Numerical Methods

(Additional Notes from Talks by PFH)

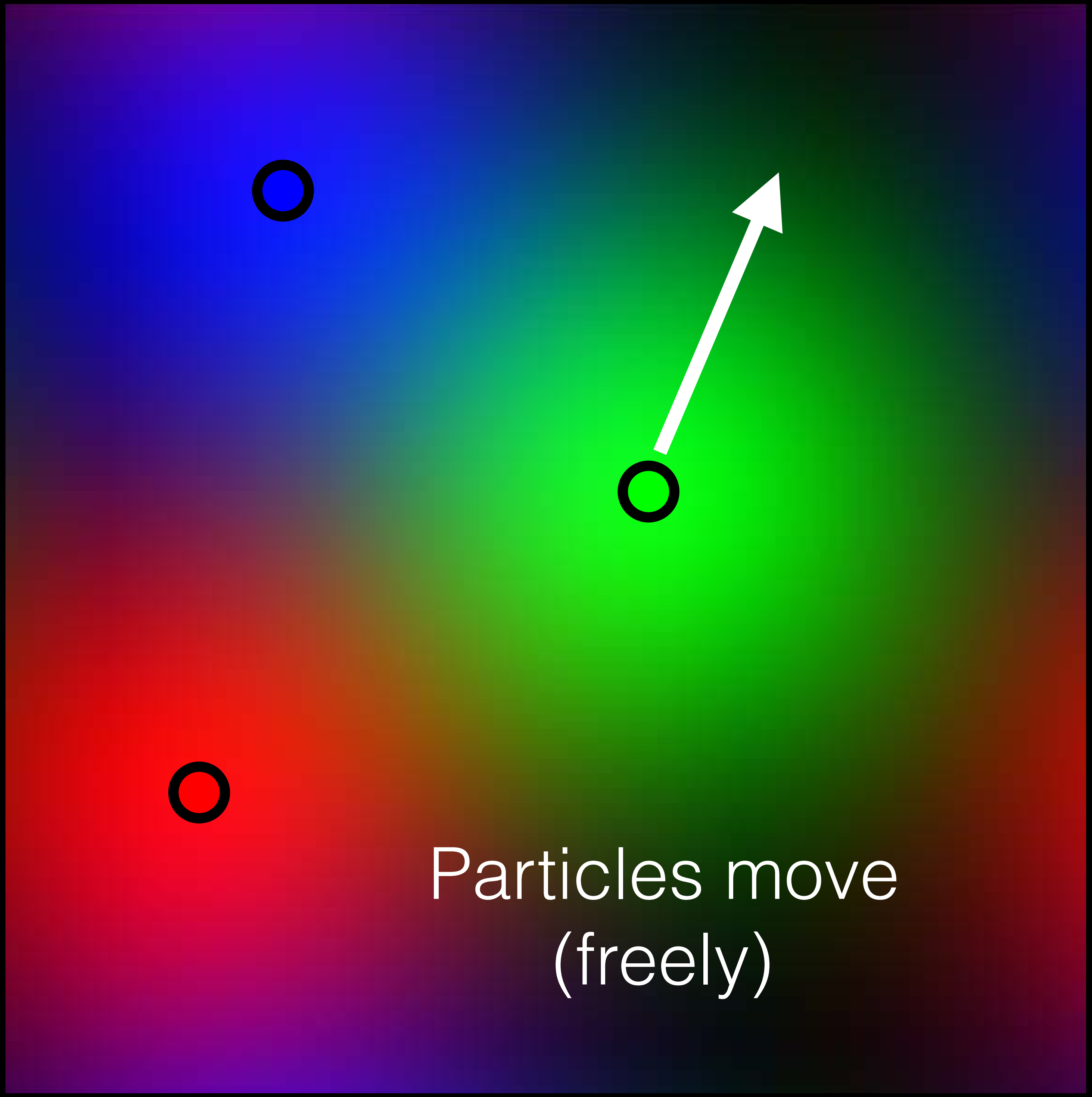
SPH

Challenge:
POPULAR METHODS FOR
HYDRODYNAMICS HAVE PROBLEMS

Lucy 77, Gingold & Monaghan 77
Reviews by: Springel 11, Price 12

Smoothed-Particle Hydrodynamics

- Lagrangian, adaptive, simple, conservative



Challenge:
POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS

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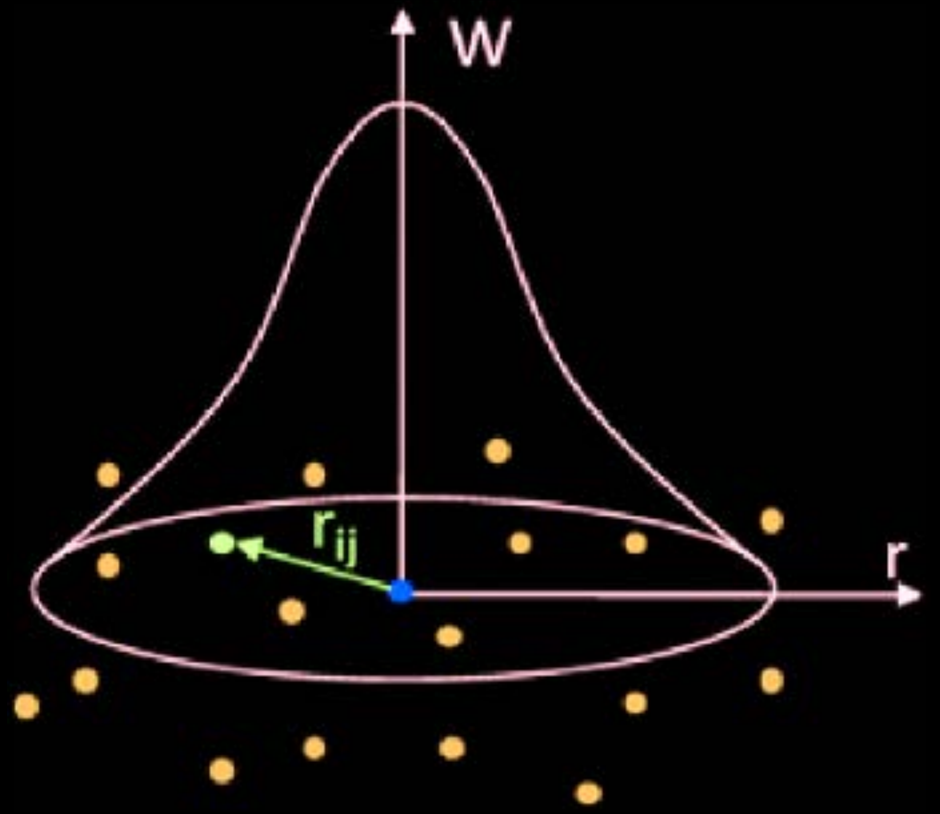
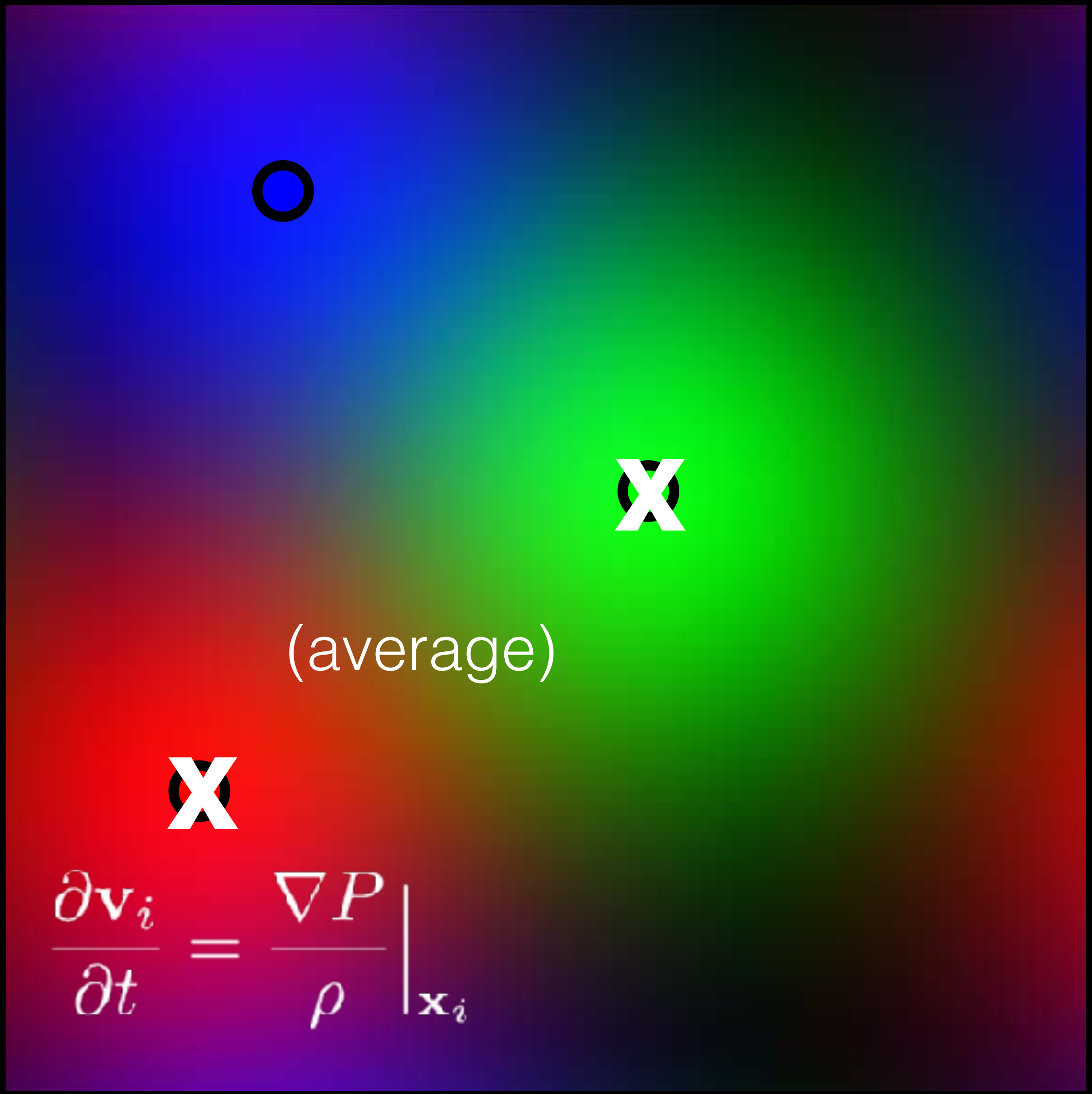
Smoothed-Particle Hydrodynamics

- No volume partition: point-like particles smoothed into fields [ok in "continuum limit"]



