

# Energy-efficient Acquisition and High-fidelity Reconstruction of Compressed Signal in Wireless Sensor Networks

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**ABSTRACT.** *This paper considers compressed acquisition and advanced reconstruction of sensor signals via compressive sensing in wireless sensor networks. Firstly, we construct a CS-based signal acquisition scheme that exploits the spatial and temporal correlations of sensor signals to reduce the energy consumption of networks. The scheme divides the network into several clusters, each node in each cluster decides whether or not to sample and transmit its signals to fusion center, with a certain probability. In order to minimum the energy consumption of the whole network, we study the energy consumption of the proposed scheme, and then work out the relationship between the number of clusters and the transmit probability. Secondly, we propose a reweighted  $l_1$  - norm minimization algorithm to reconstruct the original signal by selecting weight adaptively. The algorithm integrate the re-weighting procedure into each iteration, i.e., the weights change according to the changes of solution in each iteration process. This method promote the solution has the same sparsity structure which is present in the original signal. The simulation results show that our algorithm has a better performance in reconstruction accuracy and computation complexity.*

**Keywords:** Compressive sensing, Wireless sensor networks, Spatial and temporal correlation, Signal acquisition, Sparse signal reconstruction

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1. **Introduction.** Wireless sensor networks (WSNs) consisting of a large number of small and low-cost sensor nodes (SNs) are well-suited for various monitoring and measuring tasks in environmental, industrial, health and military applications [1, 2, 3, 4]. To accomplish these targeted applications, SNs have to collect and transmit a tremendous amount of real-time data over their lifetime. Due to the stringent energy constraints, weak computing ability and small storage capacity of SNs [5], we need to establish an efficient data acquisition and transmission scheme to reduce the cost of information acquisition and prolong the lifetime of WSNs.

As an economical data acquisition theory, compressed sensing (CS) [6, 7, 8] provides a new data acquisition approach for WSNs. Applying CS theory to traditional WSNs, SNs can realize data acquisition in a compressed way with no requirement of additional computational overhead [9, 10]. In this way, the amount of data transmission could be greatly reduced, so does the consumption of network energy. Energy has always been an important factor limiting the life cycle of WSNs. Moreover, SNs only afford the calculation of the compression part, which satisfies its characteristics of limited processing capability.

And the reconstruction part with high computational complexity is carried out on the terminal computer, which is without the limitation of computing capacity and energy.

CS has established a promising foundation for developing efficient data aggregation methods in WSNs. The conventional CS approach is achieved by random projections in the time domain, which does not suffice to efficiently recovery the signal [6]. And the new approach that exploiting the spatial and temporal correlations of multi-signal can obviously decrease the necessary data transmission quantity. Two significance frameworks for multi-signal ensembles are distribute compressive sensing (DCS) [11] and Kronecker compressive sensing (KCS) [12]. DCS was proposed to model and exploit certain types of intra-signal and inter-signal dependencies via joint sparsity models, while KCS was recently introduced to exploit more general correlation patterns by combining the possibly distinct sparsifying bases from each signal dimension into a single basis matrix. DCS and KCS have been shown to outperform single-dimensional CS approaches in terms of compression performance and sensor energy consumption. Chen et al. [13] proposed a Frchet mean based CS approach which leveraged both intra-signal and inter-signal correlation to reduce the number of samples required for reconstruction of the original signal. However, the reconstruction algorithm they proposed needed prior knowledge of the signal sparsity, which was unrealistic, and they did not analyze the energy consumption for the proposed data gathering approach. Quer et al. [14] proposed a new approach for the online recovery of large data sets in WSNs using jointly CS and Principal Component Analysis (PCA). The role of PCA was to capture the spatial and temporal characteristics of real signals. They also make their framework self-adapt to the changes in the signal statistics owing to a feedback control loop that estimates the signal construct error. But this paper neglected the procedure of compression which was taken place in the SNs.

The goal of this paper is twofold. First, considering the signals detected by a group of SNs have spatial and temporal correlations, we construct a CS-based signal acquisition scheme which takes into fully account this characteristic to reduce the networks energy consumption, and thereby, to prolong the lifetime. Second, we propose a reweighed  $l_1$ -norm minimization algorithm via GPSR (Gradient Projection for Sparse Reconstruction [15]) to reconstruct the original signal from noisy measurements, which are acquired by the former constructed CS-based signal acquisition scheme.

The remainder of this paper is organized as follows. Sect. 2 covers the mathematical background for CS and gives several necessary assumptions for a considered WSN. Sect. 3 presents an energy-efficient CS-based data acquisition scheme. Sect. 4 proposes an adaptive reweighing via GPSR algorithm. Sect. 5 provides simulation experiments that compare performance of our proposed algorithm against other algorithms. Finally, Sect. 6 concludes this paper.

**2. Preliminaries.** In this section a short mathematical background on CS is given. Then we make a brief assumption for a considered WSN.

