the idea for a sticky price economy. In that case the one-sector economy with the same average frequency of price changes is clearly defined given the parametric adjustment hazard, and the first three moments of the distribution of durations of price spells implied by the hazard can be written as a function of the frequency of price changes. A simple test for whether heterogeneity implies larger or smaller non-neutralities in an economy characterized by that adjustment hazard function amounts to verifying whether the "moments"[17] $\frac{\tau_k^2 + \sigma_k^2}{\tau_k}$ (for level shocks) and $\tau_k^2 + 3\sigma_k^2 + \frac{Skew_{g_k}[\tau]}{\tau_k}$ (for growth rate shocks) are convex or concave in the frequency of price changes. The concept underlying our simple test generalizes to the case of a parametric family with more parameters. In that case, the additional degrees of freedom combined with additional moments are used to find the parameters for the one-sector economy. After that, one can assess how cross-sectional heterogeneity affects the resulting expression for the extent of monetary non-neutrality.

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[15] We say suggest, because our results are based on limiting cases for the shocks.

[16] To make justice to their contribution, Nakamura and Steinsson (2007b) provide a lot more information on price setting behavior beyond the frequency of price changes. However, there is no information about the moments that we uncover with our analytical results.

[17] In this case these are simply functions of the frequency of price changes.

It is clear that even within the class of three-parameter families, there is literally an infinite number of models in which heterogeneity implies larger non-neutralities. At the same time, there is an infinite number of cases in which it does not. To take some natural examples, consider parametric families in which the variance and skewness of the distribution of duration of price spells are constant, i.e. do not depend on the frequency of price changes. Then, heterogeneity unequivocally implies larger non-neutralities in response to level shocks.[18] For growth rate shocks, this is also the case for all positively skewed distributions, and for negatively skewed ones as well, as long as $|Skew_{g_k}[\tau]| \leq \tau_k^3$. For strongly negatively skewed distributions ($Skew_{g_k}[\tau] \leq -\tau_k^3$), this is no longer the case.

6 Quantitative results

In this section we address the quantitative importance of the results highlighted previously. As a result of our choice of a scalar measure of non-neutrality and of our simplifying assumptions on preferences, we temporarily left aside the issues of how heterogeneity affects the shape of impulse response functions in sticky price and sticky information models, and of whether there are any interactions between strategic complementarities and heterogeneity in price setting. We now address these issues as well.

We use calibrated versions of a heterogeneous sticky price model with Taylor staggered price setting and of a heterogeneous sticky information economy based on Mankiw and Reis (2002). We compare those models with identical firms economies with the same average frequency of information updating and price changes, respectively.

We choose to calibrate the cross sectional distribution of adjustment frequencies based on data on prices analyzed by Bils and Klenow (2004). We group the different categories of goods and services reported in their paper into a number of representative frequencies of price changes, and then map each such frequency into a sector in the models. This is arguably a sensible interpretation of their data in the context of a sticky price model. However, sticky information models are at odds with such micro evidence on nominal price rigidity, since they imply continuous price adjustment. Moreover, there is hardly any direct evidence on the frequency with which firms update their information in order to set prices. In the absence of a better alternative we use the same set of statistics to calibrate the cross sectional distribution of frequencies of information updating in the Mankiw-Reis model.[19] Each model is briefly developed in a separate subsection below.

The mapping from the Bils and Klenow data to the parameters of the model is done in a way

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[18] Just note that in that case $\frac{\tau_k^2 + \sigma_k^2}{\tau_k}$ is convex in τ_k.

[19] Arguably, one advantage of using such data over an arbitrary distribution is that the frequency of price changes is likely to be at least weakly related to the frequency with which firms gather and process information that is relevant for their pricing decisions.

that is consistent with continuous time models, as in Carvalho (2006).[20] Their sample contains 350 goods and services categories. The average frequency of price changes implies that prices change on average every 2.9 months. The median frequency of price changes implies that a newly set price lasts on average for 4.3 months. Finally, the expected duration of a newly set price implied by the distribution of frequencies of price changes equals 6.6 months. The cross-sectional standard deviation of expected durations of newly set prices is 7.1 months. Bils and Klenow (2004) base their analysis on adjustment frequencies rather than on directly observed spells of price rigidity in order to avoid having to deal with censored observations. For that reason, we assume that their statistics relate to the distribution of newly set prices, and thus we use them to calibrate g_k (rather than ϕ_k).

6.1 A first pass on the effects of heterogeneity

As a first pass on the implications of heterogeneity in price setting behavior for the extent of monetary non-neutrality, we use the analytical results derived in the previous section. For the limiting cases of shocks to nominal aggregate demand, we can use the moments derived from the statistics in Bils and Klenow to compare the extent of monetary non-neutrality in a heterogeneous economy endowed with such a cross-sectional distribution for the frequency of price changes (information updating), and the extent of non-neutrality in an otherwise identical one-sector economy with the same average frequency of price changes (information updating).

For very persistent shocks to the level of nominal aggregate demand, the ratio of our measure of the extent of monetary non-neutrality in the heterogeneous economy to the same measure in the one-sector economy exceeds two: it equals $\frac{6.6 \; months}{2.9 \; months} \approx 2.3$. For very persistent shocks to the growth rate of nominal aggregate demand, the analogous ratio is much larger: it equals $\frac{6.6^2+7.1^2}{2.9^2} \approx 11.2$.

The first pass quantitative effects are quite large. For example, if we are to use a one-sector economy to match the extent of non-neutralities generated by level shocks to nominal aggregate demand in a heterogeneous economy, we need to calibrate the one-sector economy with *less than half* of the average frequency of price adjustments observed in the data. For persistent growth rate shocks we would need to calibrate the one-sector economy to feature on average one adjustment every $\sqrt{6.6^2+7.1^2}=9.69$ months rather than once every 2.9 months as in the data.

One can argue that these results are only indicative of the effects of heterogeneity in these models, for at least a couple of reasons. First, they refer to our scalar measure of monetary non-neutrality, which by definition ignores the multi-dimensional features of impulse-response functions. Second, they only hold exactly for the limiting cases for nominal aggregate demand shocks, and with the restrictions on primitives that rule out complementarity or substitutability in price setting decisions. In the next subsections we address these issues in calibrated models.

