Cheap Credit, Collateral and the Boom-Bust Cycle

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Abstract

This paper proposes a model of booms and busts in housing and non-housing consumption driven by the interplay between relatively low interest rates and an expansion of credit, triggered by further decline in interest rates and relaxing collateral requirements. When credit becomes available, households would like to borrow in order to frontload consumption, and this increases demand for housing and non-housing consumption. If the increase in the demand for housing translates into an increase in prices, then credit is fueled further, this time endogenously, both because of the wealth effect (the existing housing stock is now more valuable) and because housing can be used as collateral. Because a lifetime budget constraint still applies, even in the absence of a financial crisis, the initial expansion in housing and non-housing consumption will be followed by a period of contraction, with declining consumption and house prices. My mechanism clarifies that boom-bust dynamics will be accentuated in regions with inelastic supply of housing and muted in elastic regions. In line with qualitative predictions of my model, I provide evidence that differences in regions' elasticity of housing and initial relaxation of collateral constraints can explain most of the 2000-2006 boom and the subsequent bust in house prices and consumption across US counties. Quantitative evaluation of the model shows that reversal in the initial relaxation of collateral constraints is important in explaining the sharp decline of house prices and consumption. However, the model shows that most of the decline would have happened even without a reversal in the initial expansion of credit, albeit over a longer period of time.

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1 Introduction

During the period of 2000 to 2006, there was a decline in real interest rates followed by a rise of securitization and an easing of collateral requirements (Figure 1a). The US flow of funds during this period shows that in just seven years the stock of household mortgage liabilities more than doubled, increasing by 5.7 trillion dollars.[1] Despite a boom in housing construction, net investment of households in residential housing during this period comprised merely 2.4 trillion dollars, the other 3.3 trillion dollars of this amount is money cashed out from home equity.[2] Interestingly, as Figure 1b shows, during this period the total valule of cash-outs and the US current account deficit followed each other very closely. Turning to regional variations within the US, regions that accumulated more debt during this period experienced a larger boom in house prices and consumption which was followed by a larger bust in subsequent years (Figure 2).

This paper proposes an analysis of the economic boom and bust, where the bust is an inevitable consequence of the boom and provides empirical evidence from US counties to support this explanation. At the heart of the theory is the unsustainable increase in consumption driven by expanded credit and housing price increases that relax credit constraints. Crucially, it is the nature of this sort of increase in consumption that it must be reversed even in the absence of a financial crisis. My theory accounts not only for the boom-bust dynamics of housing wealth and consumption, but also for a central fact that has received insufficient attention: a significant fraction of the increase in consumption in many areas of the United States was financed by borrowing on housing collateral.[3] The theory thus links the decline in consumption and housing wealth in many economic sub regions to the very increase in consumption and housing wealth in the area and emphasizes that this cycle need not be driven by irrationality or exploitation by financial intermediaries. Rather the cycle results naturally from the interplay between expanding credit, consumers keen on frontloading their consumption, and the endogenous relaxation of credit constraints in a market dominated by housing collateral.

[1] From 4.7 trillion dollars in 1999 to 10.5 trillion dollars in 2006.

[2] Greenspan and Kennedy (2008) shows that the process of home equity cash-out began in the early 80s and accelerated by 1998. They estimate that since 1990, home equity extraction accounts for four-fifths of the increase in mortgage liabilities and for almost all the decline in the US households savings rate. The fact that home equity cash outs are even more important in their calculations partly springs from their definition of a cash-out which includes loans used for home improvement as well.

[3] The empirical work of Mian and Sufi (2011) is an exception which shows home-equity extraction due to rising home prices is responsible for both a large fraction of increase in household debt during the boom years as well as a rise in default rates in the years years following. However, they do not provide direct evidence on the relation between the rise in household debt and the rise in consumption during the boom years.

To be more precise, I consider an open economy with two main ingredients: the interest rate is lower than the discount rate of consumers, and households are subject to borrowing constraints with housing acting as collateral (as well as providing housing services). These two ingredients together lead to a pattern in which if it is possible to borrow, households borrow and increase their housing and non-housing consumption, and the rise in demand for housing becomes partially self-reinforcing because it increases housing prices- creating both a wealth effect and further relaxing credit constraints. However, because a lifetime budget constraint still applies, these households must reduce their housing and non-housing consumption in the future (which is anticipated), and when they do so, the dynamics play out in reverse. Given the low interest rate, they are willing to endure this period of declining consumption in return for the early consumption.

My theoretical mechanism highlights the importance of three factors in shaping how pronounced these dynamics will be. First is the expansion of credit, either because of further declines in interest rates or declines in collateral requirements that precipitate the entire boom-bust cycle in the first place. Second, is the difference between household time preference and the interest rate that determines the extent of frontloading behavior. Therefore, the lower the interest rate, the larger the boom-bust pattern induced by the same shocks. And third is the responsiveness of housing prices to the increase in demand for housing. Empirically, this is related to the elasticity of the housing supply, already emphasized and empirically exploited by Glaeser, Gyourko, and Saiz (2008), Saiz (2010) and Mian and Sufi (2011).

I show that the theoretical mechanism is quantitatively and qualitatively very different when housing supply is inelastic; an increase in housing demand leads to a rise in house prices, creating a wealth effect and relaxed credit constraints in a way that either does not happen or does not happen to the same extent with an elastic housing supply. In particular, a decline in interest rates reduces the user cost of housing, which leads to an increase in housing demand in all regions. In regions with an inelastic supply of housing, this raises the price of existing homes, which generates a wealth effect and relaxes the borrowing constraint. Relaxed borrowing constraints enable households in these regions to frontload their consumption, which results in a boom-bust cycle. On the other hand, in elastic regions, a decline in interest rates does not change house prices and therefore the borrowing constraint of households in elastic regions remains binding. However, over time, households in these regions will use the resources freed from lower interest payments to buy a larger house and increase their non-housing consumption. A decline in collateral requirements relaxes the borrowing constraint in all regions, which increases the demand for housing and non-housing in the short run. In regions with inelastic supply of housing, then, credit is further expanded, this time endogenously, because of higher house prices. This will result in a boom-bust pattern that is amplified in inelastic regions.

In order to test the qualitative implications of my model at the reduced-form quantitative level, I build on a series of studies by Mian and Sufi (2009, 2011, 2012) and Mian, Rao and Sufi (2012) and show that the basic predictions of my model are borne out by the data. In particular, I find that during the period 2000-2006, regions with more inelastic supply of housing (as measured by Saiz (2010)), and regions that experienced greater change in the fraction of loans sold to non-GSEs experienced a more rapid increase in consumption and house prices and at least 70 percent of changes in house price growth and consumption growth is attributed to these variables. These very same factors that explain the boom in house prices and consumption during 2000 to 2006 also explain a significant fraction of decline in house prices and consumption between 2006 and mid-2008. Moreover, I show regions with less elastic supply of housing and higher change in the fraction of loans sold to non-GSEs experienced higher growth in their mortgage liabilities, not only during the boom years of 2000 to 2006 but also during the downturn of mid-2006 to mid-2008. The fact that mortgage liabilities in these regions continued to grow even after the downturn in house prices and consumption suggest that a significant fraction of decline in house prices and consumption is not driven by households reducing their debt, but instead, is driven by the reduction in the amount that households can increase their debt holding. In terms of policy this is an important distinction because policies that allow households to rollover their debt can only reduce the part of the downturn that is due to the deleveraging of households.

My model also enables the analysis of the quantitative role different factors played in the boom-bust cycle of 2000-2010 in the US economy. To this purpose, I calibrate key parameters of my model for regions with different elasticities of housing and different changes in the fraction of loans sold to non-GSEs, based on static characteristics of these regions and the time series of household mortgage liabilities of these regions from 2000 to 2006. First, the parameters that results from this calibration shows a gradual decline in collateral requirements during the boom years with the most rapid decline happening between 2003 and 2004. This relaxation of collateral requirements is more extreme the more inelastic the region, and the higher the change in securitization rate in that region. These estimates resemble the findings of Lee, Mayer and Tracy (2012) on the rise of the use of second lien loans.[4] Second, I show that my model does a good job of replicating the rise in house prices and consumption for the boom years and for the beginning of the bust. Third, this exercise helps to estimate the contribution of different components to the boom and bust dynamics.

[4] This is also similar to the time series and cross section of changes in securitization rate that happened during the boom years.

In particular, the model shows that whereas most of increase in house prices during the period of 2000 to 2003 came from declining real interest rates, the boom in 2004 and 2005 was driven by declining collateral requirements. However, the model implies that the same decline in collateral requirements would have resulted in a significantly milder boom-bust in house prices and consumption if interest rates had been at the level they were in 2000. This result is mainly because with higher interest rates households would have less incentive to frontload their consumption.

In order to asses the contribution of the financial crisis to the downturn dynamics, I extend the calibration of changes in collateral requirements for the period after 2007 based on changes in the actual time series of household mortgage liabilities from 2007 to 2011 and compare the implied dynamics of housing prices and non-housing consumption with the model without a reversal in initial relaxation of borrowing constraints. First, estimated parameters show a steady decline in collateral requirements such that by 2011, most of the initial decline in collateral requirements is reversed. Second, absent a financial crisis, the model does a fairly good job at predicting the level of the decline in house prices and consumption during the bust, however, the decline happens over a longer period of time. Adding the reversal in initial decline in collateral requirements significantly helps the model predict the sharp decline in consumption and house prices. Moreover the model predicts that the initial decline in house prices and consumption will be followed by a slight recovery, but to a level that is close to the steady state of the economy without a reversal in initial relaxation of lending standards which is well below the level of house prices and consumption in 2006 (the very top of the boom years).

Finally, results of the quantitative exercise allow for the study of the impact of different policies on house prices and household consumption in different regions. In particular I compare the impact of two different policies: (i) further reductions in the real interest rate and (ii) loan modification. The policy experiment shows that lowering interest rates is not effective in increasing consumption of households living in elastic regions, whereas it does increase consumption a little in regions with an inelastic supply of housing. This result is driven by the asymmetric impact of real interest rates on house prices. On the other hand, loan modification increases consumption in all regions temporarily. However loan modification is just delaying the recovery procedure and the initial increase in consumption is followed by a decline in consumption and house prices in the years following. The effectiveness of policy in general is limited because the decline in consumption is not only driven by some households deleveraging their debt holding - as is the case in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2012) - but more importantly because the level of consumption during the boom years itself was financed by the rapid growth in household liability.

The rest of this paper proceeds as follows. The next session briefly discusses the related literature. Section 2 presents the theoretical model, which highlights the differential impact of decline in interest rates and collateral requirements in regions with different elasticities of housing supply. I discuss the data in Section 3. Section 4 outlines the reduced-form empirical evidence and relates the boom-bust cycle to variations in elasticity of housing supply as well as the rise of securitization. Section 5 extends the theoretical model to a more general supply of housing, presents the result of the calibration of the model and perform policy experiments. Section 6 concludes.

1.1 Literature Review

On the theoretical dimension, this paper is most closely related to a number of recent studies on the housing boom and bust in an incomplete-market framework in which houses, in addition to providing housing services, provide a means of collateral for households. The importance of relaxation of borrowing constraints in explaining the simultaneity in capital inflows and the rise of house prices during the boom years has been raised in Ferrero (2012). Favilukis, Ludvigson and Nieuwerburgh (2012) also emphasizes the importance of financial market liberalization and its reversal to explain the housing boom and bust, however, it argues that most of the boom and bust would have happened even in the absence of capital inflows. The independent work of Garriga, Manuelli and Peralta-Alva (2012) constructs a semi-open economy and shows a decline in interest rates in addition to the relaxation of collateral requirements that is followed by a reversal in the initial relaxation can account for the housing boom and bust.[5] Midrigan and Philippon (2011) consider the impact of a credit crunch in a cash-in-advance economy in which the main role of home equity borrowing is to provide liquidity services and therefore monetary policy is very effective in reducing the recession driven by decline in house prices. Guerrieri and Lorenzoni (2011) studies a heterogeneous-agent model with durable goods and argue an increase in credit spreads, and not a shock to credit limits, can lead to a decline in demand for durable goods.[6] The frontloading behavior of households and its interaction with the elasticity of housing is what distinguishes the mechanism of this paper from other work. Also, in terms of the results, in

[5] Their semi-open economy or segmented-financial-markets assumption assumes that a representative agent is able to use her housing stock as collateral to borrow from abroad at a rate that is lower than the marginal product of capital. Therefore a decline in mortgage rates that is not followed by a decline in marginal product of capital or a decline in collateral requirements increases the collateral value of houses.

[6] There is a larger literature incorporating housing sector (usually with heterogeneous agents) in the general equilibrium models. For example see Campbell and Hercowitz (2009), Iacoviello (2008), Jeske, Krueger and Mitman (2012) and Kiyotaki, Michaelides and Nikolov (2011).

all of the above studies the downturn in consumption and asset prices is generated by the reversal in initial credit expansion whereas in this paper the bust begins whenever there is not enough of a further decline in interest rates or in collateral requirements. At least in terms of data, it seems that both the decline in house prices and the decline in consumption predate any sign of shrinkage in the financial markets.[7] Also being written in continuous time makes this model tractable such that not only is the steady-state completely characterized, but also the transition path.

This paper naturally builds on the seminal work of Kiyotaki and Moore (1997). The literature on sudden stops also highlights the importance of collateral constraints in understanding output, asset prices and capital flows during episodes of crisis (for example see Aoki, Benigno and Kiyotaki (2007), Caballero and Krishnamurty (2001) Calvo, Coricelli and Ottonello (2012) and Mendoza (2010)). This paper complements this strand of literature by assuming financial frictions on the household side of the economy instead of on the firms side.[8, 9]

Among empirical studies of the recent financial crisis, Mian and Sufi (2009) provides evidence of the relation between an increase in securitization and the rise of household mortgage liabilities and the subsequent surge in default rates. Dagher and Fu (2011) is another related study that shows the rise in the share of independent lenders is associated with a similar pattern of mortgage liabilities and default rates.[10] Mian and Sufi (2011)

[7] For example US securitization issuance and the S&P 500 kept increasing until mid-2007. Therefore in terms of timing it seems more likely that the downturn in consumption and in house prices precipitated the financial crisis and not the other way around.

[8] In the case of the recent crisis in the US, Adrian, Colla and Shin (2012) shows much of the decline in banks' lending to firms was compensated by bond financing such that by mid-2009 US non-financial corporate sector's liabilities started to increase. On the other hand, the *NY Fed Quarterly Report on Household Debt and Credit* (available at HTTP://www.newyorkfed.org/research/national_economy/householdcredit/DistrictReport_Q22012.pdf) shows a steady decline in total household debt since the third quarter of 2008. The above evidences is suggestive that during the current crisis financial frictions on the households are more important in explaining the economic downturn. Also as is argued by Midrigan and Philippon (2011) models with financial frictions on firms have a hard time explaining the cross-section of regional variation in the data on output.

[9] Another recent strand of literature studies the impact of financial frictions when financial institutions, in addition to firms, are facing the frictions. Among the others, see Brunnermeier and Sannikov (2012), Gertler and Kiyotaki (2010) and Rampini and Viswanathan (2012). Compared to models like Bernanke, Gertler and Gilchrist (1999) which put the financial frictions only on the firms, these papers show a more persistent and non-linear impact of financial frictions on the real side of economy.

[10] In general independent lenders,as compared to banks and thrifts, have always sold a much higher fraction of their loans to non-GSEs. This is mainly driven by higher cost of capital for these lenders and their specialization in originating loans to the lower tail of the market. In data there is a very high correlation between the change in fraction of securitized loans and the share of independent lenders and it seems that the increase in the share of independent lenders was an effect of the rise of securitization which lowered financing cost significantly for these independent investors.

estimate that increasing house prices resulted in a \$1.25 trillion dollars increase in existing homeowners liabilities from 2002 to 2006. Finally Mian and Sufi (2012) and Mian, Rao and Sufi (2012) show a disproportionately larger decline in consumption and in employment in counties that had higher debt-to-income ratios by 2006. This paper complements findings of these studies in a number of dimensions. First, it shows that not only during the downturn but also during the boom years house prices and consumption are closely associated with the factors that contributed to the expansion of credit, namely variations in the elasticity of housing and changes in the securitization rate. Second, I show that regions with a less elastic supply of housing and a higher change in securitization rate, despite having a larger decline in house prices and consumption during 2006 to mid-2008, continued to have *higher* growth in their mortgage liabilities during the period of 2006 to mid-2008. These two facts together show that it is true that in comparison to 2006 the decline in demand and in employment is driven by indebted households reducing their consumption, however, the level of consumption in 2006 itself was not sustainable and was financed by home-equity extraction by existing households.[11] In addition, this paper argues theoretically and empirically that changes in interest rates and collateral requirements (proxied by changes in the securitization rate) move all the three variables of house prices, consumption and household debt together and, depending on the elasticity of housing supply, the dynamics implied by these shocks can be very different. Therefore using the elasticity of housing as an instrument does not help one estimate the causal impact of house prices on household borrowing behavior or consumption.

There is a larger literature on the relationship between housing wealth and consumption which usually finds significant, but heterogeneous, effects on housing wealth (for example see Hurst and Stafford (2004), Case, Quigley and Shiller (2005), Campbell and Cocco (2007) and Attanasio, Blow, Hamilton and Leicester (2009)). What this paper adds to that literature is that this relationship depends not only on individual characteristics, but also on the level of interest rates, on elasticity of housing in the region[12] and on the nature of the shock that is driving the both variables; meaning whether the shock is an income shock, an interest rate shock or a change in collateral requirements.

[11] In terms of employment this means that by 2006 there was too much employment in the non-tradable and construction sectors and, at some point this needed to be corrected, which can result in the long periods of adjustment associated with a high unemployment rate.

[12] Elasticity of housing determines whether the change in housing wealth is coming from the change in quantity or from the change in prices.

2 Theory

In this section I develop a model of a small, open economy with a representative household whose borrowing is constrained by the collateralizable fraction of its housing wealth. I begin by characterizing the environment and solving for the household's optimization problem, taking house prices dynamics as given. Next, I solve for the equilibrium of elastic regions and inelastic regions by endogenizing house prices. Finally, I shock the economy with surprise changes in interest rate and collateral requirements and characterize the transition path of the economy.

2.1 Setup

Consider a continuous-time, small, open economy[13] consisting of regions differing only in the total supply of land. Each region's population is normalized to one, and the representative household in region i lives there forever, has a discount rate ρ and enjoys housing consumption (h_{it}) as well as non-housing consumption (c_{it}). Region i's household preference is given by:[14]

$$\int_0^\infty e^{-\rho t}\left[log c_{it} + \eta log h_{it}\right] dt \tag{1}$$

The non-housing consumption good is the numeraire. Each unit of housing in region i is traded at price q_{it} and, in order to keep the model simple, I assume that there is no rental market for housing.

Similar to Kiyotaki and Moore (1997), I assume the only financial asset is the short term paper which has return r,[15] and the minimum holding of financial assets by the representative household (a_{it}) is constrained by fraction $\theta_i\,(< 1)$ of household housing wealth:

$$a_{it} \geq -\theta_i q_{it} h_{it} \tag{2}$$

The representative household in region i receives a stream of endowment equal to w_i and assumes there is no change in r, θ_i or w_i.

Houses are produced by a combination of land and capital according to a Leontief pro-

[13] This assumption is justified with the fact that during the period of 2000 to 2007, changes in the US current account deficit and changes in household mortgage liabilities follow each other closely. Interestingly, Ferrero (2012) shows this pattern has been common among all countries that experienced a housing boom in this period.

[14] The Cobb-Douglas aggregator is rationalized by the fact that in a cross section of data the share of housing cost in household expenditure has only small variations.

[15] As long as households assume the interest rate r is constant, this assumption is not restrictive, and any long-term contract can be replicated with a short-term contract.

duction function:[16]

$$h_{it} = min(l_{it}, \frac{k_{it}}{B}) \tag{3}$$

Capital is produced using the numeraire good and its price is equal to one. The price of one unit of land in region i at time t is q_{it}^L. Moreover I assume there is no adjustment cost for the capital used in a house.[17] Then as long as $\dot{q}_{it}^L / q_{it}^L \leq r$,[18] the Leontief production function implies that:

$$\begin{aligned} h_{it} &= l_{it} = k_{it}/B & (4) \\ q_{it} &= q_{it}^L + B & (5) \end{aligned}$$

When a household is buying a house, it receives the title for the land that is used in that house as well as the title for the house itself. Only the capital used in the house, and not the land, is subject to depreciation rate δ_k, which can be compensated for with household investment i_{it} in the house. Therefore the capital used in the house evolves according to:

$$\dot{k}_{it} = -\delta_k k_{it} + i_{it} \tag{6}$$

Given the Leontief production function for housing the amount of investment is:

$$i_{it} = \delta_k k_{it} \tag{7}$$

Therefore the representative household budget constraint is:

$$\dot{a}_{it} + q_{it}\dot{h}_{it} = w_i + r a_{it} - c_{it} - \delta_k B h_{it} \tag{8}$$

Finally, and most importantly, it is assumed that the interest rate is lower than the household's time preference rate ($r < \rho$). This assumption can be rationalized by a global saving glut hypothesis (Bernanke (2005)) or by the presence of a small fraction of the population who are more patient than others as in Guvenen (2009). But more importantly, an extension of the present model that includes agents with an income profile that is temporarily high

[16]For the quantitative exercise, I extend the housing production function to CES and show analytically that the qualitative results do not change.