**2.1. Compressed sensing.** Let  $f \in \mathbb{R}^N$  be a real-valued signal vector and it can be represented in a basis  $\Psi \in \mathbb{R}^{N \times N}$  as  $f = \Psi x$ .  $x$  is side to be  $S$ -sparse in basis  $\Psi$  if  $x$  has  $S \ll N$  non-zero entries. While for natural signals not perfectly sparse, they are said to be compressible if the energy of coefficient in  $x$  is concentrated in a relatively small set of entries, and they exhibit a power law decay [16] as  $|x_{(i)}| \leq C_p i^{-p}$ , for all  $i = 1, \dots, N$ , where  $p \geq 1$  affects the rate of decay and  $C_p$  is a constant depending only on  $p$ .

CS theory demonstrates that this kind of signals can be accurately reconstructed from  $M < N$  liner measurements  $y \in \mathbb{R}^M$  as

$$y = \Phi f = \Phi \Psi x = Ax \quad (1)$$

where  $\Phi \in \mathbb{R}^{M \times N}$  is the measurement matrix and  $A \in \mathbb{R}^{M \times N}$  is the sensing matrix.

The problem to recover the signal representation  $x$  from the measurements  $y$  is ill-posed as the number of equations  $M$  is smaller than the number of variables  $N$ . [17] shows that the solution for (1) can be achieved using the  $l_0$  - norm minimization problem. The recovery procedure corresponds to the problem given by:

$$\hat{x} = \arg \min \|x\|_0 \quad s.t. \ y = Ax \quad (2)$$

However, since (2) is an NP-hard problem, it is not feasible to solve it, especially for the reconstruction of multi-dimension signal. Instead we consider its best convex approximation, the  $l_1$  - norm minimization problem [18]:

$$\hat{x} = \arg \min \|x\|_1 \quad s.t. \ y = Ax \quad (3)$$

which can be solved efficiently via linear or quadratic programming techniques.

It is by now well-known that under the Restricted Isometry Property (RIP) of sensing matrix  $A$  and the incoherence between  $\Psi$  and  $\Phi$  [19], both (2) and (3) have the same unique solution. [20] demonstrates that a random Gauss matrix being highly incoherent with any  $\Psi$ . Therefore, we choose random Gauss matrix as the measurement matrix.

**2.2. Description of considered WSN and sensor signals.** We consider a single-sink multi-hop data gathering WSN, which consists of  $K$  battery-powered sensors, capable of acquiring, transmitting and receiving data. The sensors are densely deployed in an event area to periodically monitor physical phenomena at a pre-defined rate. And the acquired information to be sent and individually reconstructed by a fusion center (FC). In order to optimize network model to further cut down the volume of data transmitted, we propose a cluster based model for a data acquisition scheme in WSN, more details are given in section 3 below.

$f_i = (f_i^{(1)}, \dots, f_i^{(n)})^T$  denotes the sampling vector of the  $i$ th SN at a sampling instant, then  $f_i$  is an  $n$ -dimensional discrete vector which is time-related. For this signal, data at time-adjacent point are highly correlated, which, actually, is the reason why the signal can be compressed in the time domain. Furthermore, two SNs adjacent in distance share one monitoring environment, which lead to the signals of these two SNs are highly correlated as well. Thus, for WSN, especially for that of densely placed SNs, it is reasonable for us to get fully use of spatial-temporal correlations of signals to reduce the amount of data transferred via network. According to [21, 22], the network energy consumption of signal transmitting and receiving is far larger than that of signal acquisition. So it is of great significance to reduce the signal transmit quantity via network.

Actually, the signal sent by an SN is noisy. For the signal  $f_i$ , we assume that its sparsifying basis is  $\Psi_i$ , sparse representation is  $x_i$ , measurement matrix is  $\Phi_i$ , then the measurement  $y_i$  can be described as:

$$y_i = \Phi_i f_i + n_i = \Phi_i \Psi_i x_i + n_i \quad (4)$$

where  $n_i$  represents the noise level of measurement and each entry in  $n_i$  is selected as i.i.d.  $N(0, \sigma^2)$ .

It is indicated in [23] that many signals such as real-word audio and video images or biological measurements are sparse. The selection of sparsifying basis is not the emphasis of this paper. We assume that the sparsifying basis used here is proper.

### 3. Sensor signals acquisition.

**3.1. CS-based signal acquisition scheme.** Considering the spatial-temporal correlations of sensor signals, we present a CS-based signal acquisition scheme for a clustered WSN model to achieve the most information at the minimal expense of energy consumption.

Assume that  $K$  SNs are randomly and evenly deployed in the monitoring area, which covers  $L \times L m^2$ . The whole WSN is equally divided into  $W$  clusters, each cluster selects the SN having the most energy as a cluster head, as show in FIGURE.1. In an arbitrary sampling cycler, each member node independently decides whether or not to collect and transmit its signal to FC, along with sampling cycle and node ID, with probability  $p_{tx} \in (0, 1)$ . Besides, to insure the globality of monitoring information, the cluster heads with enough energy participate in sampling at every cycle. In this way, it can be achieved for SNs to sleep and work periodically; consequently, network energy can be balanced well.

