Can mathematics be heretical?

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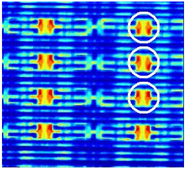
Heresy

1. Who we are
2. Superfluid quantum gravity?
3. Bouncing droplet experiments
4. Three dimensions
5. Updating Maxwell's magnetic line of force/molecular vortex
6. Consequences for Faraday's model of light
7. Conclusion – a mathematical heresy

Ross

Pure maths $\rightarrow$ cryptography $\rightarrow$ hardware reverse engineering



Cofounder of semi-invasive semiconductor testing

Apparently respectable (Professor, FRS, FREng, FInstP...)

Warning! Today's material is heretical. Do not use in Tripos!

Robert

Experimental physics

Invented the squid gyro ('Quantum navigation' now a £20m hub!)

Then research fellow at Trinity

Did a startup, which became a software company

Beginnings

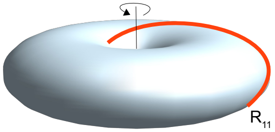
2011: Neutrinos travel faster than light! (since retracted, but ...)

So where on earth does the Lorentz contraction come from?

Could it be some kind of fluid?

But how could you get a negative dielectric coefficient?

Breakthrough?



Robert: a solution to Euler's equation that's also a solution to Dirac's!

Recall: For a lossless compressible fluid

$$\mathbf{F} = -\nabla P = \rho \frac{D\mathbf{u}}{Dt} = \rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\,\mathbf{u}\right)$$

This is Euler's equation, which describes a superfluid like ^{4}He...

Dirac's equation

Recall: For an electron

$$i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t} = (\beta mc^2 + c(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3))\psi(\mathbf{x}, t)$$

where

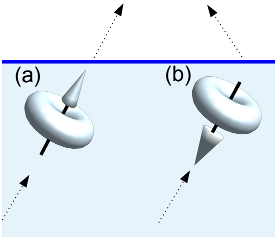
$$\alpha_i^2 = \beta^2 + I_4, \alpha_i \alpha_j + \alpha_j \alpha_k = 0, \alpha_i \beta + \beta \alpha_i = 0$$

Flux rings with a single twist obey this, so behave like electrons and positrons!

(Brady, 'The irrotational motion of a compressible inviscid fluid', arxiv 1301.7450)

Is this observable?

Rotons



Rotons are believed to be vortices in superfluid ^{4}He

You can create them by pulling an ion through the liquid at 1mm/sec

But at 40m/sec (1/6 c) you create pairs of R+ and R- rotons – which knock helium atoms from the surface in different directions

Hypothesis: the twisted flux ring model explains R+ / R- rotons

Might flux rings model the electron?

'Superfluid quantum gravity' theories: maybe the quantum vacuum is like a superfluid

'Analogue gravity': fluids have symmetries of general relativity as well as special relativity (acoustic black holes in BECs)

See Volovik 'The Universe in a Helium Droplet'

Assumed the vacuum has to be like ^{3}He as he couldn't think of a quasiparticle model for the fermion

Now we have one, maybe the quantum vacuum is like ^{4}He?

Denunciation

If quantum mechanics has an analogue basis, maybe that's why quantum computers have been stuck at 2-3 qubits for two decades!

Scott Aaronson's wager

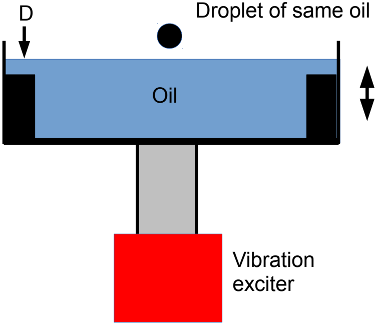
Tide of angry denunciation!