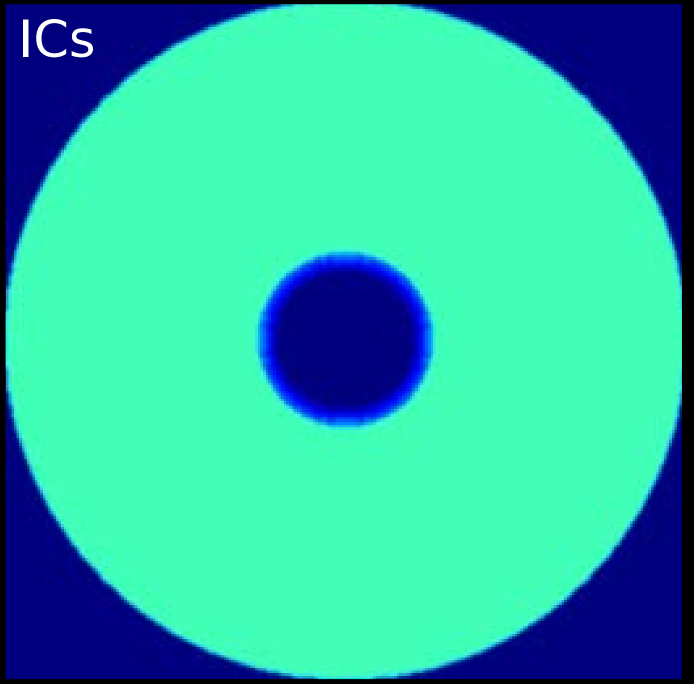


- Solve EOM at particle locations (stabilize with artificial diffusion)

Challenge:

POPULAR METHODS FOR
HYDRODYNAMICS HAVE PROBLEMS

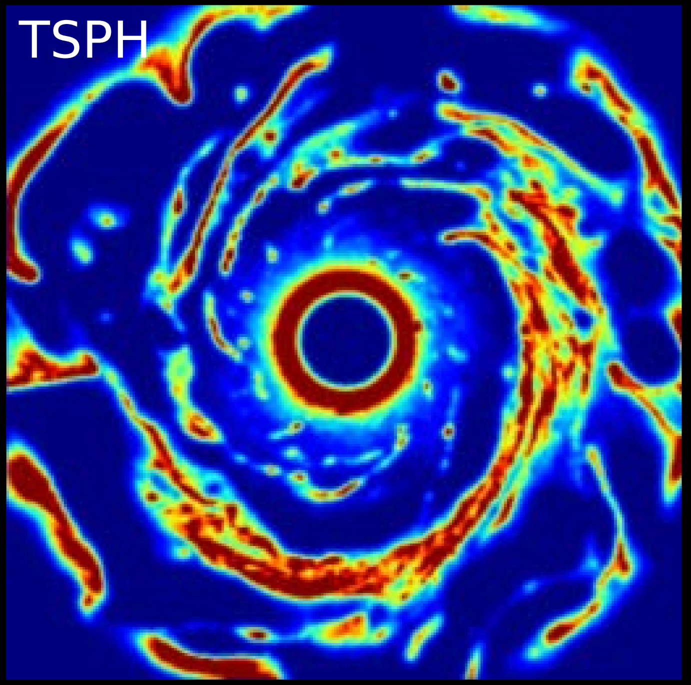
Keplerian disk/ring
(should conserve ICs)



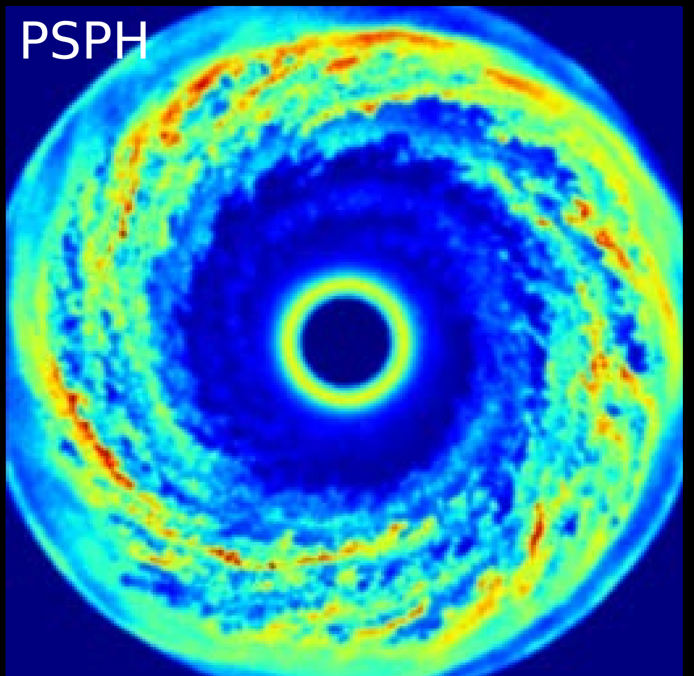
Smoothed-Particle Hydrodynamics

- Lagrangian, adaptive, simple, conservative
- Artificial diffusion terms:
  - excess diffusion, viscosity

“old” SPH
(Springel 02)
(after 20 orbits)



“new” SPH
(Hopkins 13)



Morris 97, Okamoto 03,
Cullen & Dehnen 10, Bauer & Springel 12

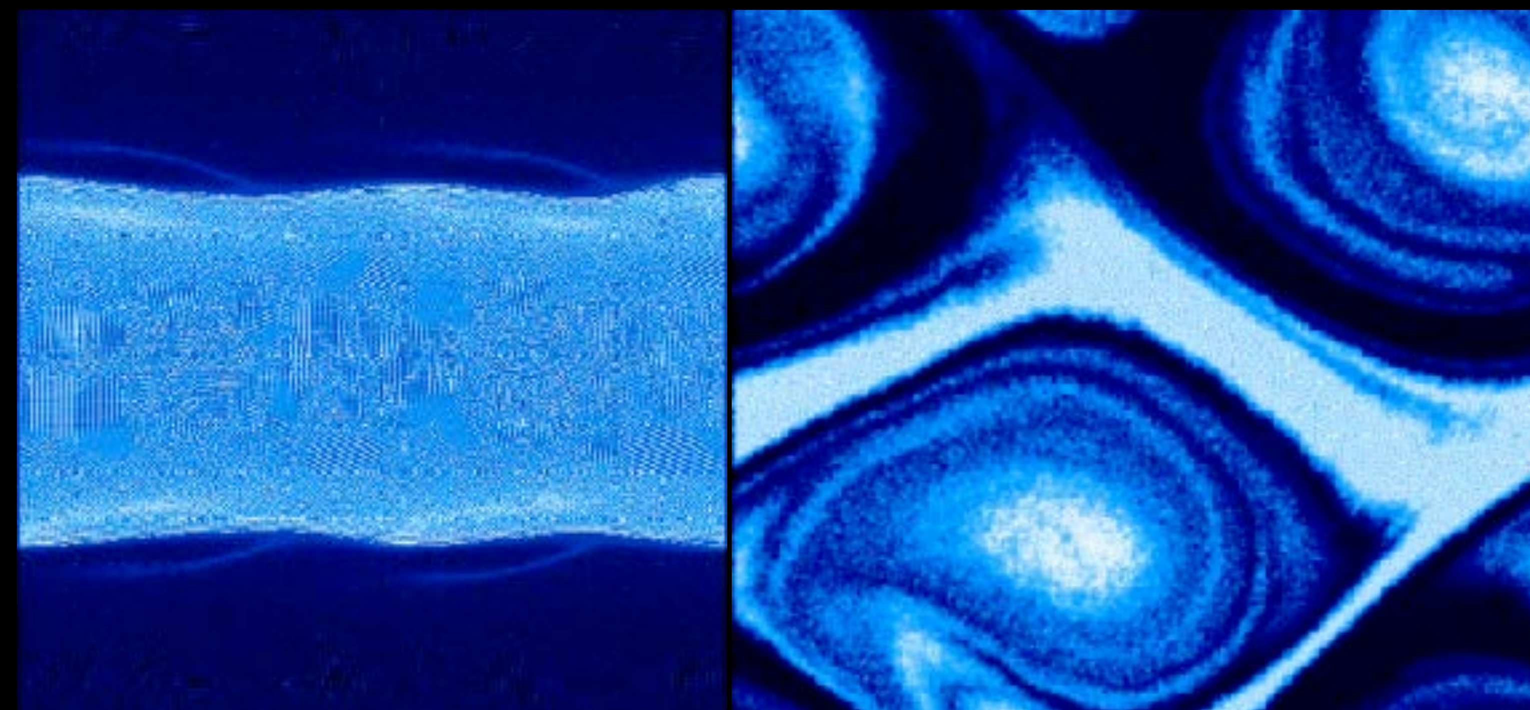
Challenge:
POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS

Ritchie & Thomas 01, Agertz 07, Price 12, Read 12

- **“Traditional SPH”**
  - GADGET/(old)GASOLINE
  - ~32 neighbors (cubic spline)
  - constant artificial viscosity
  - “density” formulation

- **“Modern SPH”**
  - P-SPH/SPHS/PHANTOM
  - ~128-500 neighbors (alt. kernels) (many people: Read, Dehnen)
  - high-order switches (Cullen+Dehnen)
  - “pressure” formulation (Hopkins, Saitoh+Makino)
  - artificial diffusion for entropy (Price, Wadsley)