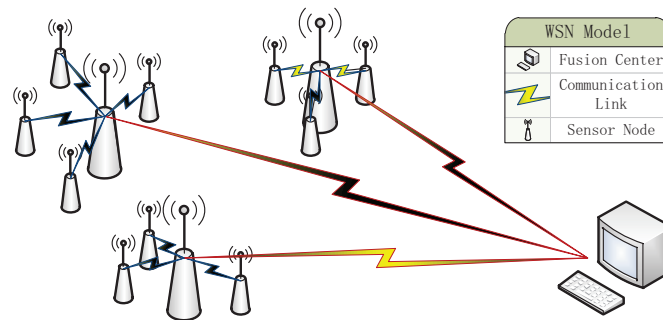


FIGURE 1. Clustered WSN Model

**3.2. Energy consumption analysis.** We use the first-order radio model [24] to analyze energy consuming of our compressed data acquisition scheme. The energy consumption of one SN is categorized into transmit message radio, receive message radio and message fuse expenditures. As shown in FIGURE.2, the transmit consumptions are mainly in operating the transmitter electronics and the transmitter amplifier; the receive consumptions are mainly in operating the receiver electronics.

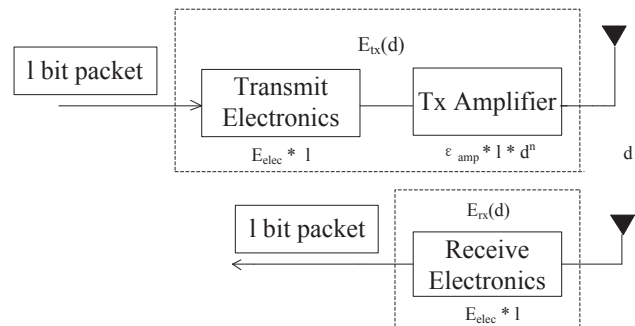


FIGURE 2. First Order Radio Model

According to the distance  $d$  between a transmitting node and a receiving node, we separately use free-space model ( $d^2$  power loss) and multi-path fading model ( $d^4$  power loss) to to calculate energy consumption for communication. Energy required to transmit

an  $l$  – bit message over a distance  $d$  is

$$E_{tx}(l, d) = \begin{cases} l \times E_{elec} + l \times \varepsilon_{fs} d^2, & d < d_0 \\ l \times E_{elec} + l \times \varepsilon_{mp} d^4, & d \geq d_0 \end{cases} \quad (5)$$

where  $E_{elec}$  is the energy needed to run the radio,  $\varepsilon_{mp}$  and  $\varepsilon_{fs}$  is the energy required to run the transmitter amplifier. When transmission distance is more than the threshold distance  $d_0$ , we use multi-path fading model to compute energy consumption, otherwise, use the free-space model. The threshold distance  $d_0$  is

$$d_0 = \sqrt{\frac{\varepsilon_{fs}}{\varepsilon_{mp}}} \quad (6)$$

Under normal circumstance, free-space model is used for intra cluster transmission, and multi-path fading model is used for the transmission between cluster heads and FC. To receive an  $l$  – bit message, energy consumption is

$$E_{rx}(l) = l \times E_{elec} \quad (7)$$

For the network model we described, each cluster covers an area of  $L^2/W$ . In order to simplify the calculation, we assume one cluster is a circle of radius  $r = L/\sqrt{\pi W}$ , and the member nodes deployed in each cluster subjects to uniformly distribution. Thus the probability density function of member nodes in one cluster is

$$\rho(x, y) = \frac{W}{L^2} \quad (8)$$

The distance between member nodes and head node in one cluster is  $d_{toCH}$ , the expectation of its square is

$$\begin{aligned} E(d_{toCH}^2) &= \iint (x^2 + y^2) \rho(x, y) dx dy \\ &= \iint x^2 \rho(r, \theta) r dr d\theta \\ &= \frac{W}{L^2} \int_0^{2\pi} d\theta \int_0^{L/\sqrt{\pi W}} r^2 dr \\ &= \frac{L^2}{2\pi W} \end{aligned} \quad (9)$$

Member nodes expend energy only in the process of transmitting message to cluster head. Thus, from (7) and (9), the energy consumption of whole network member nodes is

$$\begin{aligned} E_{CM} &= W \left( \frac{K}{W} - 1 \right) p_{tx} (lE_{elec} + l\varepsilon_{fs} d_{toCH}^2) \\ &= p_{tx} (K - W) \left( lE_{elec} + l\varepsilon_{fs} \frac{L^2}{2\pi W} \right) \end{aligned} \quad (10)$$

Head nodes consume energy in both the process of receiving message from member nodes and transmitting it to FC. Because the nodes acquire the sensor signal that has already been compressed, we do not integrate the received data on head nodes. The energy consumption of receiving message of head nodes is

$$E_{CHrx} = W \left( \frac{K}{W} - 1 \right) p_{tx} l E_{elec} \quad (11)$$

Moreover, the energy consumption of transmitting message of head nodes is

$$E_{CHtx} = W \left[ \left( \frac{K}{W} - 1 \right) p_{tx} + 1 \right] [lE_{elec} + l\varepsilon_{mp}d_{toFC}^4] \quad (12)$$

From (11) and (12), we derive the energy consumption of whole network head nodes is

$$\begin{aligned} E_{CH} &= E_{CHrx} + E_{CHtx} \\ &= 2p_{tx}(K - W)lE_{elec} + WlE_{elec} \\ &\quad + (K - W)p_{tx}l\varepsilon_{mp}d_{toFC}^4 + Wl\varepsilon_{mp}d_{toFC}^4 \end{aligned} \quad (13)$$

