Jet Propulsion Laboratory
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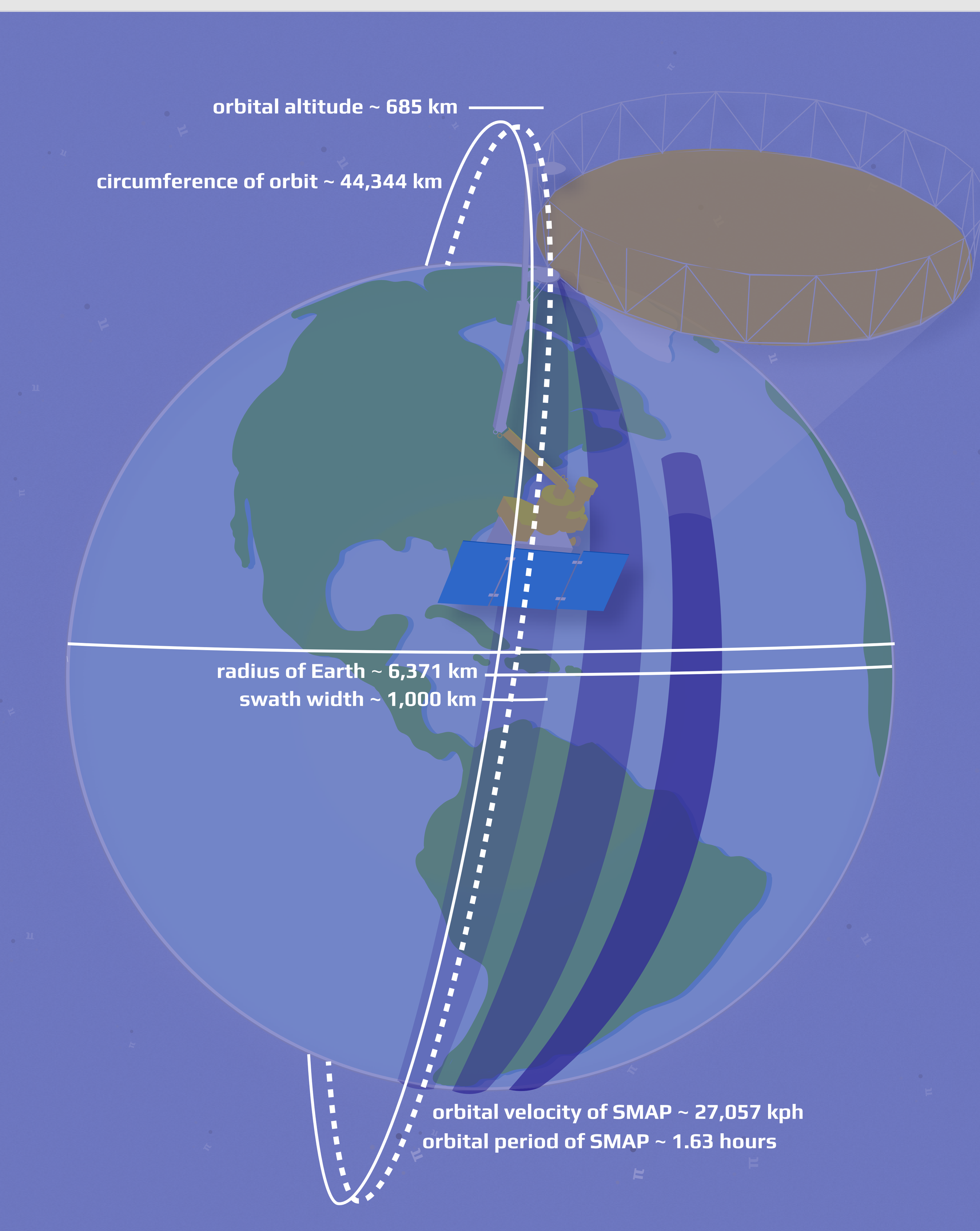
π IN THE SKY

Pretty handy, that pi, eh? Take a look at the solutions below to see if your answers match those of our NASA experts. We just might be on the lookout for a smart future scientist or engineer like yourself!