Lessons: (1) most QC people don't know much about computation, or about physics either for that matter

Lessons: (2) most physicists won't contemplate a classical (or even comprehensible) basis for QM

Heresy

Modern design of the apparatus



Air can't squeeze out from underneath the droplet quickly enough

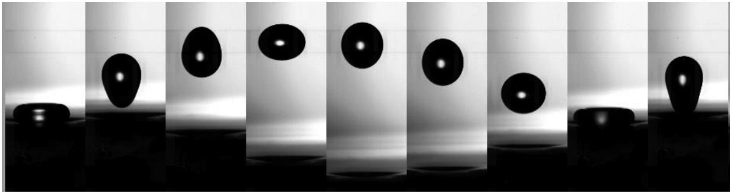
- lubricates horizontal motion

Shallow region *D* is a recent innovation to absorb energy

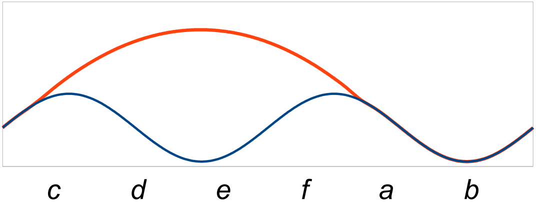
- Viscosity relatively unimportant for the phenomena of interest

The bouncing motion

Simple bouncing



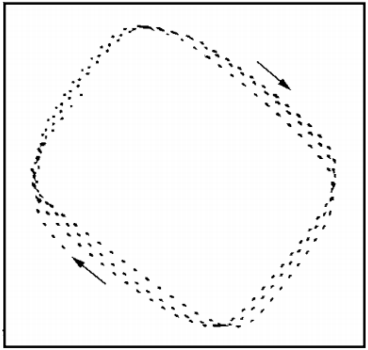
Most phenomena of interest at double period

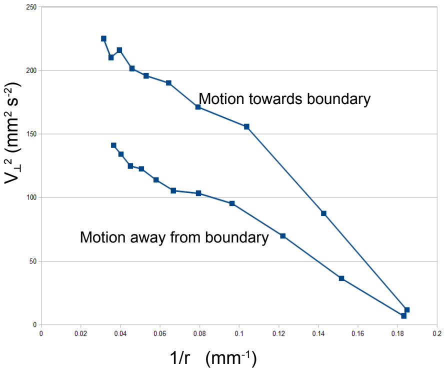


Numerical simulation at $a/g = 3.5\cos(\omega t)$

Deflection from boundary of dish

We blew up a stroboscopic photo (Protière 2006) and measured it

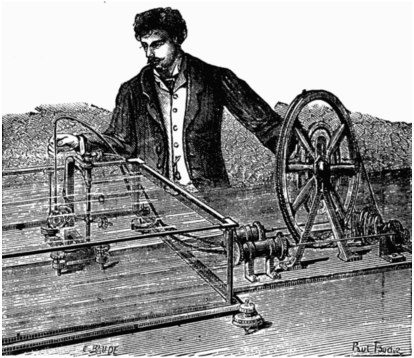




Force is inverse square near the boundary, $\frac{1}{2}mv^2 = \frac{1}{2}mv_o^2 - K/r$
Angle of incidence $\neq$ angle of reflection

Inverse square force

Carl Bjerknes predicted the inverse square force in 1875 and demonstrated it experimentally in 1880.



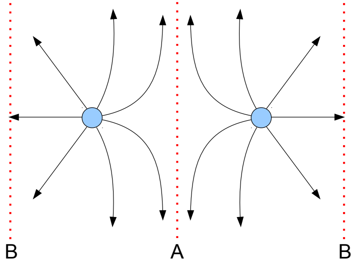
Pistons create pressure waves making bladders pulsate

Measured an inverse square force

- In-phase pulsations attract
- Antiphase pulsations repel

Reason: force $V\nabla P$ has a $\cos^2 \omega t$ term which is always positive

Flow diagram



In-phase pulsations

- Greater average flow speed near *A*
- Reduced Bernoulli pressure, so force of attraction

Droplets repelled from boundary because image droplet is antiphase.