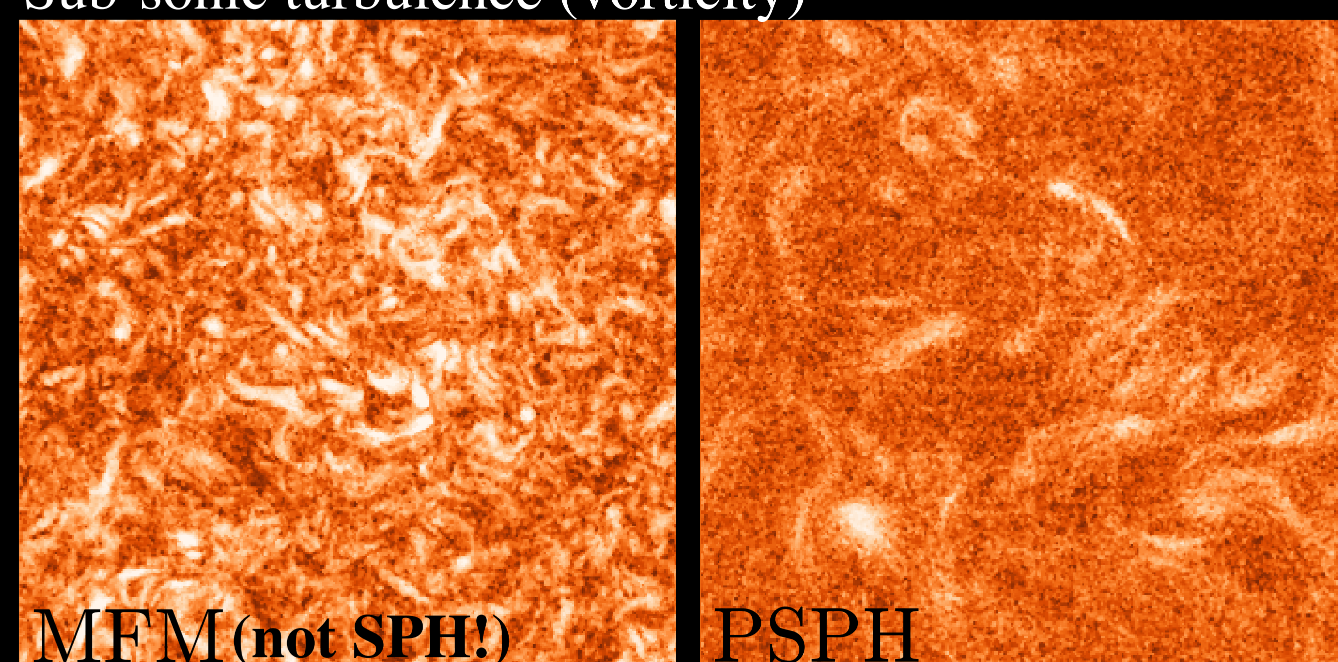
Kelvin-Helmholtz Instabilities



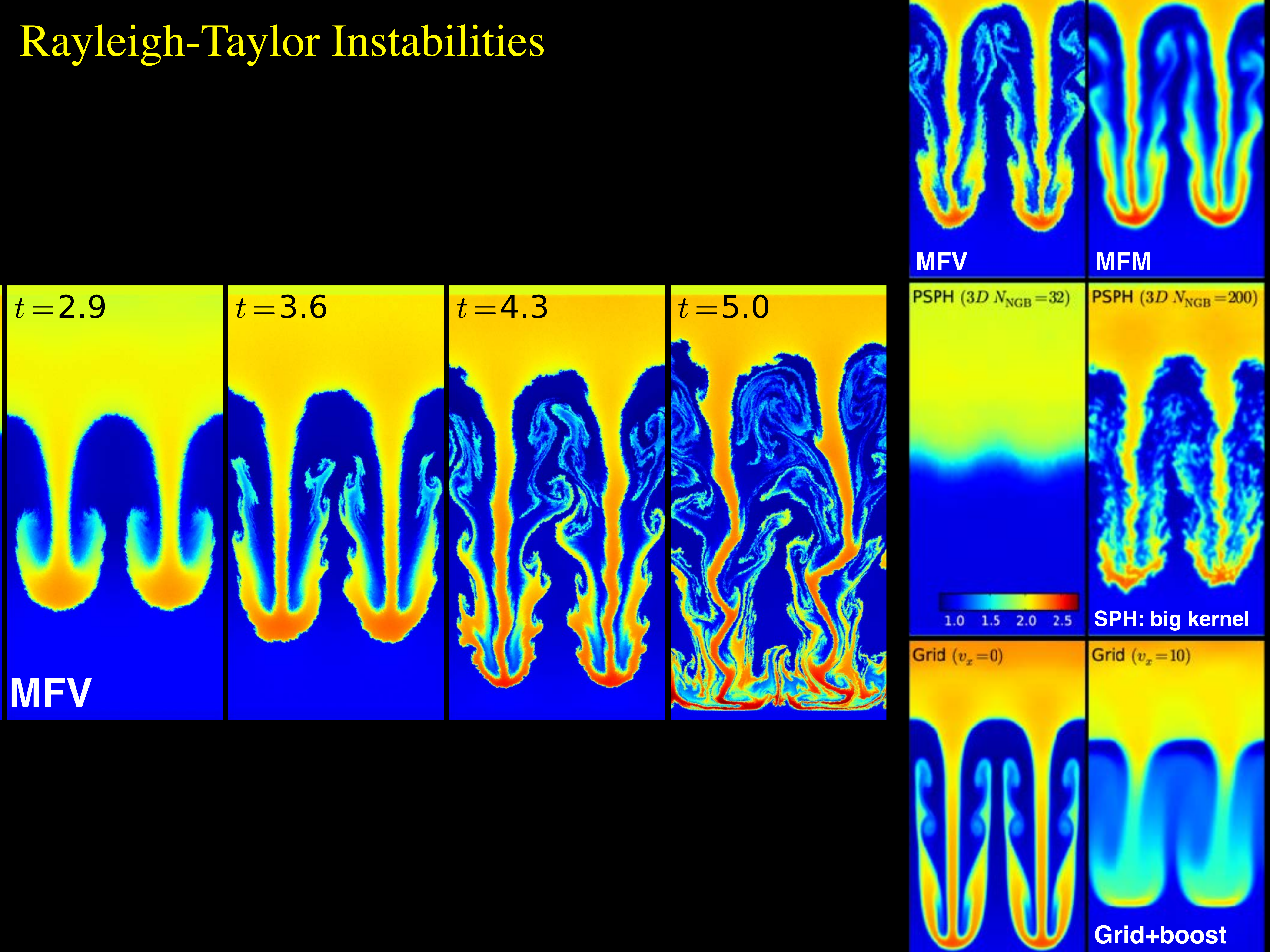
“old” SPH (Springel 02)

“new” SPH (PSPH) (Hopkins ’13): >>100 neighbors

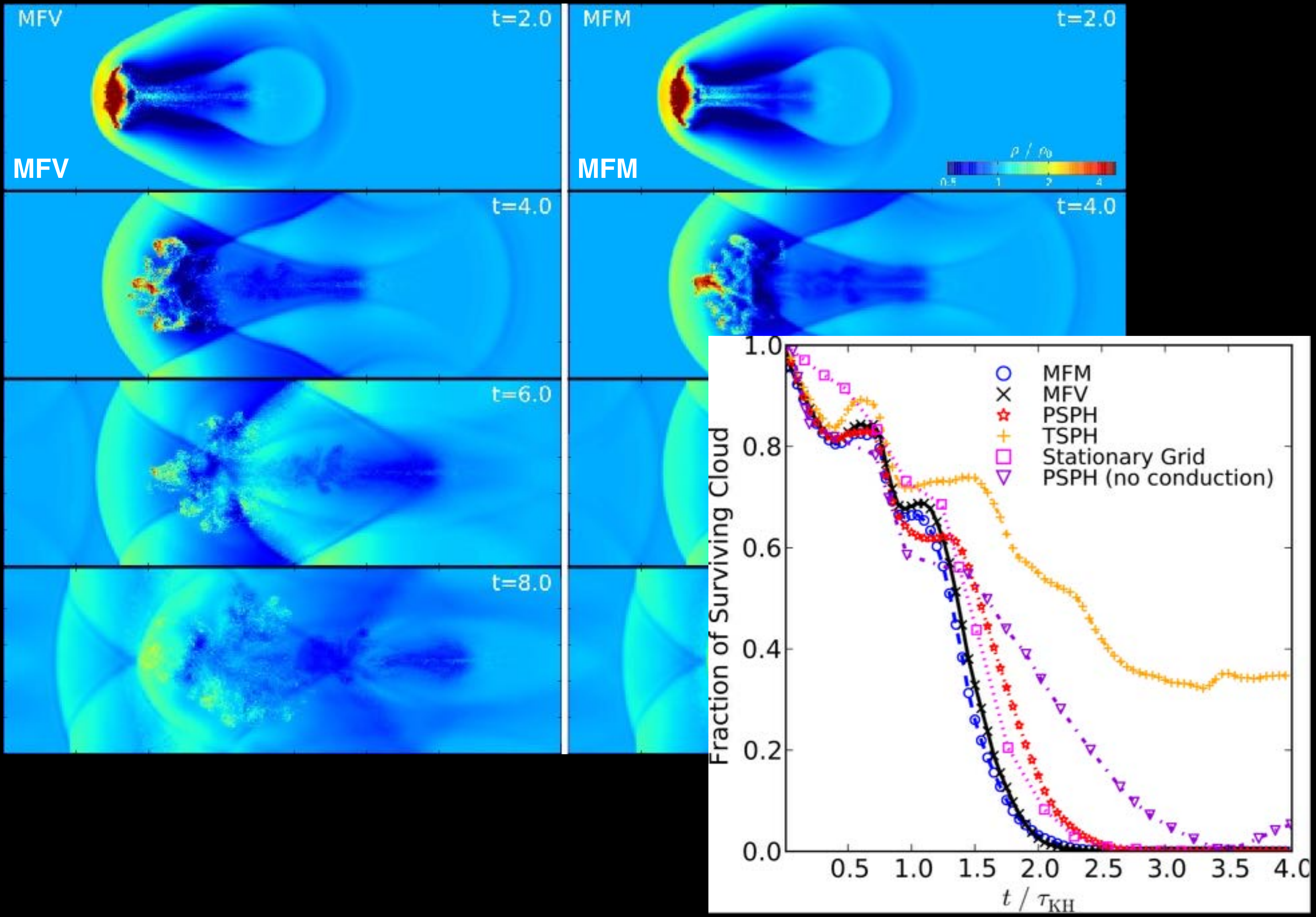
Sub-sonic turbulence (vorticity)