So, the whole network energy consumption is

$$\begin{aligned} E_{total} &= E_{CM} + E_{CH} \\ &= [3p_{tx}(K - W) + W]lE_{elec} \\ &\quad + [p_{tx}(K - W) + W]l\varepsilon_{mp}d_{toFC}^4 + \frac{p_{tx}L^2}{2\pi} \left( \frac{K}{W} - 1 \right) l\varepsilon_{fs} \end{aligned} \quad (14)$$

It can be found in (14) that the minimum value of  $E_{total}$  exists. Let us take the partial derivative with respect to  $W$ , and then make  $\frac{\partial E_{total}}{\partial W} = 0$ ; consequently, when the  $E_{total}$  is minimal, the relationship between  $W$  and  $p_{tx}$  satisfies

$$W^2 = \frac{p_{tx}KL^2l\varepsilon_{fs}}{2\pi [(1 - 3p_{tx})lE_{elec} + (1 - p_{tx})l\varepsilon_{mp}]} \quad (15)$$

By this means, if the probability of member nodes that participate in signal acquisition is determined, we can obtain the number of clusters for whole network.

Our CS-based WSNs signal acquisition scheme exploits the spatial-temporal correlations of sensor signal, which can not only reduce the amount of signal transmitted, but also balance network energy. Compared with the WSNs signal acquisition schemes which are not based on CS, or those base on CS but only use the spatial correlation or the temporal correlation, our scheme is undoubtedly more efficient in saving energy consumption.

Cluster heads receive signals of their members and transmit them to FC, and FC uses the algorithm which is proposed in Sec. 4.2 to reconstruct original signals. The entire signal acquisition process is shown in Table 1.

TABLE 1. CS-based signal acquisition scheme

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- a) Divide the whole network into clusters evenly;
  - b) In every sampling cycle, each SN checks its remaining energy, and select the ones have the most remaining energy as the cluster head in one cluster;
  - c) Cluster members independently decide whether or not to sample signals with probability  $p_{tx}$ , while the cluster heads always do sampling;
  - d) After being acquired by a SN, the original signal  $f$  is projected under matrix  $\Phi$  to yield measurement  $y$ , which is to be sent to the cluster head;
  - e) In each cluster, head node uses vectorization operator to make all the received signals into one signal  $Y$ , which is to be sent to the FC;
  - f) After receiving the signals from all the cluster heads, the FC individually reconstruct them using the adaptive reweighing via GPSR algorithm which is proposed in Sect. 4.2 to yield their sparse representations;
  - g) At last, the original signals are obtained, which are the products of sparse representations and the sparsifying basis.
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**4. Sensor signals reconstruction.** On account of WSNs' measurement signals with noise, large amount of data, and high requirements of instantaneity. Reconstruction algorithms have to fulfill the requirement of high reconstruction accuracy of multi-dimensional signals, which are measured with noise, meanwhile it has to reduce the time complexity as much as possible. This paper design an advanced  $l_1$ -norm minimization algorithm for signal reconstruction, which find a better balance between time complexity and recovery accuracy.

This section begin with an overview of the classic GPSR algorithm [15] (in Sec. 4.1), which serves as the necessary background before the discussion of our proposed algorithm. Then the proposed algorithm adaptive reweighing via GPSR is presented (in Sec. 4.2).

**4.1. The GPSR-Basic algorithm.** The key difference between  $l_1$  and  $l_0$  norms is the dependence on magnitude: larger coefficients are penalized more heavily in the  $l_1$  norm than smaller coefficients, unlike the more impartial penalization of the  $l_0$  norm [22]. To reconcile this imbalance, a new  $l_1$ -norm minimization form of (3) was designed as:

$$\min_x \quad \tau \|x\|_1 + \frac{1}{2} \|Ax - y\|_2^2 \quad (16)$$

For an n-dimensional sparse signal  $x$ , GPSR introduce vector  $u$  and  $v$  to split the variable  $x$  into its positive and negative parts:

$$x = u - v, \quad u \geq 0, v \geq 0 \quad (17)$$

where  $u_i = (x_i)_+$ ,  $v_i = (-x_i)_+$  for all  $i = 1, 2, \dots, N$ , and  $(x_i)_+ = \max\{0, x\}$ . In this way, (16) can be rewritten as the following bound-constrained quadratic program (BCQR):

$$\begin{aligned} \min_{u,v} \quad & \tau I_N^T u + \tau I_N^T v + \frac{1}{2} \|y - A(u - v)\|_2^2 \\ \text{s.t.} \quad & u \geq 0, v \geq 0 \end{aligned} \quad (18)$$

where  $I_N = [1, 1, \dots, 1]^T \in \mathbb{R}^N$ . Finally, (18) can be written in more standard BCQR form:

$$\begin{aligned} \min_z \quad & c^T z + \frac{1}{2} z^T B z^T \equiv F(z) \\ \text{s.t.} \quad & z \geq 0 \end{aligned} \quad (19)$$

where  $z = \begin{bmatrix} u \\ v \end{bmatrix}$ ,  $b = A^T y$ ,  $c = \tau I_{2N} + \begin{bmatrix} -b \\ b \end{bmatrix}$ , and  $B = \begin{bmatrix} A^T A & -A^T A \\ -A^T A & A^T A \end{bmatrix}$ . And (19) is the problem will be solved by the gradient projection (GP) algorithm.