Application: degassing oils

The secondary Bjerknes force is used to degas oils via ultrasonic vibration

Bubbles pulsate in phase, attract one another and merge

Magnitude of the secondary Bjerknes force

Our calculation for the resonant case where maximum speed $\sim c$

Secondary Bjerknes force

$$F = \alpha \frac{\text{ƀ} c}{r^2}$$

$$\alpha \sim 1$$

$$\text{ƀ} = \frac{mc^2}{\omega}$$

Compare force between electrons

$$F = \alpha \frac{\hbar c}{r^2}$$

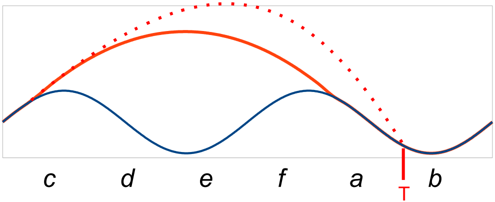
$$\alpha \approx \frac{1}{137.036}$$

$$\hbar = \frac{mc^2}{\omega}$$

The fine structure constant of the secondary Bjerknes force is two orders of magnitude larger than for an electron

ƀ is an analogue of Planck's reduced constant

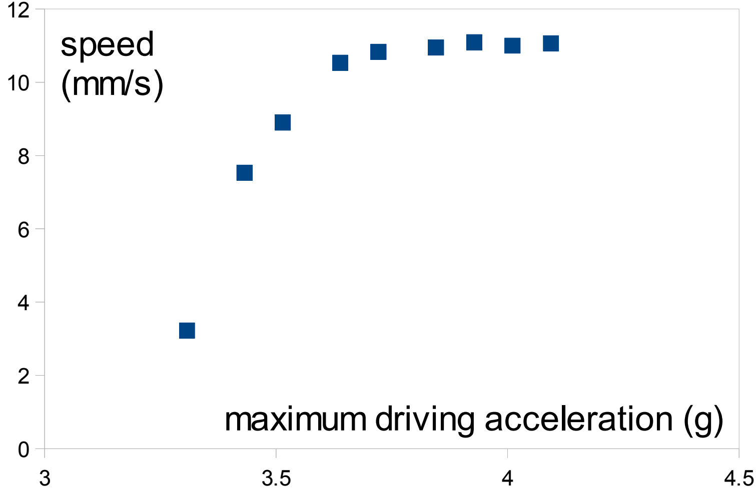
How to make a droplet go faster



Increased forcing acceleration gives greater amplitude (dotted)

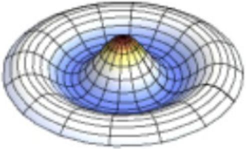
- The droplet lands later in the cycle
- The walker velocity increases

Walker speed



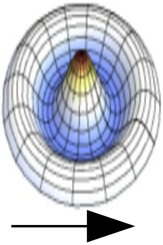
Two ways to calculate the field of a walker

Conventional approach



Each bounce excites a standing Bessel function solution to the wave equation, which decays slowly due to absorption at the boundary and band gap effects. Simulate in a computer.

Symmetry approach

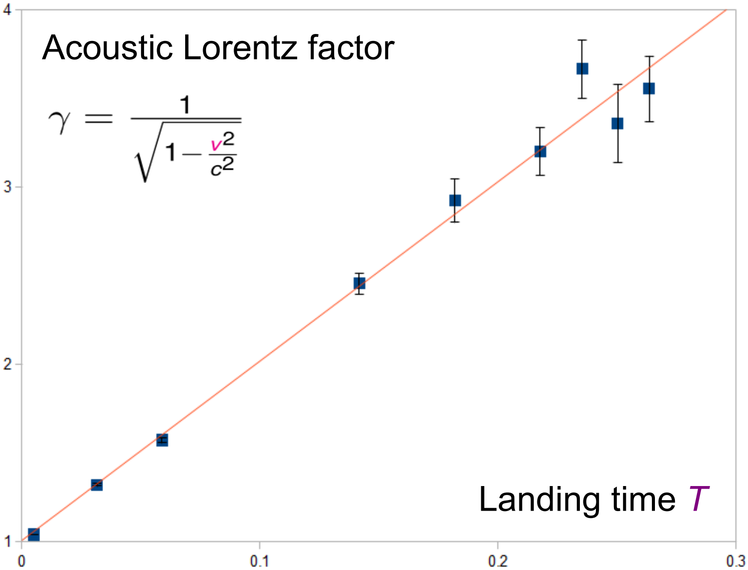


Bessel function $f(x, y, t)$ obeys wave equation. The wave equation is symmetric under Lorentz transformation, so that $f(x', y', t')$ is another solution, where