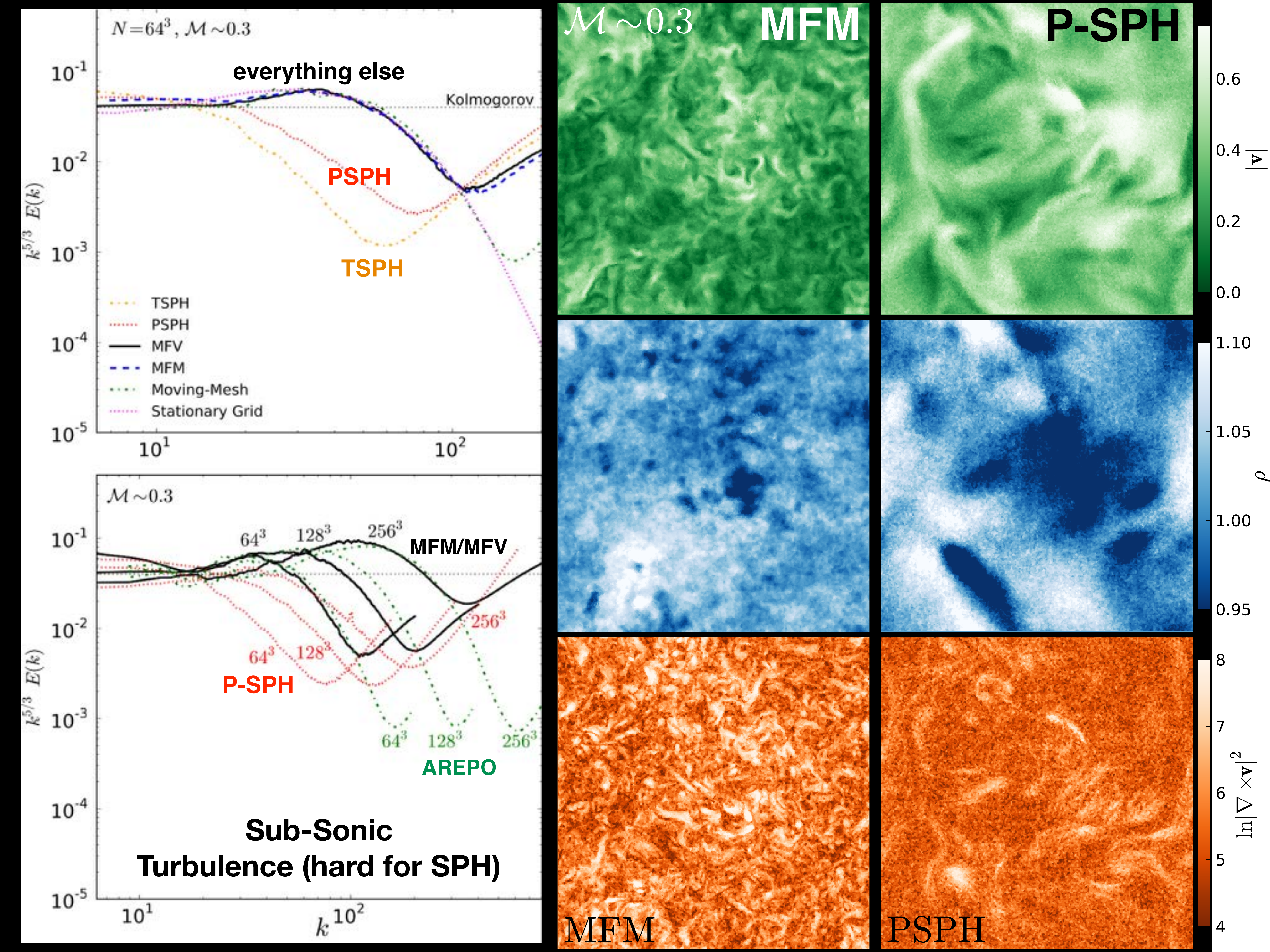


Rayleigh-Taylor Instabilities



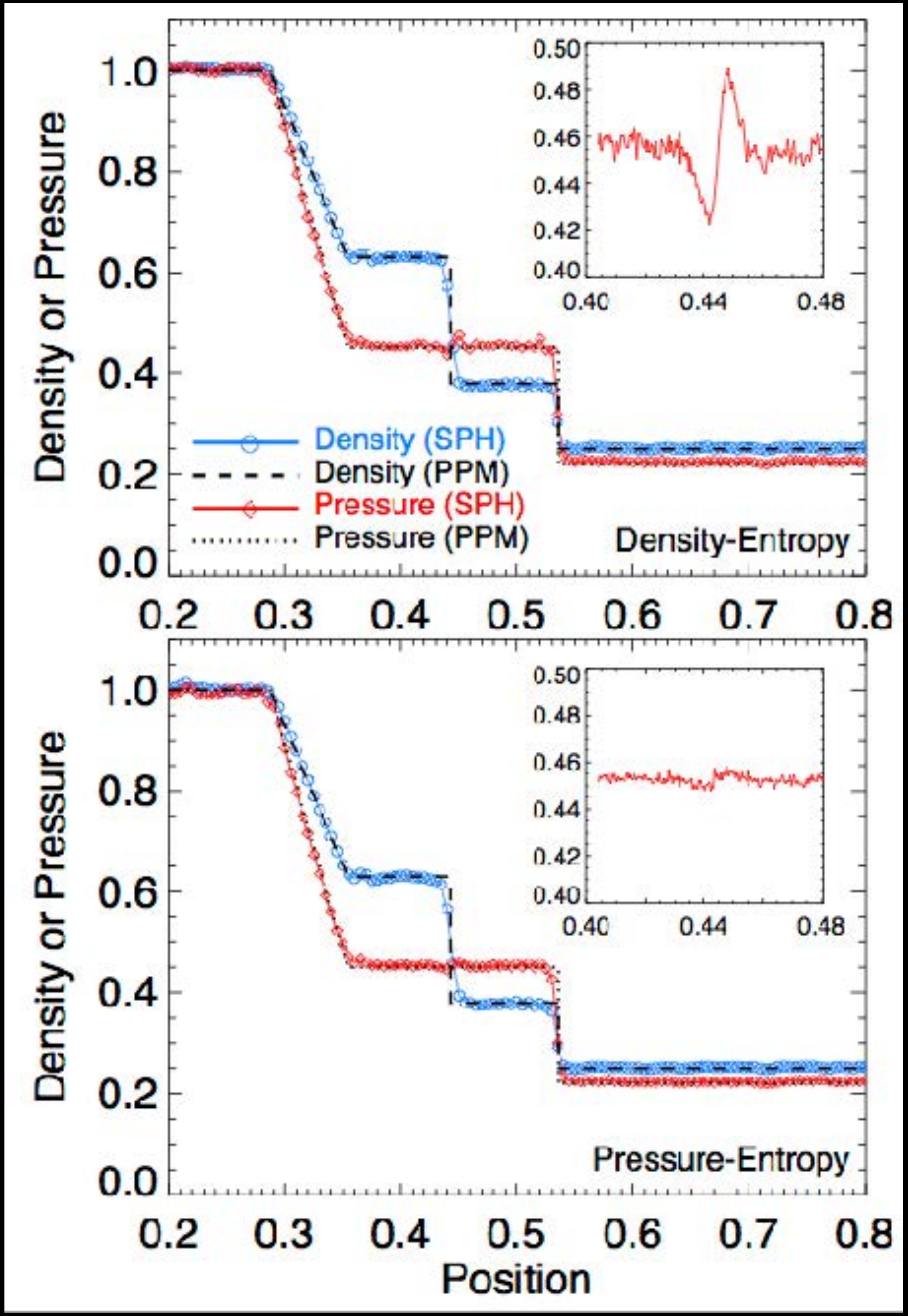
“The Blob” (Agertz et al.)

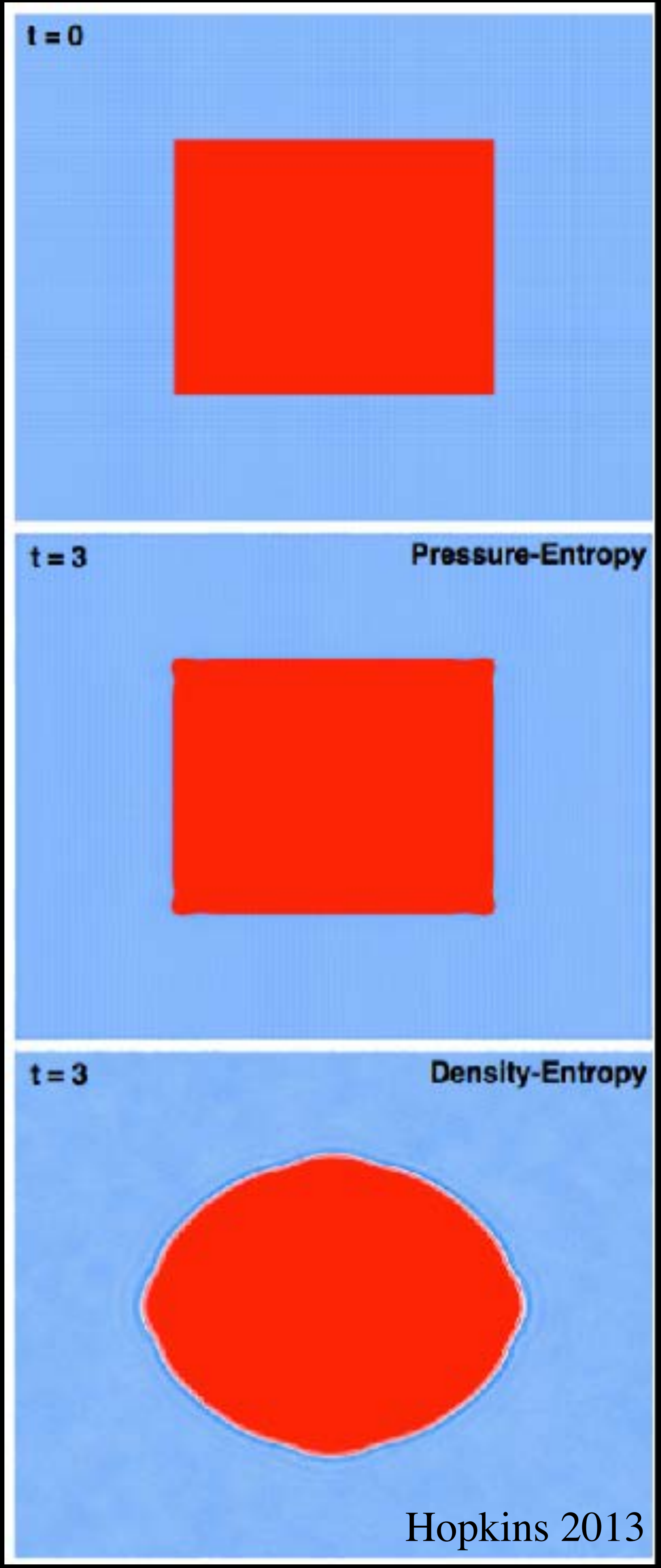




N = 64³, M ~0.3
everything else
Kolmogorov
PSPH
TSPH
TSPH
PSPH
MFV
MFM
Moving-Mesh
Stationary Grid
k^{5/3} E(k)
M ~0.3
64³
128³
256³
MFM/MFV
256³
64³
128³
P-SPH
64³
128³
256³
AREPO
Sub-Sonic
Turbulence (hard for SPH)
k
M ~0.3
MFM
P-SPH
|v|
ρ
ln|∇ × v|²
MFM
PSPH

Artificial Surface Tension (entirely fix-able!)



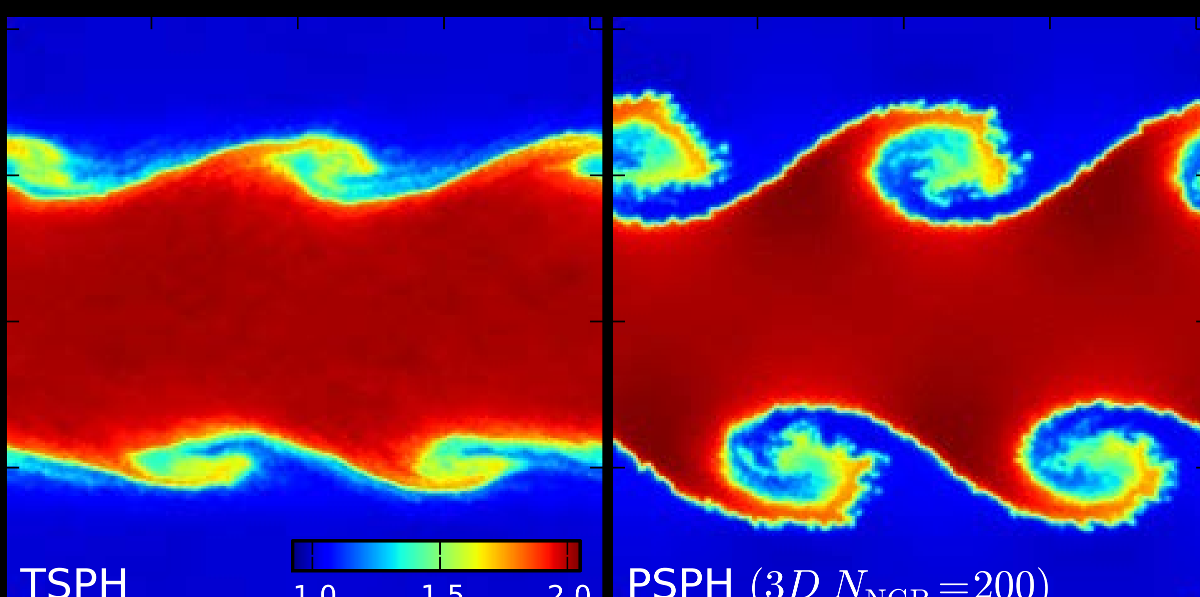


Challenge:

POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS

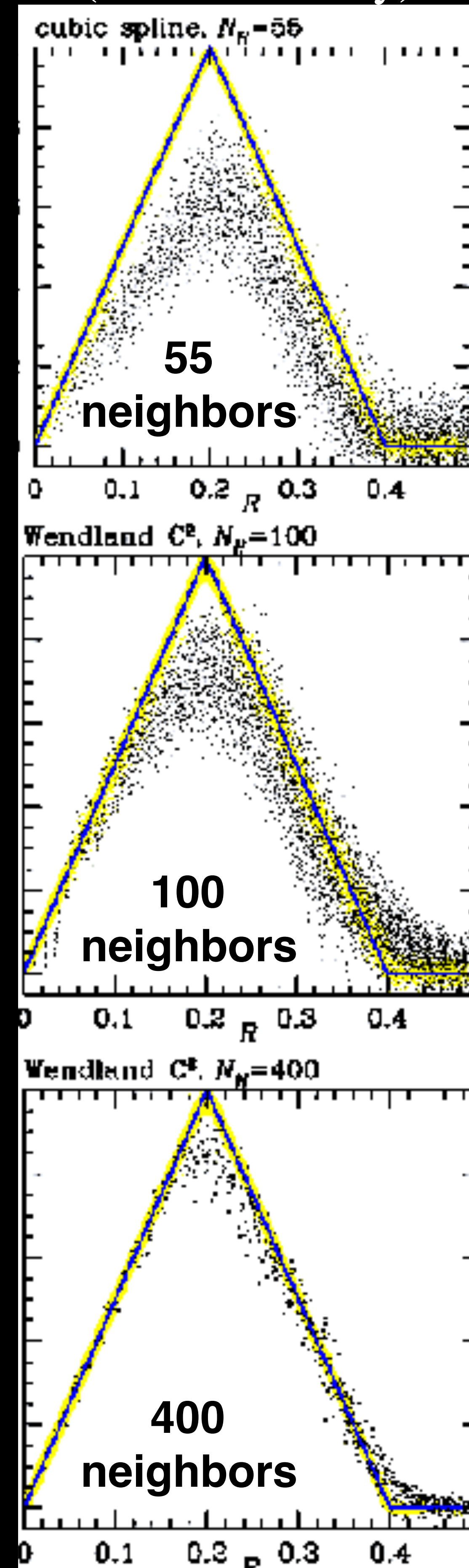
- Fundamental low-order errors:
  - converge slowly:
    "beat down" by
    increasing kernel size,
    but this is *not efficient!*

- MHD & anisotropic
  diffusion operators ill-posed



Dehnen & Aly, Rosswog, Hopkins, Tricco & Price, Read

Gresho vortex (Dehnen & Aly)



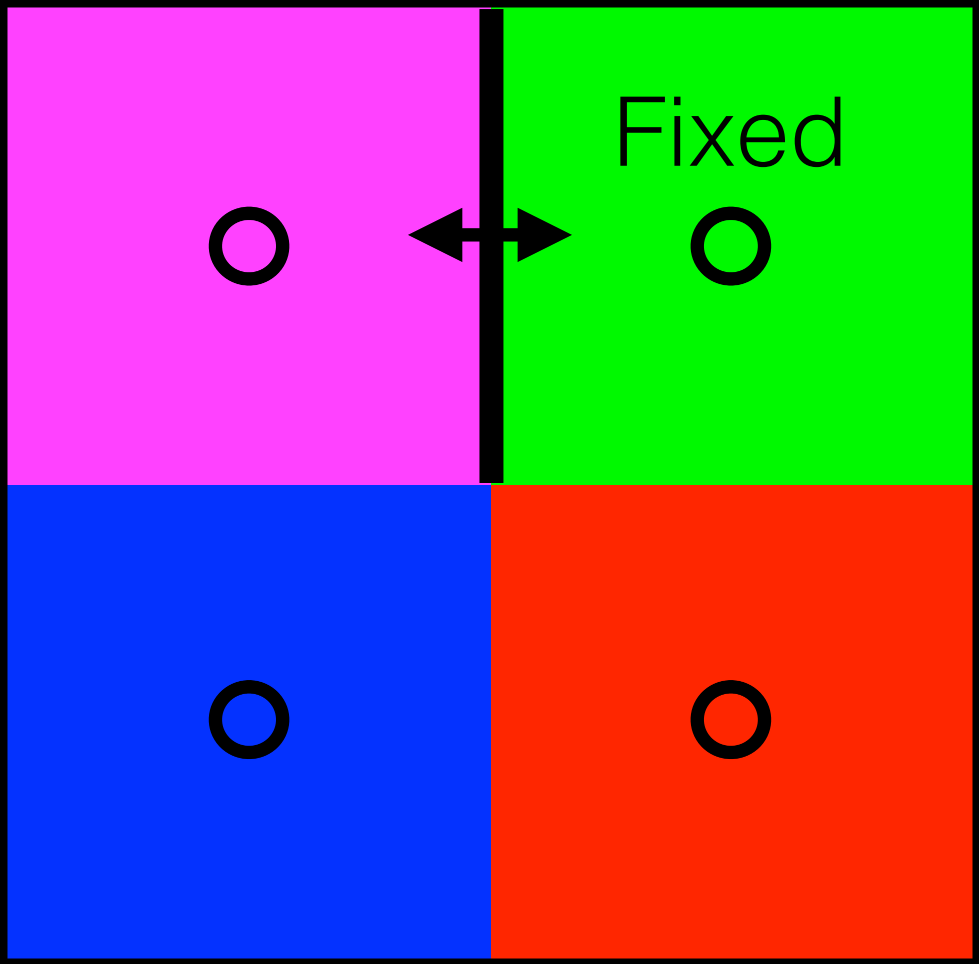
Anisotropic Conduction (MTI, HBI, Hall MRI)

AMR + Fixed-Grids

Challenge:
POPULAR METHODS FOR
HYDRODYNAMICS HAVE PROBLEMS

Berger & Colella 89 (& others)
Reviews by: Teyssier 14

Adaptive Mesh Refinement



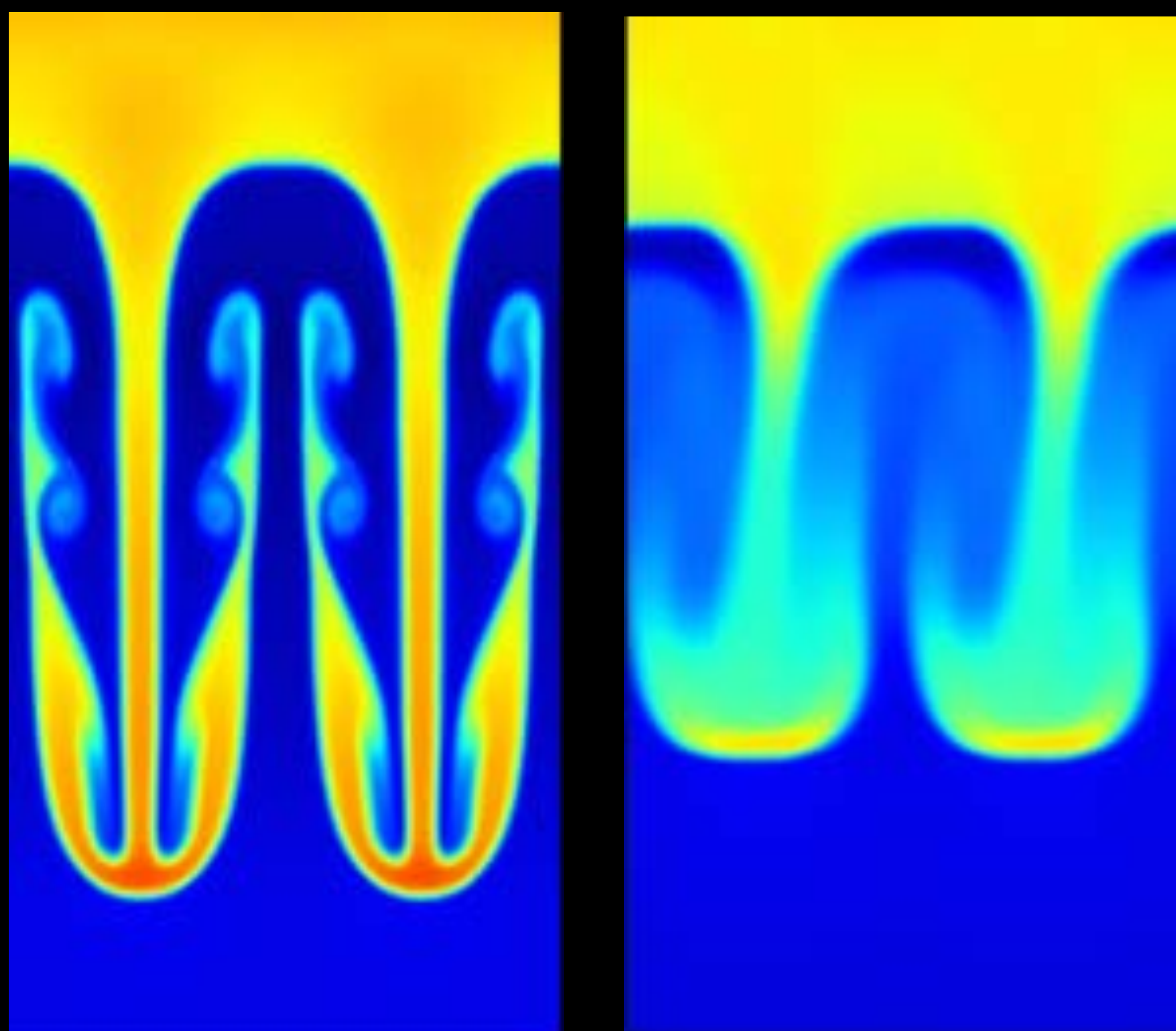
- Eulerian, well-studied, high-order
- Each cell carries conserved quantities inside volume V_i
- Solve Reimann problem between geometric faces

$$\Delta m_i = \int_{\text{cell}} \frac{\partial \rho}{\partial t} d^3\mathbf{x} = -\int_{\text{cell}} \nabla \cdot (\rho\, \mathbf{v})\, d^3\mathbf{x}$$

Adaptive Mesh Refinement (AMR)

CHALLENGE: POPULAR METHODS HAVE PROBLEMS

Rayleigh-Taylor instability
(AMR, 256^2)