[17]This is a relatively strong assumption that makes the model tractable. However, this helps to clarify the boom-bust cycle purely driven by the frontloading motivation from the boom-bust cycle induced by a temporary increase in demand for housing a la Mankiw and Weil (1989). Adding adjustment costs to this model results in larger boom-bust cycles.

[18]$\dot{q}_{it}^L / q_{it}^L > r$ is not possible because then even an investor who is not living in region i can invest in the land in region i and make more profit than buying financial assets and, therefore, there will be no lending.

(super stars) shows that $r < \rho$ is the only equilibrium steady state interest rate that arise in this economy with incomplete markets. In that framework higher income inequality results in a further decline in the interest rate.[19]

2.2 Household Problem

Region i's household problem can be written as:

$$\begin{aligned} & \underset{[c_{it}, a_{it}, h_{it}]_0^\infty}{Max} && \int_0^\infty e^{-\rho t} \left[log c_{it} + \eta log h_{it} \right] dt \\ s.t. \quad & && \dot{a}_{it} + q_{it}\dot{h}_{it} = w - c_{it} + r a_{it} - \delta_k B h_{it} \\ & && -a_{it} \leq \theta_i q_{it} h_{it} \end{aligned}$$

Defining the total wealth of the representative household as $W_{it} \equiv q_t h_{it} + a_t$, and $\delta \equiv \delta_k B$, we can rewrite the representative household problem as:

$$\begin{aligned} & \underset{[c_{it}, W_{it}, h_{it}]_0^\infty}{Max} && \int_0^\infty e^{-\rho t} \left[log c_{it} + \eta log h_{it} \right] dt \\ s.t. \quad & && \dot{W}_{it} = w - c_{it} + r \left(W_{it} - q_{it} h_{it} \right) - \delta h_{it} + \dot{q}_{it} h_{it} \\ & && W_{it} \geq (1 - \theta_i) q_{it} h_{it} \end{aligned} \tag{9}$$

Using an extension of the maximum principle for an optimal control problem with mixed constraints (see Seierstad and Sydsæter (1987)), one can form the discounted Hamiltonian as:

$$\hat{\mathcal{H}} \equiv \left[log c_{it} + \eta log h_{it} \right] + \mu_{it} \left[w - c_{it} + r \left(W_{it} - q_{it} h_{it} \right) - \delta h_{it} + \dot{q}_{it} h_{it} \right] \tag{10}$$

And associated Lagrangian is:

$$\hat{\mathcal{L}} \equiv \hat{\mathcal{H}} + \lambda_{it} \left[W_{it} - (1 - \theta_i) \, q_{it} h_{it} \right] \tag{11}$$

First order conditions can be simplified to:

[19] In fact $r < \rho$ is the general feature of most of the models with incomplete markets with shocks to the income profile of the households. For example see (Ayigari (1994) , Mendoza, Quadrini and Rios-Rull (2009)).

$$
\begin{aligned}
\frac{\dot{c}_{it}}{c_{it}} &= (r-\rho)+\frac{\lambda_{it}}{\mu_{it}} && (12)\\
\frac{\eta c_{it}}{h_{it}} &= \left(rq_{it}+\delta-\dot{q}_{it}\right)+\left[\left(1-\theta_i\right)q_{it}\frac{\lambda_{it}}{\mu_{it}}\right] && (13)\\
\lambda_{it} &\geq 0 \;\; (= \; iff\, W_{it}>\left(1-\theta_i\right)q_{it}h_{it}) && (14)
\end{aligned}
$$

Without the borrowing constraint, equation (12) is the usual Euler equation. μ_{it} is the marginal benefit of one more unit of consumption and, therefore, λ_{it}/μ_{it} is the relative marginal value of one more unit of borrowing. This equation shows that the higher the relative marginal value of borrowing, the higher the growth rate of consumption, which means the lower the ability of household to transfer resources from the future to now. In equation (13), $(rq_t+\delta-\dot{q}_t)$ is the user cost of housing in a frictionless economy and, therefore, without the borrowing constraint, consumption smoothing between non-housing goods and housing implies $(rq_t+\delta-\dot{q}_t)\,h_{it}/\eta=c_t$. However, when the borrowing constraint is binding, the representative household cannot afford the down payment for buying a house and the household's demand for housing declines in comparison with the frictionless case. The higher the required down payment for each unit of housing $((1-\theta)\,q_t)$, the higher the decline in the demand for housing.

2.3 Equilibrium Characterization

So far we have characterized the differential equations governing the optimal behavior of the representative household for a given path of prices. The final step is to add the supply side of the housing market and to find the equilibrium house prices for the given behavior of the representative household. Let us define L_i as the aggregate supply of land in region i, h_{i0} initial housing stock of the representative agent and a_{i0} as the initial holding of financial assets by the representative household in region i . In order to reduce the number of variables for the definition of the equilibrium I use the equilibrium relations (4) and (5) $(k_{it}/B=l_{it}=h_{it}, q^L_{it}=q_{it})$. Then one can define the equilibrium as follows:

Definition 1: *Equilibrium in region i is a set of choices $[c_{it},a_{it},h_{it}]_{t=0}^{\infty}$ by the representative household and a set of house prices $[q_{it}]_{t=0}^{\infty}$ such that*

- *The representative household takes $[q_{it}]_{t=0}^{\infty}$ as given and maximizes its lifetime utility, i.e., solves problem (9) with the initial condition $W_{i0}=a_{i0}+q_{i0}h_{i0}$.*

- *Total demand for land in region i does not exceed the total supply ($h_{it} \leq L_i$) with equality if and only if $q_{it} > B$.*[20]

Now in order to show main insights of the model, I consider two extreme cases for the supply of housing:

- Inelastic Supply: The supply of land in this case is very limited, such that all the land in the region has been used and the aggregate supply of housing is constant and equal to the total supply of land in the region ($h_{it} = L_i$, $i \in \{Inelastic\, Regions\}$).[21]

- Elastic Supply: In this case there is plenty of unused land and therefore the price of land, q_L is zero. This results in a constant price for houses equal to the cost of capital used for building the house ($q_{it} = B$, $i \in \{Elastic\, Regions\}$).

In characterizing the equilibrium for both regions, I use the following two lemmas that hold for both elastic regions and inelastic regions.

Lemma 1: *Suppose q_{it} is finite for all t. Then, for any value of a_{i0}, there exists t_1 such that $\lambda_{t_1} > 0$.*

Proof: see Appendix A.

Lemma 1 argues that independent of initial financial holdings of the representative household in region i (a_{i0}), there exists a time t_1 at which the household borrowing constraint binds ($\lambda_{t_1} > 0$). Intuition for this lemma is that since $r - \rho < 0$, when the household borrowing constraint does not bind, household consumption has a negative growth rate. This means the household wants to transfer as many of the resources as it can to today which results in the borrowing constraint becoming bindings.

Lemma 2: *Suppose r and θ_i are fixed. If there exists t_1 such that $\lambda_{t_1} > 0$, then $\lambda_t > 0$ for all $t \geq t_1$.*

Proof: see Appendix B.

Lemma 2 claims that in an economy without changes in r and θ_i, whenever the borrowing constraint binds, it remains binding forever. The intuition for this result is that in order for

[20] This is equivalent to the price of land being zero ($q^L_{it} = 0$)

[21] The necessary condition for this is:

$$L_i < \frac{\eta w}{(1+\eta)\,\delta + B\left[(1+\eta)\,\theta r + (1-\theta)\,\rho\right]}$$

This means the demand for housing when house prices are equal to B, or in other words the price of land is zero, should be greater than the total supply of land in region i.

a constrained borrowing constraint to become unconstrained, the representative household should either reduce its consumption or its housing stock or the growth in house prices should increase. Because of the frontloading motivation, a decline in consumption or in housing stock are not desirable for a household. The proof shows an increase in growth of house prices that leads to a transition from a constrained borrowing constraint to a relaxed borrowing constraint cannot be an equilibrium because it results in the demand for housing exceeding the supply.

Lemmas 1 and 2 together show that in the steady-state the borrowing constraint is binding. Moreover it shows that there is, at most, one point in time in which the borrowing constraint of the representative household becomes binding. Therefore in order to solve for the entire equilibrium path, we must solve the problem backwards. First, we solve for the steady-state equilibrium. Second, we characterize the transition path while the household borrowing constraint is binding. Then we characterize the transition path when the borrowing constraint does not bind. Finally, using the household's initial financial assets and the fact that house prices are a continuous function of time, we find the point in time at which the borrowing constraint becomes binding.

I now characterize the equilibrium of inelastic regions and then proceed to the equilibrium of elastic regions.

2.3.1 Equilibrium Characterization for Inelastic Regions

In regions with an inelastic supply of housing, the total supply of housing is fixed and therefore the budget constraint of the representative household reduces to:

$$\dot{a}_{it} = w_i + ra_{it} - c_{it} - \delta L_i \tag{15}$$

When the borrowing constraint is binding, equations (12) to (14) in addition to (15) reduce to:

$$\begin{aligned}
\frac{\dot{c}_{it}}{c_{it}} &= (r-\rho) + \frac{(1+\theta_i\eta)\,c_{it} - (w-(1-\theta_i)\delta L_i)}{\theta_i\,(1-\theta_i)\,L_i q_{it}} && (16)\\
\dot{q}_{it} &= rq_{it} + \frac{\delta}{\theta_i} - \frac{w-c_{it}}{\theta_i L_i} && (17)\\
a_{it} &= -\theta_i q_{it} L_i && (18)
\end{aligned}$$

Steady state can be derived by imposing $\dot{c}_{it} = \dot{q}_{it} = 0$ in equations (16) and (17).

Proposition 1: *In the steady state of inelastic region i, the household housing wealth and non-housing consumption are given by:*

$$c_{ss}^{Inelastic} = \frac{[\theta_i r + (1-\theta_i)\rho] w_i - \delta L_i (1-\theta_i)\rho}{(1+\eta)\theta_i r + (1-\theta_i)\rho} \tag{19}$$

$$(q_{ss}h_{ss})^{Inelastic} = q_{ss}^{Inelastic} L_i = \frac{\eta w_i - (1+\eta)\delta L_i}{(1+\eta)\theta_i r + (1-\theta_i)\rho} \tag{20}$$

Corollary 1: *Comparative statics with respect to the interest rate r*

$$\frac{\partial c_{ss}^{Inelastic}}{\partial r} < 0, \qquad \frac{\partial (q_{ss}h_{ss})^{Inelastic}}{\partial r} < 0 \tag{21}$$

and with respect to the maximum loan to value ratio (θ_i) *are:*

$$\frac{\partial c_{ss}^{Inelastic}}{\partial \theta_i} < 0 \qquad , \qquad \frac{\partial \left(\theta_i (q_{ss}h_{ss})^{Inelastic}\right)}{\partial \theta_i} > 0 \tag{22}$$

$$\frac{\partial (q_{ss}h_{ss})^{Inelastic}}{\partial \theta_i} \gtreqless 0 \quad if\, and\, only\, if \quad \rho - (1+\eta) r \gtreqless 0 \tag{23}$$

Equations (21) show that the lower the interest rate, the higher the housing wealth and non-housing consumption of the household. Lower interest rates reduce the user cost of housing. Since the supply of housing is fixed, house prices should increase enough to reduce demand and make it equal to supply. Taking household debt as given, lower interest rates means lower interest payments for the household, which leaves more resources for consumption. However, this effect is partly muted because in the steady-state household debt is also increasing.

Equation (22) says that as a result of an increase in θ (i.e. lower collateral requirement), the steady-state consumption of the household *declines*. The intuition for this result is that a higher θ enables the representative household to borrow more. But after the household uses up this new borrowing capacity, it cannot borrow any more, and the household ends up with a higher amount of debt which translates into higher interest payments. But higher interest payments mean fewer resources remain for non-housing consumption. The impact of an increase in θ on housing wealth (equation (23)) is more interesting: on one hand the increase in θ means a lower down-payment is required for each unit of housing, which increases demand for housing. On the other hand, because of the consumption smoothing between non-housing and housing consumption, lower non-housing consumption in the steady state lowers the demand for housing. Therefore the change in housing wealth depends on the relative importance of these two forces. The higher η is the stronger the consumption-smoothing force and, therefore, the more negative the change in housing wealth. The higher is $\rho - r$ the more important is the lower down payment in boosting the demand for housing and therefore the more positive is the change in the housing wealth. However no matter whether the steady state housing wealth increases or decreases, as a result of an increase in θ, the total borrowing capacity (and therefore the total debt in the steady state)$\theta_i \left(q_{ss}h_{ss}\right)^{Inelastic}$ increases.

After characterizing the steady-state equilibrium, now we can characterize the transition path for the representative household that begins with an initial condition (initial debt holding) that is different from the steady-state.

The next lemma shows that in inelastic regions, whenever the borrowing constraint is binding, the economy is in steady-state.

Lemma 3: *For any region i with an inelastic supply of housing, if $\lambda_{it} > 0$ then $q_{it} = q_{ss}^{Inelastic}$ and $c_{it} = c_{ss}^{Inelastic}$.*

Proof: From lemma 2 we see that once the borrowing constraint becomes binding it remains binding forever and therefore the behavior of house prices and of consumption is fully characterized by equations (16)-(18). Then from the (q_{it}, c_{it}) phase diagram in Figure 3 we see that this system of equations does not have any stable path. And the steady-state point given by $\dot{q}^{Constrained} = 0$, $\dot{c}^{Constrained} = 0$ is the only stable point in this system of equations.

When the borrowing constraint is not binding ($a_{it} > -\theta_{it}q_{it}L_i$), the household maximization problem (equations (12) to (14)) and the household budget constraint (equation (8)) reduce to:

$$\frac{\dot{c}_{it}}{c_{it}} = r - \rho \tag{24}$$

$$\dot{q}_{it} = rq_{it} + \delta - \frac{\eta c_{it}}{L_i} \tag{25}$$

$$\dot{a}_{it} = w_i - c_{it} + ra_{it} - \delta L_i \tag{26}$$

As Figure 4 illustrates, among the paths described by equations (24) and (25), there is only one path that crosses the steady state. In equilibrium the household consumption and home prices move along this path until the borrowing constraint becomes binding. Moreover, initial point (q_{i0}, c_{i0}) should be such that exactly at the time the agent is reaching the steady state point (q_{ss}, c_{ss}), the borrowing constraint should become binding. Let us define T_i as the time it takes the economy in region i to reach its steady state. Proposition 2 characterizes the equilibrium path for inelastic region i, with initial level of debt holding a_{i0}.

Preposition 2: *In the inelastic region i, starting from an initial level of debt holding $a_{i0} > -\theta_i \left(q_{ss} h_{ss}\right)^{Inelastic}$:*

- *The representative household borrowing constraint does not bind throughout the transition until the economy reaches its steady state characterized by (19), (20) and $a_{ss}^{Inelastic} = -\theta_i \left(q_{ss} h_{ss}\right)^{Inelastic}$*
- *The economy in inelastic region i reaches its steady state in a finite time ($T_i < \infty$).*
- *The representative household non-housing consumption, house prices and representative household debt-holding during the transition (i.e. $t \in [0, T_i]$) are given by:*

$$\begin{aligned} c_{it} &= c_{ss}^{Inelastic} e^{(r-\rho)(t-T_i)} && (27) \\ q_{it} &= -\frac{\delta}{r} + \frac{\eta}{\rho L_i} c_{ss}^{Inelastic} e^{(r-\rho)(t-T_i)} + \left(q_{ss}^{Inelastic} + \frac{\delta}{r} - \frac{\eta}{\rho L_i} c_{ss}^{Inelastic} \right) e^{r(t-T_i)} && (28) \\ a_{it} &= a_{i0} e^{rt} + \left(\frac{w - \delta L_i}{r} \right) \left(e^{rt} - 1 \right) + \frac{c_{ss}^{Inelastic}}{\rho} e^{(r-\rho)(t-T_i)} \left(1 - e^{\rho t} \right) && (29) \end{aligned}$$

where T_i is the solution to:

$$-\theta_i \left(q_{ss} H_{ss}\right)^{Inelastic} = a_{i0} e^{rT_i} + \left(\frac{w - \delta L_i}{r} \right) \left(e^{rT_i} - 1 \right) + \frac{c_{ss}^{Inelastic}}{\rho} \left(1 - e^{\rho T_i} \right) \quad (30)$$

Proof: The fact that representative consumer borrowing constraint does not bind throughout the transition is because the only stable point of the constrained regime is the steady state (lemma 3). Equations (27) to (29) are solutions to the first-order differential equations that result from the household maximization problem, assuming the borrowing constraint is relaxed ((24)-(26)) plus imposing the following boundary conditions:

$$\begin{aligned} c_{iT} &= c_{ss}^{Inelastic}, \; q_{iT} = q_{ss}^{Inelastic}, \\ a_{i0} &: \; given \end{aligned}$$

Finally equation (30) arises from the fact that once the household reaches the steady state the borrowing constraint should become binding: $a_{iT} = -\theta_i \left(q_{ss} H_{ss}\right)^{Inelastic}$.

Defining $BC_{i0} \equiv a_{io} + \theta_i \left(q_{ss} h_{ss}\right)^{Inelastic}$ as the initial unused borrowing capacity, we have the following comparative statics:

Corollary 2: *Comparative statics with respect to unused borrowing capacity are:*

$$\frac{\partial T_i}{\partial BC_{i0}} > 0 \ , \ \frac{\partial c_{i0}}{\partial BC_{i0}} > 0 \ , \ \frac{\partial q_{i0}}{\partial BC_{i0}} > 0$$

Proof: The right hand side of (30) is a decreasing function of T_i. Therefore an increase in a_{i0} results in an increase in T_i. Then from (27) and (28) one can see that c_{i0} and q_{i0} are increasing in T_i.

Corollary 3 shows that the larger the unused borrowing capacity, the longer it takes the economy to reach the steady state, and therefore, the economy starts from a point that is further away from the steady state. This means household consumption and house prices are initially higher. This corollary is very useful when we introduce unexpected changes to the interest rate and the maximum loan-to-value ratio into the economy.

2.3.2 Equilibrium Characterization for Elastic Regions

The main difference between elastic regions and inelastic regions is that house prices are constant in elastic regions. Since lemmas 1 and 2 hold for elastic regions as well, we follow the same steps as before and characterize the equilibrium backward : solving for the steady-state, characterizing transition while the borrowing constraint is binding, and finally solving for the whole equilibrium by characterizing the transition path when the borrowing constraint is relaxed.

The representative household utility maximization (given by equations (12) to (14)) when its borrowing constraint is binding ($\lambda_{it} > 0$) in addition to house prices being constant ($q_{it} = B$) result in:

$$\begin{aligned} (1-\theta_i)\, B \frac{\dot{c}_{it}}{c_{it}} &= -\left[\theta_i r B + (1-\theta_i)\, \rho B + \delta\right] + \frac{\eta c_{it}}{h_{it}} && (31) \\ (1-\theta_i)\, B \dot{h}_{it} &= w_i - c_{it} - \left(\theta_i r B + \delta\right) h_{it} && (32) \end{aligned}$$

Imposing steady state conditions $\dot{c} = 0$ and $\dot{h} = 0$ leads to the solution for the steady state:

Proposition 3: *In the steady state of elastic region i, the household housing wealth and non-housing consumption are given by:*

$$c_{ss}^{Elastic} = \frac{\left[\theta_i r + (1-\theta_i)\,\rho + \delta/B\right] w_i}{(1+\eta)\,(\theta_i r + \delta/B) + (1-\theta_i)\,\rho} \tag{33}$$

$$(q_{ss}h_{ss})^{Elastic} = Bh_{ss}^{Elastic} = \frac{\eta w_i}{(1+\eta)\,(\theta_i r + \delta/B) + (1-\theta_i)\,\rho} \tag{34}$$

Corollary 3: *Comparative statics with respect to the interest rate r*

$$\frac{\partial c_{ss}^{Elastic}}{\partial r} < 0, \qquad \frac{\partial\,(q_{ss}h_{ss})^{Elastic}}{\partial r} < 0 \tag{35}$$

and with respect to the maximum loan to value ratio (θ_i) *are:*

$$\frac{\partial c_{ss}^{Elastic}}{\partial \theta_i} < 0, \qquad \frac{\partial\left(\theta_i\,(q_{ss}h_{ss})^{Elastic}\right)}{\partial \theta_i} > 0 \tag{36}$$

$$\frac{\partial\,(q_{ss}h_{ss})^{Elastic}}{\partial \theta_i} \gtreqless 0 \quad if\,and\,only\,if \quad \rho - (1+\eta)\,r \gtreqless 0 \tag{37}$$

Equation 35 shows that the lower the interest rate, the higher the housing wealth. However, the impact of lower interest rates on housing wealth in the steady-state is smaller for elastic regions (in compare to its impact in inelastic regions). This is because lower interest rates reduce the user cost of housing, and households in elastic regions build larger houses. However having a larger house results in higher depreciation costs which dampens the effect of lower interest rates on housing wealth. As before, lower interest rates increase the steady-state consumption. Higher θ_i (i.e. lower collateral requirement) reduces the steady state consumption and its impact on housing wealth depends on the balance between front-loading motivation (or the importance of lower down-payments) and consumption smoothing between housing and non-housing consumption.[22]

[22] One observation is that if the depreciation cost is δqh instead of δh, the relation between the steady state housing wealth and consumption in both regions is the same and equal to:

$$q_{ss}h_{ss} = \frac{\eta}{(1+\eta)\,(r\theta + \delta) + (1-\theta)\,\rho} w$$

$$c_{ss} = \frac{\delta + r\theta + (1-\theta)\,\rho}{(1+\eta)\,(\delta + r\theta) + (1-\theta)\,\rho} w$$

One example in which the depreciation cost can be written as δqh is when housing is produced according to a Cobb-Douglas production function using capital and land. It seems that the real world is not a Leontief case since with better-quality facilities on the land the consumer can enjoy his or her housing more. On the other hand the study of Davis and Heathcote (2007) shows the share of land in the value of house is an

The following lemma characterizes the transition path of an elastic region i when the representative household borrowing constraint is binding.