$$
\begin{aligned}
x' &= \gamma(x - vt) \\
t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\
\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}
\end{aligned}
$$

A second-order scale symmetry is also involved. Decay is slow because of parametric effects.

We re-plot the original (2005) experimental results



Prediction

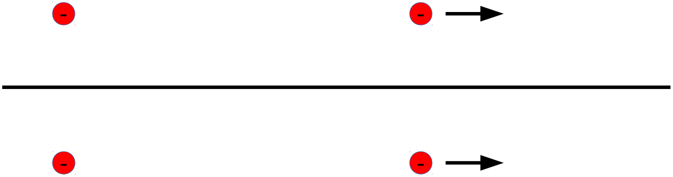
The wave equation is symmetric under Lorentz transformation

The experimental measurements suggest the wave field has the same symmetry

So we predict the inverse square secondary Bjerknes force must also be symmetric under acoustic Lorentz transformation

Borrow calculation from electromagnetism

“Inverse square force + Lorentz symmetry = Maxwell’s equations”



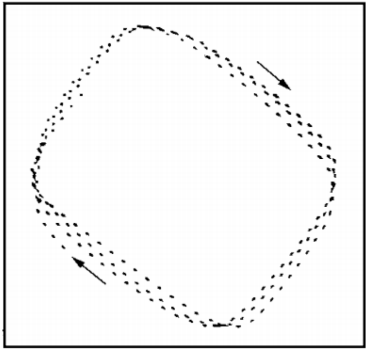
Coulomb force repels

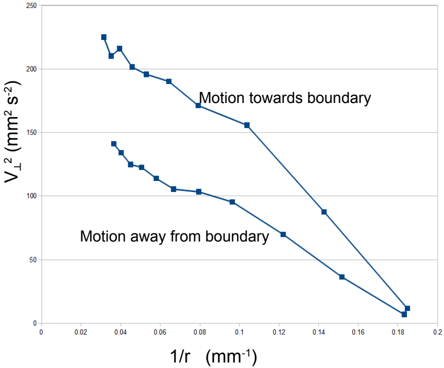
Coulomb force repels
Magnetic force attracts

In this geometry, magnetic force = $\frac{v^2}{c^2} \times$ Coulomb force

So total force reduced by factor $1 - \frac{v^2}{c^2}$

Experimental test

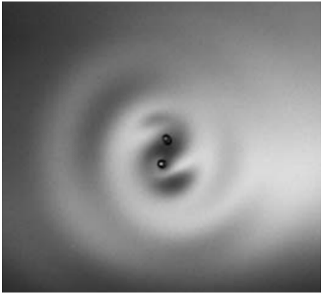




Droplet moves faster parallel to the boundary after reflection
And we see a reduced force corresponding to $v \sim 0.5c$
Consistent with our prediction of an analogue of the magnetic force

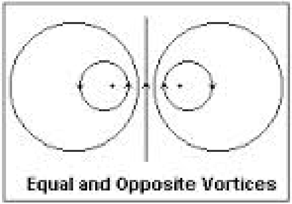
Prediction for a rotating droplet pair

Rotating droplet pair



Interaction with image in the boundary

- Static forces cancel (droplets are antiphase)
- Magnetic-like attraction remains
- Predict fine structure constant $\sim$ 1/20
- Couder observed droplet pair 'hopscotch'

