# Determining the Drag Coefficient from PITCHf/x Data

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It is argued that the drag coefficient is a useful measure of the integrity of the PITCHf/x system. A quick technique is described for extracting the drag and lift coefficients and the spin axis from PITCHf/x data.

## I. WHY DO WE CARE ABOUT DRAG COEFFICIENTS?

When a baseball travels through the air, it experiences various forces, shown in Fig. 1, and it is these forces that determine the trajectory of the baseball. The most familiar of these forces is the downward pull of gravity $F_G$. Less familiar are the aerodynamic forces, namely the drag force $F_D$ and the Magnus force $F_M$. The drag force, or in everyday language "air resistance," is due to the fact that the ball has to push the air out of the way. From a more microscopic point of view, the ball collides with air molecules, transferring energy to those molecules and losing energy in the process.[1] Thus the ball slows down. The conventional way to express the magnitude of $F_D$ is through the expression

$$F_D = \frac{1}{2}\rho A C_D v^2 \,, \tag{1}$$

where $A$ is the cross sectional area of the ball and $\rho$ is the density of the air. The direction of the drag is exactly opposite to the direction of the velocity, so that the force always retards the motion. The factor $C_D$ is called the drag coefficient. If the baseball is spinning, it also experiences the Magnus force $F_M$, which is conventionally written as

$$F_M = \frac{1}{2}\rho A C_L v^2 \,, \tag{2}$$

where $C_L$ is called the lift coefficient. The direction of the Magnus force is always perpendicular both to the velocity and the spin axis and is in the direction that the leading edge of the ball is turning.

Aerodynamics and fluid dynamics experts tell us that the drag coefficient mainly depends on the velocity of the ball. Since it is possible to extract both the velocity and $C_D$ from PITCHf/x data, as will be shown below, then the plot of $C_D$ versus velocity ought to be something like a universal curve, showing very little variation from pitch to pitch, from pitcher to pitcher, from game to game, or from venue to venue. Of course, the pitch to pitch variation in $C_D$ will have lots of fluctuations due to a variety of factors, such as small variations in the ball mass and radius, effects of spin on the drag coefficient,[2] seam orientation, and the inevitable random measurement errors.[3] However, when $C_D$ values are averaged over many pitches–e.g., a complete game–one might expect considerably less fluctuation. The drag coefficients can then serve as a useful practical tool to study measurement biases in the PITCHf/x system itself. We can ask questions such as the following:

- Are the $C_D$ values at venue A the same as those at venue B?
- Are the $C_D$ values measured in May the same as those measured in September?
- Are the $C_D$ values for pitcher X the same as those for pitcher Y?

The expectation is that the answer to all these questions is "yes."[4]

One can also extract lift coefficients for the data, do the same type of averaging, then ask the same questions. In this case, however, the expectation is that the answers are "no," the reason being that the lift coefficients depend not only on the velocity (which is measured) but on the spin (which is not measured). For a given velocity, we expect variation in $C_L$ due to unknown variations in the spin magnitude.

## II. BACKGROUND ABOUT THE PITCHF/X SYSTEM

The PITCHf/x system uses two cameras to track pitches between pitcher and batter, determining the coordinates of the ball at 1/60-sec intervals. The resulting trajectory x(t),y(t),z(t) (t is the time) is fit to a nine-parameter (or "9P") function corresponding to constant acceleration in each of the three dimensions. All quantities reported in the PITCHf/x data base, such as the pitch speed, the location of the pitch as it crosses the plate, the "break" of the pitch, etc., are derived from the fitted trajectory rather than from the original data. The nine parameters are the three initial positions $x_0$, $y_0$, and $z_0$; the three initial velocities $v_{x0}$, $v_{y0}$, $v_{z0}$; and the three accelerations $a_x$, $a_y$, and $a_z$. Here the coordinates refer to the usual PITCHf/x coordinate system, where the origin is at the point of home plate, $\hat{y}$ points towards the pitcher, $\hat{z}$ points vertically upward, and $\hat{x} = \hat{y} \times \hat{z}$ (i.e., the $x$ axis points to the catcher's right). The 9P fit is an approximation to the actual equations of motion, which are determined from gravity and the aerodynamic forces, Eqs. 1 and 2:

$$\begin{aligned}
\ddot{x} &= -KC_D v v_x \;-\; KC_L v v_y \sin\phi \\
\ddot{y} &= -KC_D v v_y \;+\; KC_L v\,(v_x \sin\phi - v_z \cos\phi) \\
\ddot{z} &= -KC_D v v_z \;+\; KC_L v v_y \cos\phi \;-\; g\,.
\end{aligned} \tag{3}$$

Here $g$ is the acceleration due to gravity (32.174 ft/s$^2$) and $K$ is a numerical factor that is discussed below. In these expressions, the spin axis is assumed to lie in the $x-z$ plane and makes an angle $\phi$ with the $x$ axis, with a sign such that $\phi = 90°$ corresponds to the spin pointing upward, along the $z$ axis. Noting that $v_y$ is negative, it is easy to see that the Magnus force makes an angle $\theta = \phi - 90°$ with the $x$ axis. Therefore $\phi = 0°$ (topspin) results in a downward acceleration, and $\phi = 90°$ (sidespin) results in an acceleration to the catcher's right, exactly as expected.

The factor $K$ is given by

$$K = \frac{1}{2}\frac{\rho A}{m}\,, \tag{4}$$

where $m$ is the mass of the ball. MLB specifies that $m$ is in the range $5 - 5\frac{1}{4}$ oz and that the circumference $C$ is in the range $9 - 9\frac{1}{4}$ inches. Moreover, the density of air is nominally 1.225 kg/m$^2$ (or 0.0767 lb/ft$^3$).[5] Therefore, $K$ can be expressed as follows:

$$K = 5.509 \times 10^{-3}\,\mathrm{ft}^{-1} \left[\frac{C}{9\frac{1}{8}\,\mathrm{in}}\right]^2 \left[\frac{m}{5\frac{1}{8}\,\mathrm{oz}}\right] \left[\frac{\rho}{0.0767\,\mathrm{lb/ft}^3}\right]. \tag{5}$$

When extracting $C_D$ from the trajectory data, it is important to use the correct value of $K$. In particular, it is important to use the correct value of $\rho$, which can vary depending on temperature, on relative humidity, and most importantly on altitude.[4] The natural variation in the ball circumference and mass will lead to variation in the inferred $C_D$ if the nominal values of circumference and mass are used.

## III. EXTRACTING $C_D$ AND $C_L$

The goal is to determine $C_D$ and $C_L$ directly from the 9P fit to the trajectory, without resorting to a more complicated fitting procedure. To do this first requires removing the effect of gravity from $a_z$, which is easily done. For future reference, we define $a_{z1}$ to be the z component of the acceleration with gravity removed:

$$a_{z1} = a_z + g\,. \tag{6}$$

Referring to Eq. 3, the next step is to unravel the effects of drag and Magnus from the various terms. In the absence of a full-scale fitting procedure, we resort to an approximation scheme. We give the result first, then discuss the nature of the approximation and estimate the size of the omitted terms. We first write down approximate expressions for the x and z components of the Magnus acceleration:

$$\begin{aligned} a_{xM} &\equiv a_x - a_y\frac{< v_x >}{< v_y >} \approx -KC_L v v_y \sin\phi \\ a_{zM} &\equiv a_{z1} - a_y\frac{< v_z >}{< v_y >} \approx KC_L v v_y \cos\phi\,, \end{aligned} \tag{7}$$

where the brackets indicate the time average over the full trajectory. We next write down approximate expressions for the x, y, and z components of the drag acceleration:

$$\begin{aligned} a_{xD} &\equiv a_x - a_{xM} \approx -KC_D v_y v_x \\ a_{yD} &\equiv a_y + a_{xM}\frac{< v_x >}{< v_y >} + a_{zM}\frac{< v_z >}{< v_y >} \approx -KC_D v_y^2 \\ a_{zD} &\equiv a_{z1} - a_{zM} \approx -KC_D v_y v_z \end{aligned} \tag{8}$$