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[20] It is detailed in the Appendix.

6.2 Sticky information

We adopt a constant adjustment hazard function specification, which is the continuous time analogue of that used by Mankiw and Reis (2002): firms only update their information sets when they receive a "Poisson signal." The hazard rate for firms in sector k is Λ_k, so that the expected duration of price plans for firms in this sector is $\tau_k = \Lambda_k^{-1}$.

There are no impediments to price adjustment, so that firms set prices at each instant to maximize current expected profits, conditional on their current (possibly outdated) information. Firms from sector k that last updated information at $t - s$ set:

$$x_k(t-s,t) = E_{t-s} \left[\frac{1+\varphi^{-1}\eta}{1+\varphi^{-1}\varepsilon} p(t) + \frac{\varphi^{-1}(\varepsilon-\eta)}{1+\varphi^{-1}\varepsilon} p_k(t) + \frac{\varphi^{-1}+\sigma}{1+\varphi^{-1}\varepsilon} y(t) \right].$$

The aggregate price level is given by:

$$p(t) = \int_{k \in K} f(k) \, p_k(t) \, dk,$$

with sectoral price indices evolving according to:

$$p_k(t) = \Lambda_k \int_{-\infty}^{t} e^{-\Lambda_k(t-s)} x_k(s,t) \, ds.$$

Our calibration assumes $\sigma = 1$, $\varphi = 0.25$, $\rho = 0$, and $\varepsilon = \eta = 8$. This yields an index of real rigidities $\theta = 0.15$, which implies that pricing decisions are strategic complements.[21] Figures 1 (a-f) display the impulse response functions (IRF) of the calibrated heterogeneous sticky information economy and its identical firms counterpart to different shocks. All cases include IRFs both with and without strategic complementarities in price setting.

From these results it is clear that heterogeneity amplifies the extent of monetary non-neutralities in a substantial way. It is also clear that strategic complementarities interact with heterogeneity to generate more persistent real effects of monetary shocks. Complementarities do make the adjustment process more sluggish in the identical firms case. With heterogeneity, however, this is even more so, according to several metrics: the recession troughs are delayed, output is lower than in the identical firms economy essentially during the whole process, and takes much longer to return to the steady state; inflation is, accordingly, also more persistent.

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[21] Woodford (2003, ch. 3) argues that values in the 0.10-0.15 range can be obtained with plausible assumptions for various sources of real rigidities.

6.3   Taylor staggered price setting

Building on Taylor (1979, 1980), in this model firms are assumed to set prices for a fixed period of time. Firms from sector k set prices for a period of length τ_k. Adjustments are uniformly staggered across time in terms of both firms and sectors. When setting prices at time t, firms from sector k choose:

$$x_k(t) = \frac{\rho}{1 - e^{-\rho\tau_k}} \int_0^{\tau_k} e^{-\rho s} E_t \left[\frac{1 + \varphi^{-1}\eta}{1 + \varphi^{-1}\varepsilon} p(t+s) + \frac{\varphi^{-1}(\varepsilon - \eta)}{1 + \varphi^{-1}\varepsilon} p_k(t+s) + \frac{\varphi^{-1} + \sigma}{1 + \varphi^{-1}\varepsilon} y(t+s) \right] ds.$$

The aggregate price level is then given by:

$$p(t) = \int_{k \in K} f(k) \, p_k(t) \, dk,$$

with

$$p(t) = \frac{1}{\tau_k} \int_0^{\tau_k} x_n(t-s) ds.$$

The calibration for σ, φ, ρ, ε, and η is the same as in the previous subsection. To solve this model with strategic complementarities, we discretize it and apply standard solution methods for discrete time linear rational expectations models. For computational reasons, we solve the model with 25 sectors. The calibration from the Bils and Klenow (2004) data is adjusted accordingly, as detailed in the Appendix. Figures 2 (a-d) display the impulse response functions of the calibrated heterogeneous economy and its one-sector counterpart to growth rate shocks with different levels of persistence. Again, we contrast cases with and without strategic complementarities in price setting. The one-sector economy is a standard Taylor staggered price setting economy with a contract length of 3 months.

From these results it is again clear that heterogeneity in this model amplifies the extent of monetary non-neutralities in a substantial way. As before, strategic complementarities interact with heterogeneity to generate more persistent real effects of monetary shocks. Complementarities do make the adjustment process more sluggish in the identical firms case, but even more so when there is heterogeneity.

7   Conclusion

This paper shows that the effects of heterogeneity in price setting behavior documented in the context of a sticky price model with Calvo pricing by Carvalho (2006), and two sticky information models by Carvalho (2008) carry over to a much larger class of sticky price and sticky information models. Moreover, in calibrated versions of a sticky information model based on Mankiw and Reis

(2002), and a sticky price model with Taylor staggered price setting, we find the quantitative effects of heterogeneity to be large. Our results suggest that heterogeneity in price setting behavior has important aggregate implications, irrespective of the nature of frictions that prevent continuous, fully informed price adjustments.

Appendix

Proofs of lemmas and propositions

Proposition 1 *Let* $\Gamma^j(t) \equiv \int_t^\infty e^{-\rho(r-t)} \left(y^j(r) - E_t y^j(r)\right) dr$, *with* $j =$ *"sp", "si". Under strategic neutrality in price setting, given an arbitrary heterogeneous sticky price economy characterized by* $\left\{f, \widetilde{\phi}_k, \Lambda_k\right\}_{k \in K}$ *and its corresponding sticky information economy:*

$$\forall t, \ \Gamma^{sp}(t) = \Gamma^{si}(t).$$

Proof. Define $\Delta_\Gamma(t) \equiv \Gamma^{si}(t) - \Gamma^{sp}(t)$. Then:

$$
\begin{aligned}
\Delta_\Gamma(t) &= \int_t^\infty e^{-\rho(r-t)} \left(y^{si}(r) - E_t y^{si}(r)\right) dr - \int_t^\infty e^{-\rho(r-t)} \left(y^{sp}(r) - E_t y^{sp}(r)\right) dr \\
&= \int_t^\infty e^{-\rho(r-t)} \left(m(r) - p^{si}(r) - E_t\left(m(r) - p^{si}(r)\right)\right) dr \\
&\quad - \int_t^\infty e^{-\rho(r-t)} \left(m(r) - p^{sp}(r) - E_t\left(m(r) - p^{sp}(r)\right)\right) dr \\
&= \int_t^\infty e^{-\rho(r-t)} \left(-p^{si}(r) + E_t p^{si}(r)\right) dr - \int_t^\infty e^{-\rho(r-t)} \left(-p^{sp}(r) + E_t p^{sp}(r)\right) dr \\
&= \int_t^\infty e^{-\rho(r-t)} \left(\left[p^{sp}(r) - p^{si}(r)\right] - E_t\left[p^{sp}(r) - p^{si}(r)\right]\right) dr.
\end{aligned}
$$

Recall that

$$
\begin{aligned}
p^{sp}(t) &= \int_{k \in K} f(k) \int_0^\infty \Lambda_k(t-s)\, \widetilde{\phi}_k(t-s,t)\, x_k^{sp}(t-s)\, ds\, dk, \\
p^{si}(t) &= \int_{k \in K} f(k) \int_0^\infty \Lambda_k(t-s)\, \widetilde{\phi}_k(t-s,t)\, x_k^{si}(t-s,t)\, ds\, dk,
\end{aligned}
$$

with

$$
\begin{aligned}
x_k^{sp}(t) &= \frac{\int_0^\infty e^{-\rho s}\, \widetilde{\phi}_k(t,t+s)\, E_t\left[p^{sp}(t+s) + \theta y^{sp}(t+s)\right] ds}{\int_0^\infty e^{-\rho s}\, \widetilde{\phi}_k(t,t+s)\, ds}, \\
x_k^{si}(t-s,t) &= E_{t-s}\left[p^{si}(t) + \theta y^{si}(t)\right].
\end{aligned}
$$

So, $\Delta_\Gamma (t) =$

$$\int_t^\infty e^{-\rho(r-t)} \int_{k\in K} \left(\begin{array}{c} \int_0^\infty \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \\ -E_t \int_0^\infty \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \end{array} \right) dF(k) \, dr$$

$$= \int_t^\infty e^{-\rho(r-t)} \int_{k\in K} \left(\begin{array}{c} \int_0^{r-t} \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \\ + \int_{r-t}^\infty \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \\ -E_t \int_0^{r-t} \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \\ -E_t \int_{r-t}^\infty \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \end{array} \right) dF(k) \, dr$$

For $s \geq r - t$, $x_k^{sp} (r-s) = E_t x_k^{sp} (r-s)$, and $x_k^{si} (r-s,r) = E_t x_k^{si} (r-s,r)$, so that:

$$\int_t^\infty e^{-\rho(r-t)} \int_{k\in K} \left(\begin{array}{c} \int_{r-t}^\infty \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \\ -E_t \int_{r-t}^\infty \Lambda_k (r-s) \widetilde{\phi}_k (r-s,r) \left[x_k^{sp} (r-s) - x_k^{si} (r-s,r) \right] ds \end{array} \right) dF(k) \, dr = 0.$$

Thus,

$$\Delta_\Gamma (t) = \int_0^\infty e^{-\rho r} \int_{k\in K} \left[\begin{array}{c} \int_0^r \Lambda_k (t+r-s) \widetilde{\phi}_k (t+r-s,t+r) \\ \{ \left[x_k^{sp} (t+r-s) - x_k^{si} (t+r-s,t+r) \right] \\ -E_t \left[x_k^{sp} (t+r-s) - x_k^{si} (t+r-s,t+r) \right] \} ds \end{array} \right] dF(k) \, dr.$$

To rewrite the expression above in a convenient way, perform the following change of variables: $r = z + w$; $s = w$, with z and w ranging from 0 to ∞. This yields:

$$\Delta_\Gamma (t) = \int_0^\infty e^{-\rho z} \int_{k\in K} \Lambda_k (t+z) \left[\begin{array}{c} \int_0^\infty e^{-\rho w} \widetilde{\phi}_k (t+z,t+z+w) \\ \{ \left[x_k^{sp} (t+z) - x_k^{si} (t+z,t+z+w) \right] \\ -E_t \left[x_k^{sp} (t+z) - x_k^{si} (t+z,t+z+w) \right] \} dw \end{array} \right] dF(k) \, dz.$$

Under strategic neutrality in price setting, $\theta = 1$. As a result, the price setting equations that determine x_k^{sp} and x_k^{si} simplify to:

$$x_k^{sp} (t) = \frac{\int_0^\infty e^{-\rho w} \widetilde{\phi}_k (t,t+w) E_t \left[m (t+w) \right] dw}{\int_0^\infty e^{-\rho w} \widetilde{\phi}_k (t,t+w) \, dw},$$

$$x_k^{si} (t,t+w) = E_t m (t+w).$$

Combining the two equations, optimal price setting implies:

$$\int_0^\infty e^{-\rho w} \widetilde{\phi}_k(t, t+w) \left[x_k^{sp}(t) - x_k^{si}(t, t+w)\right] dw = 0,$$

Finally, for all $z \geq 0$ the Law of Iterated Expectations implies:

$$\int_0^\infty e^{-\rho w} \widetilde{\phi}_k(t+z, t+z+w) E_t \left[x_k^{sp}(t+z) - x_k^{si}(t+z, t+z+w)\right] dw = 0.$$

So,

$$\Delta_\Gamma(t) = 0.$$

∎

Lemma 1 At any given time t, $\phi_k(\tau)$ gives the density of firms in sector k that will adjust prices (revise price plans) again exactly at $t + \tau$.

