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The long-period forced librations of Titan

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Abstract. A moon in synchronous rotation has longitudinal librations because of its non-spherical mass distribution and its elliptical orbit around the planet. We study the librations of Titan with periods of 14.7y and 29.5y and include deformation effects and the existence of a subsurface ocean. We take into account the fact that the orbit is not Keplerian and has other periodicities than the main period of orbital motion around Saturn due to perturbations by the Sun, other planets and moons. An orbital theory is used to compute the orbital perturbations due to these other bodies.

We numerically evaluate the amplitude of the long-period librations for many interior structure models of Titan constrained by the mass, radius and gravity field. Measurements of the librations may give constraints on the interior structure of the icy satellites.

Keywords. Titan, Dynamics, Rotation, Planetary Interiors

1. Introduction

The orbital motion of the icy moon Titan is not Keplerian, therefore Titan experiences forced librations with different frequencies. Here we study the longitudinal librations at the semi-annual period of Saturn around the Sun (14.7y) and its annual period (29.5y). These two periods are long with respect to the diurnal period (16 days). We also take into account the atmospheric torque at the Saturn semi-annual period.
We include deformation effects and the existence of a subsurface ocean. We investigate how the internal structure changes these forced librations.

2. Interior models and tides

We calculate the long-period libration for a lot of different density profiles for Titan with a liquid ocean and 3 solid layers: a solid core (c) composed of a mixture of ice and rocks, a high-pressure solid ice mantle (m), a liquid ocean (o) and an outer solid icy shell (s). We refer to the interior layer (i) as the layer composed of the solid core and the solid mantle. The layers are assumed to have an homogeneous density.

The interior models are constrained by the mass, the radius, and the moment of inertia (MOI $= 0.3431 \pm 0.0004$, Iess (2012)) of Titan.

Layer	Thickness (km)/Radius (km)	Density (kg m^{-3})
Ice shell (s)	h_s = 5-200	800-1200
Ocean (o)	r_o = 2375-2570	1000-1400
Ice mantle (m)	r_m = 2000-2550	1200-1400
Ice/Rock core (c)	r_c = 1600-2200	2430-3419

Tab. 1: The numerical range for the densities and the minimum and maximal size of the 4 layers of Titan (Baland *et al.* 2011).

We use the Clairaut equation for a synchronous satellite deformed by rotation and static tides in order to compute the flattenings and the principal moments of inertia A_l,

B_l and C_l ($A_l \leqslant B_l \leqslant C_l$) for each layer l. The polar flattening is defined as $\alpha = \frac{(a+b)/2-c}{(a+b)/2}$ where a, b and c are the radii of the principal axes of the ellipsoid ($a > b > c$) and the equatorial flattening is defined as $\beta = (a-b)/a$.

We include the elastic effects due to tides according to Van Hoolst *et al.* (2013): (1) there is a modification of the gravitational torque exerted by Saturn because of the periodic tidal bulges, (2) additional torques exist between the periodic tidal bulges and the static shape of another solid layer, and (3) there is a zonal tide.

3. Forcings

3.1. *Orbital perturbations*

In order to evaluate the orbital perturbations, we use the SAT360 orbital satellite ephemeris of Titan from R.A. Jacobson covering 400y around J2000. The orbital elements of Titan are given in an inertial frame based on the equatorial plane of Saturn at J2000. The dependence of the equation of motion to the orbital elements will be through the mean longitude $\lambda = M + \omega + \Omega$ where M, ω and Ω are the traditional orbital elements. We use the TRIP software (Gastineau & Laskar 2011) for a decomposition into a quasiperiodic series:

$$\lambda(t) = \sum \lambda_A \cos(\omega_f\, t + \phi_\lambda) \tag{3.1}$$

where λ_A is the amplitude of the orbital perturbation and ω_f its frequency.
The main perturbing frequencies here are due to the annual and semi-annual motion of Saturn around the Sun.
By using another orbital theory valid on a longer interval, the orbit of Titan can be seen to have additional long-period variations, with are much longer periods than the diurnal period (about 16d). The TASS 1.7 orbital model for Titan (Vienne & Duriez 1995) decomposes the mean longitude over 36 frequencies. The 5 largest perturbations have periods of 3263.1y, 703.5y, 14.7y, 29.5y and 914.8y.

We neglect the obliquity of Titan (0.3°, Stiles *et al.* (2008, 2010)) and the inclination of the orbit of Titan on the equatorial plane of Saturn (0.3-0.4°). Therefore only the z-component of the quantities has to be considered.
We neglect the direct torques of the bodies other than Saturn. Indirect effects of the orbital perturbations have a much larger influence on the rotational motion than these direct torques.