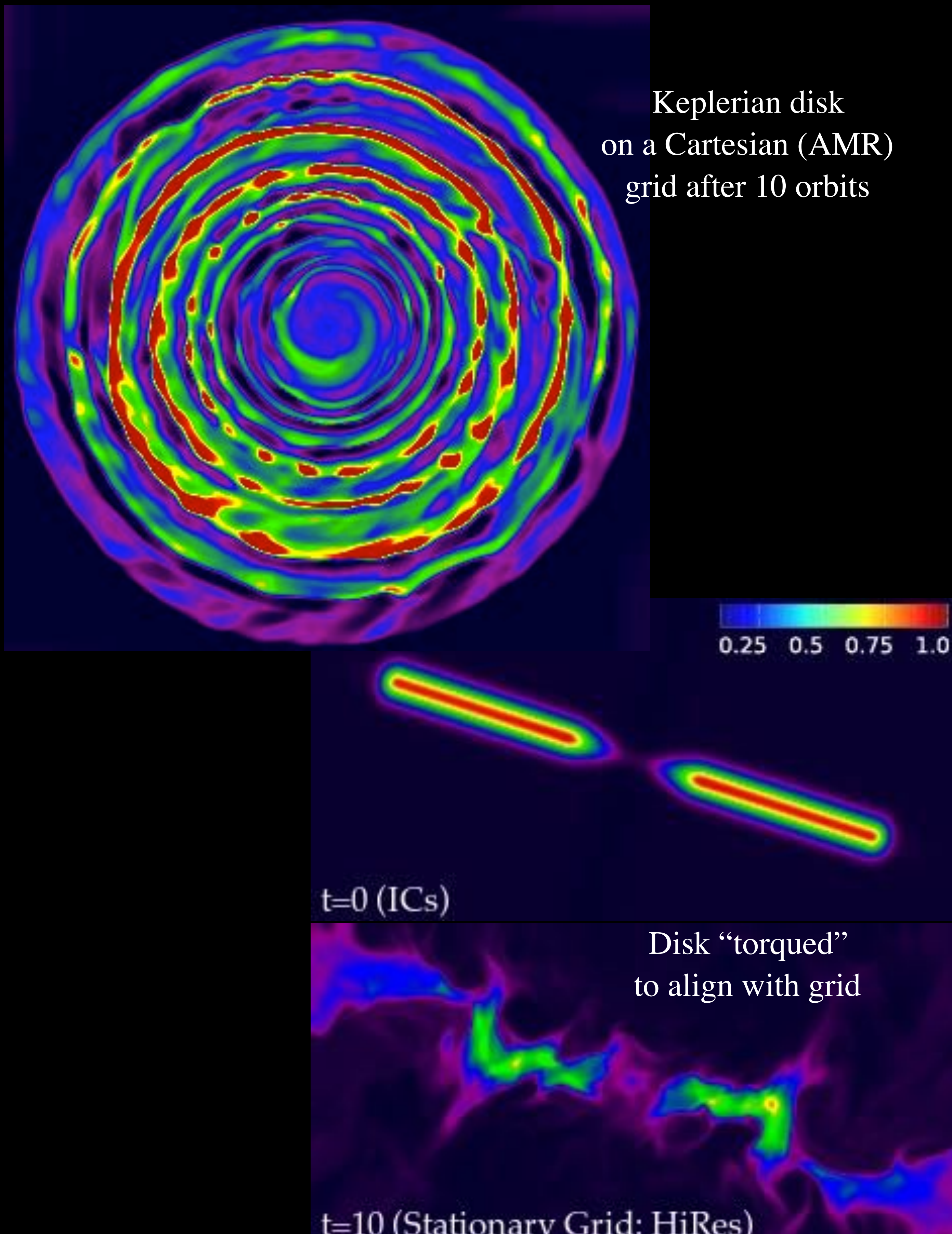
(no bulk motion) | Mach 5 boost

- Eulerian, well-studied, high-order
- Excessive mixing/diffusion when fluid moves over cells

Bryan 95, Wadsley 08,
Tasker & Bryan 08, Springel 10

Adaptive Mesh Refinement (AMR)

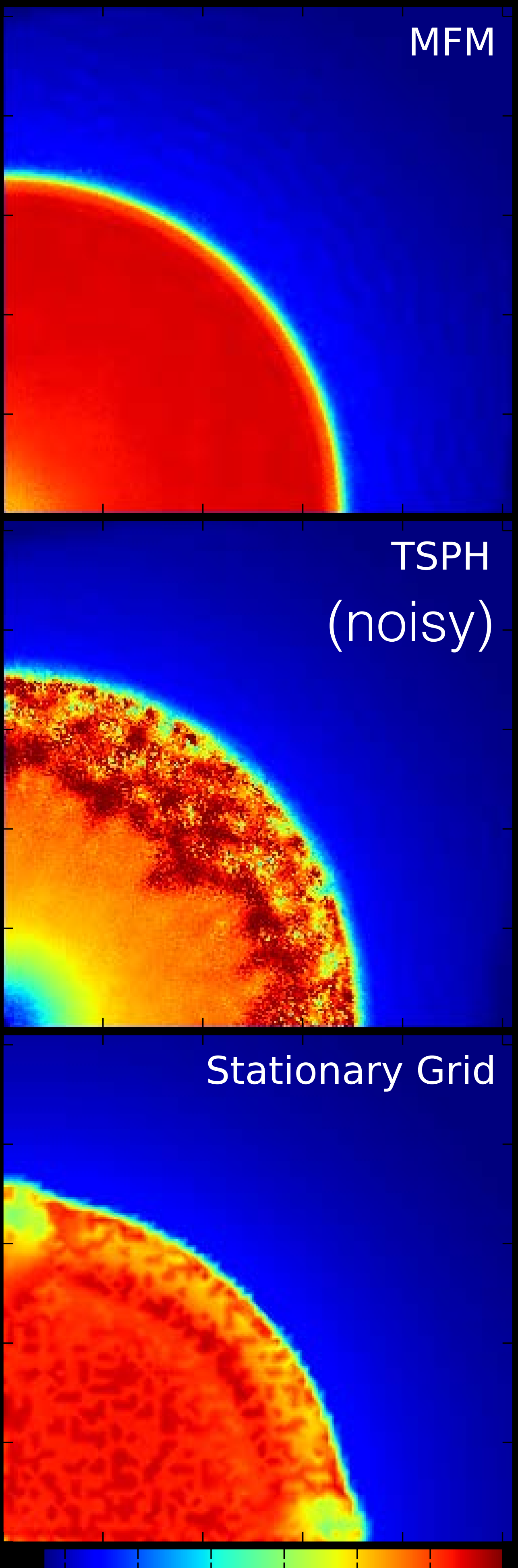
CHALLENGE: POPULAR METHODS HAVE PROBLEMS



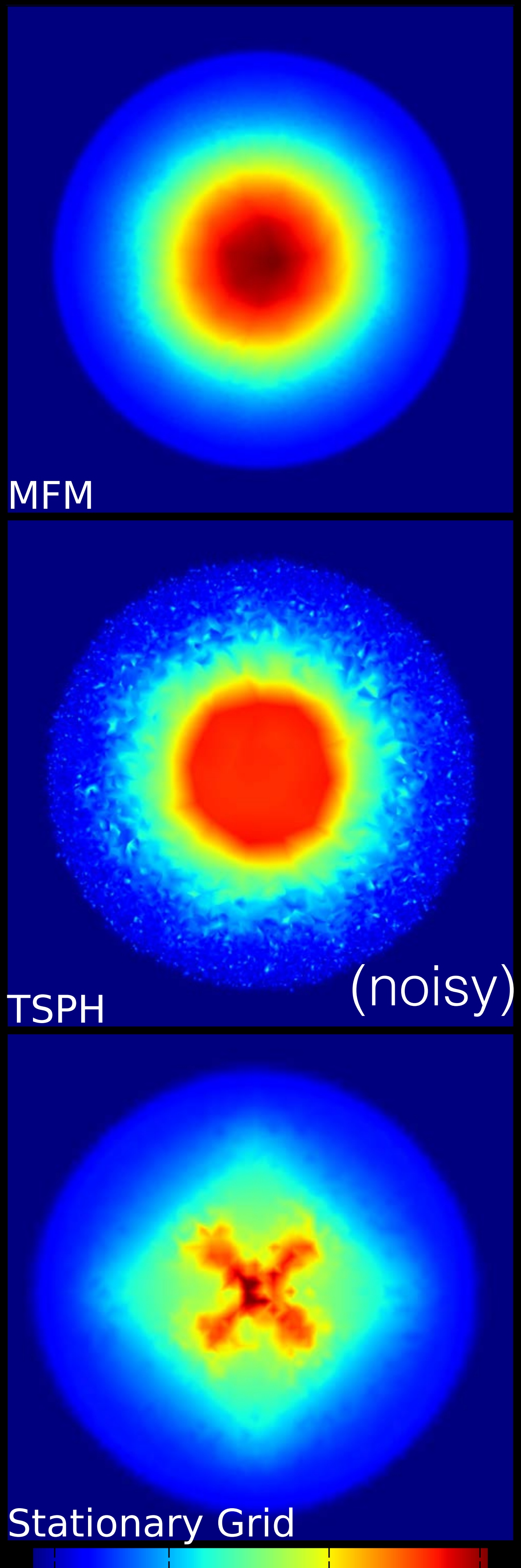
- Eulerian, well-studied, high-order
- Excessive mixing/diffusion
  when fluid moves over cells
- Geometric effects:
  - carbuncle instability (shocks)
  - loss of angular momentum
  - grid-alignment (disks)
- Also “beaten down” with resolution, but *expensive*
  - Hahn ’10: >>512^2 resolution to avoid grid-alignment

Peery & Imlay 88,
Mueller & Steinmetz 95, Hahn 10

Noh Implosion



Sedov Explosion



(grid breaks what should be spherical symmetry)

New Hybrid Lagrangian Godunov Methods (MFM/MFV/MMM)

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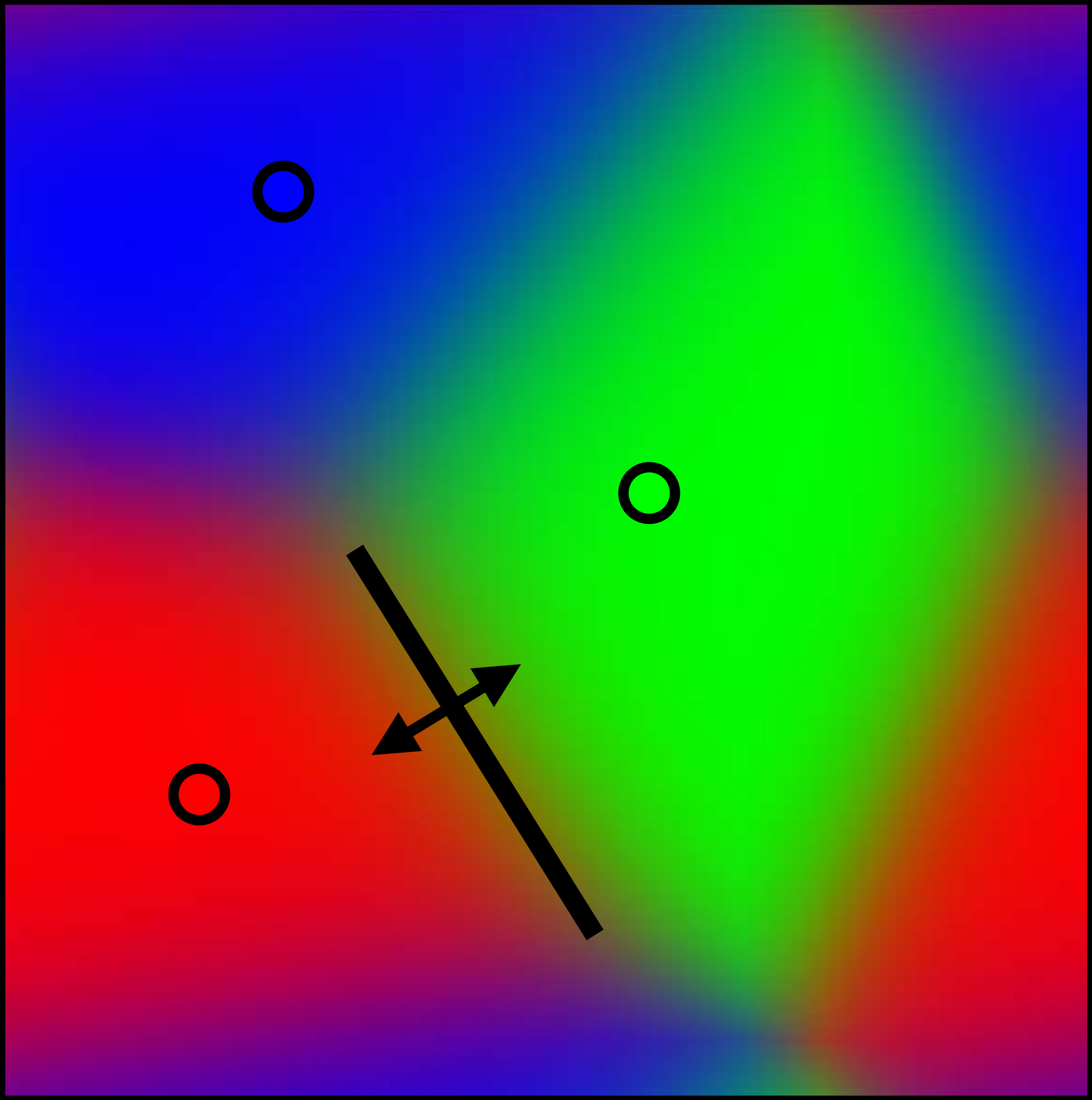