Proof. We use our stationarity assumptions, and fix $t = 0$ without loss of generality. The fraction of prices (price plans) in sector k set before time 0 that will be readjusted (revised) after τ is given by:

$$\int_\tau^\infty \int_{s=0}^\infty \Lambda_k g_k(s+t) \, ds dt.$$

Let $\Phi_k(\tau)$ be the fraction of prices (price plans) in sector k set before time 0 that will be readjusted (revised) at or before τ, i.e.:

$$\Phi_k(\tau) = 1 - \int_\tau^\infty \int_{s=0}^\infty \Lambda_k g_k(s+t) \, ds dt.$$

This is the same as:

$$\begin{aligned}
\Phi_k(\tau) &= 1 - \int_0^\infty \Lambda_k \int_\tau^\infty g_k(s+t) \, dt ds \\
&= 1 - \int_0^\infty \Lambda_k \widetilde{\phi}_k(s+\tau) \, ds \\
&= 1 - \int_0^\infty \phi_k(s+\tau) \, ds \\
&= 1 - \int_\tau^\infty \phi_k(s) \, ds.
\end{aligned}$$

The fact that $\Lambda_k = \left(\int_0^\infty \widetilde{\phi}_k(\tau) \, d\tau\right)^{-1}$ implies $\int_0^\infty \phi_k(s) \, ds = 1$, and as a result:

$$\Phi_k(\tau) = \int_0^\tau \phi_k(s) \, ds.$$

Therefore, $\phi_k(\tau)$ is the corresponding density. ∎

Lemma 2 *Let $\tau_k \equiv E_{g_k}[\tau] \equiv \int_0^\infty g_k(\tau)\tau d\tau$ denote the expected duration of price plans (price spells) for newly set price plans (prices) in sector k. Then:*

$$\tau_k = \frac{1}{\Lambda_k}.$$

Proof. Recall that:

$$\widetilde{\phi}_k(\tau) = \frac{\phi_k(\tau)}{\Lambda_k} = \int_\tau^\infty g_k(s)\,ds.$$

So,

$$\begin{aligned}
\phi_k(\tau) &= \Lambda_k \int_\tau^\infty g_k(s)\,ds \\
&= \Lambda_k(1 - G_k(\tau)),
\end{aligned}$$

where

$$G_k(\tau) = \int_0^\tau g_k(s)\,ds.$$

The fact that $\int_0^\infty \phi_k(s)\,ds = 1$ implies $\int_0^\infty \Lambda_k(1 - G_k(s))\,ds = 1$. Integrating by parts yields:

$$\Lambda_k E_{g_k}[\tau] = 1.$$

∎

Lemma 3 *For models with constant adjustment hazards:*

$$g_k(\tau) = \phi_k(\tau).$$

Proof. Multiplying both sides of the above expression by $\Lambda_k(1 - G_k(\tau))$ and using the definition of the adjustment hazard function yields:

$$h_k(\tau)\Lambda_k(1 - G_k(\tau)) = \Lambda_k g_k(\tau).$$

Now, recall that $\Lambda_k(1 - G_k(\tau)) = \phi_k(\tau)$, so that:

$$h_k(\tau)\phi_k(\tau) = \Lambda_k g_k(\tau).$$

With constant adjustment hazards - $h_k(\tau) = h_k$, say - the fact that both $\phi_k(\tau)$ and $g_k(\tau)$ are densities, and therefore integrate to unity, implies $h_k = \Lambda_k$. As a result,

$$g_k(t) = \phi_k(t).$$

∎

Proposition 2 *For a small discount rate* $(\rho \to 0)$*, the expected discounted cumulative effect on the output gap of a persistent shock* $(\gamma \to 0)$ *to the level of nominal aggregate demand equals the economy-wide expected remaining duration of price plans put in place before the shock hit:*

$$\lim_{\rho,\gamma \to 0} \Gamma = \int_{k \in K} E_{\phi_k}[\tau] dF(k),$$

where, as defined previously, $E_{\phi_k}[\tau] \equiv \int_0^\infty \phi_k(\tau)\tau dt.$

Proof.

$$\Gamma = \int_0^\infty e^{-\rho t}(m^*(t) - p(t))\,dt = \int_{k \in K}\left(\int_0^\infty e^{-(\rho+\gamma)t}[1 - \Phi_k(t)]\,dt\right)dF(k).$$

To calculate the inner integral, integrate by parts:

$$\int_0^\infty e^{-(\rho+\gamma)t}[1 - \Phi_k(t)]\,dt = \left[-(1 - \Phi_k(t))\frac{e^{-(\rho+\gamma)t}}{\rho+\gamma}\right]_{t=0}^\infty - \int_0^\infty \frac{e^{-(\rho+\gamma)t}}{\rho+\gamma}\phi_k(t)dt.$$

Note that $\Phi_k(0) = 0$, $\Phi_k(\infty) = 1$. So,

$$\int_0^\infty e^{-(\rho+\gamma)t}[1 - \Phi_k(t)]\,dt = \frac{1}{\rho+\gamma}\left[1 - \int_0^\infty e^{-(\rho+\gamma)t}\phi_k(t)dt\right].$$

Now, note that $\int_0^\infty e^{-(\rho+\gamma)\tau}\phi_k(\tau)d\tau = E_{\phi_k}\left[e^{-(\rho+\gamma)\tau}\right] = \mathcal{M}_{\phi_k}(-(\rho+\gamma))$ is the Moment Generating Function associated with the density $\phi_k(\tau)$. So, the inner integral is:

$$\int_0^\infty e^{-(\rho+\gamma)\tau}\Phi_k(\tau)d\tau = \frac{1}{\rho+\gamma}\left[1 - \mathcal{M}_{\phi_k}(-(\rho+\gamma))\right].$$

As $\rho, \gamma \to 0$, both the numerator and the denominator of the above expression go to zero. We can find the limit using l'Hopital's rule:

$$\lim_{\rho,\gamma \to 0}\int_0^\infty e^{-(\rho+\gamma)\tau}\Phi_k(\tau)d\tau = -\mathcal{M}'_{\phi_k}(0) = \int_0^\infty \tau\phi_k(\tau)d\tau = E_{\phi_k}[\tau].$$