3.2. *Atmospheric torque*

There is an exchange of Atmospheric Angular Momentum (AAM) between the surface of Titan and the dense atmosphere (by friction of winds), and therefore a coupling between these two layers. The dynamic is driven by the insolation, therefore the motion is mostly at the Saturnian semi-annual frequency. The expression for the atmospheric torque is $\Gamma_A \cos(\omega_f\, t + \phi_\lambda + \Delta\phi)$. The amplitude of the torque at the semi-annual frequency computed by Tokano & Neubauer (2005) from a Global Circulation Model (GCM) is $\Gamma_A = 1.6\,10^{17}$ kg m^2/s^2. The phase $\Delta\phi$ at the Saturnian semi-annual frequency is evaluated to be about 1.9 rad at J2000 (computed from Van Hoolst *et al.* (2009)). Richard *et al.* (2014) uses a much smaller atmospheric torque amplitude.

4. Equations of motion

We express the equations of motion of the shell and of the interior in terms of the small libration angle γ. The rotation angle ϕ is the angle between the X-axis of the inertial frame to the long axis of Titan. The difference between these two angles is equal to the mean longitude $\lambda = \phi - \gamma$.

We write the equation of conservation of angular momentum for each layer l by taking into account all the torques acting on the layers, both the external gravitational and internal torques; including the pressure torques. The two equations of rotation for the shell (s) and for the interior (i) are:

$$C_s\ddot{\gamma}_s + C_s\,\ddot{\lambda} + K_1\,\gamma_s + K_2\,\gamma_i = \Gamma_A \tag{4.1}$$

$$C_i\ddot{\gamma}_i + C_i\,\ddot{\lambda} + K_4\,\gamma_s + K_5\,\gamma_i = 0 \tag{4.2}$$

We neglect the short-periodic variations related to the orbital period of Titan since we are interested in the long-period variations. We have used the small angle approximation and we neglected dissipation inside the satellite.
The strengths of the couplings K_1, K_2, K_4 and K_5 depend on the coupling between the layers, both the static and periodic parts (Van Hoolst *et al.* 2013).

5. Long-period libration results

5.1. *Saturn annual libration*

At the Saturn annual libration (29.5y), the orbital perturbation on the mean longitude is $\lambda_{An} = 37$ as, which corresponds to 466 m on the surface of Titan.
The frequency is far away from the period of the 2 eigenmodes which vary between 58d and 1.2y and between 2.5y and 5y, depending on the interior structure of Titan. Therefore, the amplitude of libration in longitude of the icy shell (γ_s angle) is very small ($<$ 10m).
For this frequency, the rotation angle of the shell (ϕ_s) has an amplitude between 469 and 475m, depending on the interior model.

5.2. *Saturn semi-annual libration*

At the Saturn semi-annual libration (14.7y), the orbital perturbation on the mean longitude is $\lambda_{SAn} = 43$ as, which corresponds to 548 m on the surface. If there is no atmospheric torque, the corresponding libration amplitude (γ_s) of the icy shell is very small (18-48m, depending on the interior model, see Fig. 1).

This is much smaller than the semi-annual libration due to to the atmospheric torque, which can have amplitudes between 600 m and 1350 m as shown by Van Hoolst et al. (2013). Therefore, adding the orbital perturbation if the atmospheric torque is considered makes a very small difference (amplitude between 591 m and 1335 m).
For a 10 times smaller atmospheric torque (as for example in Richard *et al.* (2014)), the amplitude γ_s is about 10 times smaller (= 69-159m)
The amplitude of the shell rotation angle (ϕ_s angle) is between 837 and 1477 m.
If we do not take into account the tides (rigid planet), the amplitude γ_s is 3 times smaller (201-245m).
The libration of the ice shell may be observed in the future, but not the libration of the interior.

5.3. *The libration at the very long period*

Using the amplitudes given in the TASS orbital theory for the very long periods (3263.1y, 703.5y and 914.8y), we compute the libration amplitude for the γ_s angle. Since these

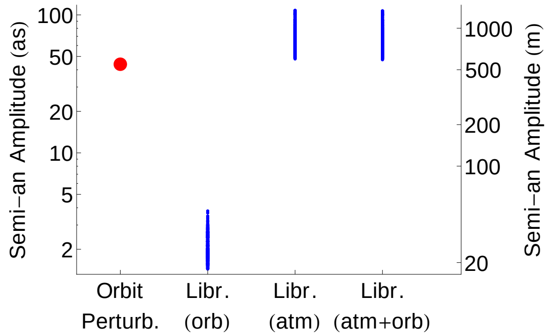

Figure 1. The amplitude of the orbital perturbation (the red point) at the Saturn semi-annual period of Saturn ($T = 14.7$y). The blue points show the libration amplitudes of the ice shell (γ_s) for different interior models by taking into account the orbital perturbation only, the atmospheric torque only, and the orbital perturbation and the atmospheric torque together. The scale is logarithmic.

periods are much longer than the periods of two eigenmodes (less than 5y), their libration amplitude is smaller than 0.1m and may be neglected.

6. Conclusion

Since the Saturn annual and semi-annual frequencies are far away from the two eigenfrequencies, there are no large amplification for the libration. The Saturn annual (29y) libration will be too small (< 10m) to be detected in a near future. The Saturn semi-annual (14.7y) libration may have a large amplitude (up to 1.5km), depending on the interior model and the value of the atmospheric torque.

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