Cross section of the APOLLO Lunar retroreflector arrays

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Contents

1. Introduction.

The Apollo Lunar retroreflector arrays have a very large cross section in order to make it possible to do laser ranging from the earth. The proposed LRO (Lunar Reconnaissance Orbiter) would like to range to the arrays from low Lunar orbit. At close range the return signal could burn out the detector because of the large cross section of the array. The question is whether it would be safe to range to the arrays at off-normal incidence angles where the cross section is lower. The cross section decreases with incidence angle on the array. Past about 17 degrees incidence angle, there can be loss of total internal reflection depending on the azimuth angle of the incidence beam. This causes an additional loss of cross section. This report gives the cross section at various incidence angles on the array in order to determine the conditions under which it would be safe to range to the array from low Lunar orbit.

2. Active reflecting area.

The cross section of an APOLLO retroreflector decreases with incidence angle ϕ as a result of the decrease in the active reflecting area. Table 1 below shows the active reflecting area of an APOLLO retroreflector vs incidence angle ϕ. Also shown is the square of the active reflecting area. The reason for showing the square is that the cross section at the center of a perfect retroreflector is proportional to the square of the area.

ϕ	Area	Area sq	Log(Area)	Log (Area sq)
0.0	1.000000	1.000000	0.000000	0.000000
5.0	0.889000	0.790321	-0.051098	-0.102196
10.0	0.717000	0.514089	-0.144481	-0.288962
15.0	0.496000	0.246016	-0.304518	-0.609037
17.0	0.408000	0.166464	-0.389340	-0.778680
20.0	0.284000	0.080656	-0.546682	-1.093363
25.0	0.107000	0.011449	-0.970616	-1.941232
28.0	0.031000	0.000961	-1.508638	-3.017277
29.0	0.013000	0.000169	-1.886057	-3.772113

Table 1. Active reflecting area and square of the active reflecting area vs incidence angle ϕ.

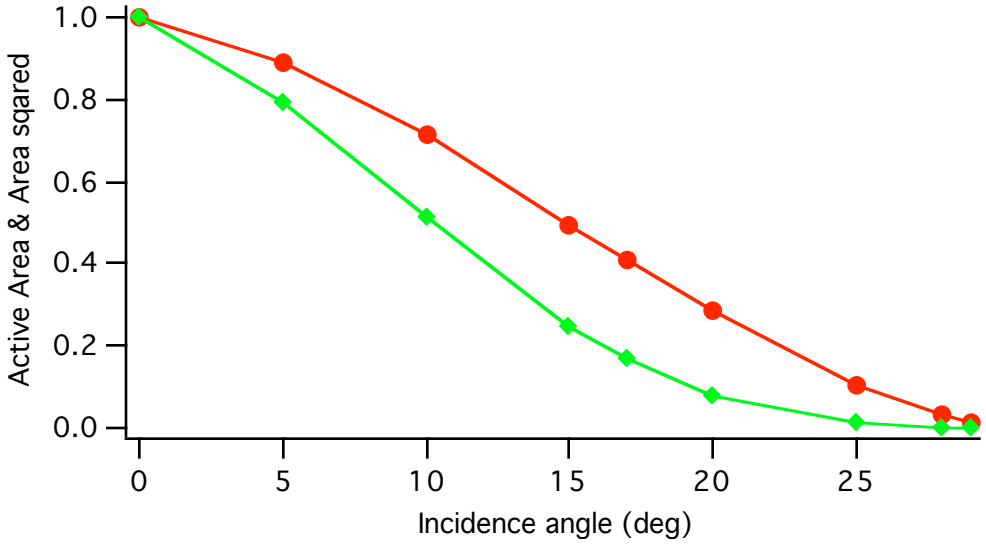
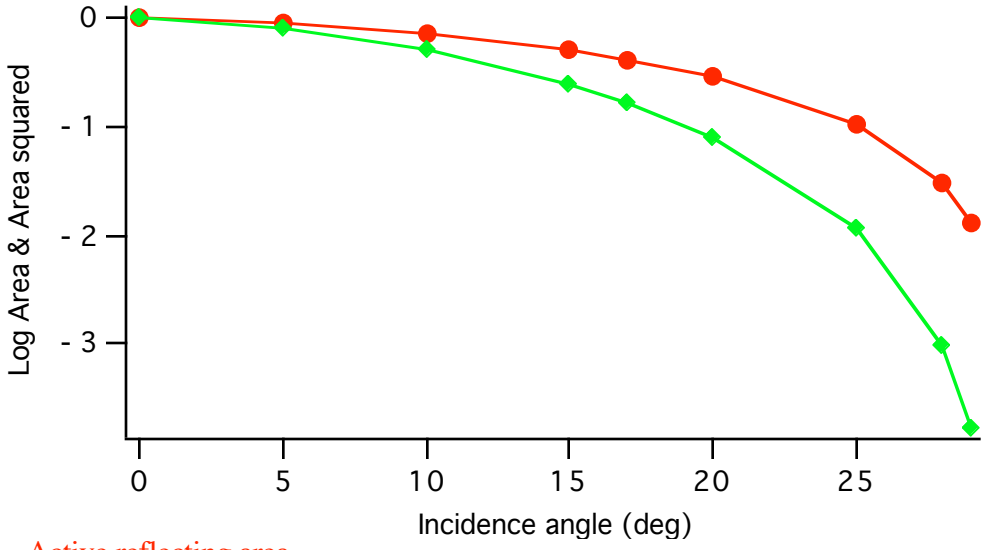
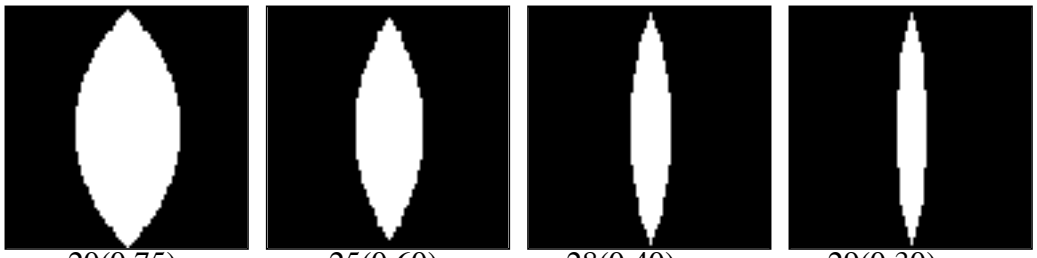


Figure 1. Active reflecting area and square of the active reflecting area vs incidence angle ϕ.

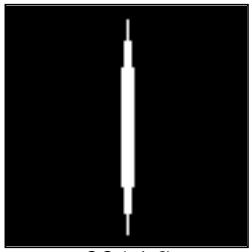


Red = Active reflecting area
Green = Square of active reflecting area
Figure 2. Log of the active reflecting area and square of the active reflecting area vs ϕ.

0(1.00) 5(1.00) 10(1.00) 15(1.00)



20(0.75) 25(0.60) 28(0.40) 29(0.30)



30(.16)

Figure 3. Shape of the active reflecting area vs incidence angle ϕ. The incidence angle is listed below each figure. The number in parenthesis is the pixel size.

In figure 3, the scale of the plot is smaller for the larger incidence angles in order to maintain better resolution. The active reflecting area become extremely narrow near the absolute cutoff angle which is a little over 30 degrees. This results in the diffraction pattern becoming very broad in the horizontal direction. This width of the diffraction pattern in each dimension is roughly inversely proportional to the width of the active reflecting area in that dimension.

The active reflecting area does not take into account diffraction and loss of total internal reflection. For this reason it is not an accurate measure of return signal strength. If the active reflecting area is multiplied by the reflection coefficient past loss of total internal reflection this gives a measure of the total reflected energy.

3. Polarization.

For a coated retroreflector the cross section is a function of the incidence angle and the magnitude and direction of the velocity aberration. Polarization effects are negligible. For an uncoated retroreflector the cross section is also a function of the polarization because the diffraction pattern depends on the polarization of the incident beam. The diffraction pattern is computed in a coordinate system where the horizontal axis is parallel to the plane of incidence and the vertical axis is perpendicular to the plane of incidence. The incident polarization state is given in the coordinate system of the diffraction pattern. For horizontal polarization the electric vector is parallel to the plane of incidence. For vertical polarization the electric vector is perpendicular to the plane of incidence.

The analysis that is presented in this report shows that the polarization effects are most significant after loss of total internal reflection. For vertical polarization the diffraction pattern tends to have a hole near the center of the pattern. Since the velocity aberration is low in Lunar orbit (about 10 microradians), the hole in the center of the pattern results in low cross section for vertical polarization. For horizontal polarization, there tends to be a bright lobe near the center of the pattern so that the cross section is higher.

4. Loss of total internal reflection.

The APOLLO retroreflectors are recessed such that the absolute cutoff angle beyond which there is no signal is a little over 30 degrees. Since they are uncoated, there is loss of internal reflection at certain incidence angles. Figure 4 below shows incidence angles where there is loss of total internal reflection.

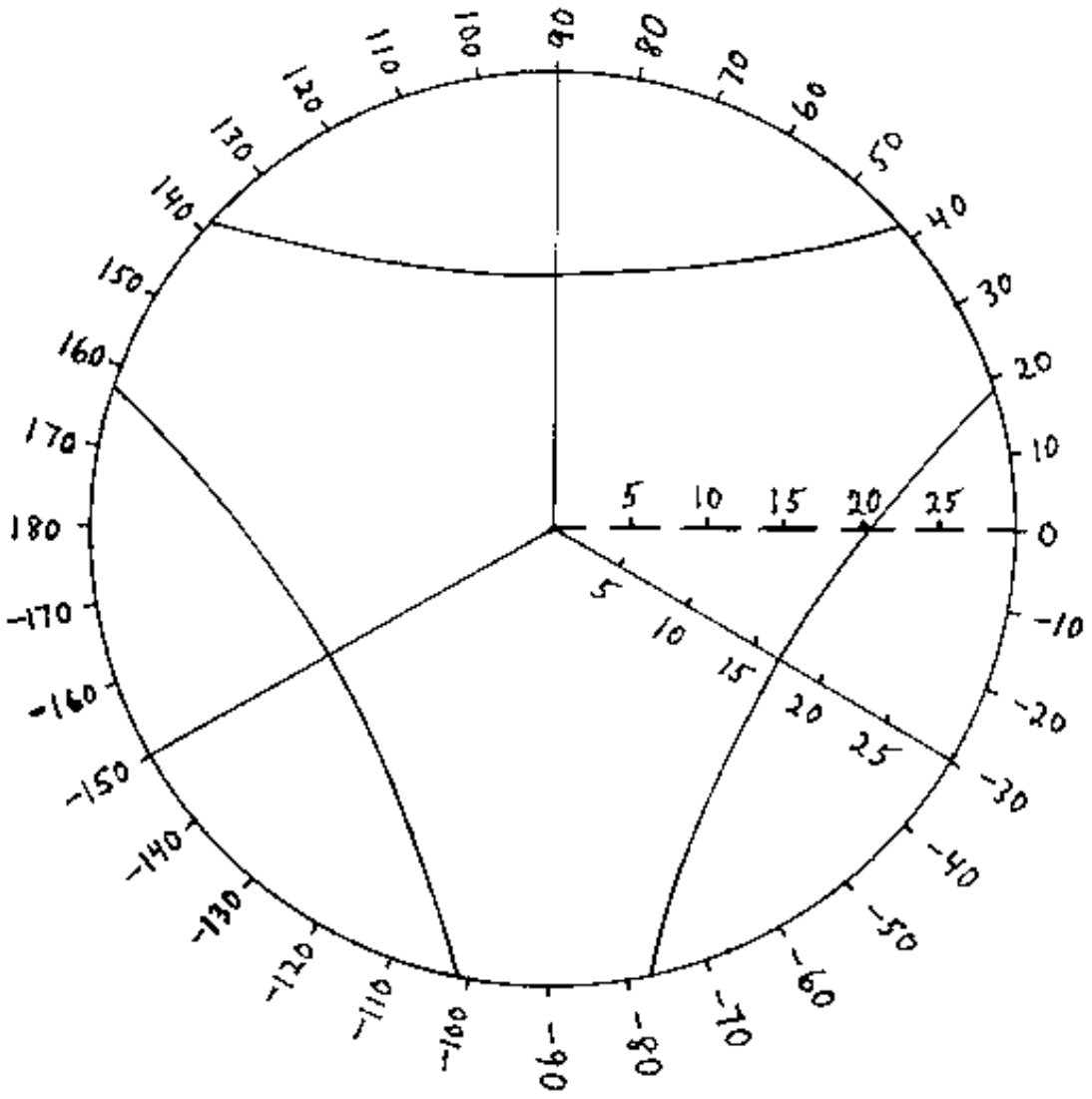


Figure 4. Total internal reflection diagram for an APOLLO retroreflector.