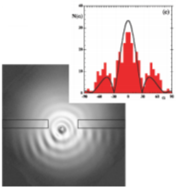


Visualising the mechanism

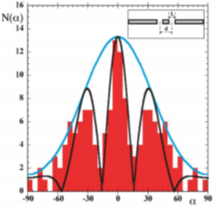
- Flow field – Bessel function J_1
- Rotates around the centre
- Attracted to image in boundary, like two vortices

Further experimental results

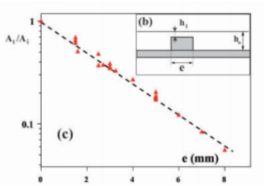
Single slit



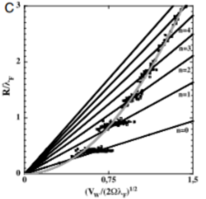
Two-slit



Tunnelling



Quantised orbits



Y Couder, E Fort ‘Single-Particle Diffraction and Interference at a Macroscopic Scale’ PRL 97 154101 (2006)
A Eddi, E Fort, F Moisi, Y Couder ‘Unpredictable tunneling of a classical wave-particle association’ PRL 102, 240401 (2009)
E Fort et al ‘Path-memory induced quantization of classical orbits’ PNAS 107 41 17515-17520 (2010)

Where do the quantum phenomena come from?

Factorise the field of a droplet

Stationary droplet

$$h = \psi \chi$$
$$\psi = R \cos(-\omega_o t)$$
$$\chi = J_0\left(\frac{\omega_o r}{c}\right)$$

Lorentz transform

$$\psi = R \cos(-\omega_o t') = R \cos(kx - \omega t)$$

where

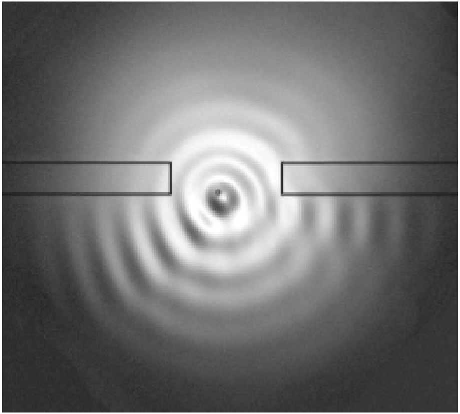
$$k = \frac{\gamma \omega_o}{c^2} v_x$$
$$\omega = \gamma \omega_o$$

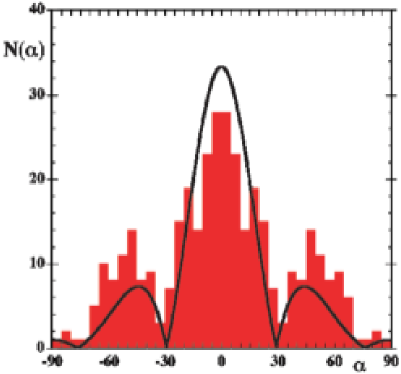
Wavelength (de Broglie!)

$$\lambda = \frac{2\pi}{k} = \frac{b}{p}$$
$$b = 2\pi \frac{mc^2}{\omega}$$

b is the same analogue of the Planck constant which we saw in the inverse square force

Single-slit diffraction





ψ modulates the amplitude of the wave field
Just like the modulation of a carrier wave
Wavelength visible in photograph, matches diffraction pattern

Klein-Gordon and Schödinger equations

$\psi = R\cos(\omega_o t)$ obeys

$$\frac{\partial^2\psi}{\partial t^2} = -\omega_o^2\psi$$

But the motion has Lorentz symmetry
so we need a Lorentz covariant equation, which is

$$\frac{\partial^2\psi}{\partial t^2} - c^2\nabla^2\psi = \omega_o^2\psi$$

the Klein-Gordon equation

the Schrödinger equation is a low-velocity approximation to this

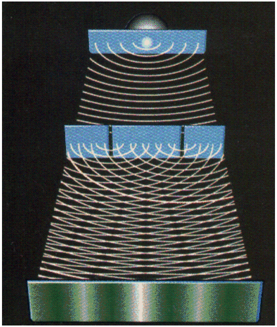
So far so good. But can this work in three dimensions?