The rationale behind this approximation is found in recognizing that for a pitched baseball, $|v_x/v_y| \ll 1$ and $|v_z/v_y| \ll 1$. In the exact expressions for the accelerations, there is a hierarchy in the order of magnitude of the various terms, depending on which velocities appear in the term. The largest are those terms proportional to $v_y^2$, the so-called "leading order" terms (LO). Next largest are those terms proportional to $v_y v_x$ or $v_y v_z$, the "next-to-leading order" terms (NLO). The smallest are those terms proportional to $v_x^2$ or $v_z^2$, the "next-to-next-to-leading order" terms (NNLO). The approximation in Eqs. 7-8 is to keep the LO and NLO terms and to neglect the NNLO terms. An upper limit for $|v_x/v_y|$ or $|v_z/v_y|$ is approximately 0.1, so we can expect the size of the omitted terms to less be than 1% of the terms retained. In the next section, we will see how well this approximation works in practice.

With these approximations, it is straightforward to find the drag and lift coefficients:

$$
\begin{aligned}
C_D &\approx \frac{\sqrt{a_{xD}^2 + a_{yD}^2 + a_{zD}^2}}{K < v_y >< v >} \\
C_L &\approx \frac{\sqrt{a_{xM}^2 + a_{zM}^2}}{K < v >^2}
\end{aligned} \tag{9}
$$

To get the time-averaged velocities, first $v_{yf}$, the $y$ component of velocity at $y_f = 1.417$ ft, is calculated. Then the total flight time $T$ is calculated. Then $a_x$ and $a_z$ are used to calculate $v_{xf}$ and $v_{zf}$. Finally, the initial and final velocities are used to calculate the time-averaged values. The particular sequence of equations is as follows:

$$
\begin{aligned}
v_{yf} &= -\sqrt{v_{y0}^2 + 2a_y(y_f - y_0)} \\
T &= \frac{v_{yf} - v_{y0}}{a_y} \\
v_{xf} &= v_{x0} + a_x T \\
v_{zf} &= v_{z0} + a_z T\,,
\end{aligned} \tag{10}
$$

and

$$< v_x >= (v_{xf} + v_{x0})/2 \tag{11}$$

and similarly for $< v_y >$ and $< v_z >$. Finally $< v >$ is found by noting that $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ and $< v >= (v_0 + v_f)/2$. The angle $\phi$ of the spin axis can be found approximately from

$$\phi = \arctan\left(\frac{a_{zM}}{a_{xM}}\right) + 90^\circ\,. \tag{12}$$

## IV. COMPARISON WITH EXACT VALUES

A total of 7763 pitches from the early part of the 2007 season for games in Toronto were analyzed. A least-squares fitting procedure was used to extract exact values of $C_D$ and $C_L$ from each of 7763 pitched-ball trajectories. The formulas in the preceding section were used to determine approximate values. The results are given in Fig. 1. We see that the approximate values are mostly within 1% of the exact values, albeit systematically low by a few tenths percent. We conclude that the approximate expressions for $C_D$ and $C_L$ are adequate for most purposes.

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[1] More precisely, the ball transfers momentum to the air molecules, losing momentum (and therefore speed) in the process.

[2] It is well known that there is a small dependence of $C_D$ on the spin. Little is known about this effect for baseball.

[3] For a study of the effect of random measurement errors on extracted valued of $C_D$, see `http://webusers.npl.uiuc.edu/ a-nathan/pob/MCAnalysis.pdf`.