Figure 4 shows the incidence angles where there is loss of total internal reflection for an APOLLO retroreflector. The back edges of the retroreflector are at θ = 90, -30, and -150 degrees. The incidence angle ϕ which is measured from the normal to the front face is shown radially on the plot up to the absolute cutoff angle of about 30 degrees. The azimuth angle θ is shown around the circumference. The cross section has been computed approximately every 5 degrees in ϕ for θ = -30, 0, and +30 degrees. The cross section at other values of θ can be obtained by symmetry.

At θ = -30 deg, the cutoff for total internal reflection is about ϕ = 17 degrees. The reason the cutoff angle is lowest at this value of θ is that the incidence angle on the face normal to the edge at θ = -30 deg is too close to normal incidence. At θ = 0 deg, the cutoff for total internal reflection is a little over ϕ = 20 degree. For values of θ between about 18 and 42 deg there is no loss of total internal reflection out to the absolute cutoff angle of 30 deg. The reason the cutoff angle for total internal reflection is largest for these azimuths is that the incidence angle on the faces next to the edge at θ = -150 deg is never close to normal incidence. For values of θ between about -77 and 18 degrees there is loss of total internal reflection once the angle ϕ passes the cutoff line shown in Figure 1. Past the limit for total internal reflection, the return is due to ordinary dielectric reflection at one of the back faces and total internal reflection at the other two faces.

Table 2 below lists the cutoff angle ϕ for total internal reflection vs the azimuth angle θ.

θ	ϕ(cutoff)
-30.0000	16.9976
-25.0000	17.0759
-20.0000	17.3150
-15.0000	17.7283
-10.0000	18.3404
-5.0000	19.1917
0.0000	20.3472
5.0000	21.9137
10.0000	24.0787
15.0000	27.2118
18.0000	29.8808
19.0000	30.9771
20.0000	32.2151
25.0000	42.8933
26.0000	47.7058
26.5000	51.6861
26.8000	55.8671
26.9000	58.4690

Table 2. Cutoff angle ϕ (deg) vs azimuth angle θ (deg).

The absolute cutoff angle for a retroreflector that is not recessed is about ϕ = 57 degrees. At this value of ϕ loss of total internal reflection occurs at about θ = 26.8 degrees. The region between θ = 26.8 and 33.2 deg would never have loss of total internal reflection in an unrecessed cube corner. The range in azimuth angles where there is no loss of total internal reflection is larger for a recessed cube because the absolute cutoff angle is smaller. The cutoff angle for total internal reflection does not depend on whether the retroreflector is recessed. It is a function only of the incidence angles θ and ϕ.

5. Diffraction and cross section.

The cross section of a retroreflector is proportional to the intensity of the diffraction pattern at the position of the receiver in the far field as determined by the magnitude and direction of the velocity aberration.

For a perfect reflector (perfect metal reflecting faces, no dihedral angle offset, perfectly flat faces), the cross section *C* at the center of the diffraction pattern is

$$C = 4\pi\left(\frac{A}{\lambda}\right)^2 \qquad (1)$$

where *A* is the reflecting area and λ is the wavelength. The diffraction calculations presented in this report have been computed in units of the cross section given by equation (1). In other words the cross section at the center of the diffraction pattern of a perfect retroreflector is unity. The results have also been converted to square meters by multiplying by the cross section given by equation (1).

The method of computing the diffraction pattern is described in Reference 1. The APOLLO retroreflectors are recessed in a conical cavity by an amount equal to the radius of the retroreflector. Recession is discussed on page 13 of Reference 1. Unfortunately, Reference 1 does not consider the case of a conical cavity. It assumes the cavity is cylindrical and the diameter of the opening is the same (or less than) the diameter of the retroreflector.

Since Reference 1 and the computer program that implements the equations in Reference 1 does not include the case of a conical recession cavity, the diffraction calculations have been done using a ray tracing program. The phase front is then integrated to give the diffraction pattern using the method described in section 5.2 “Calculation of Diffraction Patterns from an Array of Phases” on page 73 of Reference 1.

The ray tracing uses a 101 x 101 array of phases. Since the active reflecting area can become quite small at large incidence angles, the spacing between the rays is reduced by the scale factor shown in figure 3 above to preserve accuracy.

As can be seen in figure 4, loss of total internal reflection occurs most quickly at θ = -30. At θ = +30 there is no loss of total internal reflection. In order to see the range of cross sections that can occur for different values of θ. Diffraction patterns have been computed for θ = -30, 0, and +30 deg. As discussed in section 3, the cross section is most sensitive to polarization when there is loss of total internal reflection. For θ = -30 and 0 deg, the cross section has been computed for both polarizations since there is loss of total internal reflection at these orientations. For θ = +30 deg, the cross section has been computed only for vertical polarization.

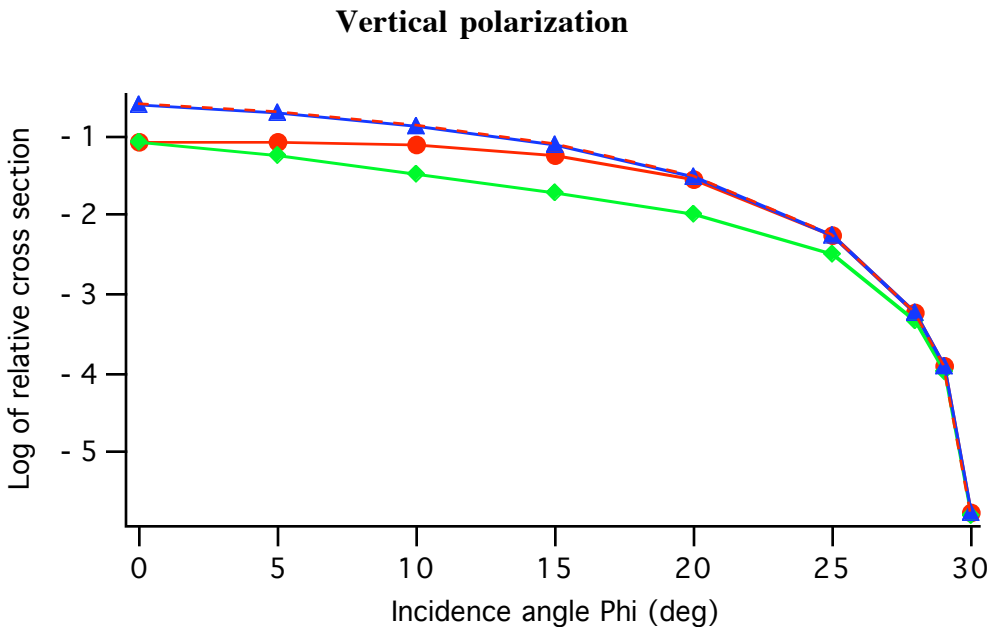
The next three sections present the cross section calculations for each value of θ and ϕ. It is difficult to know how much information to present. In principle, the cross section can be defined by giving the two dimensional cross section matrices for each value of θ and ϕ for each polarization state. This is a lot of information.

The maximum orbital velocity in low Lunar orbit is about 1.64 km/sec which is a velocity aberration of about 10.9 microradians. The velocity aberration can be much less than this depending on the angle between the line of sight and the orbital velocity. The values of cross

section at 10 microradians have been tabulated and plotted to show the variation of cross section with the incidence angles θ and ϕ. If there is reasonably symmetry in the pattern, only the points at (10,0) (horizontal axis, and (0,10) (vertical axis) microradians are given. If the patterns are asymmetrical the value at (-10,0) and (1,-10) are also tabulated. The values are given in units of equation (1), and in square meters for one, 100, and 300 retroreflectors.

The cross section at the center of the pattern and the maximum cross section have also been tabulated and plotted for reference. This gives a total of 6 points for each pattern. Since the velocity aberration can have various values up to about 11 microradians, the axes of the diffraction pattern are also tabulated from -12 to +12 microradians in 2 microradian steps. A sample plot of the axes at $\phi = 20$ degrees is given for each set of runs at a particular value of θ. This does not give the cross section at every point in the far field pattern. However, gray scale plots of each diffraction pattern are included which show the variation qualitatively at every point. This is a manageable amount of information.

6. Cross section at θ = +30 deg



Red (10,0)
Green (0,10)
Blue (0,0)
Magenta Maximum

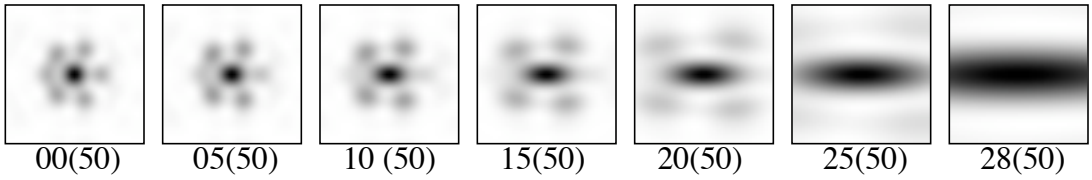
The cross section is plotted at four different points in the far field diffraction pattern. The red curve is on the horizontal axis. The Green is along the vertical axis. The blue is at the center. The magenta is the maximum and coincides with the blue curve. The polarization is perpendicular to the plane of incidence (vertical). At this orientation there is no loss of total internal reflection. The data used to plot the graph is shown in the table below in relative units.

ϕ	(10,0)	(0,10)	(0,0)	maximum
0.0	0.0845248	0.0813103	0.2482102	0.2482102
5.0	0.0833662	0.0567662	0.1973326	0.1973326
10.0	0.0799286	0.0325411	0.1375192	0.1375192
15.0	0.0578902	0.0185225	0.0769236	0.0769236
20.0	0.0273524	0.0106530	0.0310286	0.0310286
25.0	0.0055114	0.0032657	0.0057012	0.0057012
28.0	0.0005811	0.0004592	0.0005849	0.0005849
29.0	0.0001198	0.0001050	0.0001201	0.0001201
30.0	0.0000017	0.0000016	0.0000017	0.0000017

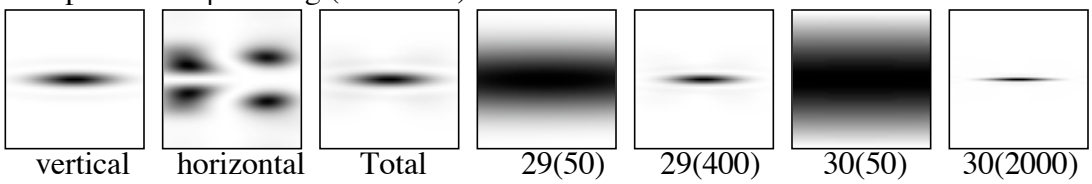
The cross section is relative to the cross section at the center of the Airy pattern. Because of polarization effects the peak is reduced by about a factor of 4 in uncoated cube corners.

Diffraction patterns at each incidence angle
θ = +30 deg, Vertical polarization

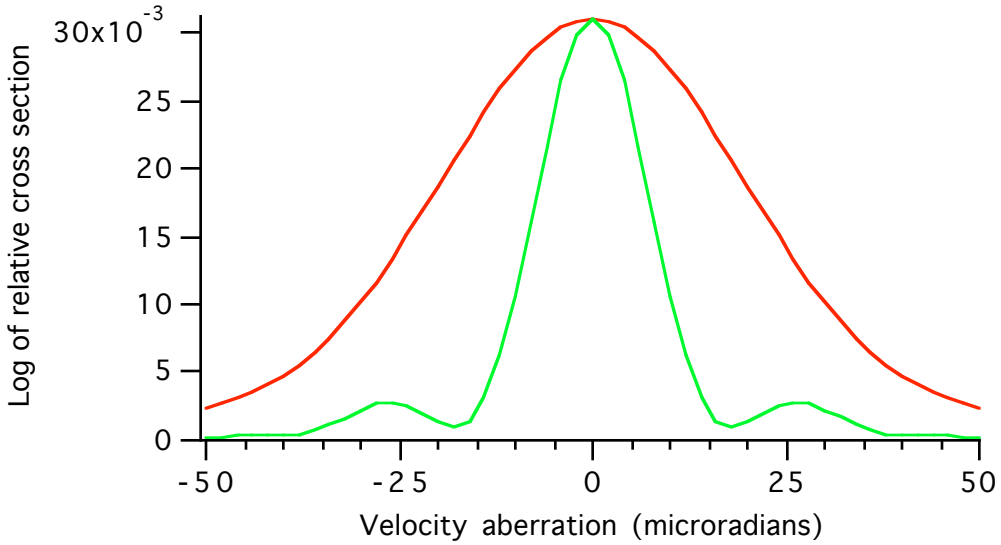
For small incidence angles, the pattern has a bright lobe at the center with six surrounding lobes. At larger incidence angles the surrounding lobes become fainter and the central spot becomes elongated in the horizontal direction. For this reason the intensity of the red curve is higher then the green curve. The intensity at the center (blue) is always highest.