Heresy

Euler’s model of light

1746 - Euler

- Light is acoustic waves in a frictionless compressible fluid
- Explains refraction, diffraction, interference

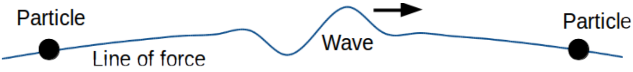


- Does not explain why light is polarised and absorbed discretely

Faraday's model of light

1846 - Faraday

- Light is waves in lines of force
- Polarised and absorbed discretely

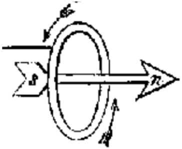


- Does not explain refraction, diffraction, interference

Maxwell’s breakthrough

1861 - Maxwell combined Euler’s and Faraday’s approaches

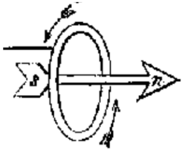
- A magnetic line of force is a ‘molecular vortex’ in a fluid-like medium



- Centrifugal forces reduce pressure near centre
- ‘tension’ along axis due to reduced pressure
- accounts for the forces between magnetic poles

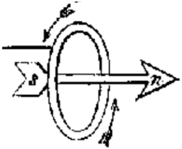
JC Maxwell. On physical lines of force. Philosophical Magazine, 21, 23(4), 1861

Maxwell’s calculation (modern notation, unit charge)



$\overline{\mathbf{p}}$ momentum per unit volume

Maxwell's calculation (modern notation, unit charge)

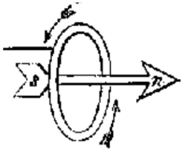


$\overline{\mathbf{p}}$ momentum per unit volume

$\mathbf{B} = \nabla \times \overline{\mathbf{p}}$ Magnetic field (definition)

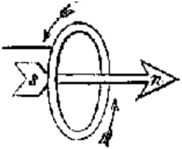
$\nabla . \mathbf{B} = 0$ Gauss's law for magnetism since $\nabla . (\nabla \times \overline{\mathbf{p}}) = 0$

Maxwell's calculation (modern notation, unit charge)



$\overline{\mathbf{p}}$	momentum per unit volume	
$\mathbf{B} = \nabla \times \overline{\mathbf{p}}$	Magnetic field (definition)	
$\nabla . \mathbf{B} = 0$	Gauss's law for magnetism	since $\nabla . (\nabla \times \overline{\mathbf{p}}) = 0$
$\phi = \int \mathbf{B} . ds$	Magnetic flux (definition)	
$\phi = \oint \overline{\mathbf{p}} . d\ell \neq 0$	Molecular vortex has flux	Stokes's theorem

Maxwell's calculation (modern notation, unit charge)



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$\phi = \int \mathbf{B}.ds$ Magnetic flux (definition)

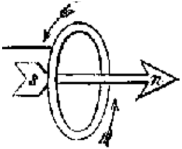
$\phi = \oint \overline{\mathbf{p}}.d\ell \neq 0$ Molecular vortex has flux — Stokes's theorem

$\mathbf{E} = -\frac{d\overline{\mathbf{p}}}{dt}$ Force per unit volume — Newton's second law

$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$ Faraday's law of induction — $\nabla \times \frac{d\overline{\mathbf{p}}}{dt} = \frac{d}{dt}(\nabla \times \overline{\mathbf{p}})$

Contraints

A molecular vortex can be any axis with mass flowing around it ($\oint \overline{\mathbf{p}}.d\ell \neq 0$)

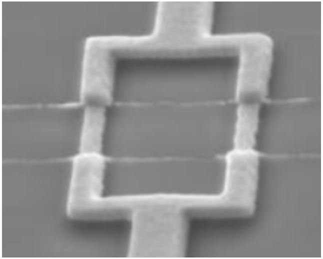


An ordinary vortex is not a good example

- Pinned to the fluid, not symmetric under Lorentz transformation
- But Maxwell's equations are symmetric under Lorentz transformation

But we now know an ordinary vortex is not the only possibility ...