New Methods Combine (some) Advantages of Both

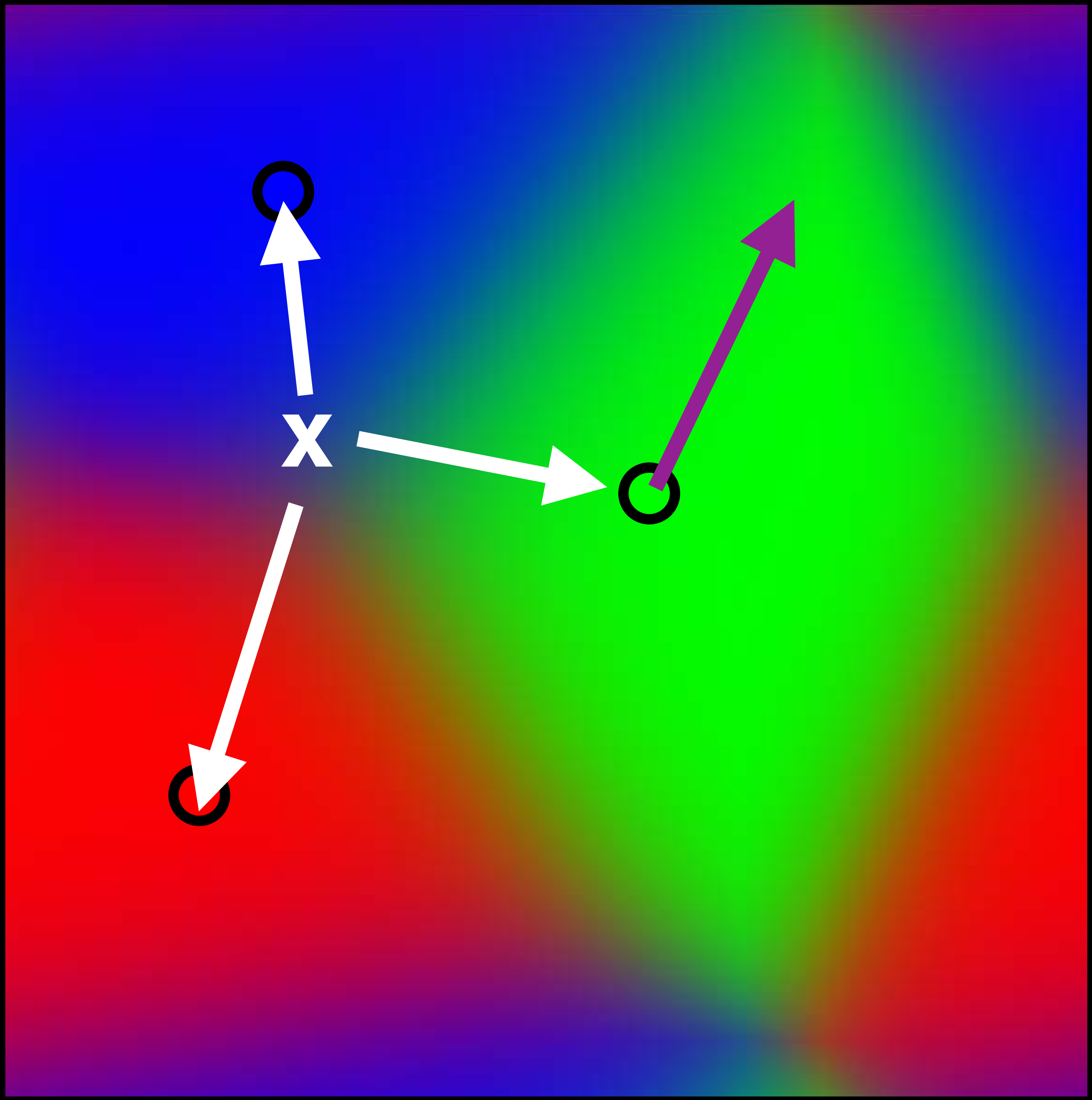
- Moving-meshes (AREPO), meshless finite-volume (GIZMO), high-order ALE methods
- Move with flow, no preferred geometry, but also accurate, high-order, and shock-capturing
- Less well-tested **!**

AREPO: Springel 2010
TESS/DISCO: Duffel 2011
FVMHD3D: Gaburov 2012
GIZMO: Hopkins 2015 (arXiv:1409.7395)

Lanson & Vila 2008
Gaburov & Nitadori 2011
PFH 2014, 2015, 2016

Challenge:

POPULAR METHODS FOR HYDRODYNAMICS HAVE PROBLEMS



- Mesh-generating points move (if desired)
- Volume is “partitioned” with a continuous kernel (MFM/MFV) or step function (moving-mesh)

$$d\mathrm{Vol}_{i,j,k} = d^3\mathbf{x}\,\frac{W(\mathbf{x}-\mathbf{x}_{i,j,k})}{\sum W_{i,j,k}}$$

Lanson & Vila 2008
Gaburov & Nitadori 2011
PFH 2014, 2015, 2016

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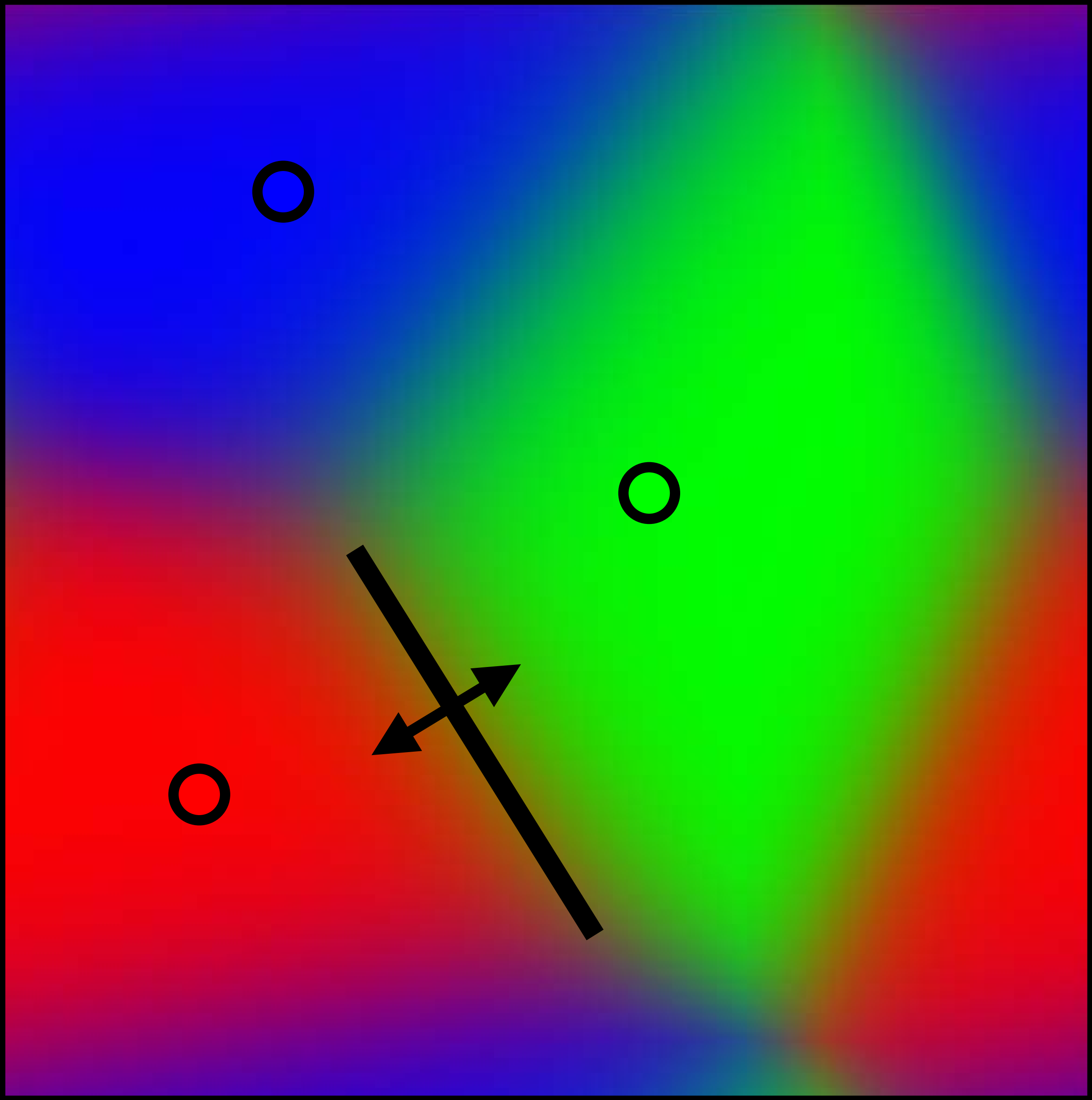