[4] There is an additional technical detail that readers need to be aware of. Namely theory actually says that $C_D$ should depend on a quantity known as the Reynold's number, which is proportional to $\rho v$. Therefore, when comparing $C_D$ values at different venues, they should be compared not at the same value of $v$ but at the same value of $\rho v$. For example, the value of $C_D$ at velocity $v$ at sea level should be the same as the value of $C_D$ at $v/0.82$ in Denver, since the air density in Denver is 0.82 of that at sea level.

[5] The value $\rho_0$=1.225 kg/m$^3$ (0.0767 lb/ft$^3$) for the air density is appropriate for sea level and a temperature of 288.16○K (15°C or 59°F). To find the air density at another temperature or elevation, use the expression $\rho = \rho_0(T/288.16)\exp(-z/26250)$, where T is the Kelvin temperature and z is elevation in ft. Note that the Kelvin temperature T is found from the Fahrenheit temperature F by T=5(F-32)/9+273.16.

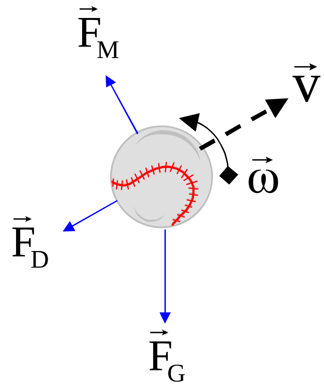

FIG. 1: Forces on a baseball in flight, including gravity ($F_G$), drag ($F_D$), and the Magnus force ($F_M$).

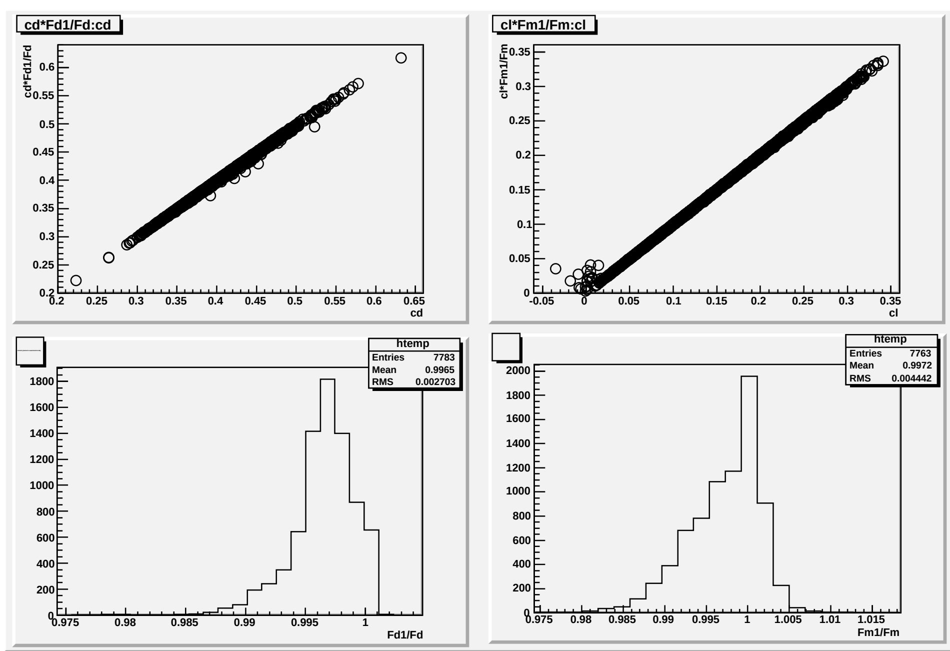


FIG. 2: Comparison between approximate and exact values of the drag and lift coefficients for actual PITCHf/x pitches. The upper panels are scatterplots of approximate vs. exact values of $C_D$ (left) and $C_L$ (right). The bottom panels are histograms of the ratios of approximate to exact values for $C_L$ (left) and $C_L$ (right).