Maxwell didn't kow magnetic flux is quantised

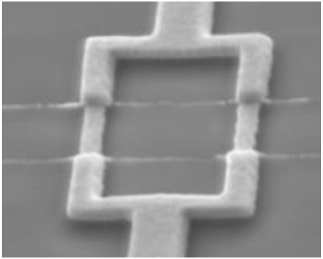


Superconductor has macroscopic 'order parameter' written Re^{iS}

When a superconducting loop encloses n quanta of magnetic flux

- Phase S advances by $2n\pi$ around the loop
- $\oint \nabla S.d\ell = 2n\pi$

Maxwell didn't kow magnetic flux is quantised



Superconductor has macroscopic 'order parameter' written Re^{iS}

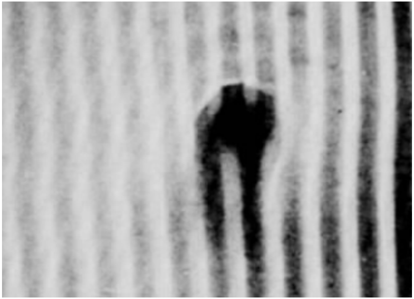
When a superconducting loop encloses n quanta of magnetic flux

- Phase S advances by $2n\pi$ around the loop
- $\oint \nabla S.d\ell = 2n\pi$
- Known in fluid mechanics as a 'phase vortex'

Berry's phase vortex experiment

1980 - Berry, Chambers, Large, Upstill, Walmsley

- Water waves past a steady vortex



- Phase of wave advances by $2n\pi$ around the centre
- $\oint \nabla S.d\ell = 2n\pi$ – defines a 'phase vortex'

Berry's phase vortex experiment

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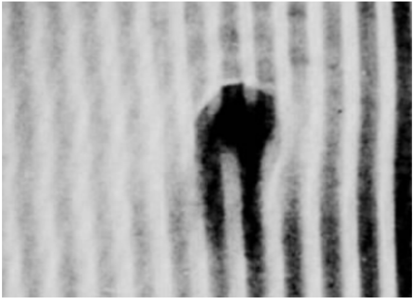
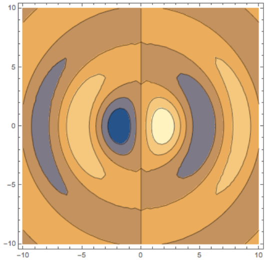


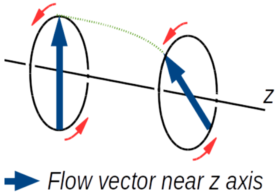
- Phase of wave advances by $2n\pi$ around the centre
- $\oint \nabla S.d\ell = 2n\pi$ – defines a 'phase vortex'
- Measured an analogue of magnetic (Aharonov-Bohm) effect

Magnetic line of force = Phase vortex

Euler's fluid to first order $\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = 0$

Solution in (r, θ, z) coords $\delta\rho_n \propto J_n(k_r r) \cos(\omega t - n\theta - k_z z)$



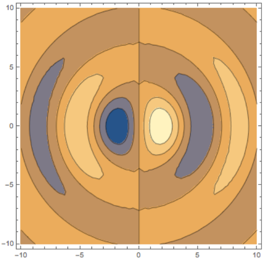


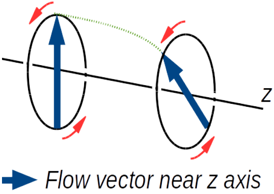
- $\oint \overline{\mathbf{p}}.d\ell \neq 0 \rightarrow$ Maxwell's equations for magnetic line of force

Magnetic line of force = Phase vortex

Euler's fluid to first order $\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = 0$

Solution in (r, θ, z) coords $\delta\rho_n \quad \propto \quad J_n(k_r r) \cos(\omega t - n\theta - k_z z)$





- $\oint \overline{\mathbf{p}}.d\ell \neq 0 \quad \rightarrow$ Maxwell's equations for magnetic line of force
- Phase $S = \omega t - n\theta - k_z z$ increases by $2n\pi \rightarrow$ flux is quantised