Lemma 4: *In elastic region i, if $\lambda_{it} > 0$ then the solution to household maximization problem (equations* (31) *and* (32)*) is a saddle path for* (h_{it}, c_{it}) *described by*

$$c_{it} = f(h_{it}) \tag{38}$$

where $f(.)$ is a strictly increasing function and $c_{ss}^{Elastic} = f(h_{ss}^{Elastic})$.

Proof: Again from lemma 2 we use the fact that once the borrowing constraint becomes binding it remains binding forever and therefore the behavior of house prices and of consumption is fully characterized by equations (31) *and* (32). Then from the (q_{it}, c_{it}) phase diagram in Figure 5 we see that this system of equations has one saddle path that passes through the steady-state.

In elastic region i, when the borrowing constraint is not binding, the household maximization problem reduces to:

$$\frac{\dot{c}_{it}}{c_{it}} = r - \rho \tag{39}$$

$$c_{it} = \frac{rB + \delta}{\eta} h_{it} \tag{40}$$

$$\dot{a}_{it} = w - \frac{(1+\eta)\left(r + \frac{\delta}{B}\right) - \rho}{r + \frac{\delta}{B}} c_{it} + r a_{it} \tag{41}$$

Using equation (40), we can see that the point (h_{th}, c_{th}) is defined as a solution to this system of equations:

$$\begin{aligned} c_{th} &= f(h_{th}) \\ c_{th} &= \frac{rB + \delta}{\eta} h_{th} \end{aligned}$$

is the only point at which the borrowing constraint can go from being relaxed to being

an increasing function of house prices which is inconsistent with the Cobb-Douglas case but is consistent with a CES production function for housing in which there is complementarity between land and capital. Assuming $h = \left[\omega_k^{1/\sigma} k^{\frac{\sigma-1}{\sigma}} + (1-\omega_k)^{1/\sigma} l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ with $0 < \sigma < 1$, depreciation cost can be written as $\delta q^{\sigma} h$ which is in between Leontief case ($\sigma = 0$) and Cobb-Douglas case ($\sigma = 1$). In the calibration exercise, I use a CES characterization. It is shown that much of the intuition from the Leontief case holds for the CES case as well.

binding.[23] Finally let us also define $a_{th} \equiv -\theta_i B h_{th}$ and $W_{i0} \equiv a_{i0} + Bh_{i0}$ as the initial wealth of the representative household in region i. Now we can characterize the full equilibrium path as follows:

Proposition 4:

- *If $W_{i0} \leq (1-\theta_i) B h_{th}$, the household borrowing constraint is binding throughout the transition, and (h_{it}, c_{it}) is the solution to equations (31) and (32) with the initial conditions:*

$$h_{i0} = \frac{W_{i0}}{(1-\theta_i) B} \ , \ c_{i0} = f(h_{i0})$$

  and throughout the transition $c_{it} = f(h_{it})$.

- *If $W_{i0} > (1-\theta_i) B h_{th}$, the household borrowing constraint does not bind initially and in finite time (T_i) the borrowing constraint becomes binding and remains binding. The equilibrium (h_{it}, c_{it}) is characterized by:*

  - *for $t \in [0, T_i]$ the borrowing constraint does not bind, and the equilibrium is the solution to equations (39) to (41) with boundary-condition equations $h_{iT} = h_{th}$, $c_{iT} = c_{th}$ and $a_{iT} = a_{th}$:*

$$\begin{aligned}
c_{it} &= c_{th} e^{(r-\rho)(t-T_i)} \\
h_{it} &= h_{th} e^{(r-\rho)(t-T_i)} \\
a_{it} &= a_{th} e^{r(t-T_i)} + \frac{w}{r}\left(e^{r(t-T_i)} - 1\right) + \left(\frac{(1+\eta)\left(r + \frac{\delta}{B}\right) - \rho}{r + \frac{\delta}{B}}\right) \frac{c_{th}}{\rho} e^{(r-\rho)(t-T_i)} \left(1 - e^{\rho t}\right)
\end{aligned}$$

    And T_i is computed with the additional boundary condition that $W_{i0} (= a_{i0} + Bh_{i0})$ is given.

  - *for $t > T_i$, the borrowing constraint is binding and the equilibrium (h_{it}, c_{it}) is characterized by the solution to equations (31) and (32) with the boundary conditions $h_{iT} = h_{th}$, $c_{iT} = c_{th}$ and $a_{iT} = a_{th}$. and $c_{it} = f(H_{it})$*

Figure 6 shows the equilibrium transition path in the elastic region. If the household initial wealth is high enough, the household borrowing constraint is relaxed for awhile, and along the transition $c_{it} = (rB + \delta) H_{it}/\eta$. As the representative household exhausts its borrowing capacity, its demand for housing and for consumption declines until it reaches the point

[23]In other word at this point the shadow value of the borrowing constraint λ is equal to zero.

(h_{th}, c_{th}). From that point forward the borrowing constraint remains binding, and it is moving on the saddle path characterized by $c_{it} = f(H_{it})$ until the household reaches the steady state.

2.4 Impacts of Unexpected Permanent Changes in the Interest Rate and Collateral Requirements

So far I have assumed that the interest rate (r) and the maximum loan-to-value ratio in each region (θ) do not change. In this section I study the impact of *unexpected permanent* changes in r and θ for elastic and inelastic regions. I maintain the assumption that households in different regions assume r and θ are fixed and, therefore, any change in r and θ is a surprise for them.[24] First, I consider the impact of a permanent decline in the interest rate and a permanent increase in the maximum loan-to-value ratio and show endogenous boom-busts arise from these shocks by themselves. Then, I consider the impact of a permanent increase in the interest rate or a permanent decline in the maximum loan-to-value ratio and show this may result in fast decline in consumption and house prices that is partly recovered after the initial decline. Finally in order to keep the text short, I assume the economy is in the steady-state before the shocks happen. The extension of results to an arbitrary initial condition is straight-forward.

2.4.1 Permanent Decline in the Interest Rate or Increase in the Maximum Loan-to-Value Ratio

Proposition 5: *Following an unexpected permanent decline in the interest rate r or increase in the maximum loan-to-value ratio θ_i in an inelastic region i, house prices q_{it} and non-housing consumption of the representative household in the region i ,c_{it}, increase discretely and the representative household borrowing constraint becomes relaxed. The initial increase in house prices and consumption is followed by a steady decline in both of them (q_{it}, c_{it}) until the economy reaches the new steady state. Throughout the transition and before reaching the steady state the borrowing constraint remains relaxed.*

The intuition for this result can be derived from equations (21) and (22). As a result of an unexpected permanent decline in the interest rate or of an increase in θ, in the new steady-state the household can rollover more debt. However, a household's level of debt holding

[24] This is a strong assumption and perhaps a more realistic assumption would be that households assume a stochastic process for r and θ. However this assumption not only makes the model tractable, but also helps to differentiate between the main mechanism of this paper (interaction between frontloading behavior and endogenous asset prices) and the precautionary saving motivation that exists in incomplete market settings such as those described in Guerrieri and Lorenzoni (2012).

before the shock and just after the shock are the same. This means that the household has some unused borrowing capacity and can therefore finance a higher level of consumption during the transition. But higher consumption also means more demand for housing which, in equilibrium, translates to higher home prices. As the representative household uses up its borrowing capacity, its consumption falls and therefore its demand for housing also declines, which results in a decline in home prices.

Figure 7 depicts the impact of a decline in interest rates. As a result of a decline in the interest rate, curves characterized by $\dot{q}^{Constrained} = 0$ and $\dot{c}^{Constrained} = 0$ shift to the right, and the new steady-state consumption and home prices are both higher than before.[25] In Figure 7 point a represents the steady-state equilibrium consumption as well as home prices for an inelastic region before a decline in the interest rate. After the interest rate decline, as a result of the wealth effect of the interest rate on home prices, the household borrowing constraint relaxes and the household can now finance a higher level of consumption by borrowing more. Therefore household consumption and home prices jump to a point on the new transition path (a') such that when the household reaches the new steady state it has used up all of its borrowing capacity.

Proposition 6: *In elastic region i, an unexpected, permanent decline in the interest rate results in a gradual increase in housing (h_{it}) and non-housing consumption (c_{it}) until the economy reaches the new steady-state. Throughout the transition the borrowing constraint remains binding.*

Figure 9 shows the impact of an unexpected decline in the interest rate in the (h_{it}, c_{it}) phase diagram. In contrast to inelastic regions, an interest rate shock does not generate a boom-bust pattern in the elastic regions. This is due to the fact that since home prices are constant, a decline in the interest rate does not lead to an immediate change in the wealth of households (in contrast to inelastic regions), and the household's borrowing constraint remains binding even after the shock. However as a result of the decline, interest payments of households decline and the freed-up resources are used to purchase a larger house as well as to increase non-housing consumption. In terms of the figure, following the decline in interest rate a constrained household housing (h_{it}) remains constant and its non-housing consumption changes discretely, which is shown as a jump from a to a' and moves along the saddle path until it reaches the new steady state.

Proposition 7: *In elastic region i, an unexpected permanent increase in the maximum loan-to-value results in a discrete increase in housing (h_{it}) and in non-housing consumption*

[25] This results from equation (21).

(c_{it}). *The initial increase in housing and in non-housing consumption is followed by a steady decline in both of them* (h_{it}, c_{it}) *until the economy reaches the new steady state.*

An increase in the maximum loan-to-value ratio enables households in elastic regions to extract more equity from the current houses that they have and to use the extra resources towards the purchase of a larger house and an increase in consumption. However as they use up their borrowing capacity their housing and non-housing consumption both decline until the economy reaches the steady state. Therefore a permanent increase in the maximum loan-to-value ratio in an elastic region generates a boom-bust cycle in housing and non-housing consumption (Figure 10).

2.4.2 Permanent Increase in the Interest Rate or Decline in the Maximum Loan-to-Value Ratio

So far I have characterized the responses of different regions to a permanent decline in interest rates or collateral requirements that are not reversed, and I show that these shocks by themselves generate a boom-bust cycle. This subsection considers the response of different regions to a permanent surprise increase in the interest rate or the collateral requirements assuming the economy is in steady-state before the shocks hit.

Proposition 5': *In inelastic region i, an unexpected permanent increase in the interest rate r or a decrease in the maximum loan-to-value ratio* (θ_i) *results in a discrete decline in house prices* (q_{it})*, housing consumption* (h_{it}) *and non-housing consumption* (c_{it})*. The initial decline in house prices, housing and non-housing consumption is followed by a steady increase in all of them* (q_{it}, h_{it}, c_{it}) *until the economy reaches its new steady state. Throughout the transition the borrowing constraint remains binding, and house prices grow at rate r.*

For the proof and a full characterization of the transition path see Appendix C.

Following an unexpected increase in the interest rate or collateral requirements house prices in inelastic regions will decline. Because the household borrowing constraint was binding before the shock hit the economy, the representative household should sell part of its stock of housing in order to reduce its debt and meet the collateral constraint.[26] However, this reduces house prices furthermore and the household needs to sell even a higher fraction of its housing stock to meet the collateral constraint. After house prices decline enough, investors who are not benefiting from the housing services of the house itself buy part of the land in the inelastic region i from the representative household. This is because of their

[26] A decline in consumption by itself cannot help a household meet its borrowing constraints because that will not change the stock of debt immediately, whereas house prices drop immediately after a shock hits the economy.

anticipation of future growth of the price of land in the region i. Because of the consumption smoothing between housing services and non-housing consumption, the representative household consumption also declines. The initial decline in consumption and housing stock increases household saving rate which enables household to buy a larger house and increase its consumption throughout the transition. In the steady state the representative household buys back all the lands that was sold to the investors and therefore $h_{ss} = L_i$. steady state house prices and consumption are also given by equations (19) and (20).

Proposition 6': *In elastic region i, an unexpected permanent increase in the interest rate results in a gradual decrease in both housing (h_{it}) and non-housing consumption (c_{it}) until the economy reaches the new steady-state.*

In elastic regions, house prices are constant. Therefore changes in the interest rate do not have any immediate impact on the household's housing wealth. However, because higher interest rates increase the user cost of housing, households decrease their stock of housing gradually until they arrive at the new steady state.

Proposition 7': *In elastic region i, an unexpected permanent decrease in the maximum loan-to-value ratio results in a discrete decline in both housing (h_{it}) and non-housing consumption (c_{it}). The initial decline in housing and non-housing consumption are followed by a gradual increase in both of them (h_{it}, c_{it}) until the economy reaches the new steady state.*

In the elastic region, as a result of an increase in collateral requirements households will need to sell a fraction of their housing stock in order to meet the new borrowing constraint. Because of the complementarity between housing and non-housing, their non-housing consumption also declines. However they gradually use the extra resources released from lower consumption to buy a larger house until the economy reaches the new steady state.

3 Data

In order to test implications of the model for the impacts of a decline in interest rates and an increase in the maximum loan-to-value ratio, I exploit the fact that there is a great deal of heterogeneity in the elasticity of the housing supply in different regions of US. In the reduced-form analysis of the next section, each county in US with a population of over 150,000 in 2000 comprises a single observation. The main reason for choosing the county as the level of aggregation (instead of MSA) is that Census contains many detailed information about the characteristics of counties. Aggregating at the state level not only reduces the number of observations considerably, but also reduces the variation of elasticity and changes in securitization rate by more than one half. The postal ZIP code level is also not a good option since much regional information is not available at the ZIP code level or its accuracy is questionable. Moreover, there are other important factors that affect the housing market at the ZIP code level such as gentrification that are not included in my model.[27]

In what follows I briefly discuss data the sources used for the empirical portion of this paper as well as their limitations.

Federal Reserve Bank of New York Consumer Credit Panel (FRBNY CCP) The FRBNY Consumer Credit Panel consists of the credit reports of anonymous and nationally-representative 5% random sample of all individuals in the US with a credit file which is released on a quarterly basis. The data begins in the first quarter of 1999.[28] FRBNY CCP contains information on the total debt holdings of individuals with its breakdown into mortgage and home equity loans, auto loans, student loans and credit cards. In order to test the predictions of the model about the dynamics of households debt in different regions and its co-movement with home prices and consumption, I have aggregated the data on the *total mortgage and home equity holdings* of individuals at the county level.[29]

Another challenge is to come up with a measure of *consumption* at the county level. Since I could not find any direct measure of consumption at the county level, I construct a measure of *car sales* at the county level using the data on auto loans of individuals. The idea here is that whenever the auto-loan holding of an individual increases by an amount larger than a threshold between two consecutive quarters,[30] it is assumed the individual has bought a new car, with the value of the car set at a constant times the change in the total auto-loan holding. Given the low interest rates car companies are charging for financing new cars, this assumption does not seem implausible when considering the sale of new cars.[31] However, using this measure as a measure of consumption presents a number of limitations. Most

[27] Guerrieri, Hartley and Hurst (2012) presents evidence in support of gentrification channel at the zip code level.

[28] In addition to this 5% primary sample, credit reports of all the other members of the family of these individuals are also available. However in order to keep the calculations straight simple, I am just using the primary sample. More information on FRBNY CCP can be find in Lee and van der Klaauw (2010). Aggregation of this data at the county level has recently been made available at: http://www.newyorkfed.org/householdcredit/

[29] Using total debt instead of mortgage debt did not really affect the results. This is partly because mortgage loan counts for almost 80% of the consumer debt. The other reason for using mortgage debt instead of total debt is that my model is silent about other forms of debt like student loan and credit cards.

[30] I used \$5000 as the threshold. But the result are robust to changes in this threshold.

[31] Of course this may underestimate the volume of cars sold on the secondary market. But on the other hand unless used cars are sold from one region to another region, we are double counting the volume of cars that are bought within a county.

importantly, as is documented by Mian, Sufi and Rao (2012) and Berger and Vavra (2012), car sales response to the financial crisis has been significantly larger than other components of consumption. Therefore the differential response of consumption in regions with different supply of housing and different changes in securitization rate during the boom period and the bust period may be overestimated. For this reason, I check the robustness of results on consumption by using employment in food services and in the retail trade sector as proxies for consumption.[32]

CoreLogic Home Price Index (HPI) For data on counties *home prices*, CoreLogic Home Price Index (HPI) is used. CoreLogic HPI has number of advantages over other indices that makes it a very good match for the purpose of my model: First of all unlike the Case-Shiller home price index it is available at the county level for most US counties on a monthly basis. Secondly, HPI is a price index constructed by the repeat-sales. Therefore one need not have concerns about the change in the characteristics of houses that are traded. Finally the fact that HPI is using the distribution of houses in its entirety for constructing the index gives HPI an advantage over the FHFA price index, which is limited to transactions involving conforming, conventional mortgages purchased or securitized by Fannie Mae or Freddie Mac. The conforming loan limit especially biases the results in the case of the large cities with many houses carrying a mortgage above the conforming-loan limit.

Home Mortgage Disclosure Act (HMDA) The HMDA, which was enacted in 1975, requires most mortgage lenders to record a number of important details about each loan applicant, such as the final decision of the lender, the loan amount, the purpose of the loan, and most importantly whether the loan has been kept on the bank balance sheet, sold to a government-sponsored enterprise (like Fannie Mae and Freddie Mac) or has been sold on the secondary market. The data is publicly available at the individual applicant level.

One of the main parameters of the model discussed in the previous part is the maximum loan-to-value ratio θ. But since I do not have a direct measure of θ,[33] I use changes in the

[32] The result for changes in employment in food services and retail trade sectors are presented in Appendix F.

[33] As it is documented in Keys, Piskorski, Seru and Vig (2012) most of the increase in the loan-to-value ratio during the boom period comes from the usage of second and third lien loans and not the first lien by itself. Therefore one needs to have a comprehensive measure of all the loans that home buyers took out to purchase or refinance a house. Another equally important point is that one must control for the quality of borrowers and their characteristics. For example it could be the case that individual LTVs are not changing, but rather, lending standards are getting relaxed. For example as it is well documented, there was a rise in number of loans with low or no documentation such that at the pick they were counting for half of the issued loans. In terms of the model this means that some households can borrow more than before, which one can think of as an increase in θ.

fraction of purchase loans within the conforming loan-size limit[34] *that were sold into non-government sponsored organizations* (non-GSEs) as a proxy for the change inθ. The idea here is that since GSEs have an explicit subsidy from the government, if there is a loan within the conforming loan limit and it is sold to non-GSEs (instead of GSEs) this is most likely because the loan had a loan-to-value ratio that is not within the criteria imposed by GSEs. Therefore the change in the fraction of loans sold to non-GSEs can be a measure of the extent to which lending standards has become more relaxed. In the empirical part, I show that the change in the fraction of loans sold to non-GSEs goes a long way towards explaining the debt accumulation of households. For more evidence on the relation between the increase in the fraction of loans sold to non-GSEs and the relaxation of lending standards, see Mian and Sufi (2009).

Of course, there are a number of limitations in using changes in the fraction of loans sold to non-GSEs as a proxy for θ. First of all even if there was no change in the lending standards to households, the fact that rating agencies began to give higher ratings to mortgage backed securitized assets (see Ashcraft, Goldsmith-Pinkham and Vickery (2010)) induces lenders to sell a higher fraction of their loans to non-GSEs and therefore this fraction may increase without an increase in θ. Another shortcoming of this measure is that it mostly captures the extensive margin of financial liberalization. For example, if the loan-to-value ratio of the pool of loans sold to non-GSEs also increases, this measure underestimates the change in θ. As long as the extensive margin of financial liberalization and the intensive margin are not perfectly correlated, this results in an underestimation of the importance of relaxed credit standards.

Another measure that I construct using the HMDA dataset is the *share of investors* in the housing market of each county. This is measured as a fraction of purchase loans that the mortgage applicant's occupancy is non-owner-occupied. Misreporting the occupancy status of applicants may result in underestimation of this measure.

Local Housing Elasticity and Land Share In the Value of House The main implication of the model is the differential response of various regions with inelastic supply of housing versus regions with elastic supply to interest rates shocks and to shocks to the maximum loan-to-value ratio. In order to test the implications of the model, I use the measure of elasticity provided by Saiz (2010). Saiz (2010)'s measure of elasticity is based on the availability of land as well as on regulatory restrictions on building new houses. Since the

[34] This is a limit set by Office of Federal Housing Enterprise Oversight and changes based on the October-to-October changes in median home price. More information on the historical limits can be find at: http://www.fhfa.gov/webfiles/860/loanlimitshistory07.pdf

Saiz (2010) measure is estimated for MSAs, I match MSAs with counties and use the average elasticity of matched MSAs for each county.[35] Finally in order to construct a measure of inelasticity, I take minus logarithm of Saiz (2010) measure and normalize it such that it has a mean of zero and a variance equal to one.

Davis and Heathcote (2007) also provides an estimate of the time series of the average share of land in the value of houses for 46 large US MSAs. In the calibration part, I use both the cross section and the time series of this data in order to estimate the supply of housing for different regions in the US.