Define the vector  $g^{(k)}$  by  $g_i^{(k)} = \begin{cases} (\nabla F(z^{(k)}))_i, & \text{if } z_i^{(k)} > 0 \text{ or } (\nabla F(z^{(k)}))_i < 0 \\ 0, & \text{else} \end{cases}$ . The pseudocode outlining the important steps are presented in Algorithm 1.

**4.2. Adaptive reweighing via GPSR.** The GPSR-Basic algorithm solves (16) for a fixed value of  $\tau$ , or the algorithm with a continuation step solves the problem for a sequence of values of  $\tau$ . The first type of GPSR algorithm performs not well in reconstruction accuracy, and the second type has an additional calculation. In order to improve signal reconstruction performance, we present a new GPSR algorithm that performs an internal adaptive reweighing in each GPSR iteration. Differ from solving (16) multiple times with different weights as GPSR do, adaptive reweighing adjusts the weights inside the algorithm. And the reweighed  $l_1$ -norm minimization form of (16) can be described as:

$$\min_x \quad \sum_{i=1}^N w_i |x_i| + \frac{1}{2} \|Ax - y\|_2^2 \quad (20)$$

**Algorithm 1**    GPSR-Basic

**Input:**  $A, y, \tau, \alpha_{\min}, \alpha_{\max}$

**Output:**  $z^*, F(z^*)$

- 1: (initialization) Give  $z^{(0)}$ , choose parameters  $\beta \in (0, 1)$  and  $\mu \in (0, 1/2)$ ; set  $k = 0$
- 2: compute  $\alpha_0 = \frac{(g^{(k)})^T(g^{(k)})}{(g^{(k)})^T B(g^{(k)})}$ , and replace  $\alpha_0$  by  $\min(\alpha_{\min}, \alpha_0, \alpha_{\max})$
- 3: (backtracking line search) Choose  $\alpha^{(k)}$  to be the first number in the sequence  $\alpha_0, \beta\alpha_0, \beta^2\alpha_0, \dots$  such that

$$F\left(\left(z^{(k)} - \alpha^{(k)}\nabla F(z^{(k)})\right)_+\right) \leq -\mu\nabla F(z^{(k)})^T \left(z^{(k)} - \left(z^{(k)} - \alpha^{(k)}\nabla F(z^{(k)})\right)_+\right)$$

And set  $z^{(k+1)} = \left(z^{(k)} - \alpha^{(k)}\nabla F(z^{(k)})\right)_+$

- 4: Terminate with approximate solution  $z^{(k+1)}$  if it satisfied  $\|\min(z, \nabla F(z))\| \leq \varepsilon$ ; otherwise set  $k \leftarrow k + 1$  and return to 2

where  $w_i > 0$  denotes the weight of  $x_i$ .

Adaptive reweighing  $w_i$  can selectively penalize the different coefficients in the solution, thus to promote the same sparsity structure in the solution that is present in the original signal. Suppose index  $i$  is a zero location of original signal, but its estimate  $\tilde{x}_i \neq 0$  is a small value, we select  $w_i$  that has a large value to encourage  $\tilde{x}_i$  toward zero in the next iteration. And if  $\tilde{x}_i = 0$  or  $\tilde{x}_i$  is a lager value, we select the weight of  $x_i$  has a small value to remain it unchanged. So, naturally, we have:

$$w_i = \begin{cases} \frac{1}{|\tilde{x}_i| + \varepsilon}, & i \in \Gamma \\ \frac{1}{\max_{i \in \Gamma} |\tilde{x}_i|}, & i \notin \Gamma \end{cases} \quad (21)$$

$\Gamma$  denotes the support of  $\tilde{x}$ . The term support is the index set of nonzero coefficients.

Due to the reality of there is no prior information of original signal, the weights are adjusted according to the solution at the previous iteration. In each GP step, we first update the solution  $x^{(k)} \leftarrow x^{(k-1)}$ , and then update the support  $\Gamma^{(k)} \leftarrow \Gamma^{(k-1)}$ , finally update the weight  $w^{(k)} \leftarrow w^{(k-1)}$ . Specifically, in the experiment, for  $\forall i \in \Gamma$ , we update  $w_i$  as:

$$w_i^{(k)} = \frac{w_i^{(k-1)}}{\gamma |x_i^{(k-1)}| + \xi} \quad (22)$$

where  $x_i^{(k-1)}$  denotes the solution from previous reweighing iteration, we fix  $\xi = 1$  and update  $\gamma \leftarrow M \frac{\|x^{(k-1)}\|_2^2}{\|x^{(k-1)}\|_1^2}$  at each reweighing iteration,  $M$  is the dimension of measurements. Suppose that  $x^{(k-1)}$  is not sparse enough, the value of  $\gamma$  is small, and thus the value of  $w_i$  is large, i.e., to punish more severely on the support of  $x^{(k-1)}$ , thus improve sparsity of solution. For  $\forall i \notin \Gamma$ , we update  $w_i$  as:

$$w_i^{(k)} = \min\left(w_i^{(k-1)}, \frac{w_i^{(k-1)}}{|x_i|}\right) \quad (23)$$

It is more flexible and reliable to separately use the formulations (22) and (23) to update the weights on different index. And this method effectively control the sparsity of the solution and greatly decrease the object function value.