Components at ϕ=28 deg (scale 200)



The incidence angle is shown below each plot in deg with the scale in microradians in parenthesis. The components of the pattern are shown at one incidence angle (28 deg). Below is a plot of the cross section along the horizontal and vertical axes for ϕ = 20 deg.



Red Horizontal (X) axis
Green Vertical (Y) axis

The following tables show the cross section in million square meters for each point in the far field pattern. The cross section at the center of the Airy pattern is 57.712 million square meters. The maximum cross section at normal incidence is .2482102 x 57.712 = 14.3247 as seen in first row below for a single cube.

θ = +30 deg, Vertical polarization

Single cube

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	4.8780968	4.6925815	14.3247117	14.3247146
5.0	4.8112317	3.2760920	11.3884627	11.3884650
10.0	4.6128408	1.8780126	7.9365106	7.9365112
15.0	3.3409603	1.0689709	4.4394162	4.4394145
20.0	1.5785622	0.6148061	1.7907231	1.7907220
25.0	0.3180740	0.1884701	0.3290278	0.3290289
28.0	0.0335365	0.0265014	0.0337558	0.0337529
29.0	0.0069139	0.0060598	0.0069312	0.0069306
30.0	0.0000981	0.0000923	0.0000981	0.0000975

100 cubes

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	487.80968	469.25815	1432.47117	1432.47146
5.0	481.12317	327.60920	1138.84627	1138.84650
10.0	461.28408	187.80126	793.65106	793.65112
15.0	334.09603	106.89709	443.94162	443.94145
20.0	157.85622	61.48061	179.07231	179.07220
25.0	31.80740	18.84701	32.90278	32.90289
28.0	3.35365	2.65014	3.37558	3.37529
29.0	0.69139	0.60598	0.69312	0.69306
30.0	0.00981	0.00923	0.00981	0.00975

300 cubes

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	1463.42905	1407.77446	4297.41350	4297.41437
5.0	1443.36951	982.82760	3416.53880	3416.53950
10.0	1383.85225	563.40377	2380.95319	2380.95336
15.0	1002.28809	320.69126	1331.82487	1331.82435
20.0	473.56867	184.44184	537.21694	537.21660
25.0	95.42221	56.54104	98.70833	98.70867
28.0	10.06094	7.95041	10.12673	10.12586
29.0	2.07417	1.81793	2.07936	2.07919
30.0	0.02943	0.02770	0.02943	0.02926

Horizontal and vertical axes of the diffraction patterns
Orientation θ = +30 deg
Vertical polarization

The tables below are in units of the cross section computed from equation (1). In these units the cross section at the center of the diffraction pattern of a perfect retroreflector is unity.

θ = +30 deg, Vertical polarization

ϕ = 0 deg

Microradians	Horizontal	Vertical
-12.0	0.0689586500	0.0614225900
-10.0	0.0844866800	0.0813218800
-8.0	0.1160606700	0.1150779600
-6.0	0.1588981500	0.1587230700
-4.0	0.2029030600	0.2029148800
-2.0	0.2359381000	0.2359563500
0.0	0.2482102500	0.2482102500
2.0	0.2359712600	0.2359562200
4.0	0.2029597200	0.2029139600
6.0	0.1589627200	0.1587201400
8.0	0.1161173800	0.1150714900
10.0	0.0845248500	0.0813102800
12.0	0.0689753400	0.0614045400

ϕ = 5 deg

Microradians	Horizontal	Vertical
-12.0	0.0640070900	0.0399439000
-10.0	0.0833225800	0.0567820600
-8.0	0.1103212900	0.0855357200
-6.0	0.1410491900	0.1224541000
-4.0	0.1696548300	0.1595505000
-2.0	0.1899716300	0.1871328900
0.0	0.1973326400	0.1973326400
2.0	0.1899995300	0.1871312400
4.0	0.1697038700	0.1595467300
6.0	0.1411080300	0.1224473600
8.0	0.1103775000	0.0855249400
10.0	0.0833662400	0.0567661500
12.0	0.0640332000	0.0399220100

ϕ = 10 deg

Microradians	Horizontal	Vertical
-12.0	0.0644088900	0.0201859000
-10.0	0.0798896900	0.0325297500
-8.0	0.0964997900	0.0540581300
-6.0	0.1124184400	0.0817125400
-4.0	0.1256599500	0.1094093300
-2.0	0.1344394500	0.1299396600
0.0	0.1375192100	0.1375192100
2.0	0.1344558400	0.1299405700
4.0	0.1256900100	0.1094115500
6.0	0.1124573200	0.0817168300
8.0	0.0965415600	0.0540654300
10.0	0.0799285800	0.0325410600
12.0	0.0644403600	0.0202020500

θ = +30 deg, Vertical polarization

ϕ = 15 deg

Microradians	Horizontal	Vertical
-12.0	0.0511989400	0.0097745200
-10.0	0.0578684400	0.0185225200
-8.0	0.0640562400	0.0316987500
-6.0	0.0693726200	0.0473296900
-4.0	0.0734622800	0.0622513500
-2.0	0.0760406200	0.0730052700
0.0	0.0769235700	0.0769235700
2.0	0.0760472500	0.0730052700
4.0	0.0734749000	0.0622513500
6.0	0.0693900000	0.0473296900
8.0	0.0640767500	0.0316987500
10.0	0.0578902400	0.0185225200
12.0	0.0512202100	0.0097745200

ϕ = 20 deg

Microradians	Horizontal	Vertical
-12.0	0.0258704500	0.0062019700
-10.0	0.0273482000	0.0106533900
-8.0	0.0286198700	0.0160331500
-6.0	0.0296496000	0.0216108300
-4.0	0.0304077000	0.0264969800
-2.0	0.0308719400	0.0298401700
0.0	0.0310285800	0.0310285800
2.0	0.0308729300	0.0298398400
4.0	0.0304096400	0.0264963900
6.0	0.0296524100	0.0216101200
8.0	0.0286234400	0.0160324900
10.0	0.0273523800	0.0106529900
12.0	0.0258750800	0.0062020500

ϕ = 25 deg

Microradians	Horizontal	Vertical
-12.0	0.0054293600	0.0025029300
-10.0	0.0055111000	0.0032652400
-8.0	0.0055788300	0.0040184500
-6.0	0.0056320500	0.0046962700
-4.0	0.0056703600	0.0052349300
-2.0	0.0056934700	0.0055815700
0.0	0.0057012200	0.0057012200
2.0	0.0056935400	0.0055816900
4.0	0.0056704900	0.0052351700
6.0	0.0056322400	0.0046966000
8.0	0.0055790800	0.0040188600
10.0	0.0055114100	0.0032657000
12.0	0.0054297300	0.0025034100

θ = +30 deg, Vertical polarization

ϕ = 28 deg

Microradians	Horizontal	Vertical
-12.0	0.0005794900	0.0004114000
-10.0	0.0005811200	0.0004592200
-8.0	0.0005824600	0.0005016300
-6.0	0.0005835100	0.0005367600
-4.0	0.0005842500	0.0005630700
-2.0	0.0005847000	0.0005793500
0.0	0.0005848500	0.0005848500
2.0	0.0005847000	0.0005793300
4.0	0.0005842600	0.0005630300
6.0	0.0005835100	0.0005367100
8.0	0.0005824700	0.0005015600
10.0	0.0005811300	0.0004591500
12.0	0.0005795000	0.0004113100

ϕ = 29 deg

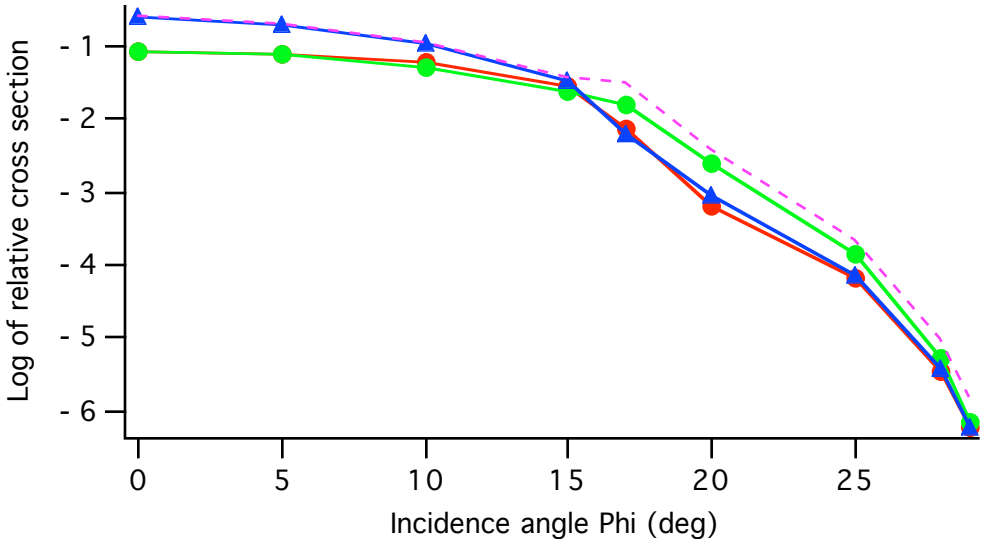
Microradians	Horizontal	Vertical
-12.0	0.0001197200	0.0000989000
-10.0	0.0001198300	0.0001050200
-8.0	0.0001199300	0.0001102500
-6.0	0.0001200000	0.0001144700
-4.0	0.0001200500	0.0001175600
-2.0	0.0001200800	0.0001194500
0.0	0.0001200900	0.0001200900
2.0	0.0001200800	0.0001194600
4.0	0.0001200500	0.0001175700
6.0	0.0001200000	0.0001144800
8.0	0.0001199300	0.0001102700
10.0	0.0001198300	0.0001050300
12.0	0.0001197200	0.0000989200

ϕ = 30 deg

Microradians	Horizontal	Vertical
-12.0	0.0000016900	0.0000016100
-10.0	0.0000016900	0.0000016400
-8.0	0.0000016900	0.0000016500
-6.0	0.0000016900	0.0000016700
-4.0	0.0000016900	0.0000016800
-2.0	0.0000016900	0.0000016900
0.0	0.0000016900	0.0000016900
2.0	0.0000016900	0.0000016900
4.0	0.0000016900	0.0000016800
6.0	0.0000016900	0.0000016700
8.0	0.0000016900	0.0000016500
10.0	0.0000016900	0.0000016400
12.0	0.0000016900	0.0000016100

7. Cross section at θ = -30 deg

Vertical polarization



Red (10,0)
Green (0,10)
Blue (0,0)
Magenta Maximum

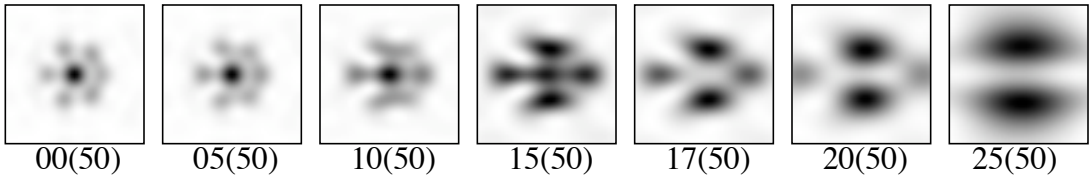
The cross section is plotted at four different points in the far field diffraction pattern. The red curve is a point along the horizontal axis at (10,0) microradians. The Green is along the vertical axis at (0,10) microradians. The blue is at the center (0,0) microradians. The magenta curve is the maximum intensity. The polarization is perpendicular to the plane of incidence (vertical). At this orientation there is loss of total internal reflection at about ϕ = 17 degrees. There is a discontinuity in the curve at this point. The data used to plot the graph is shown in the table below.