As a result, our measure of non-neutrality is:

$$\lim_{\rho,\gamma\to 0}\Gamma = \int_{k\in K} E_{\phi_k}[\tau]dF(k).$$

∎

Proposition 3 *For a small discount rate* $(\rho \to 0)$, *the expected discounted cumulative effect on the output gap of a persistent shock* $(\gamma \to 0)$ *to the level of nominal aggregate demand is given by:*

$$\lim_{\rho,\gamma\to 0}\Gamma = \frac{1}{2}\int_{k\in K}\frac{\tau_k^2 + \sigma_k^2}{\tau_k}dF(k),$$

where as defined previously $\tau_k \equiv E_{g_k}[\tau] \equiv \int_0^\infty g_k(\tau)\tau dt$ *is the expected duration of a newly set price plan by a firm in sector* k, *and* $\sigma_k^2 \equiv Var_{g_k}[\tau] \equiv \int_0^\infty g_k(\tau)(\tau - \tau_k)^2 d\tau$ *is the variance of the duration of such a plan.*

Proof. From **Proposition 2**:

$$\lim_{\rho,\gamma\to 0}\Gamma = \int_{k\in K} E_{\phi_k}[\tau]dF(k) = \int_{k\in K}\int_0^\infty \tau\phi_k(\tau)d\tau dF(k).$$

Recall that $\phi_k(\tau) = \Lambda_k(1 - G_k(\tau))$, and so our measure of non-neutrality can be written as:

$$\int_{k\in K}\Lambda_k\left(\int_0^\infty \tau(1 - G_k(\tau))d\tau\right)dF(k).$$

Integrating by parts yields:

$$\lim_{\rho,\gamma\to 0}\Gamma = \frac{1}{2}\int_{k\in K}\frac{\tau_k^2 + \sigma_k^2}{\tau_k}dF(k).$$

∎

Proposition 4 *For a small discount rate* $(\rho \to 0)$, *the expected discounted cumulative effect on the output gap of a persistent shock* $(\lambda \to 0)$ *to the growth rate of nominal aggregate demand equals (half) the cross-sectional average of the second moment of the distribution of remaining durations of price plans put in place before the shock hit:*

$$\lim_{\rho,\lambda\to 0}\Gamma = \frac{1}{2}\int_{k\in K} E_{\phi_k}[\tau^2]dF(k),$$

where $E_{\phi_k}[\tau^2] \equiv \int_0^\infty \phi_k(\tau)\tau^2 d\tau$.

Proof.

$$\Gamma = \int_0^\infty e^{-\rho t}\left(m^*\left(t\right) - p\left(t\right)\right)dt = \int_{k \in K}\left(\int_0^\infty \frac{e^{-\rho t} - e^{-(\rho+\lambda)t}}{\lambda}\left[1 - \Phi_k\left(t\right)\right]dt\right)dF\left(k\right).$$

To calculate the inner integral, integrate by parts:

$$\frac{1}{\lambda}\left(\int_0^\infty e^{-\rho t}\left[1 - \Phi_k\left(t\right)\right]dt - \int_0^\infty e^{-(\rho+\lambda)t}\left[1 - \Phi_k\left(t\right)\right]dt\right)$$

$$= \frac{1}{\lambda}\left(\left[-\left(1 - \Phi_k\left(t\right)\right)\frac{e^{-\rho t}}{\rho}\right]_{t=0}^\infty - \int_0^\infty \frac{e^{-\rho t}}{\rho}\phi_k(t)dt\right)$$

$$-\frac{1}{\lambda}\left(\left[-\left(1 - \Phi_k\left(t\right)\right)\frac{e^{(\rho+\lambda)t}}{\rho+\lambda}\right]_{t=0}^\infty - \int_0^\infty \frac{e^{-(\rho+\lambda)t}}{\rho+\lambda}\phi_k(t)dt\right).$$

Note that $\Phi_k(0) = 0$, $\Phi_k(\infty) = 1$. So, the above expression simplifies to:

$$\frac{1}{\lambda}\left[\left(\frac{1}{\rho} - \int_0^\infty \frac{e^{-\rho t}}{\rho}\phi_k(t)dt\right) - \left(\frac{1}{\rho+\lambda} - \int_0^\infty \frac{e^{-(\rho+\lambda)t}}{\rho+\lambda}\phi_k(t)dt\right)\right].$$

Using the Moment Generating Function $\mathcal{M}_{\phi_k}$, the previous expression can be written as:

$$\frac{1}{\lambda\rho}\left[1 - \mathcal{M}_{\phi_k}\left(-\rho\right)\right] - \frac{1}{\lambda\left(\rho+\lambda\right)}\left[1 - \mathcal{M}_{\phi_k}\left(-\left(\rho+\lambda\right)\right)\right]$$

$$= \frac{\left(\rho+\lambda\right)}{\lambda\rho\left(\rho+\lambda\right)}\left[1 - \mathcal{M}_{\phi_k}\left(-\rho\right)\right] - \frac{\rho}{\lambda\rho\left(\rho+\lambda\right)}\left[1 - \mathcal{M}_{\phi_k}\left(-\left(\rho+\lambda\right)\right)\right]$$

$$= \frac{\left[\rho + \lambda - \left(\rho+\lambda\right)\mathcal{M}_{\phi_k}\left(-\rho\right)\right] - \left[\rho - \rho\mathcal{M}_{\phi_k}\left(-\left(\rho+\lambda\right)\right)\right]}{\lambda\rho\left(\rho+\lambda\right)}.$$

As $\lambda \to 0$, both the numerator and the denominator in the above expression go to zero. So, we can find the limit using l'Hopital's rule:

$$\lim_{\lambda\to 0}\frac{\left[\rho + \lambda - \left(\rho+\lambda\right)\mathcal{M}_{\phi_k}\left(-\rho\right)\right] - \left[\rho - \rho\mathcal{M}_{\phi_k}\left(-\left(\rho+\lambda\right)\right)\right]}{\lambda\rho\left(\rho+\lambda\right)}$$

$$= \frac{1 - \mathcal{M}_{\phi_k}\left(-\rho\right) - \rho\mathcal{M}'_{\phi_k}\left(-\rho\right)}{\rho^2}.$$