Census Data on the income and population of each county on a yearly basis comes from USA Counties,[36] which contains a collection of data at the county level from the U.S. Census Bureau and other Federal agencies such as the IRS. Data on the aggregate value of owner-occupied homes is taken the American Community Surveys of 2000, 2005 and 2008.[37]

Anti-Predatory Lending Law Measure In order to curtail predatory lending practices, Congress enacted the Home Ownership and Equity Protection Act (HOEPA) in 1994. This legislation places some restrictions on refinance mortgages or on home equity lines with excessively high interest rates or fees. Following a rise in predatory practices, some states began to add restrictions to HOEPA usually referred to as mini-HOEPA. One of the main amendments to HOEPA was the addition of home purchase loans with high rate or high fees into the regulation. In particular according to Choi (2011) as of 2005, nineteen states included home purchase loans into anti-predatory lending legislation. For a measure of the restrictiveness of anti-predatory law, I have constructed a dummy variable that is equal to one if the state added purchase loans under the coverage of its anti-predatory lending laws and zero if the state's law regulates only refinance and equity mortgages.

4 Empirical Model

4.1 Motivation

As Figure 11 shows, a motivating fact in the data is that regions that experienced a greater boom in home prices and in consumption during the interval of 2000 to 2006 suffered from a more severe bust during the period of 2006 to 2009. The main prediction of the model in

[35]For most MSAs each MSA is matched with only one county.

[36]Available at http://www.census.gov/support/USACdata.html

[37]Available at http://factfinder2.census.gov

the previous section is that this boom-bust pattern in consumption and house prices should occur in regions with a less elastic supply of housing and in regions that experienced a greater easing of collateral constraints.

Indeed, Figure 12 shows that regions with an inelastic supply of housing on average experienced a larger boom and bust in house prices and consumption. The figure also shows that among inelastic regions, the boom-bust pattern is magnified in regions that experienced a larger change in the fraction of loans sold to non-GSEs from 2003 to 2006. The bottom graph in Figure 12 also indicates that total mortgage liability per capita in inelastic regions and regions with higher changes in the fraction of loans sold to non-GSEs grow faster during the period of 2000 to 2008. What is more important for the purpose of this paper is the fact that a significant fraction of the decline in house prices and in car sales occured between 2006 and mid-2008, a period during which households continued to increase their mortgage liabilities. This suggests that a significant fraction of the decline in house prices and consumption is not driven by the inability of households to rollover their debt, but instead, is driven by the reduction in the amount that households could increase their debt holding.

Motivated by these figures, the next subsection addresses the relation between inelasticity of housing supply and changes in the fraction of loans sold to non-GSEs and house prices, consumption and debt accumulation in a reduced-form regression framework.

4.2 The Main Results

The model in the previous section shows that a decline in interest rates leads to a boom in house prices and in consumption in regions with less elastic supply of housing which is then followed by a bust in those regions. A decline in collateral requirements (i.e. increase in θ) also results in a boom-bust in consumption and in house prices that is more extreme in regions with more inelastic supply of housing.

In what follows I divide the sample into the period of the boom from 2000 to 2006, and the period of bust from 2006 to mid-2008.[38] And I run the following regression:

[38] The reason for choosing mid-2008 is to make sure we are not capturing the impact of events that followed the bankruptcy of Lehman Brothers. This period is also prior to the period when households start to deleverage their debt-holding and therefore is more useful for the purpose of differentiating between inability to borrow more and inability to rollover the debt. Extending the period of bust to 2009 or afterward results in a larger bust and gives greater significance to the result.

$$\Delta log\,(Y_{it}) \;=\; \alpha + \beta_1 Inelasticity_i + \beta_2 \Delta Securitization\,Rate_i \qquad (42)$$
$$+\beta_3\,(Inelasticity_i \times \Delta Securitization\,Rate_i) + X_{it}\Gamma + \epsilon_i$$

where Y_{it} is a dependent variable of interest which represents either house prices,[39] or a measure of car sales in county i at time t, or total mortgage liabilities. $Inelasticity$ is based on Saiz (2010) measure of elasticity of housing supply. $\Delta Securitization\,Rate_i$ is the change in the fraction of loans sold to non-GSEs in county i from 2003 to 2006. I chose this period because the aggregate changes in the fraction of loans sold to non-GSEs has the fastest growth rate during this period.[40] The baseline controls include the growth in average income[41] of county residents during the associated period and its interaction with inelasticity, population growth and the change in fraction of homes purchased by investors.[42] The interaction terms are averaged out and, therefore, β_1 and β_2 capture the average impact of housing supply and of changes in the securitization rate on the variables of interest. In general because both personal income and the fraction of investors are influenced by the change in house prices, controlling for these two factors may result in an underestimation of the impact of changes in interest rates and the maximum loan-to-value ratio on house prices and consumption. Therefore, one would expect that estimated coefficients of β_1, β_2 and β_3 in regressions that controls for characteristics of counties would be closer to zero than their estimates in regressions without controls. In order to compute the aggregate implications of changes in interest rates and securitization rate on the growth rate of variable Y, I use estimates of β_1, β_2 and β_3 from estimation of equation (42) and compute in-sample difference between $\Delta\widehat{log(Y_{it})} - \Delta\widehat{log(Y_{lt})}$ for each county i, where l is the average predicted

[39] Since I use the same deflator (CPI deflator) for all regions, the coefficients are the same for both nominal and real house prices.

[40] The results are robust to using changes in the fraction of loans sold to non-GSEs between 2003 and 2005 or the in maximum change in fraction of loans sold to non-GSEs that the regions experienced during the period of 2003 to 2006.

[41] This is based on aggregation of IRS data on ZIP codes income at the county level.

[42] Recent studies like Bayer, Geissler and Roberts (2011) and Haughwout, Lee, Tracy, van der Klaauw and Wilbert (2011) provide evidence on the role of speculators and investors in destabilizing house prices and, therefore,I control for the share of investors to make sure the result is robust to controlling for them. However, in general there are two problems with addressing the role of investors: First, because of data availability, it is hard to distinguish between those who buy leisure homes and speculators (investors) in the housing market. Therefore one should be cautious in interpreting the results on the role of investors. In terms of the model, buying a leisure house is like increasing the housing consumption, which is a direct consequence of lower interest rates and collateral requirements. Moreover, introduction of news shocks to the model shows that investors may jump in the markets they expect house prices to grow in the future. Therefore the rise of their share can be a symptom of expectations about future house prices and not its driver.

value for counties in the lowest 10 percent of inelasticity measure and the lowest 10 percent of the change in securitization rate. Then I take the average of these differences weighted by the population of the county in 2000. It is worth to mention that this procedure may underestimate the aggregate impact of securitization. This is due to the fact that during the period of 2003 to 2006, even regions in the lowest 10 percent of the change in securitization rates experienced a more than five percent increase in the fraction of loans sold to non-GSEs. On top of this, we would also expect that the aggregate impact of changes in interest rates on mortgage liabilities growth to be underestimated. This is due to the fact that a decline in interest rates occured in all places which induces even households in elastic regions to buy a larger house and increase their debt holding. Therefore we should expect that the actual and the estimated in-sample differences for mortgage liabilities to be smaller than the aggregate changes.

4.2.1 The Boom Period of 2000 to 2006

From Figure (1a) one can see that during the period of 2000 to mid-2003, there was a steady decline of more than two percent in the long-term real interest rates, followed by more than a 20 percent increase in the fraction of loans sold to non-GSEs in the interval 2003-2006. Table 1 shows that during the boom years, house prices, consumption and mortgage liabilities of more inelastic regions and of regions that experienced a larger increase in the fraction of loans sold to non-GSEs grew faster than other regions. Not controlling for the investor shares and changes in average income, the implied aggregate impact of the interest rate and changes in securitization rate explains about 75 percent of the growth in house prices, 95 percent of the growth in car sales per capita, and 20 percent of total mortgage growth. Controlling for the share of investors and average income growth reduce the number for house prices to 70 percent and the number for consumption to 85 percent. The fact that during the boom years, the estimated in-sample difference explains a lower fraction of the change in total mortgages is consistent with the model. This is because here a decline in interest rates does not change house prices in the elastic regions, but it reduces the user cost of housing and induces households in those regions to build larger houses, thereby increasing their mortgage liabilities over time. In fact focusing on the actual in-sample differences in total mortgage growth, the estimated coefficients predict all the difference in mortgage liability growth between the most elastic regions which experienced the lowest change in securitization rate and the rest of the regions.

4.2.2 The Bust Period of 2006 to 2008

As we saw in Figure 11, most of the decline in house prices and in consumption happened in the places that experienced a boom during the period 2000-2006. In the previous section I show that most of the boom portion of the cycle can be explained by variations in the elasticity of housing and in variations in the change in securitization rate. In this part I examine to what extent the decline in house prices and in consumption can be attributed to the very same factors that created the boom: changes in fraction of loans sold to non-GSEs during the boom years and differences in the elasticity of housing supply.

The results in Table 2, shows that more inelastic regions and regions that experienced greater changes in securitization rates in the years preceding the bust years, experienced larger declines in house prices and in consumption. Interestingly, even during the bust years the total mortgage liability in these regions increased faster than other regions. In terms of the model and in line with the evidence depicted by Figure 12, this is due to the fact that households in inelastic regions and in regions that experienced a large change in securitization rates do not use up all of their borrowing capacity during the boom years, rather their borrowing capacity is exhausted over time. Table 2 also shows that on average about 35 percent of the decline in aggregate house prices and in consumption can be explained by the variations in the inelasticity measure and by changes in securitization rates during the boom years. These variables explain about 50 percent of the growth in aggregate mortgage liability, which is considerably higher than the fraction that is explained by these factors during the boom years. In terms of the model this is explained by the fact that households in elastic regions exhaust their borrowing capacity faster than their counterparts in inelastic regions. This is because decline in interest rates or in collateral requirements do not have a wealth effect in those regions and house prices remain constant. Therefore households in these regions experienced less of an expansion in their borrowing capacities.

4.3 Instrumental Variable Approach

So far we have seen that 75 percent of the variation in consumption and in house prices during the boom period and about 40 percent of the variation in consumption and in house prices during the bust is associated with variations in the elasticity of housing supply and variations in the change in securitization rate. However one concern that arises is that variations in changes in the securitization rate may not be exogenous and, in particular, increasing house prices or expectations of future growth in house prices, can induce financial institutions to relax borrowing standards and make investors of securitized assets more willing to buy these assets.

In order to address this problem, I use two sources of variations in different regions as an instrument for changes in the securitization rate: (i) variations in population characteristics of counties and (ii) variations of different states in adopting anti-predatory lending laws. In particular let us assume there is a "national securitization" shock that increases the supply of loan contracts with relaxed terms in all regions. This change in the supply has a larger impact in regions where there is a higher demand for these products. The demand for loans with more relaxed terms can be higher when the fraction of the population whose income barely covers below the required down-payment is higher. In line with this prediction and motivated by Ouazad and Rancière (2011), that shows the volume of mortgage origination to Hispanics almost doubled between 2003 and 2005 (compared to less than 40 percent increase for whites during the same period),[43] I find that the percentage of Hispanic population in a county in 2000 is positively correlated with the subsequent changes in the securitization rate.[44] As another source of variation for the changes in securitization rate, I use the fact that by the end of 2004 many states adopted new anti-predatory lending regulations which slowed down the increase in the securitization rate between 2003 and 2006. In fact Anti-predatory lending laws have been in effect since 1994 however only refinance loans were included in those laws. After the rise of predatory practices during the securitization boom, some states began to include purchase loans into the loans subject to anti-predatory lending laws. By the early 2005, nineteen states amended home purchase loans into anti-predatory lending laws. I construct an Anti-Predatory dummy that is equal to one for states that included purchase loans in their Anti-Predatory lending laws. The first column of Table 3 shows that on average states that included purchase loans in the law, experienced four percent fewer increase in securitization rates.[45]

Columns three to eight of Table 3 show the results of the same regressions as in the previous part when changes in the securitization rate are instrumented by the percentage of Hispanic population in each county. Qualitatively, the impact of the changes in securitization on house prices, on consumption and on total mortgage liability during the boom period and the bust period are the same as before: more extreme change in securitization rates result in more accumulation of mortgage debt and a larger boom in house prices and in consumption followed by a larger bust. However the estimated coefficients for the impact of the change in securitization rate on house prices, on consumption and on total mortgage liability are

[43] See Figure 18 for the time series of volume of mortgage originations among different races.

[44] State of California is among the states with the highest fraction of Hispanic population and one may concern the result are driven with observations in that state. However the following results were robust to the exclusion of counties in the state of California.

[45] Interaction of instruments with inelasticity is also used to instrument for the interaction term of inelasticity times changes in securitization rate.

significantly larger than the OLS estimates. One possible reason for this result is that IV is capturing the local average treatment effect of change in securitization, which is larger for borrowers with incomes just below the required down-payment.

4.4 Long Run Results

Table 4 shows OLS estimates relating house-price growth, car sales growth and total mortgage liability growth during the period of 2000 to mid-2008 to the *Inelasticity* measure and the change in the securitization rates during the period of 2003 to 2006. From the coefficients of *Inelasticity* in Table 4, one can see that even after the bust, house-price growth in inelastic regions is still higher than that for elastic regions which, in terms of the model, this is because steady-state house prices in inelastic regions is a decreasing function of interest rates. Therefore, lower interest rates results in permanently higher house prices in regions with less elastic supply of housing. The table also shows that inelastic regions and regions that experienced a greater changes in securitization rates accumulated greater amount of mortgage liability. In terms of aggregate impact, variations in securitization rates and in the elasticity measure can accounts for about 75 percent of the total change in household mortgage liabilities. It is interesting to see that the appreciation of home prices that occured due to the rise in securitization during the period of 2003 to 2006 is all gone by mid-2008.

5 Calibration

The reduced form evidence presented in the previous section has some important limitations. First of all it cannot distinguish between the bust that is driven purely by front-loading behavior of households and the bust resulted from the reversal of the initial decline in collateral requirements. Additionally the model reveals that the impact of a decline in collateral requirements on house prices and consumption is a function of real interest rate. Therefore the reduced-form results cannot inform us about what would have happened in the case in which there was the same decline in collateral requirements but the real interest rates differed.

In the sub-section that follows, I first extend the model to allow for a more flexible supply of housing. Then, in order to analyze implications of the extended model, I calibrate the model for three types of regions: (i) inelastic regions that experienced high change in the fraction of loans sold to non-GSEs, (ii) inelastic regions that experienced low change in the fraction of loans sold to non-GSEs and (iii) elastic regions.[46] The calibration is based on

[46] These regions are the same as those we used in constructing Figure 12.

data on actual changes in the mortgage liabilities of households in these regions and on other static characteristics of these regions. Then I compare the predictions of the model with and without a reversal in the decline in collateral requirements on house prices and on consumption. Finally the model is used to consider two sets of counterfactuals. The first set considers the counterfactuals related to past events: what would have happened if there were the same decline in the interest rate but no change in collateral requirements and what would have happened if there were the same change in collateral requirements but no change in interest rates. The second set considers two different policy choices following the tightening of credit: (i) further reduction in real interest rates and (ii) loan modification.

5.1 Extension of Housing Supply

One problem with the basic model is that assuming a fixed supply of housing in inelastic regions results an overestimation ofthe impact of a decline in interest rates and in collateral requirements on house prices and on consumption. The other problem with a fixed supply of housing is that during the boom period there was a rapid rise in activity in the construction sector even in the most inelastic regions (see Charles, Hurst, and Notowidigdo (2012)). In order to tackle this problem, I extend the model by replacing the Leontief production function for the housing sector (equation (3)) with the following CES function:

$$h = \left[\omega_k^{1/\sigma} k^{\frac{\sigma-1}{\sigma}} + (1-\omega_k)^{1/\sigma} l^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Here k and l are the capital and the land used in building a home, ω_k is the weight of capital in the housing aggregator, and σ is the elasticity of substitution between land and capital. As before I assume that there is no adjustment cost in building (or destroying) a house. Additionally I assume house producers maximize their instantaneous profit. This pins down the relation between house prices and aggregate stock of housing in region i :

$$H_{it} = \left(1-\omega_k^i\right)^{-1/(1-\sigma)} \left(\frac{q_{it}^{1-\sigma} - \omega_k^i}{q_{it}^{1-\sigma}}\right)^{\sigma/(1-\sigma)} L_i \tag{43}$$

Now, equilibrium consumption, house prices and house quantities are obtained by adding equation (43) to the first-order conditions of the household-maximization problem, given by equations (12)-(14).[47] One interesting result from the solution of the CES case is that there

[47]For the full characterization of the equilibrium in the CES case see Appendix D.

is a critical threshold price q_{cr} given by:

$$q_{cr} = \left(\frac{1-\theta}{\theta} \sigma \omega_k + \omega_k \right)^{1/(1-\sigma)} \tag{44}$$

that if the steady state price of a home falls above this threshold, the dynamics of house prices and of consumption resemble the dynamics of an inelastic region in the basic model: As a result of a decline in interest rates, household borrowing constraint becomes relaxed and remains relaxed throughout the transition, and there is a boom-bust pattern in consumption and in house prices. On the other hand if the steady-state price falls below this threshold, the economy has a saddle path similar to that of elastic regions in the basic model.

With lower interest rates, steady-state house prices increase, which leads to more regions experiencing a boom-bust cycle in response to interest rate shocks. This result is in line with the finding of Glaeser, Gottlieb and Gyourko (2012) which finds that even for elastic regions, the impact of a change in interest rates is larger when interest rates are relatively low. But more interestingly, q_{cr} is a decreasing function of θ, the collateralizable fraction of housing wealth. This means that as a result of declines in collateral requirements more regions will experience cyclical behavior in response to an interest rate or a collateral-requirement shock.

5.2 Calibrating Parameters

In order to analyze the main insights of the model, I calibrate the model for three different types of regions: Inelastic regions that experienced high change in securitization rates, Inelastic regions that experienced low change in securitization rates and elastic regions. Inelastic and elastic regions are defined as regions in the top and bottom quintiles of the inelasticity measure. High (low) change in securitization rates is defined as being above (below) the median level of change in securitization rates for inelastic regions.

5.2.1 Static Parameters

For estimating σ and ω_k, I use the database on the home prices and the share of land provided by Davis and Heathcote (2007). In particular with the CES production function for housing one can see that the relation between the share of capital (structure) and house prices is:

$$log \left(\frac{k_{it}}{q_{it} h_{it}} \right) = log\left(\omega_k\right) + (\sigma - 1)\, log\left(q_{it}\right) \tag{45}$$

Using land shares and house prices data in Davis and Heathcote (2007), I run a panel regression of the time series of average structure share in the value of house on house prices with a fixed effect for each city, and the coefficient of house prices in this regression is equal to $\sigma - 1$.[48] This results in $\sigma = 0.5$ which is in between a Cobb-Douglas production for housing ($\sigma = 1$) and the Leontief case ($\sigma = 0$). Moreover from equation (45), one can see that if we normalize the price of a reference year (say, year 2000) to one, then ω_k is equal to the share of structure in the value of house in that year. This pins down $\omega_k^{inelastic} = 0.3$ and $\omega_k^{elastic} = 0.8$. ρ is chosen equal to 6 percent in order to capture the idea that households are relatively impatient. The wage rate, w is assumed to be constant and is normalized to one. η, δ and L for each region are chosen to match the share of mortgage payments and other housing costs in household income.[49] Using data from the American Community Survey in the year 2000, median mortgage expenditure in inelastic regions is about 12 percent of the household income. This figure is equal to 8 percent for elastic regions. The median expenditure of households without a mortgage on housing is relatively constant among different regions and it is around 10 percent. This results in:

$$\begin{aligned} \eta^{Inelastic} &= 0.38\,,\ \delta^{Inelastic} = 0.078\,,\ L^{Inelastic} = 6.12 \\ \eta^{Elastic} &= 0.28\ ,\ \delta^{Elastic} = 0.044\ ,\ L^{Elastic} = 14.29 \end{aligned}$$

$\eta^{Inelastic} > \eta^{Elastic}$ is a direct consequence of the fact that in the data the share of mortgage expenditures in inelastic regions, on average, is four percent more than this share for elastic regions.

5.2.2 Dynamic Parameters (r_t, θ_{it})

Based on the yields rate on Ten-year treasury inflation-protected securities (TIPS), the real interest rate, r, is chosen to be equal to 4.3 percent in the year 2000 and gradually declining to 2.1 percent by mid-2003. For the model without a financial crisis (i.e. reversal in the initial decline in collateral requirements), I assume the interest rate remains constant from that point afterward. For the model with a financial crisis, I incorporate the fact that in response to the financial crisis real interest rates declined further (see Figure 1a) and I assume that from 2008 to 2011 real interest rates declined further by one more percentage point to 1.1 percent.

Using data from the NY Fed Consumer Credit Panel on total mortgage liability for households in different regions, I compute the time series of changes in total mortgage liability per capita for different regions. For the model without a financial crisis I use the time series

[48] I used the period of 1995 to 2005 in the panel regression.

[49] In terms of the model, here I assume all the expenses other than mortgage payments is the depreciation cost of capital (δk).

of household mortgage liabilities from 2000 to 2006 to calibrate the time series of θ_{it} for each region and I assume from 2006 onward that the maximum loan-to-value ratio in that region (θ_{it}) remains at its 2006 level. For the model with a financial crisis, I extend the calibration of θ_{it} in order to match changes in household mortgage liability in the period 2007-2010. It is important to mention that no information about the time series of house prices or consumption is used in calibrating the parameters of the model and therefore the performance of the model can be evaluated upon matching those time series.

Finally it is assumed whenever there is a change in the interest rate or in the maximum loan-to-value ratio households are surprised.[50]

5.3 Calibration Results

In this section, I compare the performance of the model with a financial crisis and without a financial crisis with the actual data. It should be mentioned that the parameters of the model without a financial crisis and with a financial crisis are the same for the time period before the year 2007 and therefore, by construction, the predictions of the two models for this period are the same.