ANSWER KEY



1. Find the **circumference (C) of Earth** at the equator.

   circumference = 2πr

   C = 2π(6,371 km) ~ **40,300 km**

2. Divide circumference by twice the swath width to find the number of **spacecraft orbits** required to image the whole Earth.

   40,300 / 2,000 ~ **20 orbits**

3. Compute **SMAP's orbital period** (the length of time it takes the spacecraft to make one full orbit around Earth).

   orbital period = c / V_c

   Find the **circumference (C) of the orbit**.

   C = 2π(6,371km + 685km) ~ 44,334km

   Compute the **orbital velocity (V_c) of SMAP.**

   $$V_c = \sqrt{\frac{gm}{r}}$$

   $$\sqrt{\frac{(6.67 \times 10^{-11}\ m^3/s^2kg)(5.976 \times 10^{24}kg)}{(6,371\ km + 685\ km)}} \sim \mathbf{27,057\ km/hr}$$

   Plug in circumference and orbital velocity to find the **orbital period**.

   44,334 km / 27,057 kph ~ **1.63 hours**

4. Multiply number of orbits by orbital period to find the number of days it takes SMAP to image Earth.

   20 orbits x 1.63 hours per orbit ~ **32.77 hours (1.37 days)**

1.37 days*

* In reality, there is some swath overlap, so mapping Earth's surface takes 2 days near the poles and 3 days near the equator.