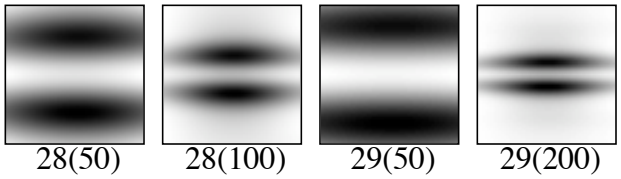
ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	0.0844867	0.0813219	0.2482102	0.2482102
5.0	0.0777084	0.0740974	0.1910439	0.1910439
10.0	0.0622524	0.0502930	0.1090571	0.1090571
15.0	0.0275213	0.0236460	0.0336651	0.0362466
17.0	0.0071335	0.0161781	0.0064166	0.0297362
20.0	0.0006610	0.0024908	0.0008880	0.0036328
25.0	0.0000663	0.0001352	0.0000724	0.0002074
28.0	0.0000036	0.0000054	0.0000037	0.0000099
29.0	0.0000006	0.0000007	0.0000006	0.0000015

The cross section is relative to the cross section at the center of the Airy pattern. Because of polarization effects the peak is reduced by about a factor of 4 in uncoated cube corners. The headings of columns 2, 3, and 4 are the position in the far field diffraction pattern in microradians.

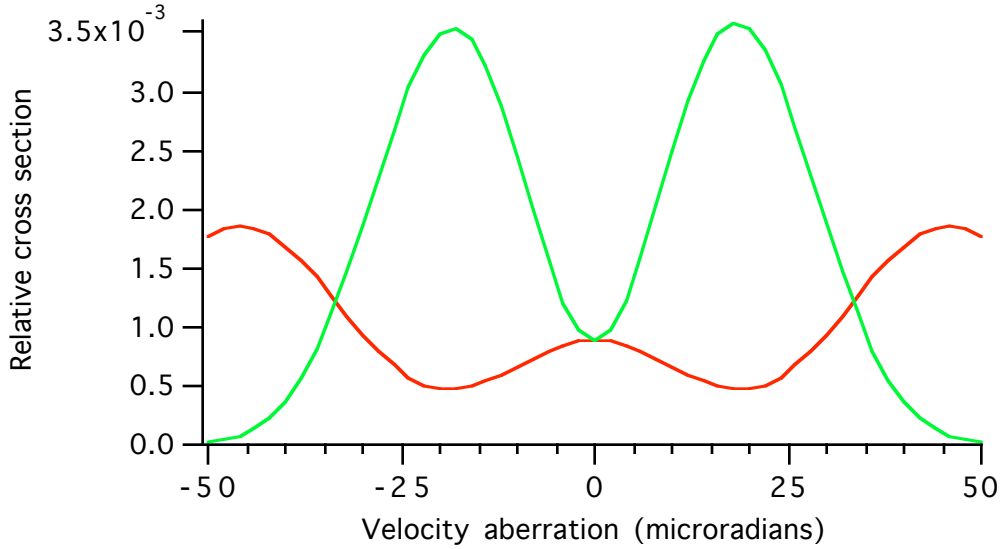
Diffraction patterns at each incidence angle
θ = -30 deg, Vertical polarization

For small incidence angles, the pattern has a bright lobe at the center with six surrounding lobes. Past the loss of total internal reflection at 17 degrees the pattern has two lobes aligned with the vertical axis. For this reason the intensity of the green curve is higher than the red and blue curves in the plot above.





The incidence angle in degrees is shown below each pattern with the scale in microradians in parenthesis. For large incidence angles the pattern is too large to be shown on a 50 x 50 microradian scale. Below is a plot of the cross section along the horizontal and vertical axes for ϕ = 20 deg.



Red Horizontal (X) axis
Green Vertical (Y) axis

The following tables show the cross section in million square meters for each point in the far field pattern. The cross section at the center of the Airy pattern is 57.712 million square meters. The maximum cross section at normal incidence is .2482102 x 57.712 = 14.3247 as seen in first row below for a single cube.

θ = -30 deg, Vertical polarization

Single cube

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	4.8758980	4.6932510	14.3247117	14.3247146
5.0	4.4847086	4.2763105	11.0255291	11.0255291
10.0	3.5927117	2.9025106	6.2939054	6.2939060
15.0	1.5883098	1.3646584	1.9428809	2.0918650
17.0	0.4116887	0.9336708	0.3703149	1.7161367
20.0	0.0381476	0.1437491	0.0512483	0.2096568
25.0	0.0038263	0.0078027	0.0041784	0.0119695
28.0	0.0002078	0.0003116	0.0002135	0.0005742
29.0	0.0000346	0.0000404	0.0000346	0.0000866

100 cubes

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	487.58980	469.32510	1432.47117	1432.47146
5.0	448.47086	427.63105	1102.55291	1102.55291
10.0	359.27117	290.25106	629.39054	629.39060
15.0	158.83098	136.46584	194.28809	209.18650
17.0	41.16887	93.36708	37.03149	171.61367
20.0	3.81476	14.37491	5.12483	20.96568
25.0	0.38263	0.78027	0.41784	1.19695
28.0	0.02078	0.03116	0.02135	0.05742
29.0	0.00346	0.00404	0.00346	0.00866

300 cubes

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	1462.76940	1407.97530	4297.41350	4297.41437
5.0	1345.41259	1282.89316	3307.65873	3307.65873
10.0	1077.81350	870.75317	1888.17161	1888.17179
15.0	476.49293	409.39752	582.86426	627.55951
17.0	123.50661	280.10124	111.09448	514.84101
20.0	11.44429	43.12473	15.37448	62.89704
25.0	1.14789	2.34080	1.25351	3.59084
28.0	0.06233	0.09349	0.06406	0.17227
29.0	0.01039	0.01212	0.01039	0.02597

Cross section in million square meters for one, 100, and 300 cubes.

Horizontal and vertical axes of the diffraction patterns
Orientation θ = -30 deg
Vertical polarization

The tables below are in units of the cross section computed from equation (1). In these units the cross section at the center of the diffraction pattern of a perfect retroreflector is unity.

θ = -30 deg, Vertical polarization

ϕ = 0 deg

Microradians	Horizontal	Vertical
-12.0	0.0689753400	0.0614045400
-10.0	0.0845248500	0.0813102800
-8.0	0.1161173800	0.1150714900
-6.0	0.1589627200	0.1587201400
-4.0	0.2029597200	0.2029139600
-2.0	0.2359712600	0.2359562200
0.0	0.2482102500	0.2482102500
2.0	0.2359381000	0.2359563500
4.0	0.2029030600	0.2029148800
6.0	0.1588981500	0.1587230700
8.0	0.1160606700	0.1150779600
10.0	0.0844866800	0.0813218800
12.0	0.0689586500	0.0614225900

ϕ = 5 deg

Microradians	Horizontal	Vertical
-12.0	0.0647774500	0.0609463000
-10.0	0.0777410900	0.0741031500
-8.0	0.1013127800	0.0972928400
-6.0	0.1314234700	0.1278034400
-4.0	0.1612001500	0.1589613100
-2.0	0.1830359900	0.1823545900
0.0	0.1910439000	0.1910439000
2.0	0.1830150800	0.1823504500
4.0	0.1611634000	0.1589539100
6.0	0.1313793800	0.1277944100
8.0	0.1012706600	0.0972842900
10.0	0.0777083600	0.0740973900
12.0	0.0647578800	0.0609455300

ϕ = 10 deg

Microradians	Horizontal	Vertical
-12.0	0.0538074600	0.0468424600
-10.0	0.0622719100	0.0502930200
-8.0	0.0738372400	0.0601726200
-6.0	0.0865893300	0.0752272500
-4.0	0.0981376100	0.0916102700
-2.0	0.1061790000	0.1042880300
0.0	0.1090571100	0.1090571100
2.0	0.1061705300	0.1042880300
4.0	0.0981221100	0.0916102700
6.0	0.0865693900	0.0752272500
8.0	0.0738160000	0.0601726200
10.0	0.0622524200	0.0502930200
12.0	0.0537920900	0.0468424600

θ = -30 deg, Vertical polarization

ϕ = 15 deg

Microradians	Horizontal	Vertical
-12.0	0.0262449400	0.0255492800
-10.0	0.0275272100	0.0236459600
-8.0	0.0291541400	0.0240309100
-6.0	0.0308475100	0.0264295300
-4.0	0.0323177900	0.0297578000
-2.0	0.0333137100	0.0325716000
0.0	0.0336651000	0.0336651000
2.0	0.0333118700	0.0325716000
4.0	0.0323143000	0.0297578000
6.0	0.0308427200	0.0264295300
8.0	0.0291485300	0.0240309100
10.0	0.0275213100	0.0236459600
12.0	0.0262392700	0.0255492800

ϕ = 17 deg

Microradians	Horizontal	Vertical
-12.0	0.0077385100	0.0201002300
-10.0	0.0071328600	0.0161661500
-8.0	0.0067547400	0.0126317700
-6.0	0.0065486600	0.0098307900
-4.0	0.0064554000	0.0078868600
-2.0	0.0064231100	0.0067740500
0.0	0.0064165500	0.0064165500
2.0	0.0064232600	0.0067773100
4.0	0.0064557000	0.0078931500
6.0	0.0065490900	0.0098396600
8.0	0.0067553000	0.0126425800
10.0	0.0071335400	0.0161781500
12.0	0.0077392900	0.0201126200

ϕ = 20 deg

Microradians	Horizontal	Vertical
-12.0	0.0005919600	0.0028746100
-10.0	0.0006610300	0.0024491400
-8.0	0.0007309000	0.0019911500
-6.0	0.0007942300	0.0015560400
-4.0	0.0008445500	0.0011987300
-2.0	0.0008768700	0.0009657100
0.0	0.0008880100	0.0008880100
2.0	0.0008768700	0.0009763300
4.0	0.0008445500	0.0012193500
6.0	0.0007942300	0.0015854700
8.0	0.0007309000	0.0020277200
10.0	0.0006610300	0.0024908200
12.0	0.0005919600	0.0029191800

θ = -30 deg, Vertical polarization

ϕ = 25 deg

Microradians	Horizontal	Vertical
-12.0	0.0000637200	0.0001585100
-10.0	0.0000662600	0.0001385700
-8.0	0.0000684100	0.0001185200
-6.0	0.0000701200	0.0001002500
-4.0	0.0000713600	0.0000855600
-2.0	0.0000721200	0.0000759500
0.0	0.0000723700	0.0000723700
2.0	0.0000721200	0.0000751800
4.0	0.0000713600	0.0000840500
6.0	0.0000701200	0.0000980400
8.0	0.0000684100	0.0001156800
10.0	0.0000662600	0.0001351900
12.0	0.0000637200	0.0001546800

ϕ = 28 deg

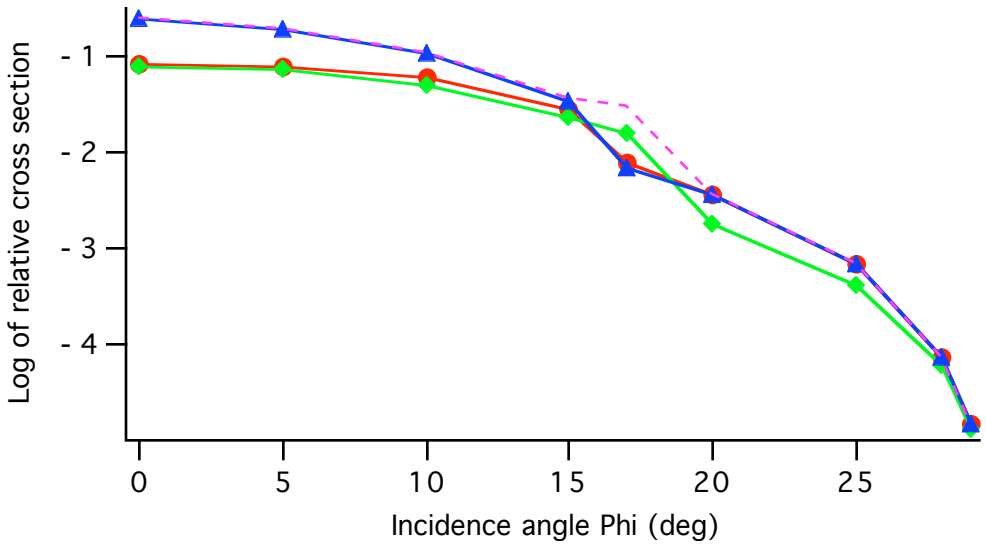
Microradians	Horizontal	Vertical
-12.0	0.0000035900	0.0000061900
-10.0	0.0000036100	0.0000055200
-8.0	0.0000036200	0.0000049100
-6.0	0.0000036400	0.0000044000
-4.0	0.0000036500	0.0000040000
-2.0	0.0000036500	0.0000037500
0.0	0.0000036600	0.0000036600
2.0	0.0000036500	0.0000037200
4.0	0.0000036500	0.0000039400
6.0	0.0000036400	0.0000043000
8.0	0.0000036200	0.0000047900
10.0	0.0000036100	0.0000053700
12.0	0.0000035900	0.0000060100