Rewrite  $c$  as  $c = wI_{2N} + \begin{bmatrix} -A^T y \\ A^T y \end{bmatrix}$ , the pseudocode of our proposed adaptively reweighing via GPSR algorithm is presented in Algorithm 2.

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**Algorithm 2** Adaptive Reweighing via GPSR
 

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**Input:**  $A, y, \alpha_{\min}, \alpha_{\max}$

**Output:**  $z^*, w^*, F(z^*)$

- 1: (initialization) Give  $z^{(0)}, w^{(0)}$ , choose parameters  $\beta \in (0, 1)$  and  $\mu \in (0, 1/2)$ ; set  $k = 0$
  - 2: compute  $\nabla F(z^{(k)}) = Bz^{(k)} + c$
  - 3: compute  $\alpha_0 = \frac{(g^{(k)})^T (g^{(k)})}{(g^{(k)})^T B(g^{(k)})}$ , and replace  $\alpha_0$  by  $\min(\alpha_{\min}, \alpha_0, \alpha_{\max})$
  - 4: (backtracking line search) Choose  $\alpha^{(k)}$  to be the first number in the sequence  $\alpha_0, \beta\alpha_0, \beta^2\alpha_0, \dots$  such that
 
$$F\left((z^{(k)} - \alpha^{(k)}\nabla F(z^{(k)}))_+\right) \leq -\mu\nabla F(z^{(k)})^T \left(z^{(k)} - (z^{(k)} - \alpha^{(k)}\nabla F(z^{(k)}))_+\right)$$
 And set  $z^{(k+1)} = (z^{(k)} - \alpha^{(k)}\nabla F(z^{(k)}))_+$
  - 5: update the support  $\Gamma$
  - 6: update the weight  $w$ : if  $i \in \Gamma$ , let  $w_i^{(k)} = \frac{w_i^{(k-1)}}{\gamma|z_i^{(k-1)}| + \xi}$ , otherwise  $w_i^{(k)} = \min\left(w_i^{(k-1)}, \frac{w_i^{(k-1)}}{\max|z_i|}\right)$
  - 7: Terminate with approximate solution  $z^{(k+1)}$  if it satisfied  $\|\min(z, \nabla F(z))\| \leq \varepsilon$ ; otherwise set  $k \leftarrow k + 1$  and return to 2
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The main computational cost at each step of adaptive reweighing via GPSR algorithm comes from a  $|2N| \times |2N|$  multiplication in step 2 for computing  $\nabla F(z^{(k)})$ , and the calculation in step 4 for  $\alpha^{(k)}$ . It is worth noting that the matrixes  $B$  and  $A^T y$  stay the same in each iteration, which can be pre-computed at the start of the algorithm. And we perform a backtracking line search for  $\alpha^{(k)}$  until a sufficient decrease is obtained in  $F$ . In the simulation experiments, it is found that in most cases a proper  $\alpha^{(k)}$  can be searched quickly. Occasionally it requires many more searches to meet a  $\alpha^{(k)}$ , here jump out of the liner search and reset  $\alpha_0 = 1$  will not reduce the performance of reconstruction algorithm, and thereby reduce the algorithm complexity.

What is noteworthy is that our proposed adaptive reweighing via GPSR algorithm does not require prior knowledge of the sparsity of the signal. This is no doubt in line with that in reality, which the degrees of sparseness of a signal are generally unknown. Considering that, our algorithm is more practical than other algorithms demand the degrees of sparseness such as OMP [25], CoSaMP [26], etc.

**5. Performance comparison: reconstruction algorithms.** In this section, we present some experiments to evaluate the performance of our proposed adaptive reweighing via GPSR algorithm, which we will call ARGP, in terms of the computational cost and the reconstruction accuracy. We show that, in comparison with GPSR-Basic [15], GPSR-BB [15], ARW-H (a state-of-the-art algorithm) [27] and the classic greedy OMP [25] algorithm, solving the recovery problem using ARGP yields significantly higher quality signal reconstruction, at a small computational cost.

**5.1. Simulation setting.** We reconstruct the 0-1 sparse signal to demonstrate the performance of the algorithms. In addition to the images can be directly expressed as 0-1 signals, many natural signals can be expressed as 0-1 signals using binary coding. The measurement of the *i*th SN  $y_i$  is generated according to (4), with  $\sigma^2 = 10^{-4}$ .

In GPSR-Basic and GPSR-BB, we set  $\beta = 0.5$ ,  $\mu = 0.1$ , and  $\tau = 0.1\|A^T y\|_\infty$  as suggested in [14]. And in ARW-H, we set  $\tau = \sigma\sqrt{\log N}$  as suggested in [26]. For our proposed ARGp, we initialize all the weights with a value of  $\tau = 0.3\|A^T y\|_\infty$  for which the solution is a zero vector, and the other parameters in ARGp are as same as those in GPSR series algorithms.