Figure 13 shows the results of the calibration of the model for inelastic regions that experienced high change in securitization rates. In order to match the time series of total mortgage liability between 2000 and 2006, $\theta_t^{Inelastic,HighSec}$ remained constant at 0.6 until 2003, when it began to steadily increase to 0.97 by 2006.[51] The model without a financial crisis predicts slightly more than a 60 percent increase in house prices for these regions compared to about an 85 percent appreciation that occured in the data. Non-housing consumption in the model also replicates the time series of car sales during the boom years. However, since car sales have been more volatile than other components of consumption, the model perhaps overestimates the consumption boom.[52] Even in the model without any reversal of the initial decline in collateral requirements, house prices and consumption be-

[50] At least for interest rates this does not seem unreasonable. This is because the baseline interest rate used for calibration is the yield on 10-year TIPS, which its movements are usually assumed to be a surprise for the market. On the other hand, the assumption that households assume interest rates and collateral requirements remain constant forever is a relatively strong assumption. In general the role of expectations about interest rates, growth rates and collateral requirements is an important dimension that in the future work it ought to be incorporated into the model.

[51] Since for each region in the model there is only one representative household, one should think of changes in θ as capturing both the extensive margin of adjustment(people who have been excluded from the lending market are now able to borrow) and the intensive margin (controlling for the quality of the borrower loans have more relaxed terms).

[52] Replacing the Cobb-Douglas assumption for housing and non-housing consumption with a CES function with complementarities between housing and non-housing can magnify the boom-bust in house prices and can dampen the boom-bust in non-housing consumption.

gin to decline by the time that decline in the interest rate or collateral requirements slows down. The decline in house prices and in non-housing consumption predicted by the model without a financial crisis is smoother than what is revealed in the data. This is due to the fact that in the model without a surprise, during the transitional period, the borrowing constraints of households in inelastic regions are unconstrained and therefore non-housing consumption declines at the rate $\rho - r$, which is about 4 percent in the model. As the time series of mortgage liabilities shows, it is important to notice that the decline in consumption is not happening because of households inability to roll-over their debt. This decline happens because the level of consumption during the boom is financed by borrowing more, and households realize they cannot increase their debt holding forever. Therefore as they see their untapped borrowing capacity decline, they reduce their consumption-which also leads to lower house prices. Extending the calibration of the model to match changes in mortgage liability from 2007 to 2010 significantly improves the performance of the model in predicting the rapid decline in house prices and in consumption that one observes in the data. The model also predicts that as a result of the financial crisis maximum the loan-to-value ratio in inelastic regions with high change in securitization rate declined by 0.3 to 0.67 which is close to its level in 2000. There are two reasons for the impact of an increase in collateral requirements on house prices and on consumption. First, as a result of decline in θ, the total amount that households can borrow throughout their lifetime declines. This is both because of the direct impact of lower loan-to-value ratios and because of the indirect impact of a lower θ on house prices. This induces households to reduce their consumption in order to smooth their consumption for the rest of their lives. Second, if the increase in collateral requirements is high enough, the current debt holding of the household may well exceed the maximum amount that a household can borrow. In this case, on top of consumption smoothing motivation, the household should give up a higher fraction of its housing stock to meet the new borrowing constraint. Only in this case, households deleverage their debt holding. Moreover in the cases that households are forced to deleverage their debt holdings, part of the decline in consumption and in house prices will be recovered in the following years.[53] This is because households' deleveraging results in a "fire sale" of houses. But after they reduce their debt, households begin to increase their housing stock and consumption.

Calibration of the model for inelastic regions with changes in the securitization rate that are lower than the median for inelastic regions, results in a time path for $\theta_t^{Inelastic,LowSec}$ that starts out at 0.6, and, by 2003, it increases only slightly to 0.65. From 2003 to 2006, $\theta_t^{Inelastic,LowSec}$ increases by another 0.17 units. The time paths of house prices and of consumption are similar to the previous case except that for these regions the model correctly

[53]This is related to proposition 5' in the model.

predicts both the timing of increase in house prices and the level of house prices growth (see Figure 14).

Calibration of the model for the most elastic regions results in lower estimates of changes in collateral requirements in comparison with changes in collateral requirements in inelastic regions (Figure 15). In elastic regions, a decline in interest rates does not lead to a boom in consumption. This is because house-price change is insignificant and therefore household borrowing constraint remains binding. The model predicts that the impact of a financial crisis on consumption and house prices in elastic regions is less severe than this impact for inelastic regions. In fact the model fails in capturing the level of decline in consumption and house prices that happened for elastic regions in the data. This is, partly, due to an assumption of the model that is more problematic for elastic regions: The model assumes that households can disinvest the capital used in their house and pay back their debt. This assumption is less problematic for the model without a financial crisis since the adjustment in housing stock is happening slowly. But for the model with a financial crisis, it is more realistic to assume the stock of existing houses cannot decline, and instead of house quantities, house prices should adjust. This can help the model to predict the sharp decline in house prices and in consumption even in elastic regions.

5.4 Past Events Counterfactual

After testing the performance of the model, in this section I want to consider two informative counterfactuals about past events: first, what was happening for the house prices and consumption if there was the same decline in the real interest rate but there has been no change in collateral requirements.[54] Second, what was the impact of the same decline in collateral requirements if there was not a decline in the real interest rate during the period of 2000 to 2003.[55] In order to simplify the comparison, in the following graphs I just show the time path of consumption and of house prices for inelastic regions that experienced high change in securitization rate and for elastic regions.

The model predicts only 30 percent increase in house prices of inelastic regions if there was not a decline in collateral requirements compared to more than 60 percent increase when

[54] In models with endogenous collateral requirements like Rampini and Viswanathan (2012) decline in real interest rates, themselves, results in a decline in collateral requirements. Therefore the way one should think about this policy is that in contrast to market forces, financial regulation is preventing banks from relaxing their standards. For example in the early 90s, in response to rising house prices, Hong Kong Commissioner of Banking restricted loan-to-value ratios.

[55] Perhaps the main reason that collateral requirements get relaxed was the fast appreciation of house prices which was fueled by declining interest rates. Therefore one should think of this experiment as an "upper bound" on the impact of changes in collateral requirements by themselves

decline in interest rates was followed by decline in collateral requirements. The growth in consumption would have been 60 percent less if there was no decline in collateral requirements. The model predicts that absent a decline in collateral requirements, decline in house prices and consumption would have started by mid-2003. In terms of the model, decline in consumption and house prices were postponed to mid-2006 as a result of a continuous decline in collateral requirements, which led to a gradual increase in the steady-state borrowing capacity of households. Finally as is emphasized before, decline in interest rates, by itself, does not generate a boom-bust in elastic regions.

The model shows that if there was not a decline in interest rates, the impact of the same decline in collateral requirements on house prices and consumption was significantly milder. The reason for this is that with interest rates closer to the rate of time preference ρ, households have less motivation to frontload and distribute the new borrowing capacity more evenly over their life time. The other channel through which the real interest rate influences the impact of a decline in collateral requirements is through its impact on the steady-state house prices. Lower collateral requirements results in larger debt holding in the steady state and therefore a larger interest payments. This reduces demand for consumption and housing services. On the other hand lower collateral requirements makes housing more affordable and increases demand for housing. Whether house prices in the steady-state increase or decrease depends on the interest rate. The main message from this experiment is that the impact of collateral requirements on consumption and house prices depends crucially on the level of interest rates.

5.5 Policy Experiment: Interest Rate Cuts versus Loan Modification

The next step is to compare the prediction of the model with a financial crisis with two scenarios: First, in response to the financial shock, there is an even stronger monetary policy that reduces real interest rates by another 50 basis points. Second, households are given more time to deleverage and the decline in the maximum loan-to-value ratio occurs over a longer period of time. In particular I assume the same decline that occured in θ during the period of 2008 to 2011 to occur during the period of 2008 to 2013. Of course the model abstracts from monetary policy or a micro-foundation for collateral requirements and, therefore, one should think of these policy experiments as qualitative exercises that can highlight some of the mechanisms of the model.

The policy experiment (Figure 17) shows that lower interest rates is not effective in increasing consumption of households living in elastic regions, whereas it does increase consumption in regions with inelastic supply of housing. This result is driven by the asymmetric impact of real interest rates on house prices in regions with elastic supply of housing and regions with inelastic supply of housing. On the other hand, loan modification increases the consumption in all regions temporarily. However loan modification delays the recovery procedure and the initial increase in consumption is followed by a decline in consumption and house prices in the following years. The main reason that in this framework effectiveness of policy is limited is because the decline in consumption is not only driven by some households deleveraging their debt holding, but more importantly because the level of consumption during the boom years, itself, was financed by the fast growth in household liabilities.

6 Conclusion

During the period from 2000 to mid-2008 the stock of US household liabilities more than doubled. During the same period house prices and consumption experienced a boom-bust pattern that is magnified in regions with a more inelastic supply of housing and in regions with higher change is securitization rate during the boom years.

The purpose of this paper is to provide an economic framework that can help in understanding the increase in liabilities of households as well as the swing in house prices and in consumption. At the heart of the theory is an unsustainable increase in consumption driven both by expanded access to credit and the endogenous increase in house prices that relax credit constraints. My theoretical mechanism highlights the importance of low interest rates and of elasticity of housing supply to explain how pronounced the dynamics implied by a credit expansion will be. Reduced-form empirical evidence supports the predictions of the model and shows that variations in the elasticity of the supply of housing and changes in securitization rates during the boom years can explain most of the increases and declines in house prices and consumption during the boom years (2000-2006) and bust years (2006-mid-2008). The quantitative exercise illustrates the importance of the reversal in the initial relaxation of credit standards to explain the precipitous decline in consumption and house prices. However, the model constructed in this paper shows that even without a reversal in credit standards, most of the decline would have taken place, but over a longer period of time.

From a broader perspective, this paper is also related to two recent strands of literature. First, this paper is related to the literature on macroprudential policy (see Hanson, Kashyap, and Stein (2011)) and shows the interaction between interest rates and collateral constraints for the macroeconomy. The model shows that this interaction is more pronounced during periods of low interest rates. The model is also suggestive that an impatient policy maker has more incentives for financial deregulation which can results in excessive fluctuations in the economy. This paper is also related to the recent literature on the distributional impacts of monetary policy like Piazzesi and Schneider (2012), Coibion, Gorodnichenko, Kueng and Silvia (2012) and Brunnermeier and Sannikov (2012). In particular the model implies that impacts of lower interest rates and financial deregulation can be very different for households in different regions.

There are a number of important theoretical dimensions that are currently beyond the scope of this paper. First, one can study the role of expectations about future interest rates, collateral requirements and growth rates in this economy.[56] Second, the model abstract from savers in the economy. My preliminary results of the inclusion of households with temporary high incomes shows that income inequality can be an important factor in explaining the decline in real interest rates and the boom-bust in house prices and consumption.[57] Third, any welfare implications of the boom-bust cycles within this framework needs a further study.[58] Fourth, understanding the micro-foundation of changes in collateral requirements contributes to a better understanding of the boom-bust cycles caused by an expansion of credit.

Also from the empirical point of view, there are a number of extensions that I should conduct. First of all the logic of the model is applicable to the European countries that experienced a surge in capital inflows and a housing boom and bust (like Spain and Ireland). The boom-bust cycle of housing market in US coastal areas in mid-80s to mid-90s is another related episode that can be used for testing the model.[59] Secondly, a better measure of changes in lending standards and a better measure of consumption can be very useful for a better testing of the model. Also for the quantitative exercise, addition of adjustment costs for housing seems to be of a first order of importance.

[56] My preliminary result shows expectations about future growth can also generate very long-lasting periods of boom followed by a bust.

[57] The relation between rise in inequality and higher household leverages is also discussed in Kumhof and Ranciere (2010).

[58] In particular the framework of this paper is similar to the ones in Jeane and Korinek (2010). However in contrast to their framework, during the transition periods the borrowing constraint of households in inelastic regions is relaxed and therefore Pigouvian taxation is not necessarily welfare improving.

[59] Interestingly, during this period on one hand there was a decline in real interest rates and deregulation of financial institutions in the US. On the other hand this period also experienced a rise in household mortgage liabilities and in the US current account deficit.

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Table 1: The Boom Period of 2000 to 2006

	House Prices Growth between 2000 and 2006			Car Sales Growth between 2000 and 2006			Total Mortgage Liabilities Growth between 2000 and 2006		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Inelasticity	0.18***	0.16***	0.06***	0.07***	0.06***	0.05	0.10***	0.08***	0.02
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)
Change in Securitization Fraction 03_06	1.94***	1.63***	0.64***	0.74***	0.68**	0.78**	1.22***	0.95***	0.64*
	(0.29)	(0.26)	(0.24)	(0.28)	(0.28)	(0.37)	(0.29)	(0.27)	(0.36)
Inelasticity X Change in Securitization Fraction 03_06	0.99***	0.72***	0.08	0.13	0.09	0.02	0.75***	0.54**	0.18
	(0.21)	(0.21)	(0.19)	(0.25)	(0.24)	(0.29)	(0.27)	(0.27)	(0.31)
Implied Aggregate Impact	0.44	0.4	0.18	0.2	0.18	0.18	0.23	0.19	0.09
Percentage of Total	76	69	31	95	86	86	22	18	8
Controls	N	Y	Y	N	Y	Y	N	Y	Y
State Fixed Effect	N	N	Y	N	N	Y	N	N	Y
Regression Type	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
Observations	323	323	323	323	323	323	323	323	323
R-squared	0.43	0.55	0.84	0.18	0.19	0.40	0.29	0.37	0.51

Notes: This table presents estimates of the impact of variations in the elasticity of housing supply and changes in the securitization rate on house prices, car sales and total mortgage growth during the years of 2000 to 2006. *Inelasticity* is based on Saiz (2010) measure of the elasticity of housing. The *Securitization Fraction* is computed as the fraction of purchase loans sold to non-GSEs. Baseline controls include the growth of average income between 2000 and 2006 and its interaction with inelasticity measures, population growth during this period, and the change in the fraction of homes purchased by investors in this period. Each county with a population greater than 150,000 in 2000 is one unit of observation. Robust standard errors are below coefficients in parentheses, and asterisks denote significance levels (***=1%, **=5%, *=10%).

Table 2: The Bust Period of 2006 to 2008

	House Prices Growth between 2006 and mid-2008			Car Sales Growth between 2006 and mid-2008			Total Mortgage Liabilities Growth between 2006 and mid-2008		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Inelasticity	-0.02***	-0.01**	-0.00	-0.04***	-0.04***	-0.03**	0.02**	0.03***	0.03**
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Change in Securitization Fraction 03_06	-0.71***	-0.61***	-0.38***	-0.33**	-0.36**	-0.07	0.57***	0.61***	0.41***
	(0.08)	(0.08)	(0.09)	(0.15)	(0.15)	(0.17)	(0.10)	(0.10)	(0.12)
Inelasticity X Change in Securitization Fraction 03_06	-0.37***	-0.28***	-0.11	-0.29**	-0.35***	0.07	0.17*	0.18*	0.04
	(0.06)	(0.06)	(0.07)	(0.13)	(0.14)	(0.13)	(0.10)	(0.11)	(0.11)
Implied Aggregate Impact	-0.08	-0.07	-0.05	-0.09	-0.08	-0.08	0.1	0.12	0.11
Percentage of Total	42	37	26	38	33	33	43	52	48
Controls	N	Y	Y	N	Y	Y	N	Y	Y
State Fixed Effect	N	N	Y	N	N	Y	N	N	Y
Regression Type	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
Observations	323	323	323	323	323	323	323	323	323
R-squared	0.32	0.44	0.78	0.10	0.13	0.39	0.33	0.42	0.57

Notes: This table presents estimates of the impact of variations in the elasticity of housing supply and changes in the securitization rate on house prices, car sales and total mortgage growth during the years of 2006 to mid-2008. *Inelasticity* is based on Saiz (2010) measure of the elasticity of housing. The *Securitization Fraction* is computed as fraction of purchase loans sold to non-GSEs. Baseline controls include the growth in average income between 2006 and 2008 and its interaction with inelasticity measure, population growth during this period and the fraction of homes purchased by investors in 2006. Each county with a population greater than 150000 in 2000 is one unit of observation. Robust standard errors are below coefficients in parentheses, and asterisks denote significance levels (***=1%, **=5%, *=10%).

Table 3: Instrumental Variable Approach

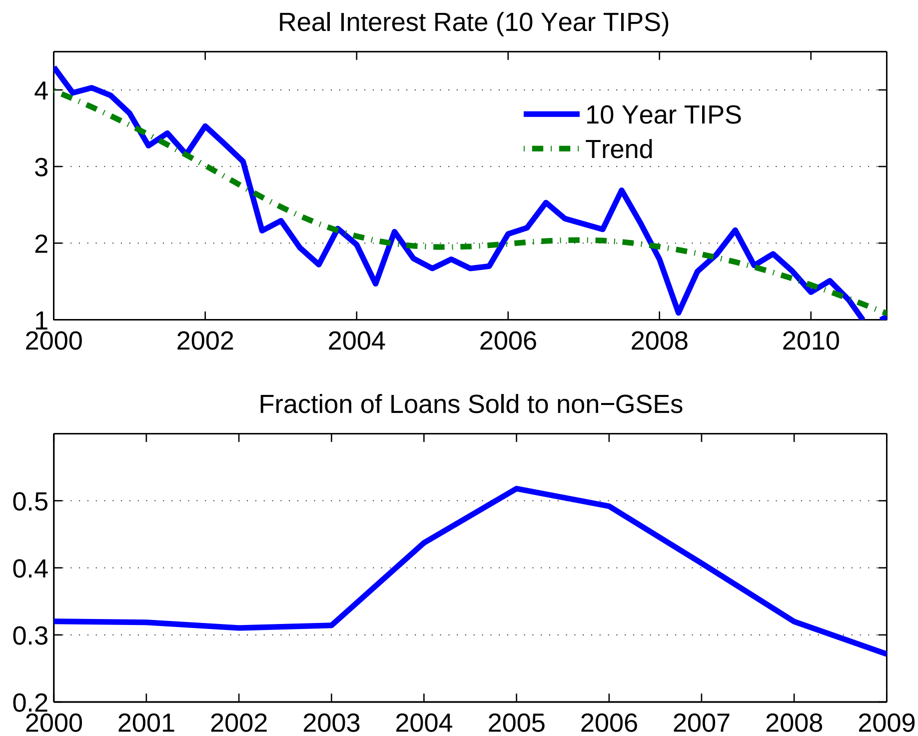
	Change in Securitization Fraction 03_06	Inelasticity X Change in Securitization Fraction 03_06	House Prices	Car Sales	Total Mortgage Liabilities	House Prices	Car Sales	Total Mortgage Liabilities
			Growth between 2000 and 2006			Growth between 2006 and mid-2008		
	(1)	(2)	(4)	(5)	(6)	(7)	(8)	(9)
Percentage Hispanics	0.17***	0.05*						
	(0.03)	(0.03)						
Percentage Hispancs X Inelasticity	-0.01	0.24***						
	(0.02)	(0.03)						
Anti-Predatory Dummy	-0.04***	0.00						
	(0.01)	(0.01)						
Anti-Pred X Inelasticity	-0.00	-0.05***						
	(0.01)	(0.01)						
Inelasticity	0.02***	-0.01	0.13***	0.02	0.06*	0.00	-0.04***	-0.01
	(0.00)	(0.01)	(0.02)	(0.03)	(0.04)	(0.01)	(0.01)	(0.02)
Change in Securitization Fraction 03_06			2.73***	2.27***	1.37	-1.26***	-0.46	1.68***
			(0.67)	(0.70)	(0.95)	(0.25)	(0.35)	(0.36)
Inelasticity X Change in Securitization Fraction 03_06			0.83*	0.32	-0.70	-0.29	-0.69**	0.20
			(0.50)	(0.60)	(0.59)	(0.20)	(0.30)	(0.22)
Controls	Y	Y	N	Y	Y	N	Y	Y
State Fixed Effect	N	N	N	N	N	N	N	N
Regression Type	OLS	OLS	IV	IV	IV	IV	IV	IV
Observations	323	323	323	323	323	323	323	323
R-squared	0.24	0.21	0.22		0.05	0.02	0.06	0.34

Notes: This table replicates the regressions of column (2), (5) , (8) in Tables 1 and 2 by using the fraction of Hispanic population and an Anti-Predatory Lending Laws dummy and their interactions with *Inelasticity* as instruments for the changes in securitization rate and its interaction with *Inelasticity*. Here, the Anti-Predatory dummy is equal to one for states that include purchase loans in Anti-Predatory Lending laws. *Inelasticity* is based on Saiz (2010) measure of elasticity of housing. Securitization Fraction is computed as fraction of purchase loans sold to non-GSEs. The change in securitization rate for the years of 2003 to 2006 is computed. The baseline controls include the growth of the average income and its interaction with inelasticity measure, population growth and the fraction of homes purchased by investors in the corresponding period. Each county with the population greater than 150000 in 2000 is a unit of observation. Robust standard errors are below coefficients in parentheses, and asterisks denote significance levels (***=1%, **=5%, *=10%).