ϕ = 29 deg

Microradians	Horizontal	Vertical
-12.0	0.0000005500	0.0000008100
-10.0	0.0000005600	0.0000007400
-8.0	0.0000005600	0.0000006800
-6.0	0.0000005600	0.0000006300
-4.0	0.0000005600	0.0000005900
-2.0	0.0000005600	0.0000005700
0.0	0.0000005600	0.0000005600
2.0	0.0000005600	0.0000005600
4.0	0.0000005600	0.0000005800
6.0	0.0000005600	0.0000006200
8.0	0.0000005600	0.0000006600
10.0	0.0000005600	0.0000007200
12.0	0.0000005500	0.0000007900

Cross section for orientation θ = -30 deg

Horizontal polarization



Red	(10,0)
Green	(0,10)
Blue	(0,0)
Magenta	Maximum

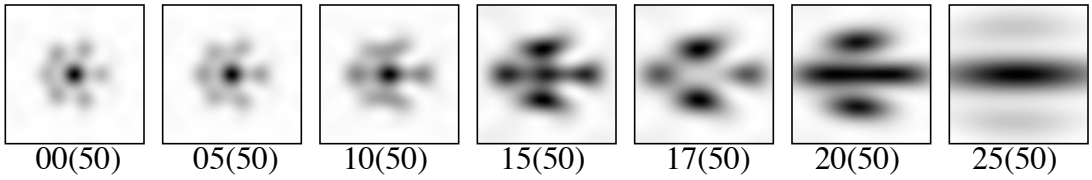
The cross section is plotted at four different points in the far field diffraction pattern. The red curve is along the horizontal axis. The Green is along the vertical axis. The blue is at the center. The magenta curve is the maximum intensity. The polarization is parallel to the plane of incidence (horizontal). At this orientation there is loss of total internal reflection at about ϕ = 17 degrees. There is a discontinuity in the curve at this point. The data used to plot the graph is shown in the table below.

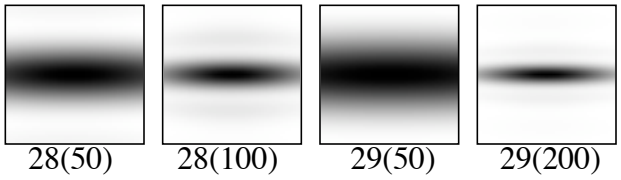
ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	0.0845248	0.0813103	0.2482102	0.2482102
5.0	0.0778593	0.0742156	0.1913347	0.1913347
10.0	0.0626553	0.0505818	0.1097288	0.1097288
15.0	0.0279160	0.0238702	0.0341407	0.0367033
17.0	0.0076725	0.0158627	0.0067445	0.0299777
20.0	0.0035835	0.0017644	0.0035424	0.0036196
25.0	0.0006931	0.0004152	0.0007083	0.0007083
28.0	0.0000733	0.0000590	0.0000737	0.0000737
29.0	0.0000148	0.0000131	0.0000148	0.0000148

The cross section is relative to the cross section at the center of the Airy pattern. Because of polarization effects the peak is reduced by about a factor of 4 in uncoated cube corners.

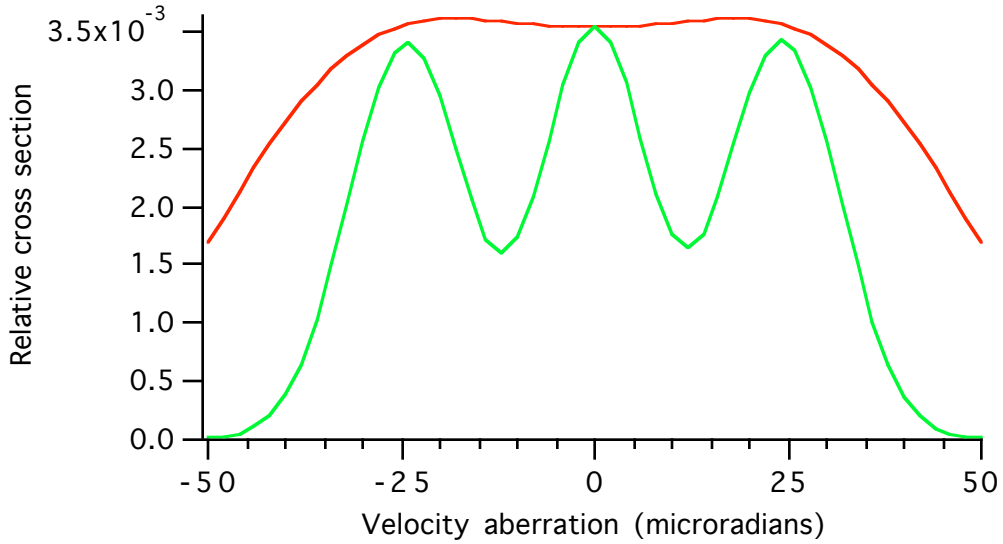
Diffraction patterns at each incidence angle
θ = -30 deg, Horizontal polarization

For small incidence angles, the pattern has a bright lobe at the center with six surrounding lobes. Past the loss of total internal reflection at 17 degrees the pattern has three lobes elongated along the horizontal axis. Because the velocity aberration is low (10 microradians or less), the cross section is higher for horizontal polarization than for vertical polarization.





The incidence angle is shown below each pattern with the scale in microradians in parentheses. For large incidence angles the pattern is too large to be shown on a 50 x 50 microradians plot. Below is a plot of the cross section along the horizontal and vertical axes for ϕ = 20 deg.



Red Horizontal (X) axis
Green Vertical (Y) axis

The following tables show the cross section in million square meters for each point in the far field pattern. The cross section at the center of the Airy pattern is 57.712 million square meters. The maximum cross section at normal incidence is .2482102 x 57.712 = 14.3247 as seen in first row below for a single cube.

θ = -30 deg, Horizontal polarization

Single cube

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	4.8780968	4.6925815	14.3247117	14.3247146
5.0	4.4934174	4.2831321	11.0423118	11.0423118
10.0	3.6159638	2.9191778	6.3326705	6.3326711
15.0	1.6110887	1.3775974	1.9703287	2.1182204
17.0	0.4427955	0.9154684	0.3892387	1.7300753
20.0	0.2068110	0.1018271	0.2044391	0.2088967
25.0	0.0400002	0.0239620	0.0408774	0.0408786
28.0	0.0042303	0.0034050	0.0042534	0.0042551
29.0	0.0008541	0.0007560	0.0008541	0.0008547

100 cubes

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	487.80968	469.25815	1432.47117	1432.47146
5.0	449.34174	428.31321	1104.23118	1104.23118
10.0	361.59638	291.91778	633.26705	633.26711
15.0	161.10887	137.75974	197.03287	211.82204
17.0	44.27955	91.54684	38.92387	173.00753
20.0	20.68110	10.18271	20.44391	20.88967
25.0	4.00002	2.39620	4.08774	4.08786
28.0	0.42303	0.34050	0.42534	0.42551
29.0	0.08541	0.07560	0.08541	0.08547

300 cubes

ϕ	(10,0)	(0,10)	(0,0)	Maximum
0.0	1463.42905	1407.77446	4297.41350	4297.41437
5.0	1348.02521	1284.93963	3312.69353	3312.69353
10.0	1084.78915	875.75333	1899.80116	1899.80134
15.0	483.32661	413.27923	591.09861	635.46611
17.0	132.83864	274.64053	116.77161	519.02259
20.0	62.04331	30.54813	61.33172	62.66902
25.0	12.00006	7.18861	12.26323	12.26357
28.0	1.26909	1.02150	1.27601	1.27653
29.0	0.25624	0.22681	0.25624	0.25641

Cross section in million square meters for one, 100, and 300 cubes.

Horizontal and vertical axes of the diffraction patterns
Orientation θ = -30 deg
Horizontal polarization

The tables below are in units of the cross section computed from equation (1). In these units the cross section at the center of the diffraction pattern of a perfect retroreflector is unity.

θ = -30 deg, Horizontal polarization

ϕ = 0 deg

Microradians	Horizontal	Vertical
-12.0	0.0689586500	0.0614225900
-10.0	0.0844866800	0.0813218800
-8.0	0.1160606700	0.1150779600
-6.0	0.1588981500	0.1587230700
-4.0	0.2029030600	0.2029148800
-2.0	0.2359381000	0.2359563500
0.0	0.2482102500	0.2482102500
2.0	0.2359712600	0.2359562200
4.0	0.2029597200	0.2029139600
6.0	0.1589627200	0.1587201400
8.0	0.1161173800	0.1150714900
10.0	0.0845248500	0.0813102800
12.0	0.0689753400	0.0614045400

ϕ = 5 deg

Microradians	Horizontal	Vertical
-12.0	0.0648564200	0.0610382900
-10.0	0.0778266200	0.0742098900
-8.0	0.1014248000	0.0974317400
-6.0	0.1315793700	0.1279882300
-4.0	0.1614087300	0.1591954000
-2.0	0.1832936700	0.1826278700
0.0	0.1913347000	0.1913347000
2.0	0.1833145800	0.1826320100
4.0	0.1614454800	0.1592027900
6.0	0.1316234600	0.1279972700
8.0	0.1014669200	0.0974402900
10.0	0.0778593500	0.0742156400
12.0	0.0648759900	0.0610390500

ϕ = 10 deg

Microradians	Horizontal	Vertical
-12.0	0.0541233300	0.0471151400
-10.0	0.0626358000	0.0505817800
-8.0	0.0742706300	0.0605219000
-6.0	0.0871025900	0.0756739200
-4.0	0.0987264900	0.0921652700
-2.0	0.1068244700	0.1049277200
0.0	0.1097288100	0.1097288100
2.0	0.1068329400	0.1049277200
4.0	0.0987419800	0.0921652700
6.0	0.0871225300	0.0756739200
8.0	0.0742918700	0.0605219000
10.0	0.0626553000	0.0505817800
12.0	0.0541387000	0.0471151400

θ = -30 deg, Horizontal polarization

ϕ = 15 deg

Microradians	Horizontal	Vertical
-12.0	0.0266099500	0.0258015000
-10.0	0.0279101100	0.0238701800
-8.0	0.0295603500	0.0242764000
-6.0	0.0312785000	0.0267372000
-4.0	0.0327708800	0.0301442500
-2.0	0.0337825400	0.0330225100
0.0	0.0341407400	0.0341407400
2.0	0.0337843800	0.0330225100
4.0	0.0327743700	0.0301442500
6.0	0.0312832800	0.0267372000
8.0	0.0295659600	0.0242764000
10.0	0.0279160200	0.0238701800
12.0	0.0266156200	0.0258015000

ϕ = 17 deg

Microradians	Horizontal	Vertical
-12.0	0.0083476700	0.0198051700
-10.0	0.0076731700	0.0158507200
-8.0	0.0072281800	0.0124048400
-6.0	0.0069629100	0.0097701300
-4.0	0.0068232400	0.0080139800
-2.0	0.0067613300	0.0070471500
0.0	0.0067445000	0.0067445000
2.0	0.0067611800	0.0070504200
4.0	0.0068229500	0.0080202800
6.0	0.0069624700	0.0097789900
8.0	0.0072276200	0.0124156500
10.0	0.0076725000	0.0158627100
12.0	0.0083469000	0.0198175700