Next, using l'Hopital's rule once more to take the limit of the above expression as $\rho \to 0$, yields:

$$\lim_{\rho\to 0}\frac{1 - \mathcal{M}_{\phi_k}\left(-\rho\right) - \rho\mathcal{M}'_{\phi_k}\left(-\rho\right)}{\rho^2} = \frac{1}{2}\mathcal{M}''_{\phi_k}\left(0\right) = \frac{1}{2}\int_0^\infty \tau^2\phi_k(\tau)dt = \frac{1}{2}E_{\phi_k}[\tau^2].$$

The order in which the limits are taken does not matter in this case, and as a result our measure of non-neutrality is equal to:

$$\lim_{\rho,\lambda\to 0}\Gamma = \frac{1}{2}\int_{k\in K}E_{\phi_k}[\tau^2]dF(k).$$

■

Proposition 5 *For a small discount rate $(\rho \to 0)$, the expected discounted cumulative effect on the output gap of a persistent shock $(\lambda \to 0)$ to the growth rate of nominal aggregate demand equals*

$$\lim_{\rho,\lambda\to 0}\Gamma = \frac{1}{6}\int_{k\in K}\tau_k^2 + 3\sigma_k^2 + \frac{Skew_{g_k}[\tau]}{\tau_k}dF(k),$$

where $Skew_{g_k}[\tau] = \int_0^\infty g_k(\tau)(\tau-\tau_k)^3 d\tau$ is the skewness of the distribution of the duration of a newly set price plan (price) by a firm in sector k.

Proof. From **Proposition 4**:

$$\lim_{\rho,\lambda\to 0}\Gamma = \frac{1}{2}\int_{k\in K}E_{\phi_k}[\tau^2]dF(k) = \frac{1}{2}\int_{k\in K}\int_0^\infty \phi_k(\tau)\tau^2 d\tau dF(k).$$

Recall that $\phi_k(\tau) = \Lambda_k(1 - G_k(\tau))$, and thus our measure of non-neutrality can be written as:

$$\lim_{\rho,\lambda\to 0}\Gamma = \int_{k\in K}\frac{\Lambda_k}{2}\left(\int_0^\infty \tau^2(1-G_k(\tau))d\tau\right)dF(k).$$

Integrating by parts and recalling that $\Lambda_k^{-1} = \tau_k$ yields:

$$\lim_{\rho,\lambda\to 0}\Gamma = \frac{1}{6}\int_{k\in K}\tau_k^2 + 3\sigma_k^2 + \frac{Skew_{g_k}[\tau]}{\tau_k}dF(k),$$

where $Skew_{g_k}[\tau] = \int_0^\infty g_k(\tau)(\tau-\tau_k)^3 d\tau$. ■

Cross sectional distribution based on Bils and Klenow (2004)

We group the different categories of goods and services listed in the appendix in Bils and Klenow (2004) into a number of representative frequencies of price changes, and then map each such frequency into a sector in the models. Recall that we use the data to calibrate frequencies of information updating in the sticky information model as well, although the data refers to frequencies of price changes.

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For the sticky information model, we simply group all items that display the same monthly frequency of adjustment. This results in 142 sectors.[22] We take α_k, the monthly frequency of adjustment reported for the categories identified with sector k, and compute the expected duration of price plans according to $\tau_k = \frac{-1}{\ln(1-\alpha_k)}$. The underlying rate of arrival of information updating dates in the continuous time constant hazard model is then such that $\alpha_k = 1 - e^{-\Lambda_k}$. That is, $\Lambda_k = -\ln(1-\alpha_k) = \tau_k^{-1}$. Finally, we set each sectoral weight equal to the sum of the CPI weights for the categories that have that given frequency of price changes, and renormalize so that sectoral weights add up to one. This approach yields the sample statistics presented in Table 1.

Table 1: Moments of the Distribution of the Frequency of Price Changes

Description	Formula	Months
"Inverse average frequency" duration of price ridigity	$\left(\overline{\Lambda}\right)^{-1}, \overline{\Lambda} = \sum_{k=1}^{142} f(k)\Lambda_k$	2.9
"Median frequency based" duration of price ridigity*	$\Lambda_{med}^{-1} = \frac{-1}{\ln(1-\alpha_{med})}$	4.3
Average duration of price rigidity**	$\overline{\tau} = \sum_{k=1}^{142} f(k)\tau_k,\ \tau_k = \Lambda_k^{-1}$	6.6
Standard deviation of durations of price rigidity***	$\left(\sum_{k=1}^{142} f(k)(\tau_k - \overline{\tau})^2\right)^{1/2}$	7.1

Obs: Based on the statistics reported by Bils and Klenow (2004). *α_{med} denotes the weighted median frequency of price changes in their data. *Technically, τ_k is the expected duration of price spells in sector k. So, this is actually the cross-sectional average of the expected durations of price spells.**This is the cross-sectional standard deviation of the expected sectoral durations of price spells.

For the Taylor pricing model our approach is slightly different, because with complementarities the model has to be solved numerically, and the state space grows fast in the number of sectors (quadratically). As a result, solving the model with as many sectors becomes infeasible. To circumvent this problem we construct the distribution of contract lengths from the BK data in a slightly different way. We discretize the model to apply standard methods for solving discrete time linear rational expectations models, taking a time period to be one month. We consider contract lengths which are multiples of one month, and aggregate the goods and services categories so that the ones which have an expected duration of price spells (τ_k as computed above) between zero and one month (inclusive) are assigned to the one month contract length sector; the ones with an expected duration of price spells between one (exclusive) and two months (inclusive) are assigned to the two month contract length sector, and so on. The sectoral weights are aggregated

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[22]The dynamics of this economy is identical to that of the economy calibrated directly with the original 350 categories analyzed by Bils and Klenow (2004). This grouping by common frequencies simply makes computations more efficient.

accordingly. We proceed in this fashion until the sector with contract lengths of 24 months. Finally, we aggregate all the remaining categories, which have mean durations of price rigidity between 24 and 80 months, into a sector with 25-month contracts.[23] This gives rise to 25 sectors with an "inverse average frequency duration of price rigidity" of 3.45 months, an average contract length of 6.7 months, and a standard deviation of contracts of 5.6 months.