We use mean squared error (MSE) to measure the reconstruction error. The MSE is computed as:

$$MSE = \left(\frac{1}{n}\right) \|\hat{x} - x\|_2^2 \tag{24}$$

where  $\hat{x}$  denotes the estimate of  $x$ . And we use CPU times to measure the computational cost of algorithms. Our simulation environment is MATLAB R2010a.

**5.2. Fidelity of the signal reconstruction algorithms.** In our first experiment, we consider a typical CS scenario, where the goal is to reconstruct a length- $n$  sparse signal from  $m$  measurements. Here,  $n = 4096$ ,  $m = 1024$ , and the original signal  $x$  contains 160 randomly placed  $\pm 1$  spikes. And we will call GPSR-Basic, the monotone version of GPSR-BB, and the nonmonotone version of GPSR-BB as GPSR series algorithms, the adaptive reweighting via homotopy algorithm as ARW-H.

FIGURE.3 shows that the original signal, the estimate obtained by solving (20) using the ARGp, and the estimates obtained by solving (16) using the ARW-H and GPSR series algorithms. It is indicated in FIGURE.3 that ARGp not only does an excellent job at locating the spikes but also exhibits a much lower MSE which is  $3.16 \times 10^{-5}$  with respect to the original signal, and the GPSR series algorithms and ARW-H have larger MSEs which are  $2.3 \times 10^{-4}$  and about  $2.45 \times 10^{-3}$ , respectively. On behalf of all the other algorithms, ARW-H which have the smallest MSE is compared with ARGp to indicate more details about their recovery signals. As shown in FIGURE.4, the estimate obtained by ARGp is almost as same as the original signal, and ARW-H get a poor performance in locating of spikes and evaluating the values of spikes.

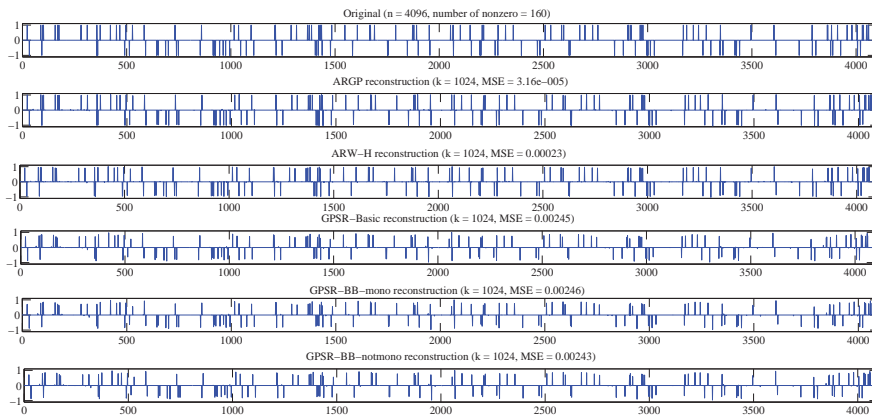


FIGURE 3. Original signal and reconstruction results

In FIGURE.5, we plot the evolution of the objective functions versus iteration number, for ARGp and both GPSR series algorithms. And it also shows the change in MSEs

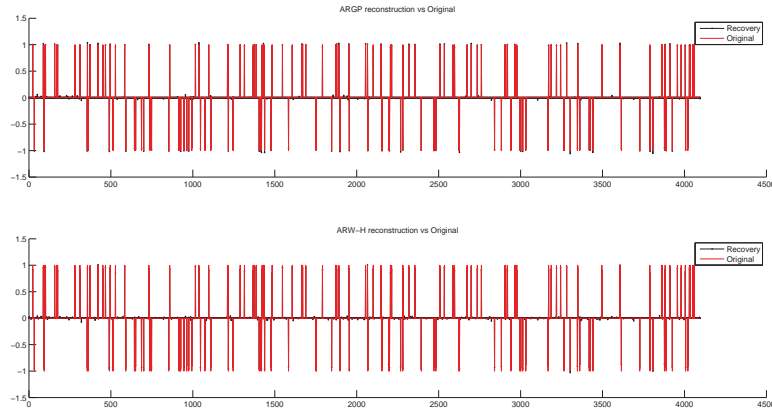


FIGURE 4. Comparison of the recovery performances of ARGP and ARW-H

versus iteration number. We observed that the objective function and MSE of ARGP are clearly smaller than those of both GPSR series algorithms.