ϕ = 20 deg

Microradians	Horizontal	Vertical
-12.0	0.0035966400	0.0015993300
-10.0	0.0035835000	0.0017282600
-8.0	0.0035705100	0.0020787000
-6.0	0.0035590000	0.0025611700
-4.0	0.0035500300	0.0030478100
-2.0	0.0035443500	0.0034074200
0.0	0.0035424100	0.0035424100
2.0	0.0035443500	0.0034166200
4.0	0.0035500300	0.0030656600
6.0	0.0035590000	0.0025866500
8.0	0.0035705100	0.0021103500
10.0	0.0035835000	0.0017643500
12.0	0.0035966400	0.0016379100

θ = -30 deg, Horizontal polarization

ϕ = 25 deg

Microradians	Horizontal	Vertical
-12.0	0.0006864400	0.0003308500
-10.0	0.0006930700	0.0004182000
-8.0	0.0006985300	0.0005066000
-6.0	0.0007028000	0.0005874000
-4.0	0.0007058600	0.0006522200
-2.0	0.0007077100	0.0006940700
0.0	0.0007083200	0.0007083200
2.0	0.0007077100	0.0006933800
4.0	0.0007058600	0.0006508700
6.0	0.0007028000	0.0005854400
8.0	0.0006985300	0.0005040800
10.0	0.0006930700	0.0004151900
12.0	0.0006864400	0.0003274500

ϕ = 28 deg

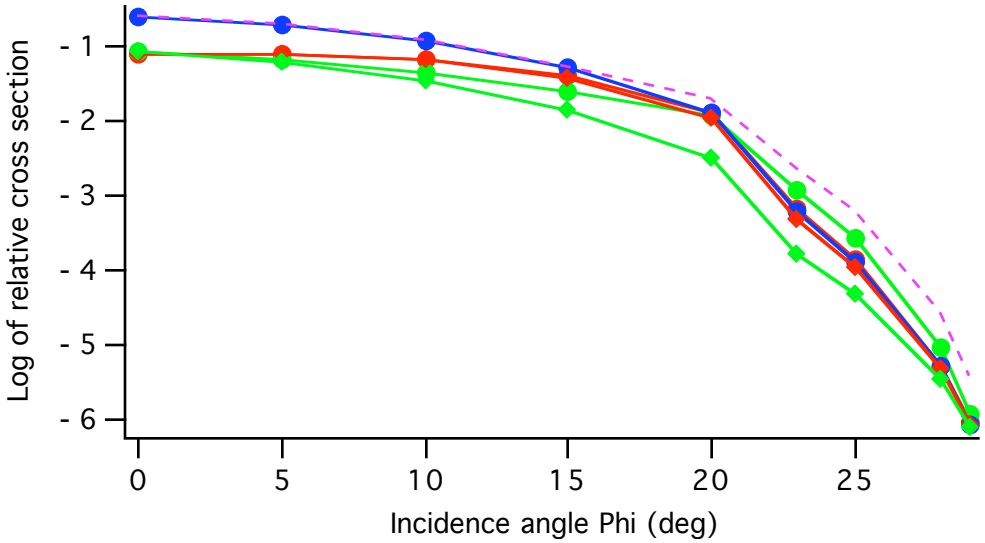
Microradians	Horizontal	Vertical
-12.0	0.0000731100	0.0000535800
-10.0	0.0000733000	0.0000591500
-8.0	0.0000734500	0.0000640800
-6.0	0.0000735700	0.0000681600
-4.0	0.0000736600	0.0000712200
-2.0	0.0000737100	0.0000731000
0.0	0.0000737300	0.0000737300
2.0	0.0000737100	0.0000730700
4.0	0.0000736600	0.0000711600
6.0	0.0000735700	0.0000680800
8.0	0.0000734500	0.0000639700
10.0	0.0000733000	0.0000590100
12.0	0.0000731100	0.0000534200

ϕ = 29 deg

Microradians	Horizontal	Vertical
-12.0	0.0000147600	0.0000124300
-10.0	0.0000147700	0.0000131200
-8.0	0.0000147900	0.0000137100
-6.0	0.0000147900	0.0000141800
-4.0	0.0000148000	0.0000145300
-2.0	0.0000148000	0.0000147400
0.0	0.0000148100	0.0000148100
2.0	0.0000148000	0.0000147300
4.0	0.0000148000	0.0000145200
6.0	0.0000147900	0.0000141700
8.0	0.0000147900	0.0000136900
10.0	0.0000147700	0.0000131000
12.0	0.0000147600	0.0000124100

8. Cross section at θ = 0 deg

Vertical polarization



Red	circle = (10,0), diamond = (-10,0)
Green	circle = (0,10), diamond = (0,-10)
Blue	(0,0)
Magenta	Maximum

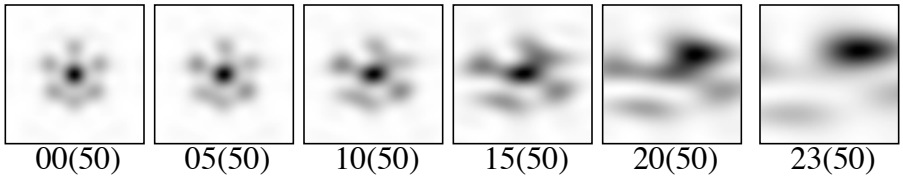
The cross section is plotted at six different points in the far field diffraction pattern. The red curves are points along the horizontal axis. The Green are along the vertical axis. The blue is at the center. The magenta curve is the maximum intensity. The polarization is perpendicular to the plane of incidence (vertical). At this orientation there is loss of total internal reflection at about ϕ = 20 degrees. There is a discontinuity in the curve at this point. The data used to plot the graph is shown in the table below.

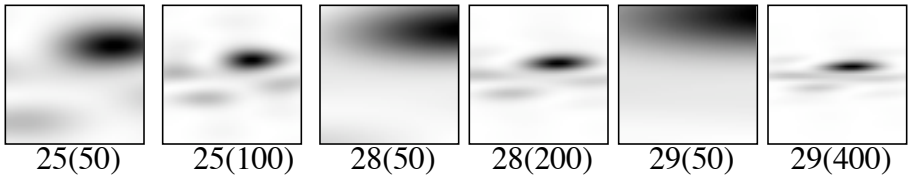
ϕ	(10,0)	(0,10)	(0,0)	(-10,0)	(0,-10)	Maximum
0.0	0.0813103	0.0845248	0.2482102	0.0813219	0.0844867	0.2482102
5.0	0.0772902	0.0673489	0.1938179	0.0766350	0.0638974	0.1938179
10.0	0.0688122	0.0439264	0.1222230	0.0661030	0.0345126	0.1222230
15.0	0.0418371	0.0256163	0.0539718	0.0386546	0.0142296	0.0539718
20.0	0.0131337	0.0118743	0.0132831	0.0110896	0.0031401	0.0197206
23.0	0.0006670	0.0011923	0.0005988	0.0004779	0.0001618	0.0023607
25.0	0.0001372	0.0002722	0.0001248	0.0001069	0.0000464	0.0005999
28.0	0.0000054	0.0000094	0.0000052	0.0000050	0.0000036	0.0000271
29.0	0.0000009	0.0000012	0.0000009	0.0000009	0.0000008	0.0000038

The cross section is relative to the cross section at the center of the Airy pattern. Because of polarization effects the peak is reduced by about a factor of 4 in uncoated cube corners.

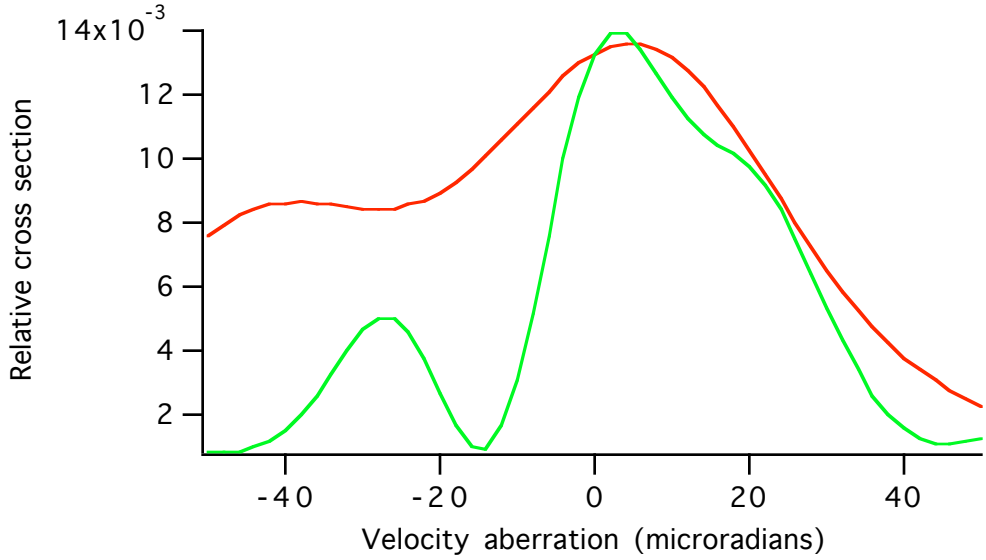
Diffraction patterns at each incidence angle
θ = 0 deg, Vertical polarization

For small incidence angles, the pattern has a bright lobe at the center with six surrounding lobes. Past the loss of total internal reflection at 20 degrees the pattern becomes asymmetrical with a bright lobe above and slightly to the right of center. The intensity of the red points along the horizontal axis do not vary much but the green point above center (dots) is much larger than the green point below center (diamonds).





The incidence angle in degrees is shown below each pattern with the scale in microradians in parenthesis. For large incidence angles the pattern is too large to be shown on a 50 x 50 microradian scale. Below is a plot of the cross section along the horizontal and vertical axes for ϕ = 20 deg. The plots are asymmetrical.



Red Horizontal (X) axis
Green Vertical (Y) axis

The following tables show the cross section in million square meters for each point in the far field pattern. The cross section at the center of the Airy pattern is 57.712 million square meters. The maximum cross section at normal incidence is .2482102 x 57.712 = 14.3247 as seen in first row below for a single cube.

θ = 0 deg, Vertical polarization

Single cube

ϕ	(10,0)	(0,10)	(0,0)	(-10,0)	(0,-10)	Maximum
0.0	4.6925815	4.8780968	14.3247117	4.6932510	4.8758980	14.3247146
5.0	4.4605735	3.8868410	11.1856222	4.4227605	3.6876479	11.1856194
10.0	3.9712910	2.5350812	7.0537360	3.8149376	1.9917918	7.0537372
15.0	2.4145035	1.4783684	3.1148215	2.2308350	0.8212189	3.1148238
20.0	0.7579723	0.6852898	0.7665945	0.6400032	0.1812215	1.1381162
23.0	0.0384939	0.0688100	0.0345580	0.0275806	0.0093378	0.1362425
25.0	0.0079181	0.0157092	0.0072025	0.0061694	0.0026778	0.0346209
28.0	0.0003116	0.0005425	0.0003001	0.0002886	0.0002078	0.0015628
29.0	0.0000519	0.0000693	0.0000519	0.0000519	0.0000462	0.0002210

100 cubes

ϕ	(10,0)	(0,10)	(0,0)	(-10,0)	(0,-10)	Maximum
0.0	469.25815	487.80968	1432.47117	469.32510	487.58980	1432.47146
5.0	446.05735	388.68410	1118.56222	442.27605	368.76479	1118.56194
10.0	397.12910	253.50812	705.37360	381.49376	199.17918	705.37372
15.0	241.45035	147.83684	311.48215	223.08350	82.12189	311.48238
20.0	75.79723	68.52898	76.65945	64.00032	18.12215	113.81162
23.0	3.84939	6.88100	3.45580	2.75806	0.93378	13.62425
25.0	0.79181	1.57092	0.72025	0.61694	0.26778	3.46209
28.0	0.03116	0.05425	0.03001	0.02886	0.02078	0.15628
29.0	0.00519	0.00693	0.00519	0.00519	0.00462	0.02210

300 cubes

ϕ	(10,0)		(0,10)	(0,0)	Maximum	
0.0	1407.77446	1463.42905	4297.41350	1407.97530	1462.76940	4297.41437
5.0	1338.17204	1166.05229	3355.68667	1326.82816	1106.29438	3355.68581
10.0	1191.38729	760.52436	2116.12081	1144.48127	597.53754	2116.12116
15.0	724.35105	443.51051	934.44646	669.25050	246.36568	934.44715
20.0	227.39170	205.58695	229.97835	192.00096	54.36645	341.43486
23.0	11.54817	20.64301	10.36739	8.27417	2.80134	40.87275
25.0	2.37543	4.71276	2.16074	1.85082	0.80335	10.38626
28.0	0.09349	0.16275	0.09003	0.08657	0.06233	0.46885
29.0	0.01558	0.02078	0.01558	0.01558	0.01385	0.06631

Cross section in million square meters for one, 100, and 300 cubes.