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[23]The total weight of these categories is approximately 2%.

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Figures 1(a-f): Sticky Information (Mankiw and Reis, 2002)

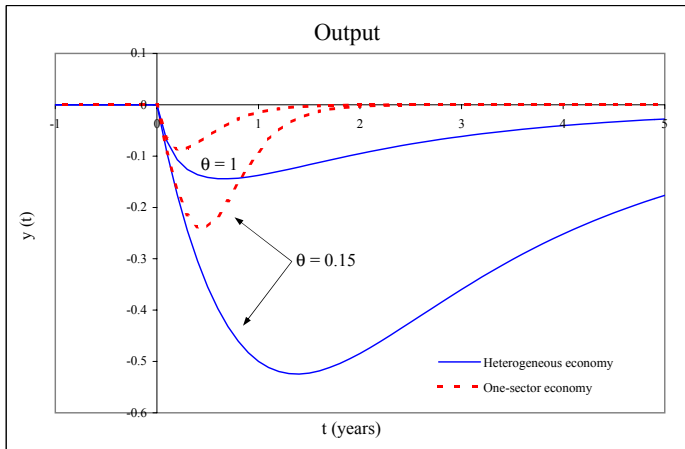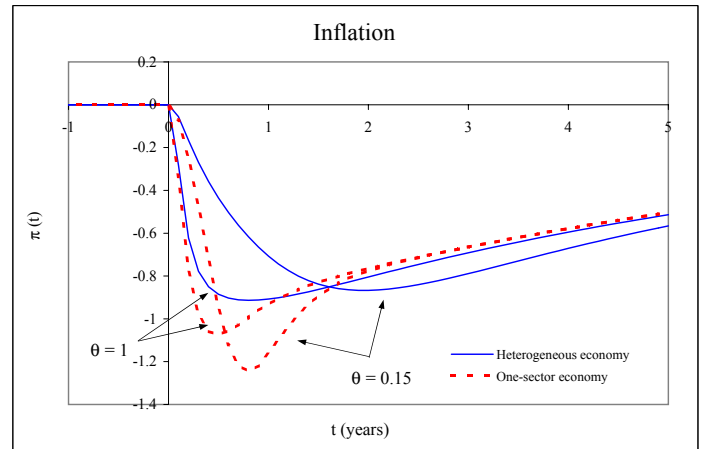## Persistent Growth Rate Shock: λ = 0.14 (half life = 5 years)

Figure 1a

Figure 1b

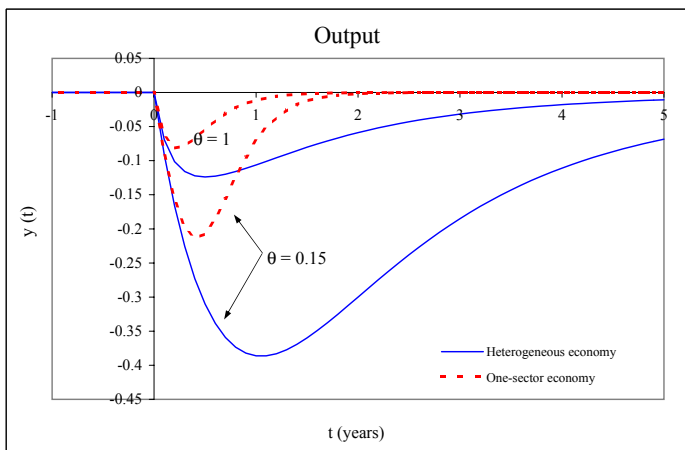## Temporary Growth Rate Shock: λ = 0.7 (half life = 1 year)
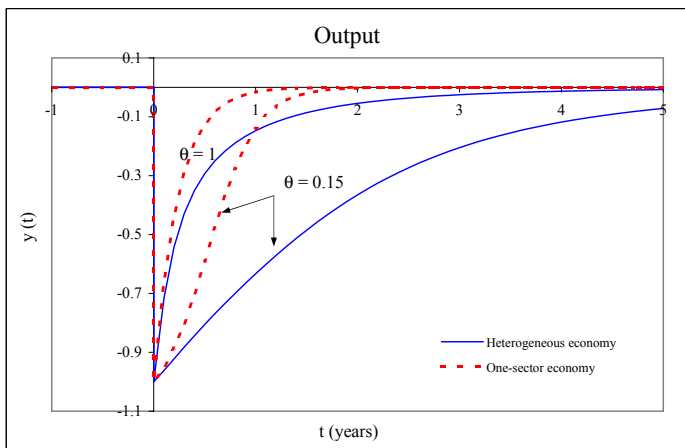