1. Find the **circumference (C)** of the wheel.

   circumference = πd

   C = π(50 cm) ~ **157.1 cm**

2. Multiply the circumference by the number of wheel rotations to find the **distance traveled**.

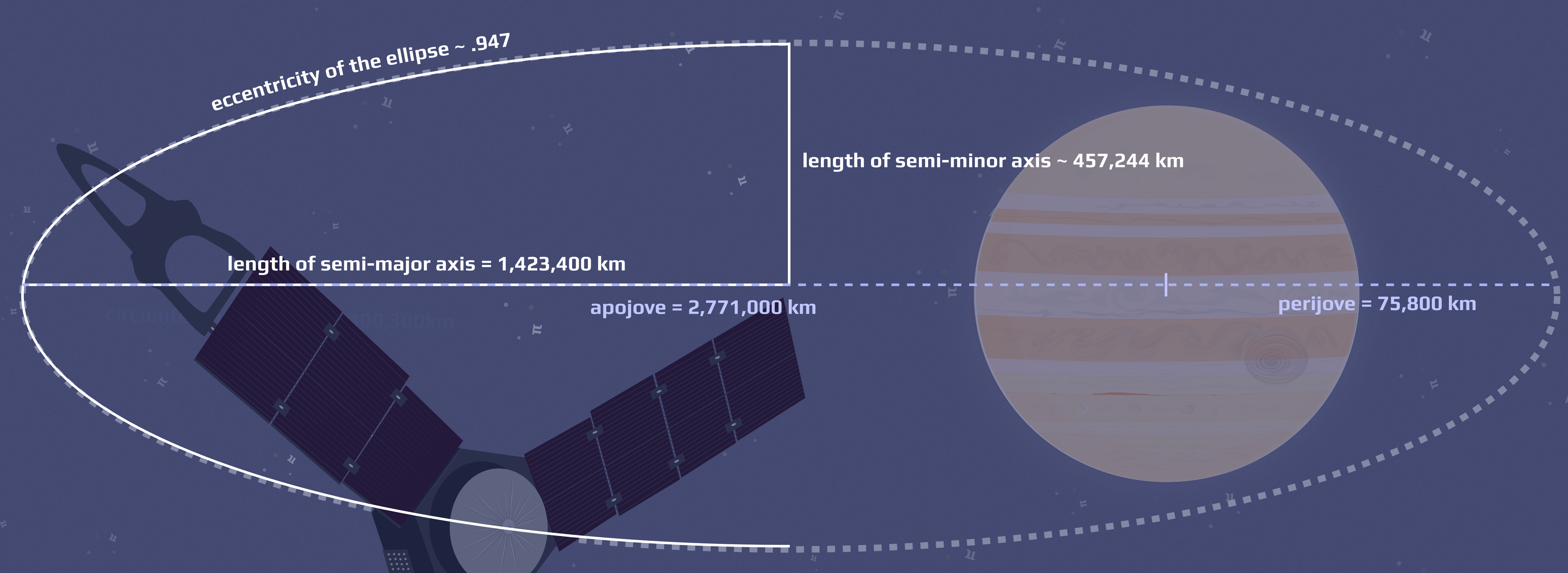
   157.1 x 3689.2 ~ **579,573.32 cm**

5.8 km

1. Divide the visual odometry measurement by the circumference of the wheel and subtract from 100% to find the **slippage percent**.

   1 - (143 cm / 157.1 cm) ~ **9%**

9% slippage



1. Find the **perimeter of the ellipse (P)** (use any available formula -- we'll use Ramanujan's approximation).

   $$P \sim \pi \left[\ 3(a + b) - \sqrt{(3a + b)(a + 3b)}\ \right]$$

   Compute **length of semi-major axis (a)**.

   a = (75,800 km + 2,771,000 km) / 2 = **1,423,400 km**

   Compute **length of semi-minor axis (b)**.

   $$e = c / a \ = \ \sqrt{a^2 - b^2} / a \ \Rightarrow \ b = a\sqrt{(1 - e^2)}$$

   Compute the **eccentricity (e) of the ellipse**.

   e = rA - rP / rA + rP

   2,771,000 - 75,800 / 2,771,000 + 75,800 ~ **.947**

   Substitute values and compute.

   b = 1,423,400km $\sqrt{(1 - .947^2)}$ ~ **457,244 km**

   P ~ **6,304,701 km**

6,300,000 km

1. Compute the **volume of Cassini's hydrazine tank**.

   Convert the tank's radius to centimeters.

   14in x 2.54cm/in ~ **35.56 cm**

   volume = $\frac{4}{3}\pi r^3$

   $\frac{4}{3}\pi(35.56\ cm)^3$ ~ **188,353.57 cm^3**

2. Compute the **volume of hydrazine in the tank** at launch.

   0.69(188,353.57 cm^3) ~ **129,963.97 cm^3**

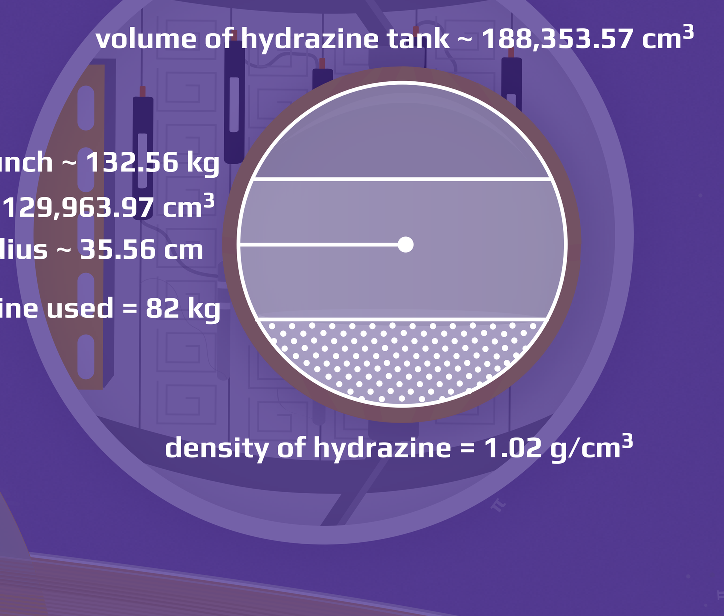
3. Compute the **amount of hydrazine in tank at launch.**

   mass = density x volume

   (1.02 g/cm^3)(129,963.97 cm^3) ~ **132,563 g**

3. **Subtract** the amount of hydrazine used from the amount at launch.

   132.56 kg - 82 kg ~ **51 kg**



51 kg