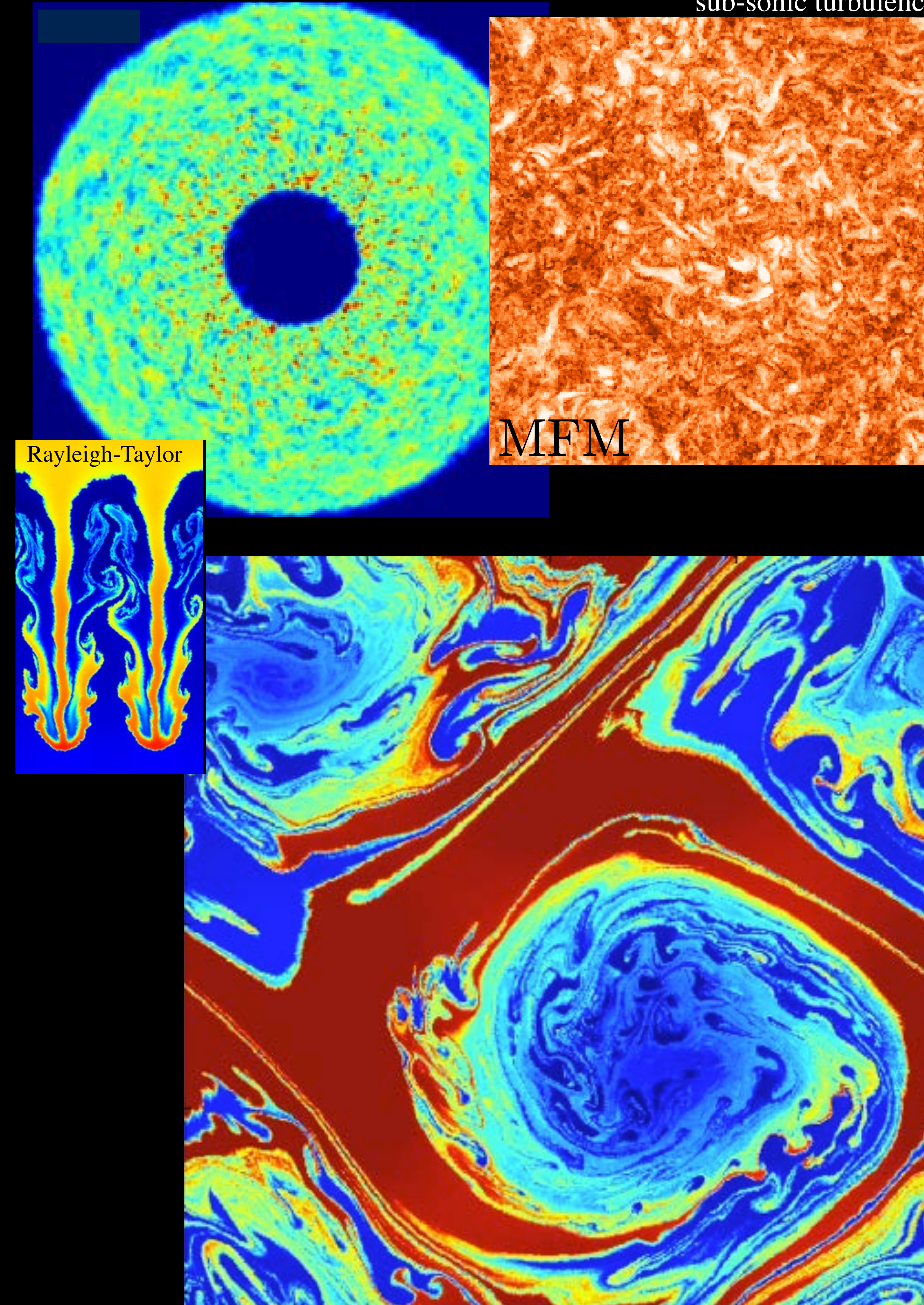
- Integrate EOM over volume: equivalent to Reimann problem at "effective face" (quadrature)

$$\Delta m_i = \int_{\rm vol} \frac{\partial \rho}{\partial t} d^3\mathbf{x} = -\int_{\rm vol} \nabla \cdot (\rho\, \mathbf{v})\, d^3\mathbf{x}$$

New Methods Combine (some) Advantages of Both:

(BUT REMAIN LESS WELL-TESTED)

- Moving-meshes (AREPO), meshless finite-volume (GIZMO), high-order ALE methods
- Move with flow, no preferred geometry, but also accurate, high-order, and shock-capturing
- Grid noise is more severe



AREPO: Springel 2010
TESS/DISCO: Duffel 2011
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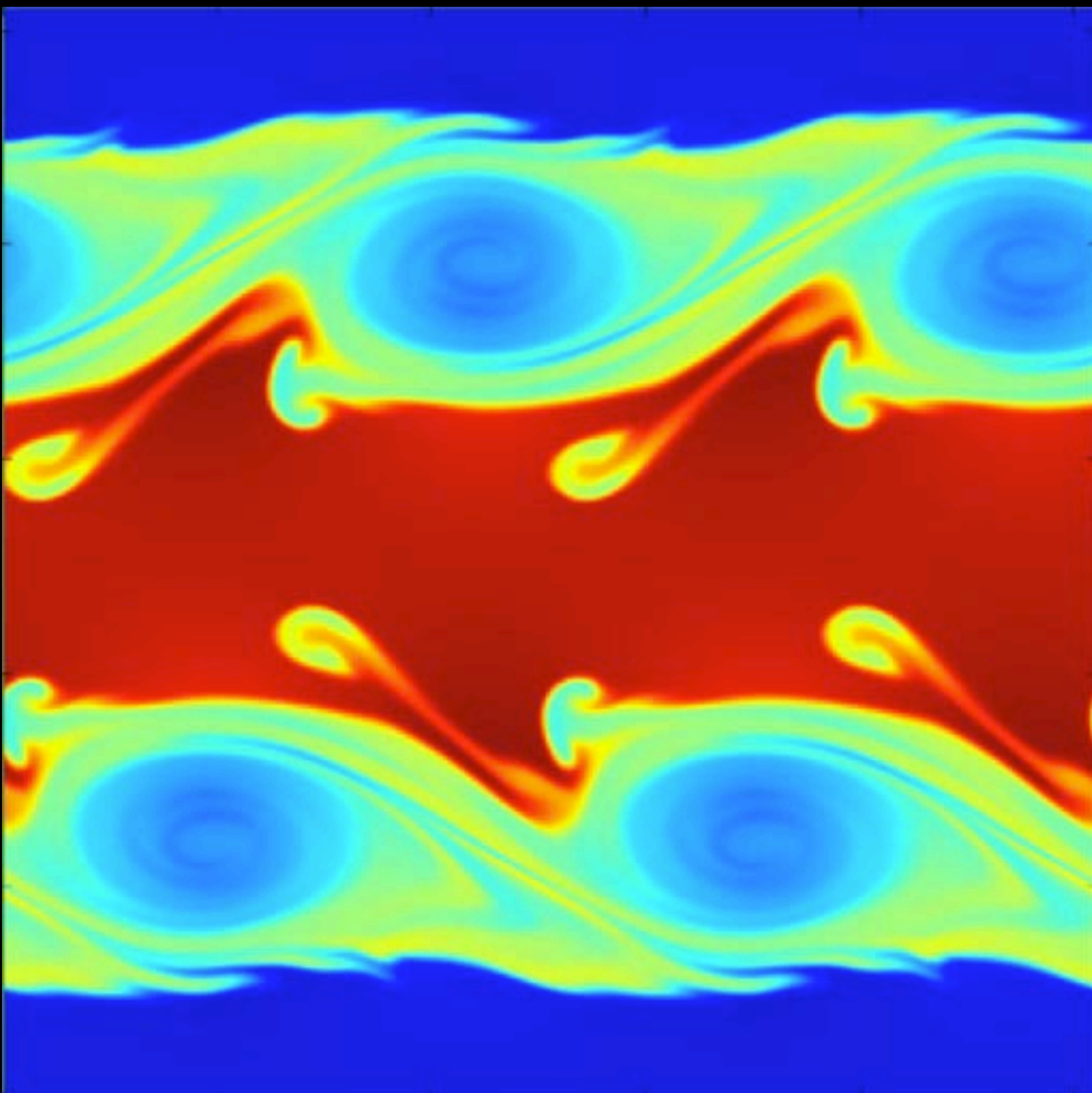
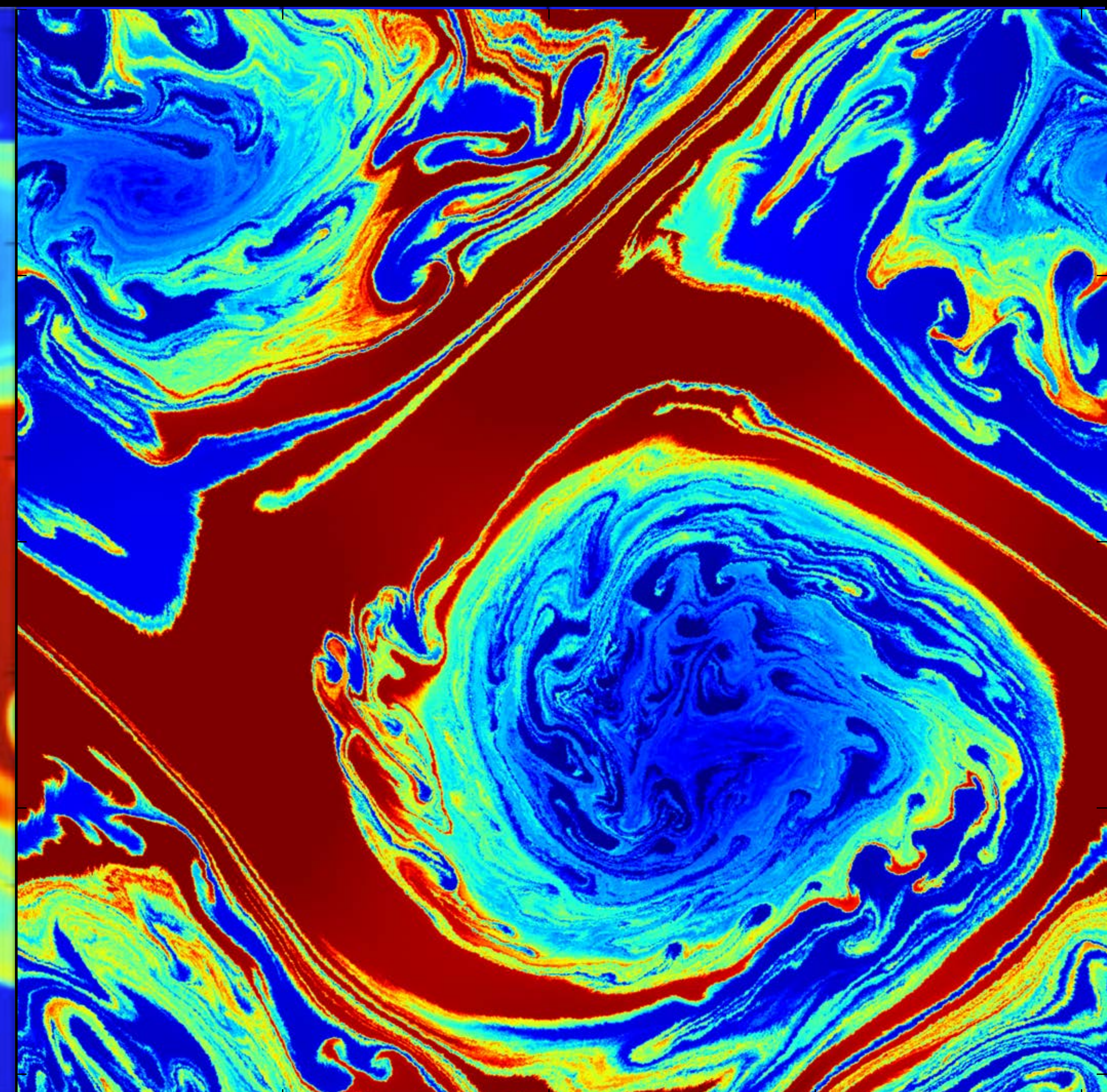
GIZMO: Hopkins 2015

GIZMO: New Meshless Methods & Fluid Mixing

(www.tapir.caltech.edu/~phopkins)



Cartesian Grid



Meshless Finite Volume

Hopkins 2015 (arXiv:1409.7395)

Summary

SPH:

Neighbor number: is it worth it?

Hot halos: numerical mixing

MHD: need *even more* neighbors

Anisotropic diffusion: fundamental barriers

Moving-mesh, meshless Godunov:

"Grid noise": Mach <~ 0.01 problems

MHD: div-cleaning corrupts weak-field ($v_A < 0.01 * v_{turb,thermal}$)

AMR:

Angular momentum/grid alignment