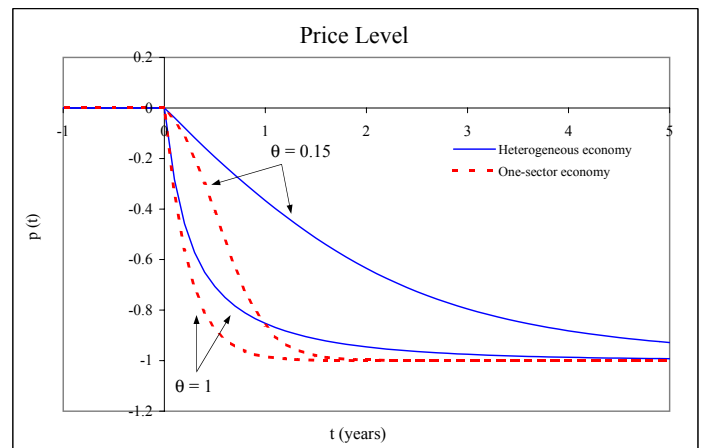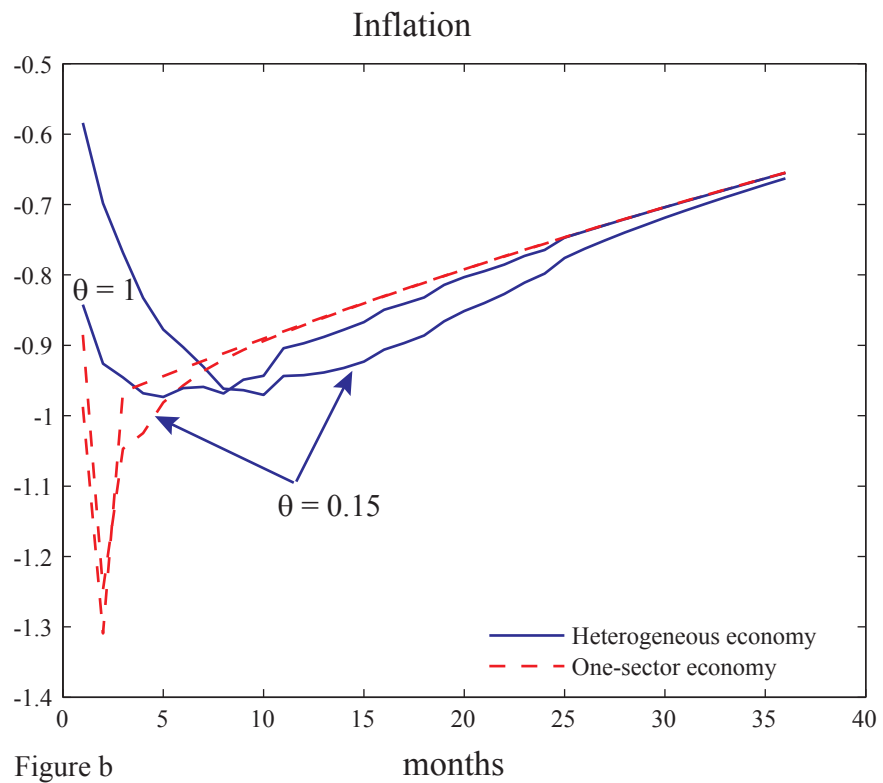Figure 1c
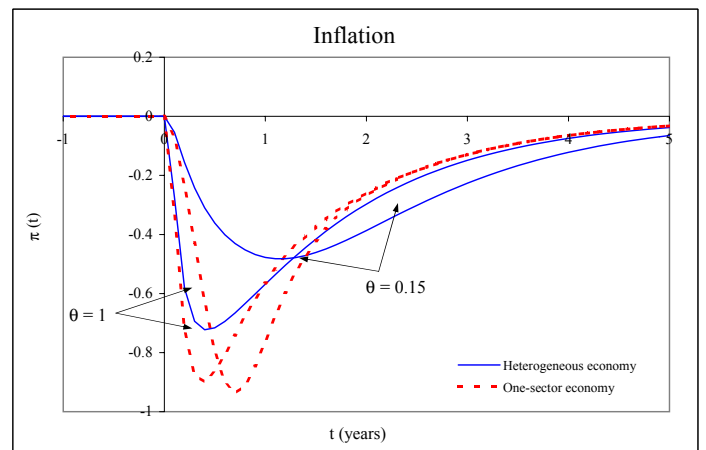
Figure 1d

Permanent Level Shock: γ = 0

Figure 1e

Figure 1f

Figures 2 (a-b): Taylor Pricing
Growth Rate Shock: Half life = 5 years

Output

θ = 1

θ = 0.15

Heterogeneous economy
One-sector economy

months

Figure a

Inflation

θ = 1

θ = 0.15

Heterogeneous economy
One-sector economy

months
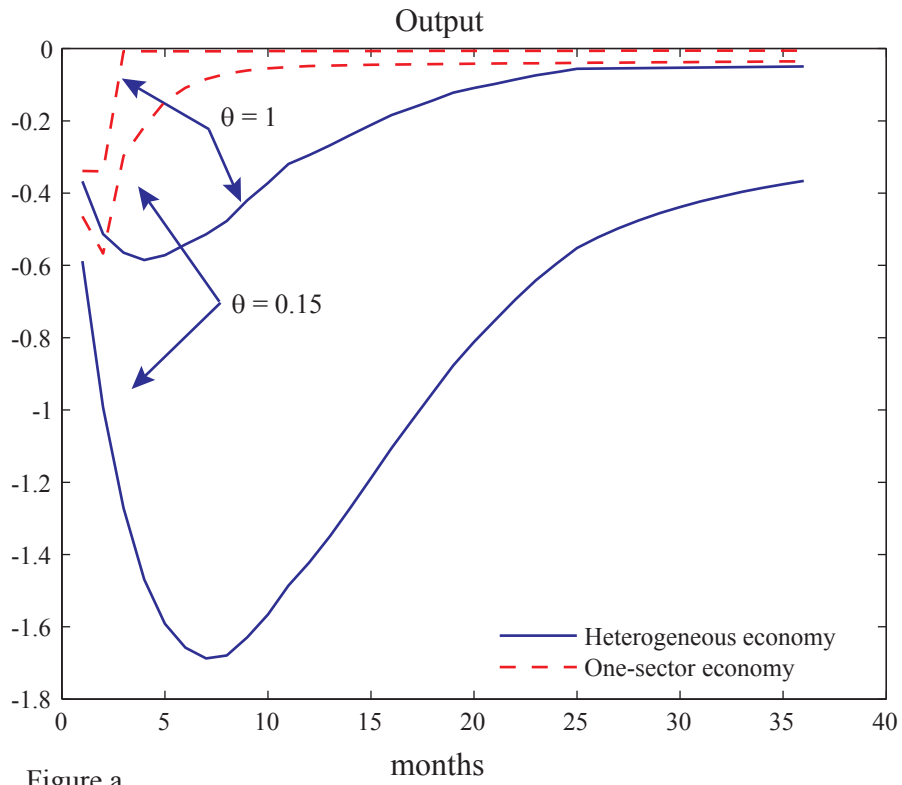
Figure b

Figures 2 (c-d): Taylor Pricing
Growth Rate Shock: Half life = 1 year

Output

θ = 1

θ = 0.15

— Heterogeneous economy
- - - One-sector economy

months

Figure c

Inflation

θ = 1

θ = 0.15

— Heterogeneous economy
- - - One-sector economy

months

Figure d