Jet Propulsion Laboratory
California Institute of Technology

Answer Key

# π IN THE SKY 3

How did you do exploring 'round the solar system with pi? Are you on your way to becoming a NASA scientist or engineer? Check your answers below and find out!

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## Hazy Halo

**What percentage of Titan's makeup by volume is atmospheric haze?**

1. Use the formula for volume of a sphere to compute Titan's total volume

$$V = \frac{4}{3}\pi r^3$$

$$V_{total} = \frac{4}{3}\pi(2{,}575\ km + 600km)^3 = \frac{4}{3}\pi(3{,}175\ km)^3 = \frac{4}{3}\pi(32{,}005{,}984{,}375\ km^3) \approx 134{,}066{,}353{,}845\ km^3$$

2. Compute the volume of Titan's solid body

$$V_{solid} = \frac{4}{3}\pi(2{,}575\ km)^3 = \frac{4}{3}\pi(17{,}073{,}859{,}375\ km^3) \approx 71{,}518{,}814{,}908\ km^3$$

3. Subtract the volume of the solid body from the total volume of Titan

$$V_{atmosphere} \approx 134{,}066{,}353{,}845\ km^3 - 71{,}518{,}814{,}908\ km^3 \approx 62{,}547{,}538{,}937\ km^3$$

4. Find the ratio of the volume of the atmosphere to the total volume of Titan

$$(62{,}547{,}538{,}937\ km^3 / 134{,}066{,}353{,}845\ km^3) \cdot 100\% \approx 47\%$$

**What is the surface area that a future spacecraft to Titan would map?**

1. Use the formula for surface area of a sphere to compute Titan's surface area

$$A = 4\pi r^2$$

$$A = 4\pi(2{,}575\ km)^2 = 4\pi(6{,}630{,}625\ km^2) \approx 83{,}322{,}891\ km^2$$

## Round Recon

**How far does the Mars Reconnaissance Orbiter (MRO) travel in one orbit?**

1. Use the formula for circumference of a circle to find the distance MRO travels in one orbit

$$C = \pi d$$

$$C = \pi(6{,}752\ km + 255\ km + 320\ km) = \pi(7{,}327\ km) \approx 23{,}018\ km$$

**How long does it take MRO to complete one orbit?**

1. Divide the distance that MRO travels in one orbit by its average speed

$$time = distance\ /\ rate$$

$$time \approx 23{,}018\ km\ /\ 3.42\ \tfrac{km}{sec} \approx 6{,}730\ sec = 112\ min \approx 1.87\ hrs$$

**How many orbits does MRO complete in one Earth day?**

1. Divide the hours in one Earth day by the hours per orbit

$$24\ hours\ /\ 1.87\ hours\ per\ orbit = 12.8\ orbits$$

## Sun Screen

1. Use the ratio of areas formula to compute the percentage dip in sunlight reaching Earth during a transit of Mercury

$$B\% = 100\left(\frac{\pi r^2}{\pi R^2}\right)$$

$$B\% = 100\left(\frac{\pi(6\ arcseconds)^2}{\pi(954.5\ arcseconds)^2}\right)$$

$$B\% \approx 100\left(\frac{\pi(36\ arcseconds^2)}{\pi(911{,}070\ arcseconds^2)}\right)$$

$$B\% \approx 100\left(\frac{113\ arcseconds^2}{2{,}862{,}212\ arcseconds^2}\right)$$

$$B\% \approx 100\,(0.0000395) \approx 0.00395\%$$

2. Multiply the amount of solar energy that reaches the top of Earth's atmosphere by the percentage dip in sunlight during a transit of Mercury

$$0.00395\% \cdot 1{,}360.8\ \tfrac{w}{m^2} \approx 0.05\ \tfrac{w}{m^2}$$

## Gravity Grab

**Solution approach:** The change in velocity can be found by subtracting the velocity of Juno after deceleration ($v$) from Juno's velocity at closest approach (57.98 km per second). Once we know $a$, we can plug it into the equation for total orbital energy and solve for $v$. So:

1. Use the given formula for orbital period to find the semi-major axis of Juno's orbit ($a$)

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \rightarrow a = \sqrt[3]{\frac{\mu T^2}{4\pi^2}}$$

$$a = \sqrt[3]{\frac{(126{,}686{,}536\ \tfrac{km^3}{sec^2})(53.5 \cdot 24 \cdot 60 \cdot 60\ sec)^2}{4\pi^2}} \approx 4{,}092{,}939\ km$$

2. Use the given formula for total orbital energy – and the semi-major axis of Juno's orbit ($a$) found above – to find the velocity of Juno after deceleration ($v$)

$$\frac{-\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \rightarrow v = \pm\sqrt{2\left(\frac{\mu}{r} - \frac{\mu}{2a}\right)}$$

$$v = \pm\sqrt{2\left(\frac{126{,}686{,}536\ \tfrac{km^3}{sec^2}}{76{,}006\ km} - \frac{126{,}686{,}536\ \tfrac{km^3}{sec^2}}{2(4{,}092{,}939\ km)}\right)} \approx 57.47\ \tfrac{km}{sec}$$

3. Subtract the velocity ($v$) of Juno after deceleration from Juno's velocity at closest approach

$$57.98\ \tfrac{km}{sec} - 57.47\ \tfrac{km}{sec} \approx 0.51\ \tfrac{km}{sec}\ \text{or}\ 510\ \tfrac{m}{sec}$$

**Note:** The above calculations assume that Juno's change in velocity ($\Delta v$) is instantaneous, while in reality, it takes time for the spacecraft's main engine to provide the required amount of thrust. Because of this, the thrusting begins before perijove and continues after perijove, so it is less efficient than the $\Delta v$ represented in this problem. The actual $\Delta v$ during Juno's orbit insertion on July 4, 2016, will be closer to 540 m/s.