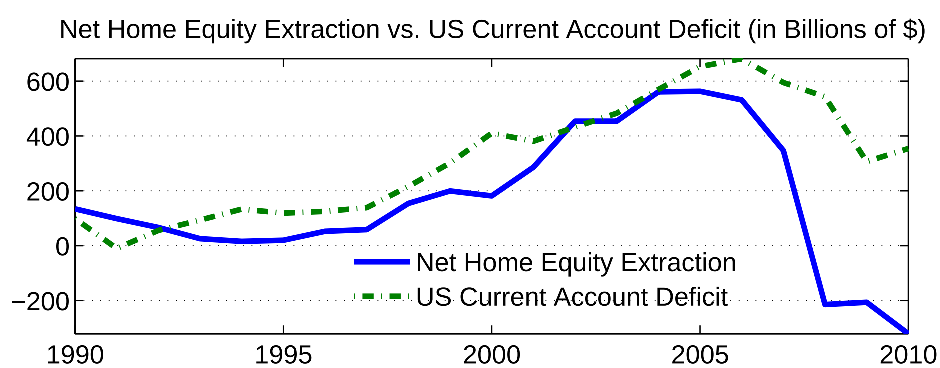
Table 4: Long Differences: 2000 to mid-2008

	House Prices Growth between 2000 and mid-2008			Car Sales Growth between 2000 and mid-2008			Total Mortgage Liabilities Growth between 2000 and mid-2008		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Inelasticity	0.13***	0.12***	0.06***	-0.01	-0.01	-0.02	0.15***	0.14***	0.06
	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)
Change in Securitization Fraction 03_06	0.26	0.16	-0.17	0.18	0.19	0.53**	2.76***	2.49***	1.70***
	(0.20)	(0.16)	(0.16)	(0.20)	(0.19)	(0.24)	(0.43)	(0.37)	(0.45)
Inelasticity X Change in Securitization Fraction 03_06	0.05	-0.04	-0.16	-0.40*	-0.36*	-0.02	1.37***	1.11***	0.37
	(0.16)	(0.13)	(0.16)	(0.21)	(0.19)	(0.22)	(0.37)	(0.32)	(0.38)
Implied Aggregate Impact	0.29	0.27	0.12	0.09	0.07	0.06	0.51	0.48	0.33
Percentage of Total	116	108	48	-64	-50	-43	31	29	20
Controls	N	Y	Y	N	Y	Y	N	Y	Y
State Fixed Effect	N	N	Y	N	N	Y	N	N	Y
Regression Type	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
Observations	323	323	323	323	323	323	323	323	323
R-squared	0.25	0.45	0.80	0.14	0.18	0.41	0.41	0.52	0.65

Notes: This table presents estimates of the impact of variations in the elasticity of housing supply and changes in the securitization rates on house prices, car sales and total mortgage growth during the years of 2000 to mid-2008. *Inelasticity* is based on Saiz (2010) measure of elasticity of housing. The *Securitization Fraction* is computed as fraction of purchase loans sold to non-GSEs. Baseline controls include the growth of average income between 2000 and 2008 and its interaction with the inelasticity measure, population growth during this period and the fraction of homes purchased by investors in 2006. Each county with population greater than 150,000 in 2000 is one unit of observation. Robust standard errors are below coefficients in parentheses, and asterisks denote significance levels (***=1%, **=5%, *=10%).



(a) Decline in Interest Rates and the Rise of Securitization



(b) Home Equity Extraction and Current Account

Figure 1: Expansion of Credit During 2000 to 2006

Notes: In Figure 1a,10-Year TIPS contains quarterly yields on treasury-inflation-protected securities (TIPS). Data are obtained from J. Huston McCulloch, http://www.econ.ohio-state.edu/jhm/ts/ts.html. HP filter with $\lambda = 400$ is used for the calculation of the interest rate trend. Fraction of Loans sold to non-GSEs are fraction of purchase loans that mortgage originators sold to an institution other than government sponsored organizations like Fannie Mae and Freddie Mac. In Figure 1b, Net Home Equity Extraction is defined as the change in the total mortgage liabilities of household minus the net investment of households in residential housing. Data are obtained from US Flow of Funds.

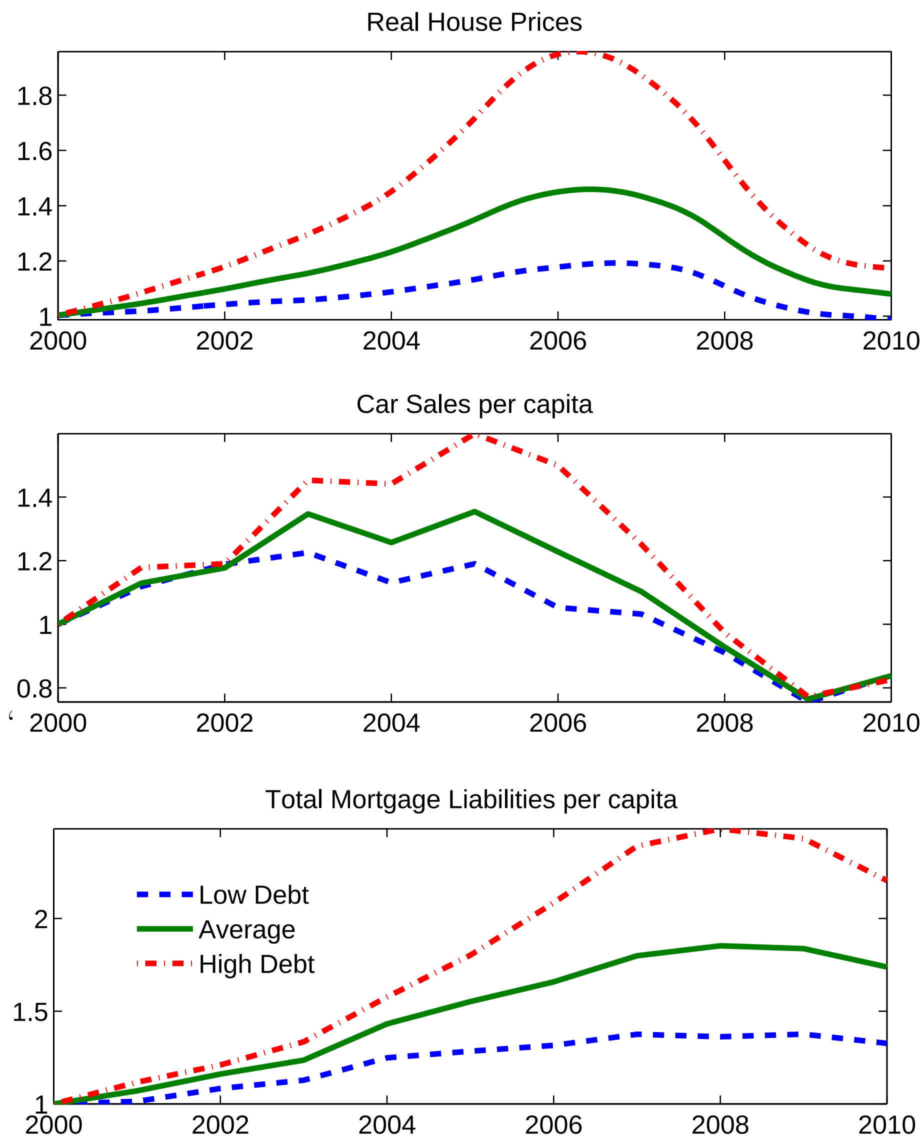


Figure 2: High Debt Growth versus Low Debt Growth Regions Dynamics
Notes: This figure shows the differential dynamics of house prices and car sales per capita for regions that experienced high and low growth in mortgage liabilities. In the above figures, high and low debt counties are defined to be the top and bottom quintile of counties (with more than 150,000 population in 2000) based on the growth in mortgage liabilities per capita between 2000 and mid-2008, and, the graphs show the average for each region as well as the average for all counties with a population of more than 150,000 in 2000. House prices are based on CoreLogic HPI. Car Sales per capita and Total Mortgage Liabilities are based on FRBNY CCP data.

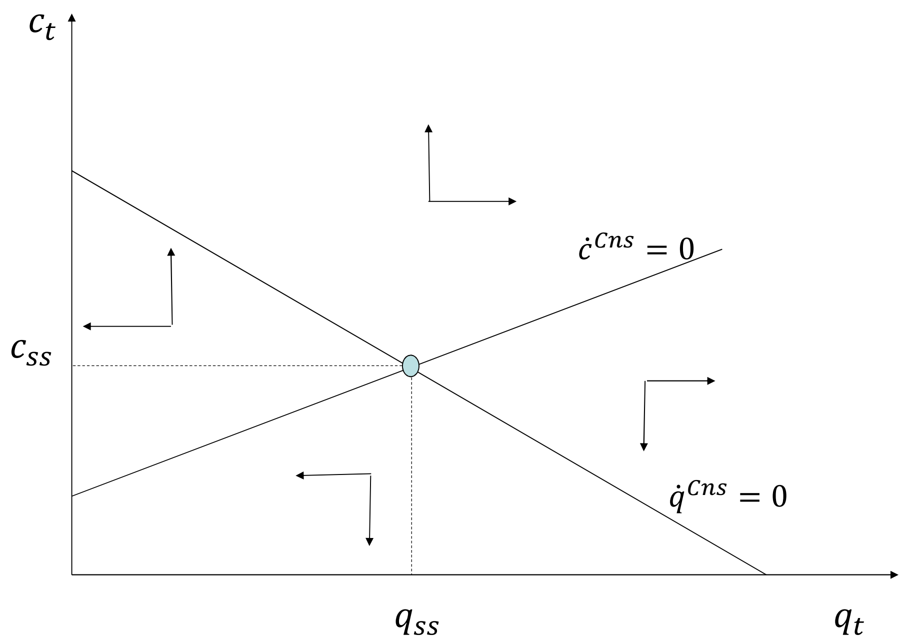


Figure 3: The phase diagram of (q_{it}, c_{it}) for an inelastic region when the borrowing constraint is binding. This graph is based on equations (16) and (17).

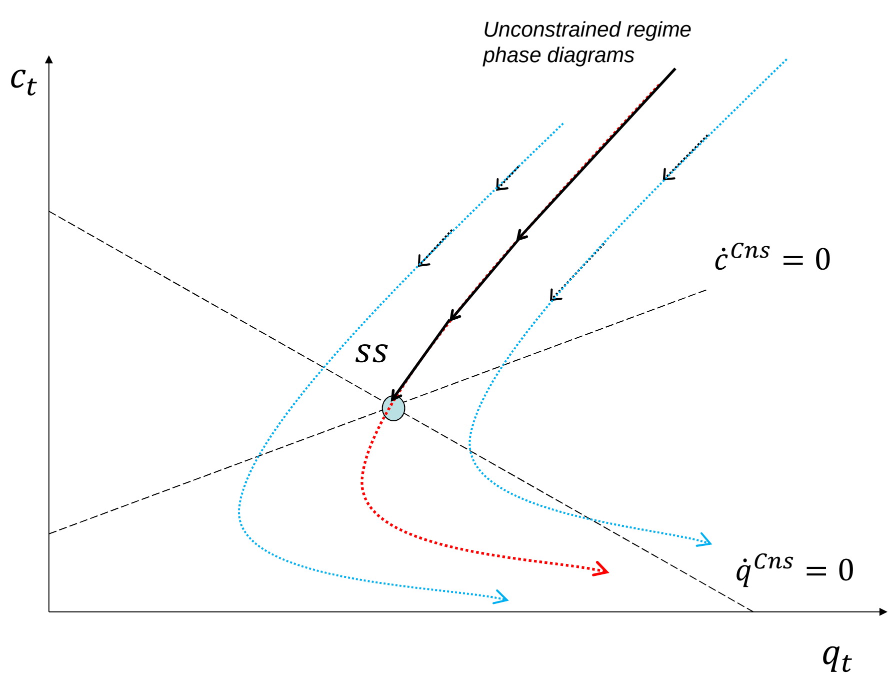


Figure 4: The equilibrium transition path for an inelastic region.

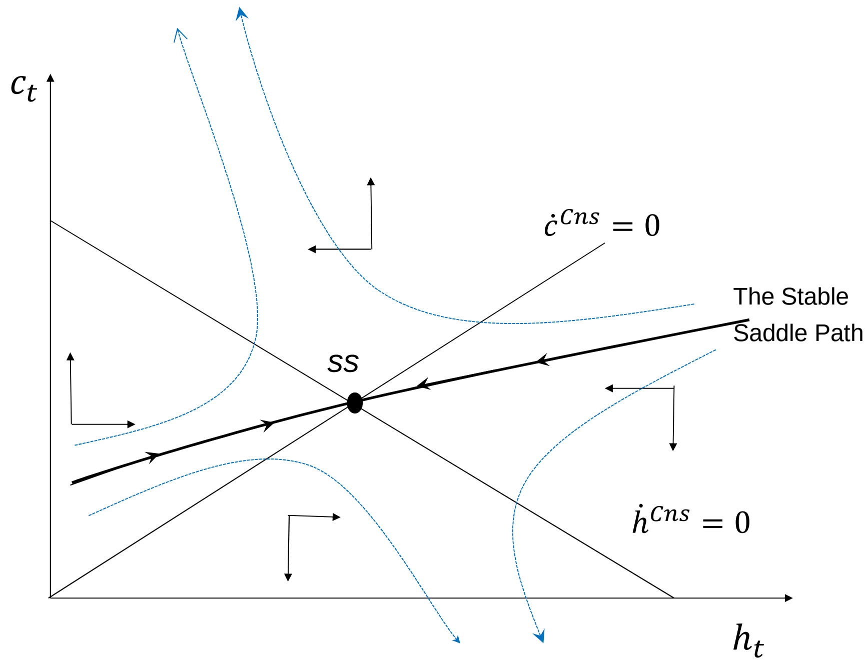


Figure 5: The phase diagram for (h_{it}, c_{it}) in elastic region when the borrowing constraint is binding. The saddle path is the solution to equations 31 and 32.

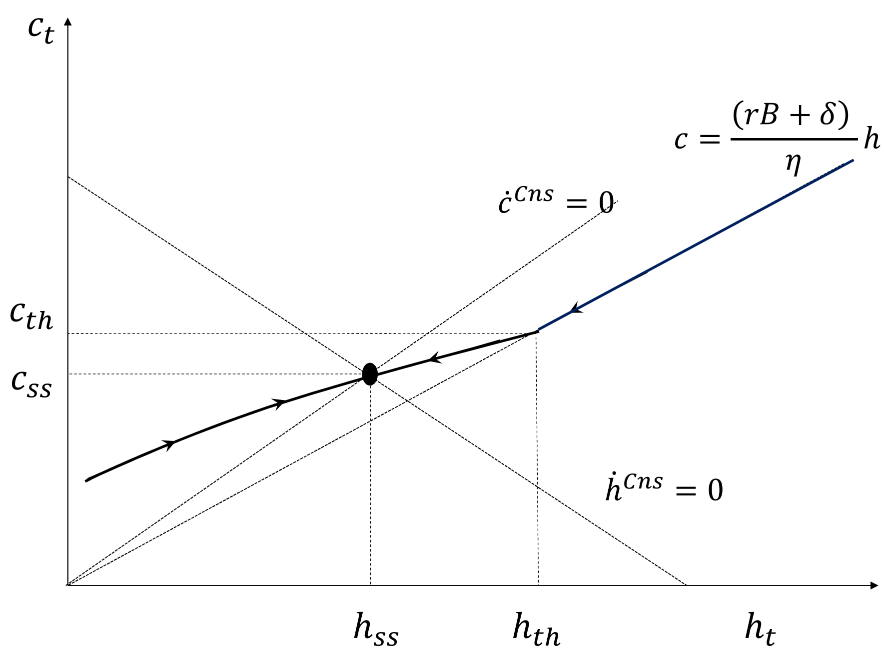


Figure 6: Equilibrium transition path for the elastic region

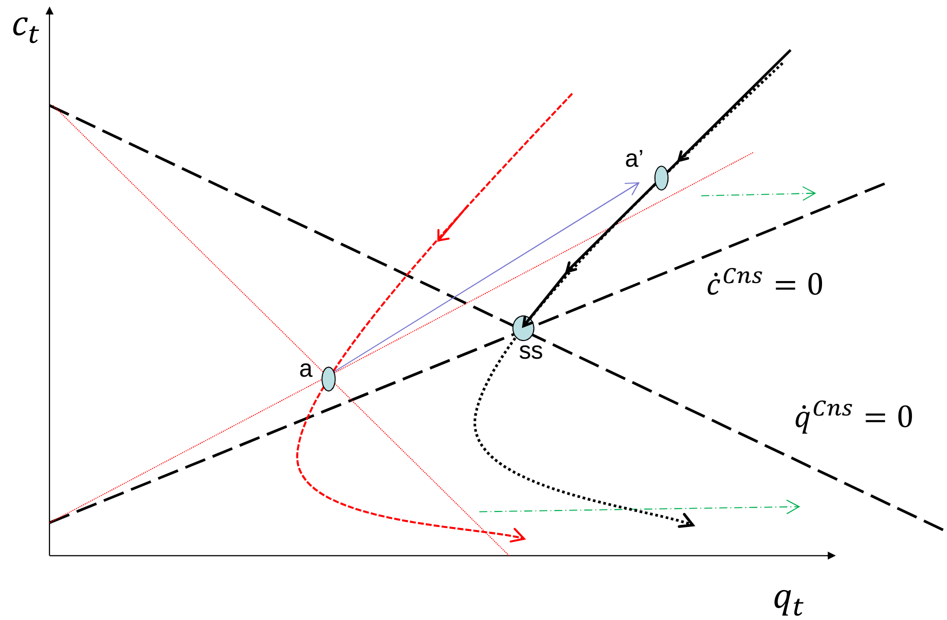


Figure 7: The Impact of an unexpected permanent decline in the interest rate in an inelastic region

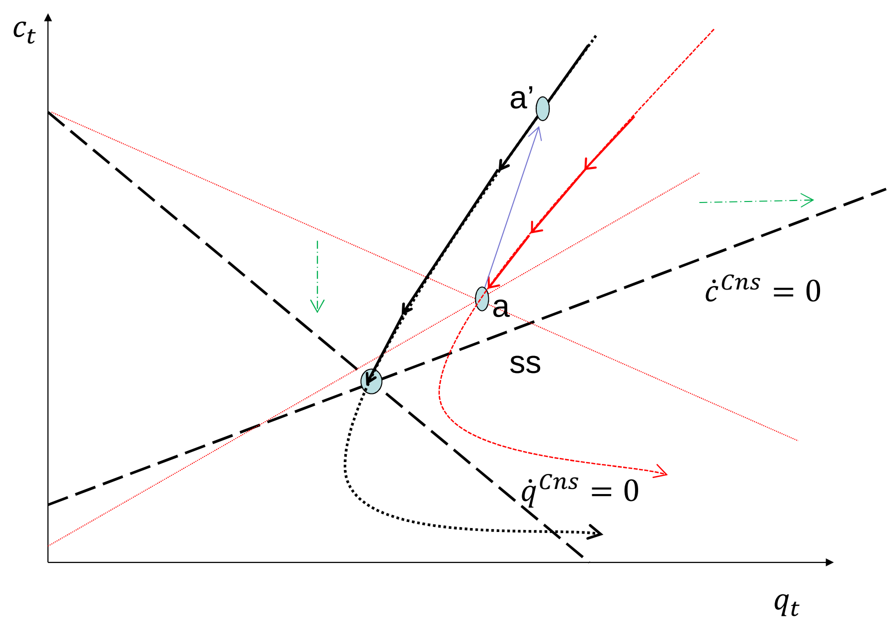


Figure 8: The Impact of an unexpected permanent increase in the maximum loan-to-value ratio in an inelastic region

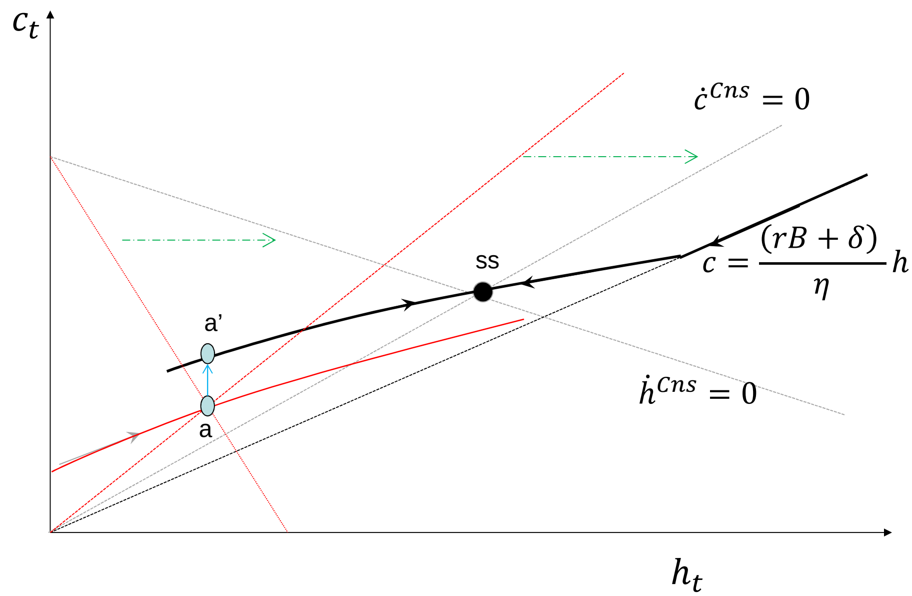


Figure 9: The impact of decline in interest rate in an elastic region

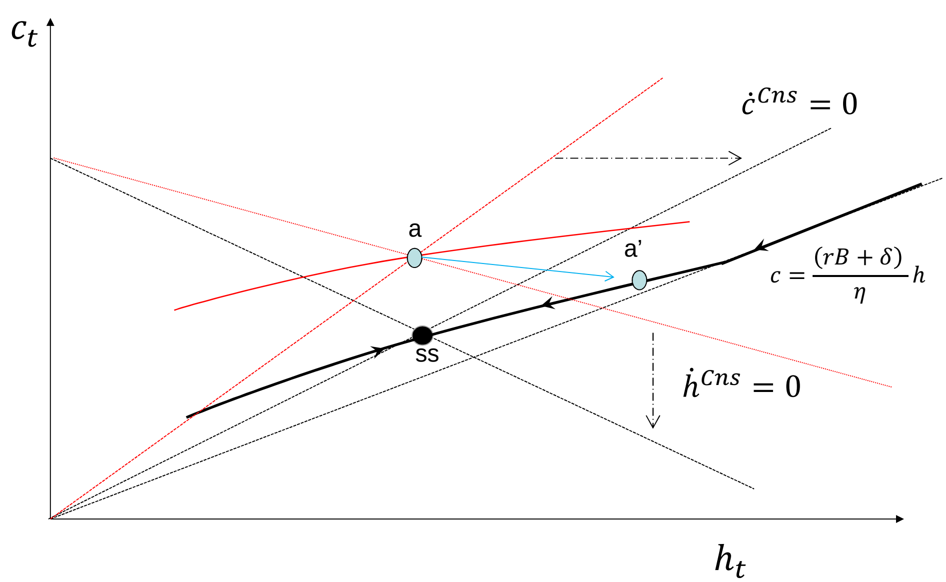


Figure 10: The impact of an increase in the maximum loan-to-value ratio in an elastic region