Horizontal and vertical axes of the diffraction patterns
Orientation θ = +00 deg
Vertical polarization

The tables below are in units of the cross section computed from equation (1). In these units the cross section at the center of the diffraction pattern of a perfect retroreflector is unity.

θ = 0 deg, Vertical polarization

ϕ = 0 deg

Microradians	Horizontal	Vertical
-12.0	0.0614225900	0.0689586500
-10.0	0.0813218800	0.0844866800
-8.0	0.1150779600	0.1160606700
-6.0	0.1587230700	0.1588981500
-4.0	0.2029148800	0.2029030600
-2.0	0.2359563500	0.2359381000
0.0	0.2482102500	0.2482102500
2.0	0.2359562200	0.2359712600
4.0	0.2029139600	0.2029597200
6.0	0.1587201400	0.1589627200
8.0	0.1150714900	0.1161173800
10.0	0.0813102800	0.0845248500
12.0	0.0614045400	0.0689753400

ϕ = 5 deg

Microradians	Horizontal	Vertical
-12.0	0.0581705100	0.0505897900
-10.0	0.0766350100	0.0638973800
-8.0	0.1033892200	0.0895597600
-6.0	0.1347052600	0.1235492100
-4.0	0.1644540300	0.1580477700
-2.0	0.1858917400	0.1839049200
0.0	0.1938178500	0.1938178500
2.0	0.1862939400	0.1849756200
4.0	0.1652638200	0.1600843400
6.0	0.1358427500	0.1263501500
8.0	0.1045668900	0.0928466000
10.0	0.0772901700	0.0673488500
12.0	0.0575200800	0.0538887500

ϕ = 10 deg

Microradians	Horizontal	Vertical
-12.0	0.0532317000	0.0255019500
-10.0	0.0661030100	0.0345125900
-8.0	0.0810058800	0.0517442900
-6.0	0.0961210200	0.0744085000
-4.0	0.1092959700	0.0974049400
-2.0	0.1184917100	0.1148870400
0.0	0.1222230200	0.1222230200
2.0	0.1198797400	0.1175976900
4.0	0.1118469400	0.1026218200
6.0	0.0993958100	0.0817120700
8.0	0.0843786700	0.0605023200
10.0	0.0688121700	0.0439264500
12.0	0.0544588300	0.0347111900

θ = 0 deg, Vertical polarization

ϕ = 15 deg

Microradians	Horizontal	Vertical
-12.0	0.0343946800	0.0093470900
-10.0	0.0386546000	0.0142295600
-8.0	0.0429667100	0.0223936300
-6.0	0.0469817300	0.0325153800
-4.0	0.0503482900	0.0425344200
-2.0	0.0527556600	0.0502461900
0.0	0.0539718400	0.0539718400
2.0	0.0538712300	0.0530670600
4.0	0.0524482500	0.0480887200
6.0	0.0498149600	0.0405668600
8.0	0.0461836000	0.0324774300
10.0	0.0418370900	0.0256162800
12.0	0.0370922100	0.0210973000

ϕ = 20 deg

Microradians	Horizontal	Vertical
-12.0	0.0105747000	0.0016697300
-10.0	0.0110895600	0.0031400700
-8.0	0.0116098200	0.0052225500
-6.0	0.0121122900	0.0076209600
-4.0	0.0125733900	0.0099738000
-2.0	0.0129704900	0.0119411300
0.0	0.0132831100	0.0132831100
2.0	0.0134940400	0.0139083800
4.0	0.0135902300	0.0138795300
6.0	0.0135634500	0.0133771500
8.0	0.0134105900	0.0126359300
10.0	0.0131337200	0.0118743000
12.0	0.0127398300	0.0112387200

ϕ = 23 deg

Microradians	Horizontal	Vertical
-12.0	0.0004553600	0.0001222800
-10.0	0.0004778600	0.0001617700
-8.0	0.0005020200	0.0002225500
-6.0	0.0005270200	0.0003003400
-4.0	0.0005520000	0.0003909500
-2.0	0.0005761800	0.0004911000
0.0	0.0005987800	0.0005987800
2.0	0.0006191100	0.0007129100
4.0	0.0006365500	0.0008324000
6.0	0.0006505900	0.0009551900
8.0	0.0006608400	0.0010772600
10.0	0.0006670300	0.0011923300
12.0	0.0006690100	0.0012921600

θ = 0 deg, Vertical polarization

ϕ = 25 deg

Microradians	Horizontal	Vertical
-12.0	0.0001033900	0.0000385700
-10.0	0.0001069400	0.0000464300
-8.0	0.0001106000	0.0000569800
-6.0	0.0001142800	0.0000700200
-4.0	0.0001179200	0.0000855500
-2.0	0.0001214600	0.0001037200
0.0	0.0001248300	0.0001248300
2.0	0.0001279600	0.0001490800
4.0	0.0001308100	0.0001765300
6.0	0.0001333300	0.0002068300
8.0	0.0001354700	0.0002391700
10.0	0.0001372000	0.0002721900
12.0	0.0001384900	0.0003040800

ϕ = 28 deg

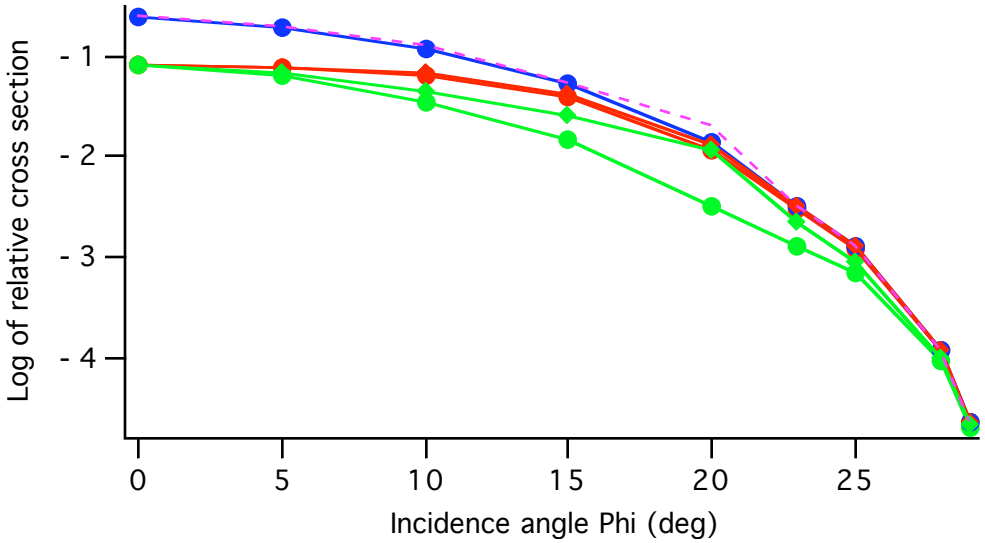
Microradians	Horizontal	Vertical
-12.0	0.0000050200	0.0000034100
-10.0	0.0000050500	0.0000035800
-8.0	0.0000050900	0.0000037800
-6.0	0.0000051200	0.0000040300
-4.0	0.0000051500	0.0000043400
-2.0	0.0000051900	0.0000047300
0.0	0.0000052200	0.0000052200
2.0	0.0000052500	0.0000058200
4.0	0.0000052800	0.0000065400
6.0	0.0000053000	0.0000073800
8.0	0.0000053300	0.0000083600
10.0	0.0000053600	0.0000094400
12.0	0.0000053800	0.0000106200

ϕ = 29 deg

Microradians	Horizontal	Vertical
-12.0	0.0000008900	0.0000008000
-10.0	0.0000009000	0.0000008100
-8.0	0.0000009000	0.0000008200
-6.0	0.0000009000	0.0000008300
-4.0	0.0000009000	0.0000008400
-2.0	0.0000009000	0.0000008700
0.0	0.0000009000	0.0000009000
2.0	0.0000009000	0.0000009400
4.0	0.0000009000	0.0000010000
6.0	0.0000009000	0.0000010600
8.0	0.0000009000	0.0000011400
10.0	0.0000009000	0.0000012300
12.0	0.0000009000	0.0000013400

Cross section for orientation θ = +00 deg

Horizontal polarization



Red	circle = (10,0), diamond = (-10,0)
Green	circle = (0,10), diamond = (0,-10)
Blue	(0,0)
Magenta	Maximum

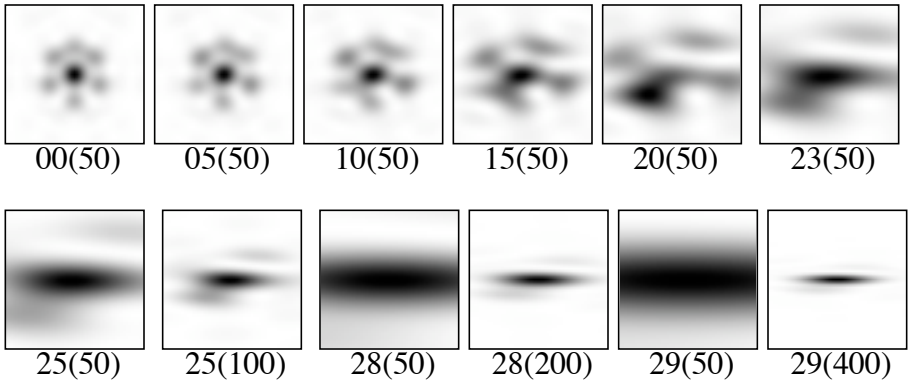
The cross section is plotted at six different points in the far field diffraction pattern. The red curves are points along the horizontal axis. The Green are along the vertical axis. The blue is at the center. The magenta curve is the maximum intensity. The polarization is parallel to the plane of incidence (horizontal). At this orientation there is loss of total internal reflection at about ϕ = 20 degrees. The behavior past this angle shows a small discontinuity but it is not as pronounced as for vertical polarization. The data used to plot the graph is shown in the table below.

ϕ	(10,0)	(0,10)	(0,0)	(-10,0)	(0,-10)	Maximum
0.0	0.0813219	0.0844867	0.2482102	0.0813103	0.0845248	0.2482102
5.0	0.0767511	0.0639944	0.1941105	0.0774062	0.0674458	0.1941105
10.0	0.0665093	0.0347233	0.1229556	0.0692185	0.0441372	0.1229556
15.0	0.0392006	0.0144291	0.0546970	0.0423831	0.0258158	0.0546970
20.0	0.0113698	0.0032208	0.0135863	0.0134139	0.0119551	0.0197402
23.0	0.0029660	0.0012547	0.0032248	0.0031554	0.0022860	0.0032808
25.0	0.0012189	0.0006806	0.0012724	0.0012492	0.0009063	0.0012726
28.0	0.0001144	0.0000898	0.0001152	0.0001147	0.0000956	0.0001152
29.0	0.0000227	0.0000199	0.0000228	0.0000227	0.0000204	0.0000228

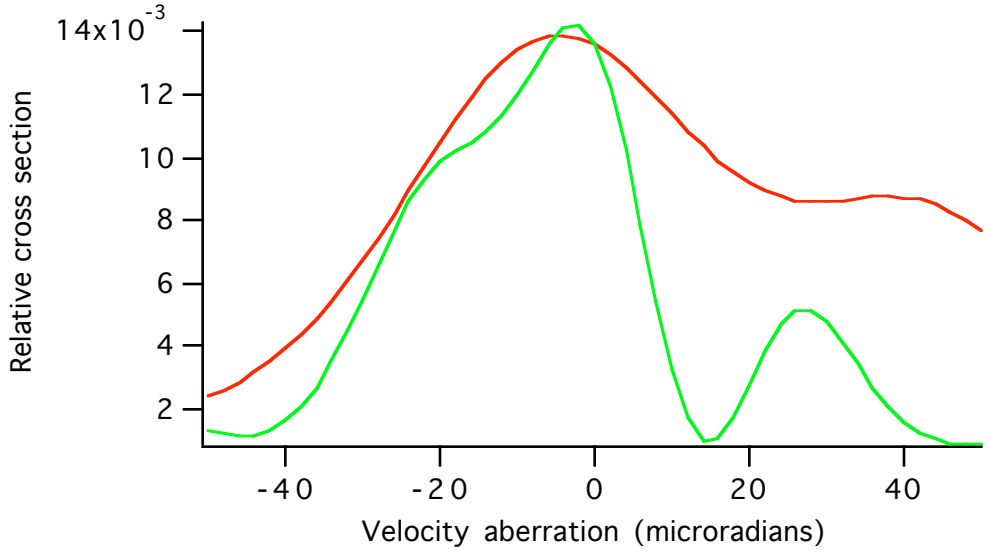
The cross section is relative to the cross section at the center of the Airy pattern. Because of polarization effects the peak is reduced by about a factor of 4 in uncoated cube corners.