Magnetic line of force = Phase vortex

Euler's fluid to first order $\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = 0$

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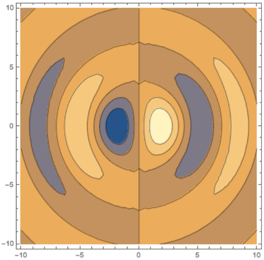

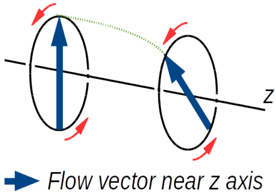

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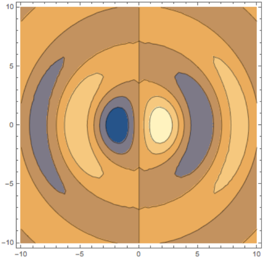
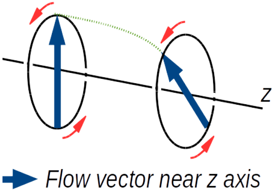
- Phase $S = \omega t - n\theta - k_z z$ increases by $2n\pi$ $\rightarrow$ flux is quantised
- Obeys wave equation to first order $\rightarrow$ Lorentz covariant

Heresy

Circularly polarised light

Amplitude modulated wavepacket in a line of force

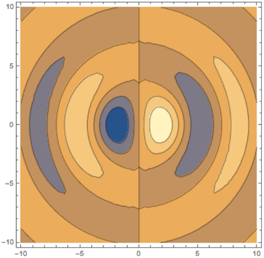


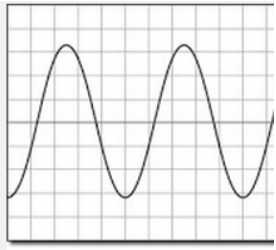


- Chiral symmetry - same as circularly polarised light
- Small amplitude except near axis – absorbed discretely
- Obeys Maxwell's equations to first order

Linearly polarised light

$$\delta\rho_1 + \delta\rho_{-1} \quad \propto \quad J_1(k_r r)\cos(\omega t - k_z z)\cos(\theta - \theta_o)$$

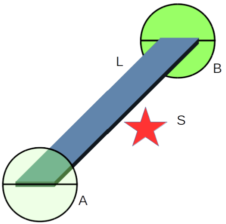




Oscillating dipole parallel to $\theta = \theta_o$

- Linearly polarised light is amplitude modulation
- Fourier components have various values of k_z

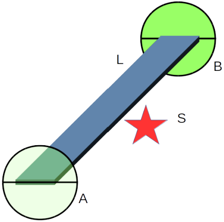
Bell tests



Bell test experiment

- 100% correlation when polarisers are parallel

Bell tests



Bell test experiment

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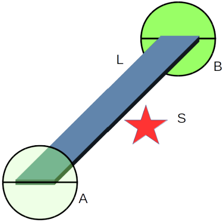
- $\cos(2\phi)$ correlation when polarisers are at angle ϕ

Bell tests



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- cos(2ϕ) correlation when polarisers are at angle ϕ

Clauser, Horne, Shimony and Holt (1969)

- assumed polarisation 'carried by and localised within' a photon
- Not true on Faraday's model
- CHSH concluded that cos(2ϕ) is impossible

Conclusion - a mathematical heresy

Euler (1746) may be right

- Light is waves in a frictionless compressible fluid
- Guided by Faraday/Maxwell lines of force

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Our heretical programme

- Derive all of physics from Euler's equation for a compressible fluid
- Or prove Euler wrong by failing in any area you choose

Read the papers!

Latest: ‘Maxwell’s fluid model of magnetism’. Out on arxiv tomorrow: (edited) now on http://arxiv.org/abs/1502.05926

Droplets: ‘Why bouncing droplets are a pretty good model of quantum mechanics’, arxiv 1401.4356

Rotons: ‘The irrotational motion of a compressible inviscid fluid’, arxiv 1301.7540