Figure 11: The Boom-Bust in House Prices and Consumption

Note: The graph in the top panel shows the correlation between house prices growth between 2000 and 2006 and house prices growth between 2006 and 2009 for counties with more than 150,000 population in 2000. The size of circles is proportional to the population of the corresponding county in 2000. The graph in the bottom panel replicates the same graph for car sales growth. The solid line represents the OLS regression line

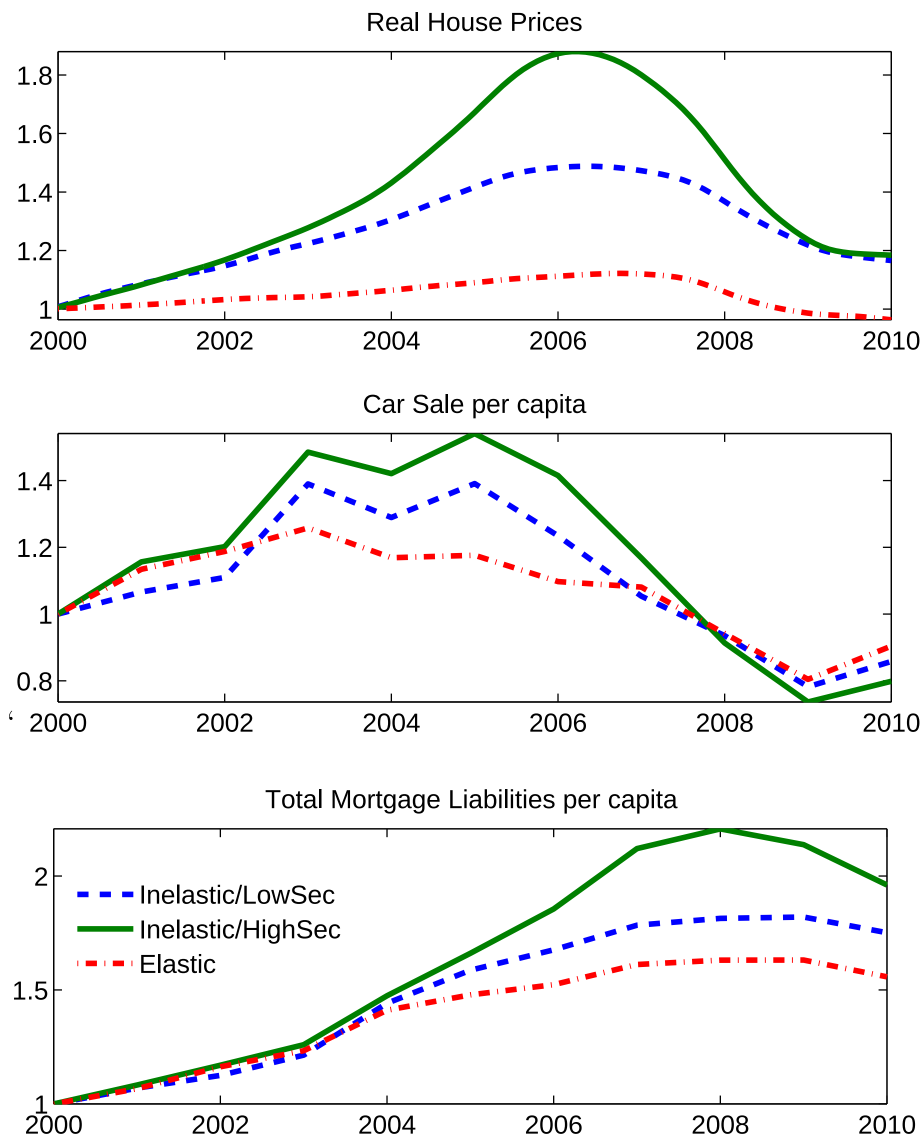


Figure 12: Differential Dynamics of House Prices, Consumption and Total Mortgage Liabilities in Different Regions

Notes: This figure shows the differential dynamics of house prices and car sales per capita for regions with different elasticities of housing and with different changes in the fraction of loans sold to non-GSEs. In the above figures, Inelastic (Elastic) regions are counties in the top (bottom) 20 percent distribution of inelasticity measure based on Saiz (2010). "Inelastic/Low Sec" ("Inelastic/High Sec") are inelastic counties in which the change in the fraction of loans sold to non-GSEs during 2003 to 2006 is less (more) than the median for Inelastic regions.

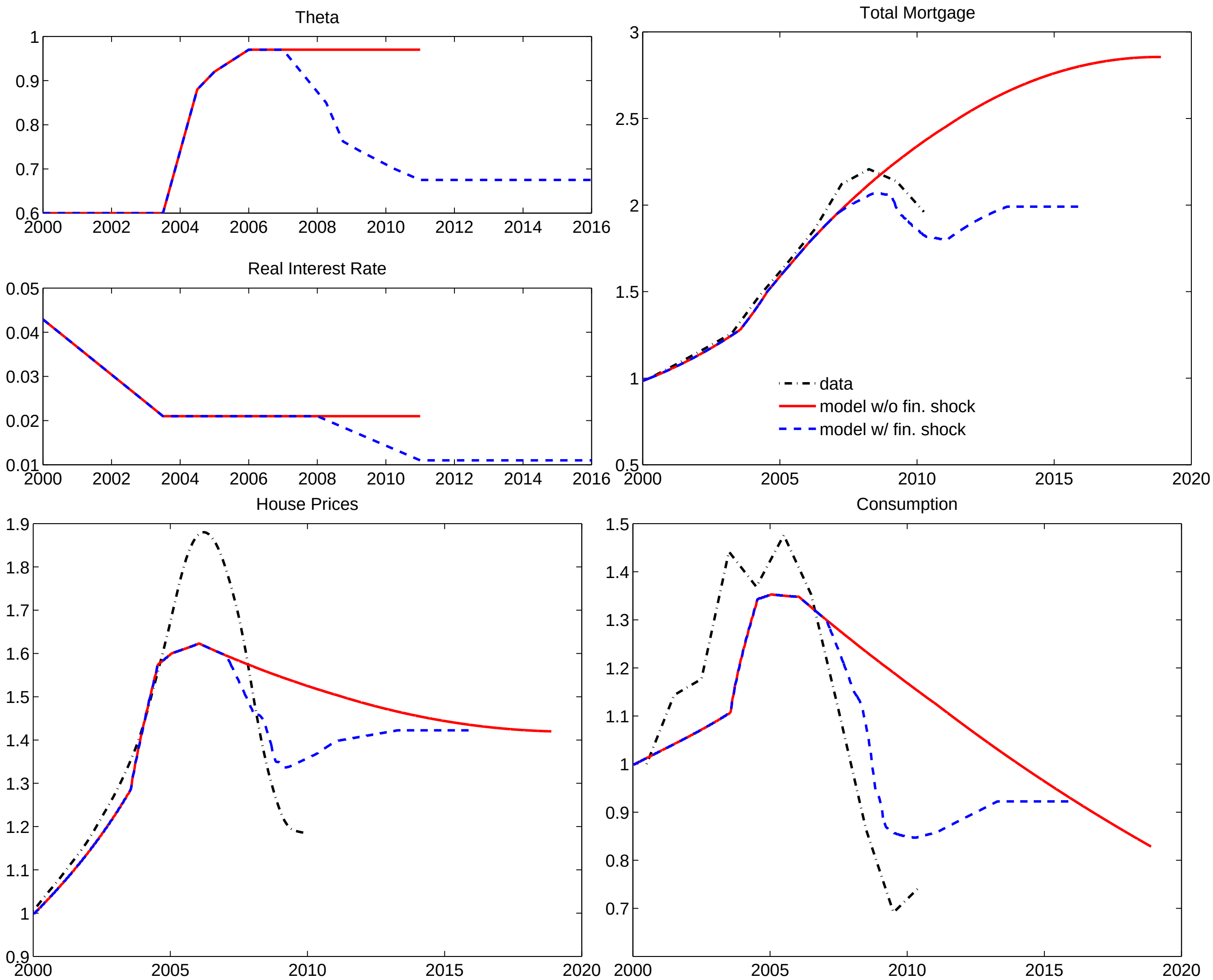


Figure 13: Inelastic Regions with High Change in Securitization Rate

Notes: This figure presents results of the calibration of the model without a financial crisis and the model with a financial crisis for inelastic regions that experienced high change in securitization rate and compares it with the time series of actual data on the average of house prices, total mortgage liabilities per capita and car sales per capita (as a proxy for consumption) in those regions. Inelastic Regions with High Change in Securitization Rate refers to regions in the top quintile of Inelasticity measured for which the change in securitization rate during the period of 2003 to 2006 has been more than the median of the change in securitization rate for inelastic regions.

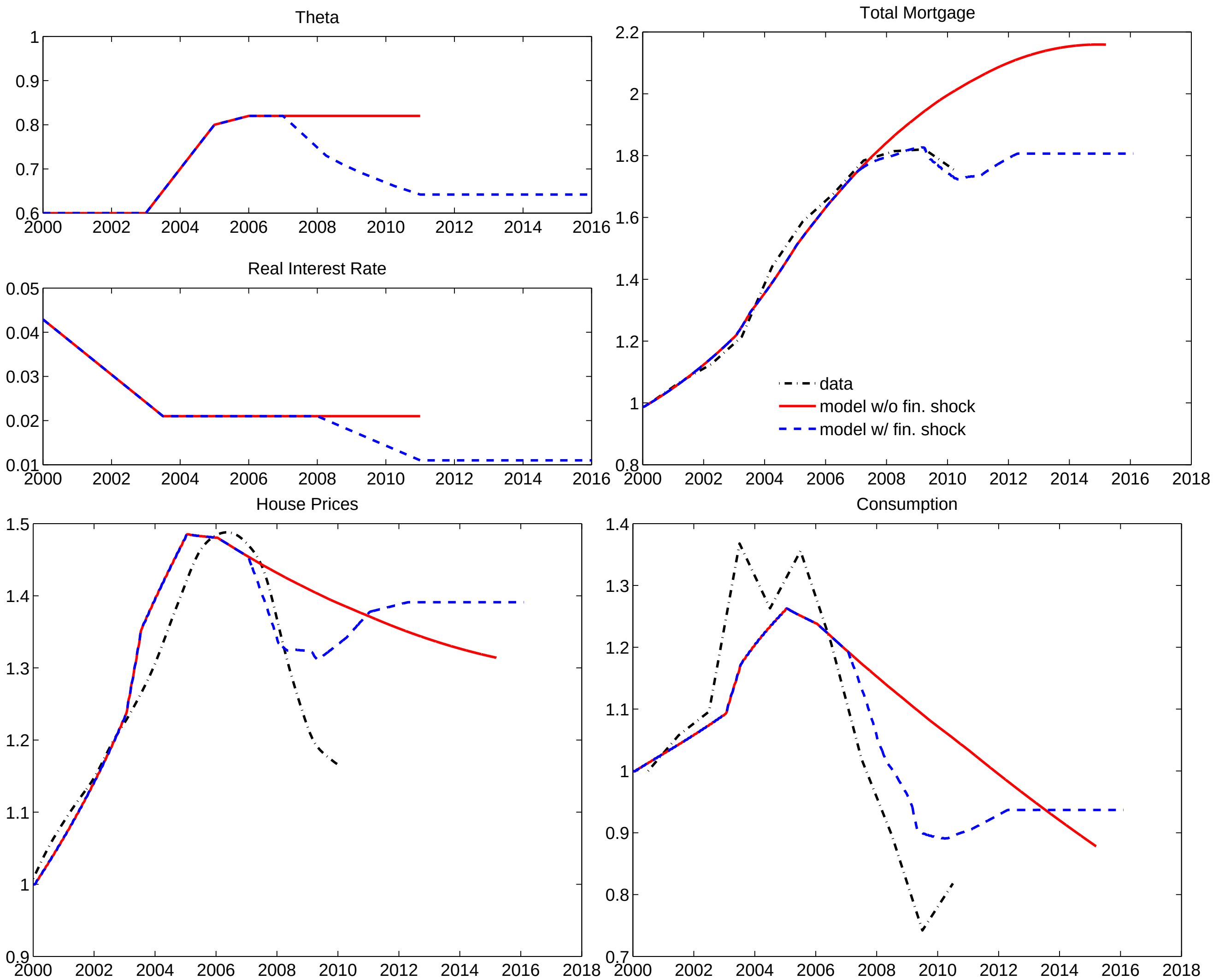


Figure 14: Inelastic Regions with Low Change in Securitization Rate

Notes: This figure presents the results of the calibration of the model without a financial crisis and the model with a financial crisis for inelastic regions that experienced low change in securitization rate and compares it with the time series of actual data on the average of house prices, total mortgage liabilities per capita and car sales per capita (as a proxy for consumption) in those regions. Inelastic Regions with Low Change in Securitization Rate refers to regions in the top quintile of Inelasticity measure for which the change in securitization rate during the period of 2003 to 2006 has been less than the median of the change in securitization rate for inelastic regions.

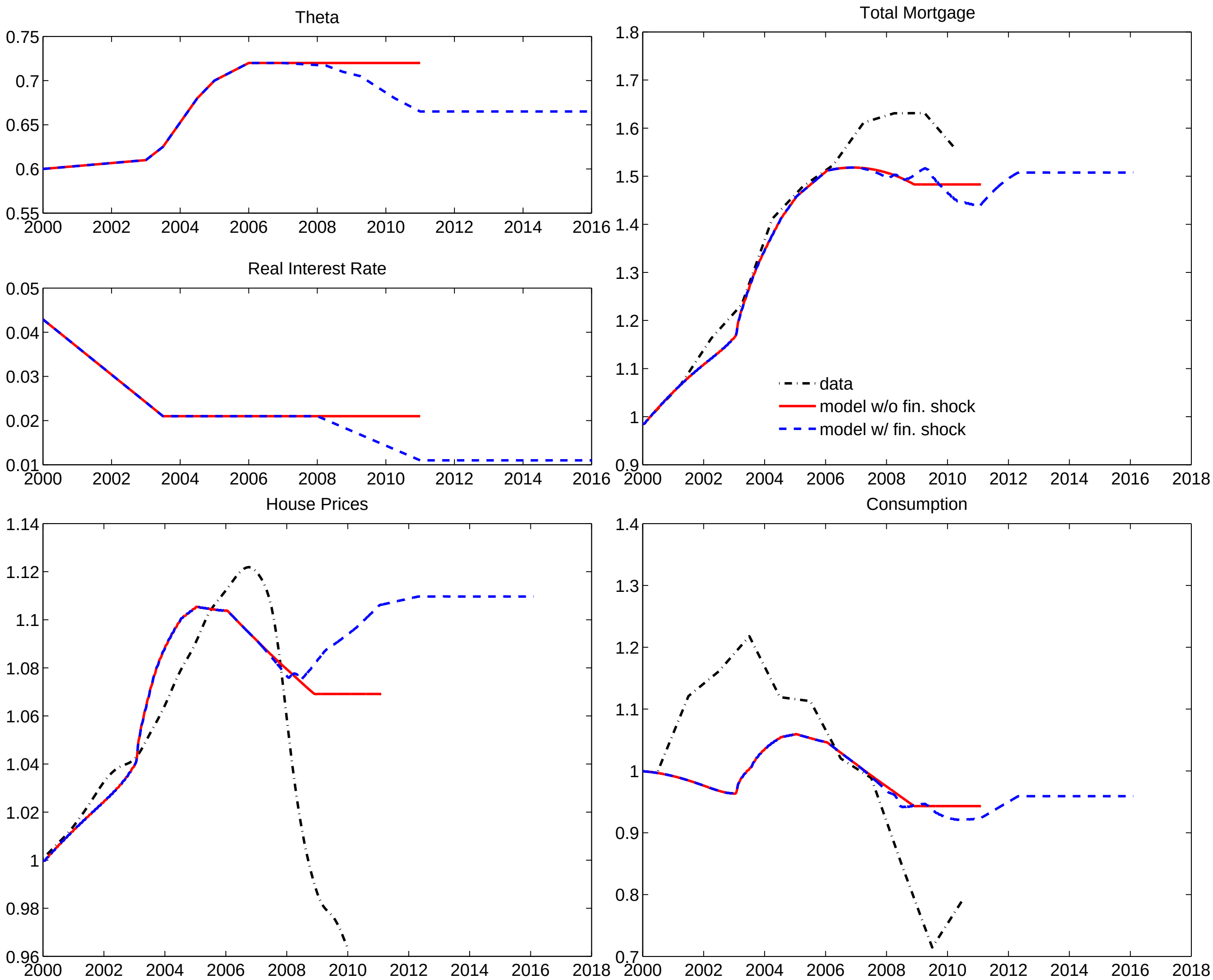


Figure 15: Elastic Regions

Notes: This figure presents the results of the calibration of the model without a financial crisis and the model with a financial crisis for Elastic regions and compares it with the time series of actual data on the average of house prices, total mortgage liabilities per capita and car sales per capita (as a proxy for consumption) in those regions. Elastic Regions refers to regions in the bottom quintile of Inelasticity measure.

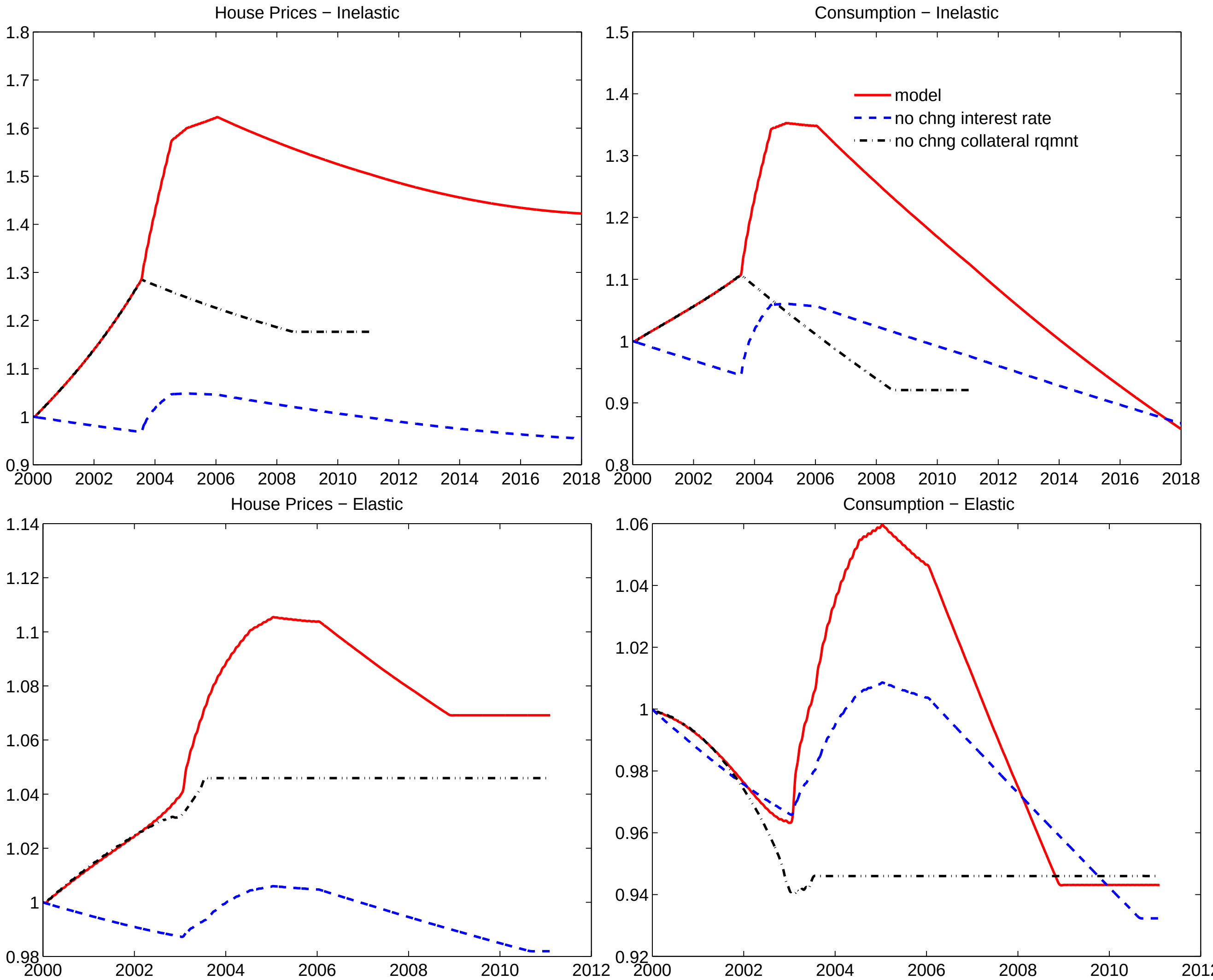


Figure 16: Counterfactual of Past Policies

Notes: This figure compares the predictions of the model without a financial crisis (baseline model) with two hypothetical scenarios; first, the interest rate remains at its level in 2000 but the decline in collateral requirements is the same as in the baseline model. Second, there is no decline in collateral requirements but decline in inters rates is the same is in the baseline model. Results are presented for inelastic regions that experienced high change in securitization rate and for elastic regions.