Diffraction patterns at each incidence angle
θ = 0 deg, Horizontal polarization

For small incidence angles, the pattern has a bright lobe at the center with six surrounding lobes. Around the loss of total internal reflection at 20 degrees the pattern becomes irregular. There is a bright lobe in the lower left part of the pattern. For this reason the green curve is higher for negative values (the diamonds) than for positive values (dots). From 25 degrees on the pattern is elongated along the horizontal axis. This makes the cross section higher than for vertical polarization where there is no central lobe.



The incidence angle is shown below each pattern with the scale in microradians in parentheses. For large incidence angles the pattern is too large to be shown on a 50 x 50 microradians plot. Below is a plot of the cross section along the horizontal and vertical axes for ϕ = 20 deg. The plot is very asymmetrical.



Red Horizontal (X) axis
Green Vertical (Y) axis

The following tables show the cross section in million square meters for each point in the far field pattern. The cross section at the center of the Airy pattern is 57.712 million square meters. The maximum cross section at normal incidence is .2482102 x 57.712 = 14.3247 as seen in first row below for a single cube.

θ = 0 deg, Horizontal polarization

Single cube

ϕ	(10,0)	(0,10)	(0,0)	(-10,0)	(0,-10)	Maximum
0.0	4.6932510	4.8758980	14.3247117	4.6925815	4.8780968	14.3247146
5.0	4.4294609	3.6932460	11.2025088	4.4672681	3.8924333	11.2025071
10.0	3.8383860	2.0039517	7.0960159	3.9947394	2.5472469	7.0960147
15.0	2.2623458	0.8327325	3.1566743	2.4460143	1.4898819	3.1566754
20.0	0.6561741	0.1858789	0.7840928	0.7741432	0.6899530	1.1392445
23.0	0.1711738	0.0724113	0.1861097	0.1821045	0.1319297	0.1893410
25.0	0.0703452	0.0392788	0.0734328	0.0720939	0.0523044	0.0734466
28.0	0.0066023	0.0051825	0.0066484	0.0066196	0.0055173	0.0066513
29.0	0.0013101	0.0011485	0.0013158	0.0013101	0.0011773	0.0013147

100 cubes

ϕ	(10,0)	(0,10)	(0,0)	(-10,0)	(0,-10)	Maximum
0.0	469.32510	487.58980	1432.47117	469.25815	487.80968	1432.47146
5.0	442.94609	369.32460	1120.25088	446.72681	389.24333	1120.25071
10.0	383.83860	200.39517	709.60159	399.47394	254.72469	709.60147
15.0	226.23458	83.27325	315.66743	244.60143	148.98819	315.66754
20.0	65.61741	18.58789	78.40928	77.41432	68.99530	113.92445
23.0	17.11738	7.24113	18.61097	18.21045	13.19297	18.93410
25.0	7.03452	3.92788	7.34328	7.20939	5.23044	7.34466
28.0	0.66023	0.51825	0.66484	0.66196	0.55173	0.66513
29.0	0.13101	0.11485	0.13158	0.13101	0.11773	0.13147

300 cubes

ϕ	(10,0)	(0,10)	(0,0)	(-10,0)	(0,-10)	Maximum
0.0	1407.97530	1462.76940	4297.41350	1407.77446	1463.42905	4297.41437
5.0	1328.83827	1107.97380	3360.75264	1340.18042	1167.72998	3360.75212
10.0	1151.51579	601.18552	2128.80476	1198.42181	764.17407	2128.80442
15.0	678.70373	249.81975	947.00228	733.80428	446.96458	947.00263
20.0	196.85223	55.76366	235.22784	232.24297	206.98589	341.77334
23.0	51.35215	21.72338	55.83292	54.63135	39.57890	56.80230
25.0	21.10355	11.78364	22.02983	21.62816	15.69132	22.03399
28.0	1.98068	1.55476	1.99453	1.98587	1.65518	1.99539
29.0	0.39302	0.34454	0.39475	0.39302	0.35320	0.39440

Cross section in million square meters for one, 100, and 300 cubes.

Horizontal and vertical axes of the diffraction patterns
Orientation θ = +00 deg
Horizontal polarization

The tables below are in units of the cross section computed from equation (1). In these units the cross section at the center of the diffraction pattern of a perfect retroreflector is unity.

θ = 0 deg, Horizontal polarization

ϕ = 0 deg

Microradians	Horizontal	Vertical
-12.0	0.0614045400	0.0689753400
-10.0	0.0813102800	0.0845248500
-8.0	0.1150714900	0.1161173800
-6.0	0.1587201400	0.1589627200
-4.0	0.2029139600	0.2029597200
-2.0	0.2359562200	0.2359712600
0.0	0.2482102500	0.2482102500
2.0	0.2359563500	0.2359381000
4.0	0.2029148800	0.2029030600
6.0	0.1587230700	0.1588981500
8.0	0.1150779600	0.1160606700
10.0	0.0813218800	0.0844866800
12.0	0.0614225900	0.0689586500

ϕ = 5 deg

Microradians	Horizontal	Vertical
-12.0	0.0576060400	0.0539655300
-10.0	0.0774062100	0.0674458200
-8.0	0.1047242700	0.0929824900
-6.0	0.1360474500	0.1265375400
-4.0	0.1655130200	0.1603238500
-2.0	0.1865749900	0.1852539100
0.0	0.1941104700	0.1941104700
2.0	0.1861727900	0.1841832100
4.0	0.1647032300	0.1582872800
6.0	0.1349099700	0.1237366000
8.0	0.1035466000	0.0896956600
10.0	0.0767510500	0.0639943500
12.0	0.0582564600	0.0506665800

ϕ = 10 deg

Microradians	Horizontal	Vertical
-12.0	0.0547805000	0.0348667600
-10.0	0.0692184700	0.0441372000
-8.0	0.0848773100	0.0608183400
-6.0	0.0999843200	0.0821659100
-4.0	0.1125111300	0.1032140800
-2.0	0.1205945100	0.1182924500
0.0	0.1229555800	0.1229555800
2.0	0.1192064800	0.1155818000
4.0	0.1099601500	0.0979972000
6.0	0.0967095300	0.0748623400
8.0	0.0815045200	0.0520603200
10.0	0.0665093000	0.0347233400
12.0	0.0535533600	0.0256575100

θ = 0 deg, Horizontal polarization

ϕ = 15 deg

Microradians	Horizontal	Vertical
-12.0	0.0375768900	0.0212279000
-10.0	0.0423830700	0.0258157900
-8.0	0.0467871500	0.0327914000
-6.0	0.0504684700	0.0410213400
-4.0	0.0531404900	0.0486792400
-2.0	0.0545880100	0.0537562200
0.0	0.0546970200	0.0546970200
2.0	0.0534724500	0.0509353600
4.0	0.0510405300	0.0431249400
6.0	0.0476352400	0.0329698600
8.0	0.0435702500	0.0227076000
10.0	0.0392005800	0.0144290700
12.0	0.0348793600	0.0094777000

ϕ = 20 deg

Microradians	Horizontal	Vertical
-12.0	0.0130106300	0.0112810600
-10.0	0.0134139300	0.0119550600
-8.0	0.0136988200	0.0127707500
-6.0	0.0138581200	0.0135724700
-4.0	0.0138896200	0.0141301400
-2.0	0.0137963000	0.0141977000
0.0	0.0135863400	0.0135863400
2.0	0.0132727600	0.0122304500
4.0	0.0128727800	0.0102244100
6.0	0.0124069600	0.0078162700
8.0	0.0118980500	0.0053573700
10.0	0.0113697700	0.0032208200
12.0	0.0108455000	0.0017120800

ϕ = 23 deg

Microradians	Horizontal	Vertical
-12.0	0.0030984800	0.0019683200
-10.0	0.0031553700	0.0022859800
-8.0	0.0031981200	0.0026168100
-6.0	0.0032264300	0.0029190100
-4.0	0.0032401900	0.0031466900
-2.0	0.0032395300	0.0032582800
0.0	0.0032248100	0.0032248100
2.0	0.0031965700	0.0030363000
4.0	0.0031555800	0.0027048900
6.0	0.0031027500	0.0022636700
8.0	0.0030391700	0.0017614900
10.0	0.0029660300	0.0012547100
12.0	0.0028846300	0.0007976600

θ = 0 deg, Horizontal polarization

ϕ = 25 deg

Microradians	Horizontal	Vertical
-12.0	0.0012351000	0.0007769100
-10.0	0.0012491900	0.0009063200
-8.0	0.0012601700	0.0010323600
-6.0	0.0012680000	0.0011429200
-4.0	0.0012726400	0.0012260400
-2.0	0.0012741100	0.0012715100
0.0	0.0012724100	0.0012724100
2.0	0.0012676000	0.0012261700
4.0	0.0012597500	0.0011350900
6.0	0.0012489400	0.0010061500
8.0	0.0012352800	0.0008502300
10.0	0.0012189200	0.0006806400
12.0	0.0011999900	0.0005114800

ϕ = 28 deg

Microradians	Horizontal	Vertical
-12.0	0.0001144000	0.0000877200
-10.0	0.0001146900	0.0000956400
-8.0	0.0001149100	0.0001026000
-6.0	0.0001150800	0.0001083000
-4.0	0.0001152000	0.0001124400
-2.0	0.0001152500	0.0001148000
0.0	0.0001152500	0.0001152500
2.0	0.0001151900	0.0001137200
4.0	0.0001150700	0.0001102400
6.0	0.0001149000	0.0001049400
8.0	0.0001146700	0.0000980200
10.0	0.0001143800	0.0000897600
12.0	0.0001140400	0.0000804900

ϕ = 29 deg

Microradians	Horizontal	Vertical
-12.0	0.0000227100	0.0000193500
-10.0	0.0000227300	0.0000203600
-8.0	0.0000227500	0.0000212200
-6.0	0.0000227600	0.0000219200
-4.0	0.0000227700	0.0000224100
-2.0	0.0000227800	0.0000227000
0.0	0.0000227800	0.0000227800
2.0	0.0000227800	0.0000226300
4.0	0.0000227700	0.0000222600
6.0	0.0000227600	0.0000216800
8.0	0.0000227400	0.0000209000
10.0	0.0000227300	0.0000199300
12.0	0.0000227000	0.0000188100

9. Conclusions and recommendations.

The analyses in this report show that the there are large variations in the cross section of the APOLLO Lunar retroreflectors for various incidence angles and polarizations. It should be possible to limit laser ranging to situations where the cross section is at a safe level. The cross section can vary very rapidly with incidence angle. This is primarily a result of the fact that the retroreflectors are recessed. This reduces the cutoff angle to a little over 30 degrees. If the retroreflectors were not recessed the loss of active reflecting area would occur over an interval of about 57 degrees instead of 30 degrees so the rate of change of cross section would be only about half as great. There is also a fairly rapid change in cross section when total internal reflection is lost. However, the change does not occur in a discontinuous way. The ordinary reflection coefficient is still high just after loss of total internal reflection.

The tables of cross section in square meters show that the cross section with 100 retroreflectors has a maximum cross section of about a half billion square meters. The 300 cube array has a maximum cross section of about 1.4 billion square meters. Between about 25 and 28 degrees incidence angle the cross section drops to a level comparable to most low earth satellites depending on whether there is loss of total internal reflection.

In order to make use of the information in this report it will be necessary to determine the orientation of the arrays on the Lunar surface so that the incidence angles θ and ϕ of the laser beam on the array can be determined. Once the incidence angles are known the cross section can be estimated from the tables in this report.

10. References

1. Method of Calculating Retroreflector -Array Transfer Functions, David A. Arnold, Smithsonian Astrophysical Observatory Special Report 382.