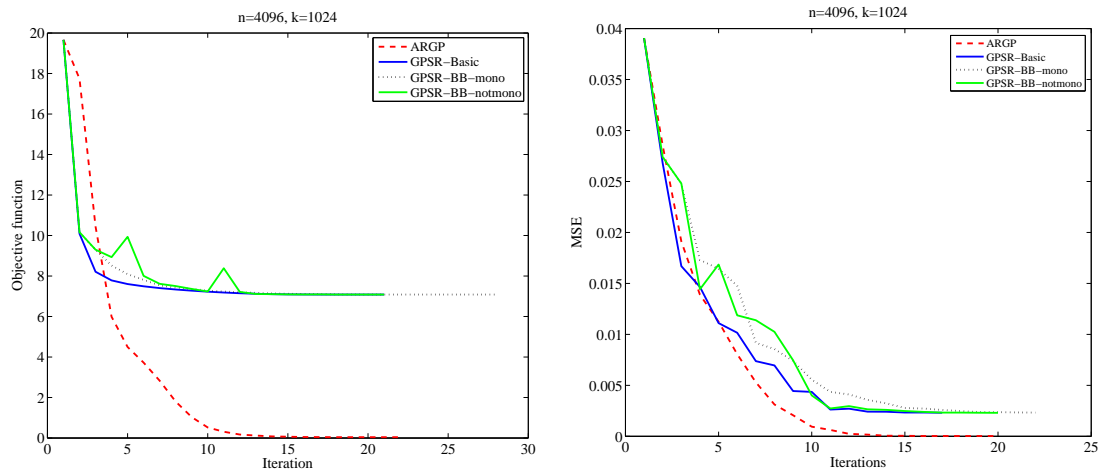


FIGURE 5. Objective function and MSE plotted against iteration number

TABLE 2. Average CPU times and MSEs of several algorithms

	CPU time (s)	MSE
ARGP	0.7004	3.4364e-005
ARW-H	19.8901	2.3652e-004
GPSR-BASIC	0.6053	2.6065e-003
GPSR-BB-mono	0.5117	3.0561e-003
GPSR-BB-nonmono	0.4493	2.6106e-003

Table 2 reports average CPU times (over ten experiments) required by ARGP, ARW-H and the GPSR series algorithms as well as the average reconstruction MSE with respect to the original signal. The results in this table show that, the MSE for ARGP is clearly much smaller than other algorithms. Although the CPU time for ARGP pertains to the same order as the GPSR series algorithms CPU time, the MSE for ARGP is about two

order of magnitude smaller than the GPSR series algorithms. We think it is worthy to exchange a slight CPU time increase for a significant MSE decrease.

**5.3. Efficiency of the signal reconstruction algorithms.** Next, we compare the computational efficiency of ARGP algorithm against OMP, often regarded as a classical greedy algorithm that obtains the estimates by solving (2). Due to the real case, the degrees of sparseness of a signal are unknown and unfixed, the comparison of the reconstruct performance against different degrees of sparseness is necessary. In our experiment, we consider a ranger of degrees of sparseness: the number of nonzero spikes in  $x$  ranges from 5 to 250. For each value of the number of nonzero spikes, we generate a random measurements set and obtain the reconstruction signal from it. And then we compute average MSE and average CPU time, over the ten runs.

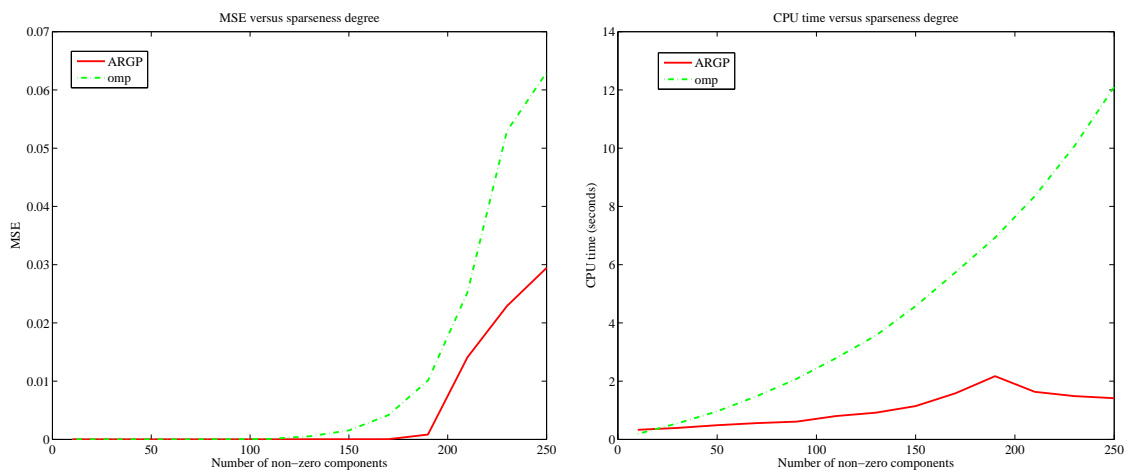


FIGURE 6. Average MSE and CPU times plotted against the number of nonzero components of  $x$

FIGURE.6 plots the average reconstruction MSE and the average CPU times as a function of the number of nonzero components in  $x$ . We observe that our purposed ARGP in generally obtain exact reconstructions for the number of nonzero spikes in  $x$  up to 200, and the OMP method starting to degrade earlier and faster. It is clearly that our ARGP algorithm is faster and more accurate than OMP algorithm.

**6. Conclusions.** In this paper, we consider compressed acquisition and advanced reconstruction of sensor signals with CS for WSNs. Firstly, we construct an energy-efficient CS-based signal acquisition scheme, which exploits the spatial and temporal correlations of sensor signals to reduce the energy consumption of networks. By analyzing the energy consumption of proposed scheme, we work out the relationship of the scheme parameters, the number of clusters and the transmit probability. Secondly, we propose a reweighted  $l_1$  - norm minimization algorithm to reconstruct the original signals from the measurements, which are acquired by the proposed signal acquisition scheme. The algorithm performs an internal adaptive reweighing in each iteration, so that the weights can be adjusted to selectively penalize different coefficients in the solution. The simulation results show that the proposed algorithm can significant enhance the reconstruct performance.

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