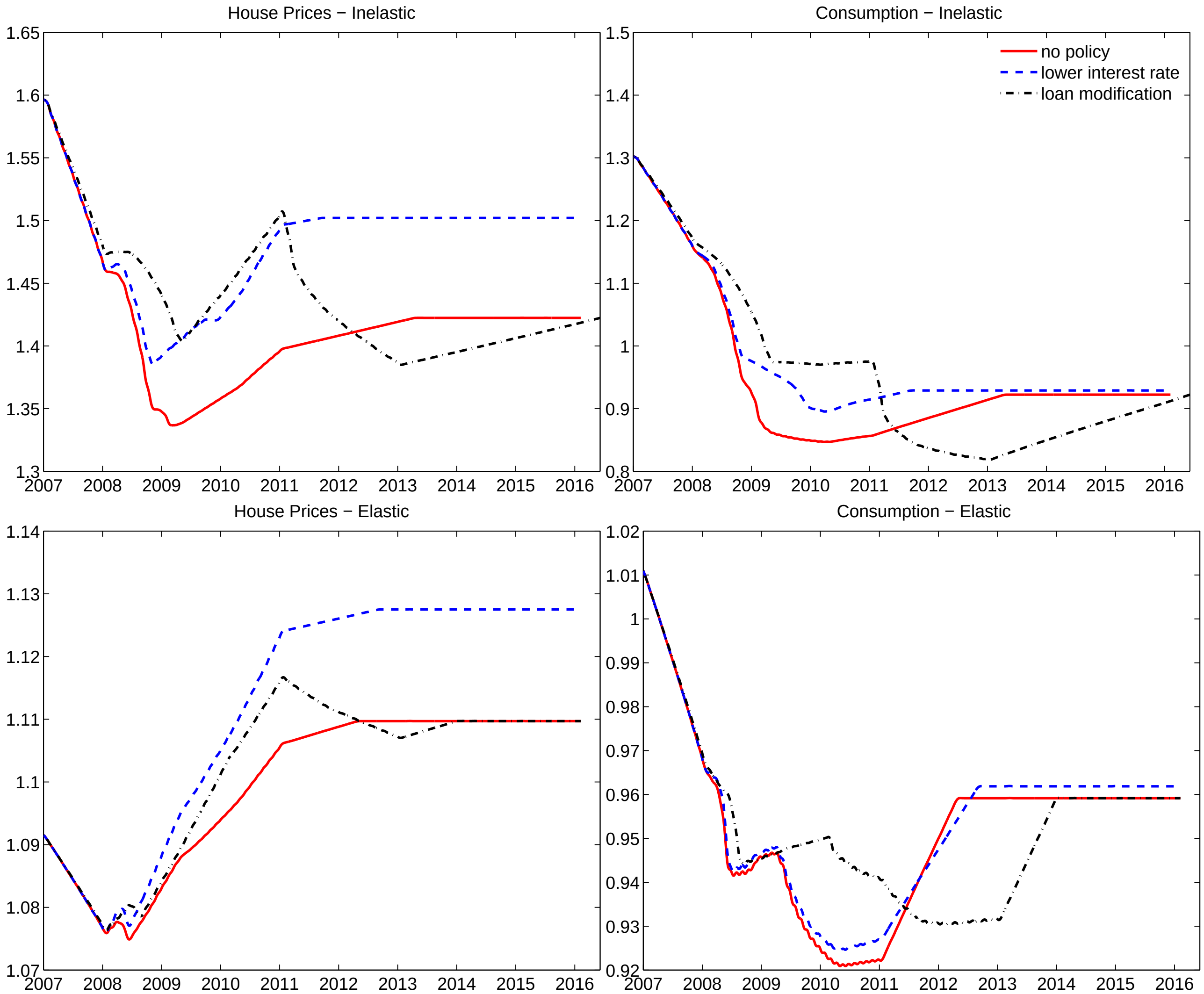


Figure 17: Policy Experiment

Notes: This figure compares the predictions of the model with a financial crisis with two scenarios; (i) during the period of 2008 to 2011, the interest rate declines by 1.5 percent as opposed to 1 percent in the baseline model with a financial crisis. (ii) Increase in collateral requirements is happening over a longer period of time. The result are presented for inelastic regions that experienced high change in securitization rate and for elastic regions.

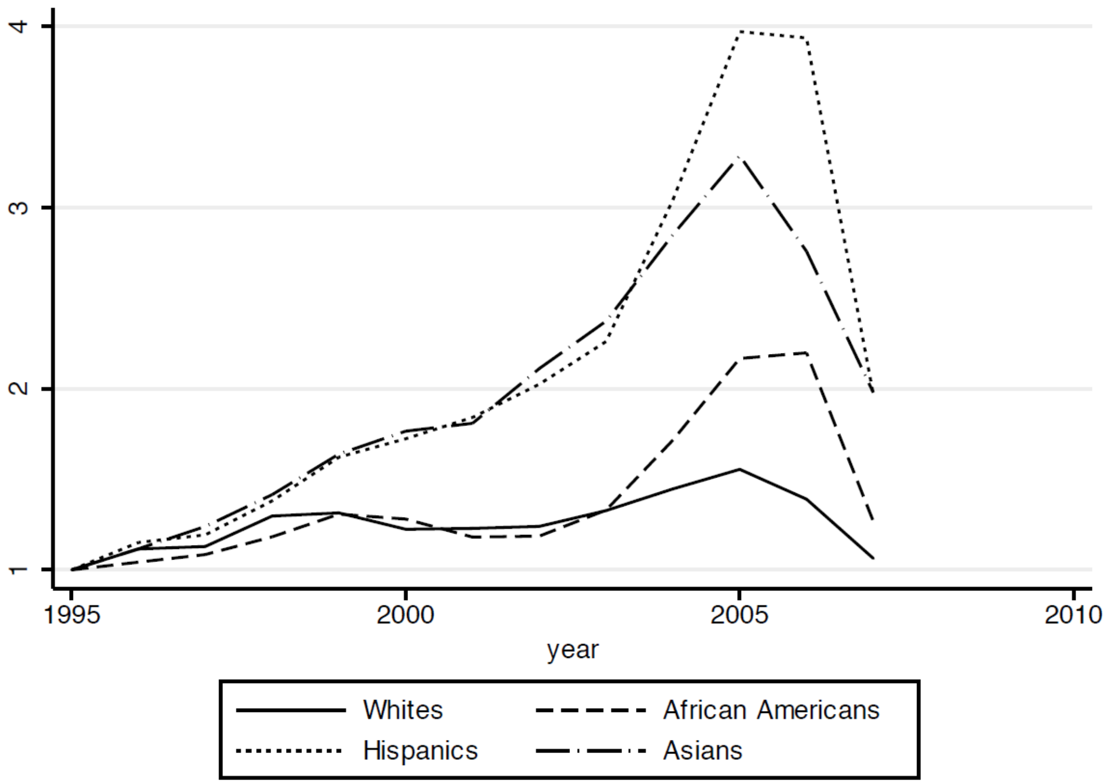


Figure 18: Volume of Mortgage Originations by Race

Notes: The volume of 1995 mortgage originations for each race is normalized to one. Calculations are based on the HMDA dataset. Source:Ouazad and Rancière (2011)

Appendix

A: Proof of Lemma 1

First I show in elastic regions ($q_{it} = B$), there cannot be an equilibrium in which the borrowing constraint never binds.

Proof by contradiction: Let assume there is an equilibrium in which the borrowing constraint is always relaxed. Imposing $\lambda_{it} = 0$ in the first order conditions, leads to:

$$\begin{aligned} \dot{c}_{it} &= (r-\rho)\, c_{it} \\ h_{it} &= \frac{\eta c_{it}}{rB+\delta} \\ \mu_{it} &= \frac{1}{c_{it}} \end{aligned}$$

And the wealth dynamics can be written as:

$$\begin{aligned} \dot{W}_{it} &= w - c_{it} + r\left(W_{it} - Bh_{it}\right) - \delta h_{it} \\ &= w + rW_{it} - c_{it} - (rB+\delta)\, h_{it} \\ &= w + rW_{it} - (1+\eta)\, c_{it} \end{aligned}$$

This has the solution:

$$W_{it} = \frac{w}{r}\left(e^{rt}-1\right) + \frac{(1+\eta)\, c_{i0}}{\rho}\left(e^{(r-\rho)t} - e^{rt}\right) + W_{i0}e^{rt}$$

imposing transversality condition $\underset{t\to\infty}{lim}\left[e^{-\rho t}\mu_{it}W_{it}\right] = \frac{\frac{w}{r}+W_{i0}-\frac{(1+\eta)c_{i0}}{\rho}}{c_{i0}} = 0$, pins down c_{i0}:

$$c_{i0} = \frac{\rho}{1+\eta}\left(\frac{w}{r} + W_{i0}\right)$$

Using this in the relation for W_{it}, results in:

$$W_{it} = -\frac{w}{r} + \left(\frac{w}{r} + W_{i0}\right) e^{(r-\rho)t}$$

Therefore $\underset{t\to\infty}{lim} W_{it} = -\frac{w}{r} < 0$. But this is violating the collateral constraint $W_{it} > (1-\theta_i)\, Bh_{it}$. Therefore in elastic regions, independent of the initial wealth of the representative household, there is no equilibrium in which the borrowing constraint does not become binding.

Now for inelastic region, if the borrowing constraint never binds (independent of the time path for house prices) we have:

$$\dot{c}_{it} = (r - \rho)\, c_{it}$$

Then we show as long as there is no bubble in house prices, at some point land prices become zero and therefore at that point afterward the economy is characterized with the equations of the elastic economy. This is because as long as the borrowing constraint is not binding and housing supply is fixed, house prices are characterized by:

$$\dot{q}_{it} = rq_{it} + \delta - \frac{\eta c_{it}}{L}$$

Since c_{it} is declining and $\underset{t\to\infty}{lim} c_{it} = 0$ we have $\underset{t\to\infty}{lim} \frac{\dot{q}}{q} = r$ which is inconsistent with no-bubble condition in house prices.

Therefore at some time (T) land prices becomes zero and house prices are $q_{it} = B$, from that point afterward ($\forall t \geq T$). Since from that point afterward the economy is exactly the same as an economy with elastic housing supply, we have shown that independent of the W_{iT}, there cannot be any equilibrium in which borrowing constraint never becomes binding.

B: Proof of Lemma 2

Extended Lagrangian of the representative household in an inelastic region can be written as:

$$\hat{\mathcal{H}} = [logc_t + \eta logh_t] + \mu_t\left[w - c_t + r\left(W_t - q_t h_t\right) - \delta B h_t + \dot{q}_t H_t\right] + \lambda_t\left[W_t - (1-\theta)\, q_t h_t\right] \quad (46)$$

Imposing the fixed supply of housing ($h_t = L$) after taking the first order conditions results in:

$$\begin{aligned}
\hat{\mathcal{H}}_c: &\quad \tfrac{1}{c_t} - \mu_t && = 0 && (47)\\
\hat{\mathcal{H}}_H: &\quad \tfrac{\eta}{L} - \mu_t\left[rq_t + \delta - \dot{q}_t\right] && -\lambda_t(1-\theta)q_t = 0 && (48)\\
\hat{\mathcal{H}}_W: &\quad \mu_t r + \lambda_t && = \rho\mu_t - \dot{\mu}_t && (49)\\
&\quad \left[W_t - (1-\theta)\, q_t L\right]\lambda_t && = 0 && (50)\\
&\quad \lambda_t && \geq 0 && (51)
\end{aligned}$$

An extended Maximum principle (see Seierstad and Sydsæter (1987)) makes sure λ_t is piecewise continuous. Now I want to prove that if $\exists t | \lambda_t > 0 \Rightarrow \lambda_{t'} > 0, \forall t' > t$.

The proof is by contradiction: let assume there is a point of discontinuity in λ_t for which $\lambda_{t^+} = 0$ but $\lambda_{t^-} > 0$. $(t^- < t^+)$

As long as q_t is continuous[60] and c_t is finite, from equation 48 we can see λ_t is finite. finite λ_t in addition to equation 49 results in $\dot{\mu}_t$ being finite and therefore μ_t is continuous. continuous μ_t plus equation 47 leads to c_t being continuous. Therefore without loss of generality we can write:

$$\begin{aligned} q_{t^-} &= q_{t^+} = q_t \\ c_{t^-} &= c_{t^+} = c_t \\ \mu_{t^-} &= \mu_{t^+} = \mu_t \end{aligned}$$

Now taking the difference of 48 for t^- and t^+, results in:

$$\dot{q}_{t^-} - \dot{q}_{t^+} = (1-\theta)\, q_t \frac{\lambda_{t^-} - \lambda_{t^+}}{\mu_t} > 0 \tag{52}$$

Equation 52, says for the borrowing constraint to become relaxed, it should be the case that there is a decline in house prices growth. This decline, increases the user cost of housing and reduces households demand for housing to a point that even they become unconstrained they do not demand more housing. Now I show this condition contradicts with the borrowing constraint when household budget constraint is added.

From the budget constraint we have:

$$\dot{W}_t = w - c_t + r\,(W_t - q_t L) - \delta B L + \dot{q}_t L$$

We can also define borrowing capacity as $C_t = W_t - (1-\theta)\, q_t L$ and the derivative of borrowing capacity w.r.t time is:

$$\dot{C}_t = \dot{W}_t - (1-\theta)\, \dot{q}_t L$$

[60] A negative jump in q_t is inconsistent with households maximization problem and cannot be an equilibrium. A positive jump in q_t also can be ruled out by assuming international investors do not benefit from housing services but can hold a piece of land and resell it in the future. This assumption do not change any other result in the model because as long as there is no change in r and θ, $\dot{q} < r$ and therefore these agents never invest in housing. This assumption just excludes possibility of positive jump in q from the expectation of households and makes the equilibrium unique.

Then taking the difference for t^- and t^+,

$$\begin{aligned}\dot{C}_{t^+}-\dot{C}_{t^-} &= \left(\dot{W}_{t^+}-(1-\theta)\,\dot{q}_{t^+}L\right)-\left(\dot{W}_{t^-}-(1-\theta)\,\dot{q}_{t^-}L\right)\\ &= \theta\left(\dot{q}_{t^+}-\dot{q}_{t^-}\right)L<0\end{aligned}$$

But because the borrowing constraint is binding for $t=t^-$(or $\dot{C}_{t^-}=0$),this result in $\dot{C}_{t^+}<0$ which contradicts with the borrowing constraint becoming relaxed.

C: Proof of Proposition 5' and Characterization of "Fire Sales" in Inelastic Regions

If $a_0<-\theta(q_{ss}H_{ss})$

households should sell enough land to investors to make sure they are satisfying the borrowing constraint.

let us call a_{0^-} debt holding before the shock.

then $a_{0^+}=a_{0^-}+q_{0^+}\left(h_{0^-}-h_{0^+}\right)$

Also the borrowing constraint will be binding:

$$a_{0^+}=-\theta q_{0^+}h_{0^+}$$

the economy goes directly to the steady state.

Characterizing the transition:

$$\begin{aligned}\frac{\dot{q}_{it}}{q_{it}} &= r &(53)\\ (1-\theta_i)\,q_{it}\dot{h}_{it} &= w-c_{it}-\delta h_{it} &(54)\\ (1-\theta_i)\,q_{it}\frac{\dot{c}_{it}}{c_{it}} &= \frac{\eta c_{it}}{h_{it}}+(1-\theta_i)\,q_{it}\,(r-\rho)-\delta &(55)\end{aligned}$$

Let assume the economy reaches the steady state at T. During the transition, house prices are characterized by

$$q_{it}=q_i^{ns}(t-T)=q_{ss}e^{r(t-T)},t\in[0,T] \quad (56)$$

Substituting 56, in 54 and 55, and using $h_{iT}=h_{ss}=L_i$, shows the solution to 54 and

55, is a unique saddle path characterized by $c_{it} = c_i^{ns}(t-T)$ and $h_{it} = h_i^{ns}(t-T)$, in which:

$$\begin{aligned} \dot{c}_i^{ns} &> 0, t \in [0,T] \\ \dot{h}_i^{ns} &> 0, t \in [0,T] \end{aligned}$$

The only remained unknown is the time it takes the economy to reach the steady state (T).

This is pinned down by the boundary condition at $t = 0^+$.

$$q_i^{ns}(-T)\left[(1-\theta)\, h_i^{ns}(-T) - L_i\right] = a_{0^-}$$

D: Characterization of the Model with Extended Housing Supply

The extended Lagrangian in this case has the form:

$$\hat{\mathcal{L}} = [logc_t + \eta logH_t] + \mu_t\left[w - c_t + r\left(W_t - q_t h_t\right) - \delta\omega_k q_t^{\sigma} h_t + \dot{q}_t h_t\right] + \lambda_t\left[W_t - (1-\theta)\, q_t h_t\right] \tag{57}$$

so the system of equations for $\lambda = 0$ is:

$$\begin{cases} \frac{\dot{c}_t}{c_t} & = r - \rho \\ \eta c_t & = \left[r q_t + \delta\omega_k q_t^{\sigma} - \dot{q}_t\right] H_t \\ & = (1-\omega_k)^{-1/(1-\sigma)} L\left(\frac{q^{1-\sigma}-\omega_k}{q^{1-\sigma}}\right)^{\sigma/(1-\sigma)}\left[r q_t + \delta\omega_k q_t^{\sigma} - \dot{q}_t\right] \end{cases} \tag{58}$$

and if $\lambda > 0$

$$\begin{cases} (1-\theta)\, q_t h_t \frac{\dot{c}}{c} & = \eta c_t - \left[\theta r q_t + (1-\theta)\,\rho q_t + \delta\omega_k q_t^{\sigma} - \dot{q}_t\right] h_t \\ (1-\theta)\, q_t \dot{h}_t - \theta \dot{q}_t h_t & = w - c_t - r\theta q_t h_t - \delta\omega_k q_t^{\sigma} h_t \end{cases} \tag{59}$$

Adding the relation between q and h given by (43), (59) reduces to:

$$\begin{cases} (1-\theta)\, q_t h_t \frac{\dot{c}}{c} & = \eta c_t - \left[\theta r q_t + (1-\theta)\,\rho q_t + \delta\omega_k q_t^{\sigma} - \dot{q}_t\right] h_t \\ \left((1-\theta)\frac{\sigma\omega_k}{q^{1-\sigma}-\omega_k} - \theta\right)\dot{q}_t h_t & = w - c_t - r\theta q_t h_t - \delta\omega_k q_t^{\sigma} h_t \end{cases} \tag{60}$$

This system of equations has a stable saddle path if $q_{ss} < q_{cr}$ given by equation (44), and do not have any stable point other than the steady state if $q_{ss} > q_{cr}$.

F: Employment in Food Services and Retail Trade Sectors

Table 5

	Food Services and Retial Trade Growth between								
	2000 and 2006			2006 and 2009			2000 to 2009		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Inelasticity	0.01	0.01**	0.00	0.00	0.00	0.01*	0.01	0.02**	0.02**
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Change in Securitization Fraction 03_06	0.23**	0.10*	-0.05	-0.14**	-0.03	-0.08	0.03	0.04	-0.17*
	(0.10)	(0.05)	(0.07)	(0.06)	(0.06)	(0.08)	(0.12)	(0.08)	(0.10)
Inelasticity X Change in Securitization Fraction 03_06	0.11	-0.01	-0.02	-0.08	-0.03	0.07	-0.03	-0.08	0.03
	(0.09)	(0.06)	(0.07)	(0.05)	(0.05)	(0.08)	(0.10)	(0.08)	(0.10)
Controls	N	Y	Y	N	Y	Y	N	Y	Y
State Fixed Effect	N	N	Y	N	N	Y	N	N	Y
Regression Type	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
Observations	321	321	321	323	323	323	321	321	321
R-squared	0.02	0.63	0.72	0.01	0.20	0.36	0.00	0.64	0.70

Notes: This tables presents estimates of the impact of variations in the elasticity of housing supply and changes in securitization rate on growth in employment in food services and retail trade sectors during the years of 2000 to 2006, 2006 to 2009 and 2000 to 2009. Baseline controls include the growth of the average income and its interaction with inelasticity measure and population growth during the associated years and the fraction of houses purchased by investors in 2004. Each county with population greater than 150000 in the year 2000 is one uint of observation. Robust standard errors are below coefficients in parentheses, and asterisks denote significance levels (***=1%, **=5%, *=10%).