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# Stochastic Corn Yield Response Functions to Nitrogen for Corn after Corn, Corn after Cotton, and Corn after Soybeans

**Christopher N. Boyer, James A. Larson, Roland K. Roberts, Angela T. McClure, Donald D. Tyler, and Vivian Zhou**

Deterministic and stochastic yield response plateau functions were estimated to determine the expected profit-maximizing nitrogen rates, yields, and net returns for corn grown after corn, cotton, and soybeans. The stochastic response functions were more appropriate than their deterministic counterparts, and the linear response stochastic plateau described the data the best. The profit-maximizing nitrogen rates were similar for corn after corn, cotton, and soybeans, but relative to corn after corn, the expected corn yield plateaus increased by 12% and 16% after cotton and soybeans, respectively. Expected net returns increased for corn after cotton and soybeans relative to corn after corn.

*Key Words:* corn, linear response stochastic plateau, net returns, nitrogen, quadratic response stochastic plateau

**JEL Classifications:** C12, D24, Q12

Rotating corn with various crops has long been understood to increase corn yields and can reduce optimal nitrogen (N) fertilization rates relative to continuous corn production (Bullock, 1992). These yield increases are commonly referred to as corn "rotation effects" (Bullock, 1992). The vast majority of the corn rotation literature has focused in the Corn Belt area of the United States, where rotating corn with soybeans, oats, wheat, and alfalfa has been a common practice for years. For example, experimental studies have found that corn grown after soybeans has 3–20% higher yields than continuous corn in this area (Bullock and Bullock, 1994; Crookston et al., 1991; Hennessy, 2006; Lauer, Porter, and Oplinger, 1997; Lund, Carter, and Oplinger, 1993; Mallarino, Ortiz-Torres, and Pecinovsky, 2004; Meese et al., 1991; Pedersen and Lauer, 2002, 2003; Peterson and Varvel, 1989; Pikul, Hammack, and Riedell, 2005; Ruffo, Bullock, and Bollero, 2004; Singer, Chase, and Karlen, 2003; Stanger, Lauer, and Chavas, 2008; Varvel and Wilhelm, 2003; Wilhelm and Wortmann, 2004). The reduction in optimal N fertilization rates as a result of rotating corn after soybeans in the Corn Belt

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has ranged widely but has been as high as 60 lb/acre (Bullock and Bullock, 1994; Chase and Duffy, 1991; Hennessy, 2006; Kanwar, Colvin, and Karlen, 1997; Paulson and Babcock, 2010).

The effects of growing corn after soybeans and cotton on optimal N fertilization rates, yields, and net returns for corn are not well documented for the mid-South region of the United States (Alabama, Kentucky, Louisiana, Mississippi, and Tennessee). Edwards, Thurlow, and Eason (1988) showed corn yields grown after soybeans to range from 0–12% higher than continuous corn yields in Alabama. Howard, Chambers, and Lessman (1998) found corn yields after soybeans to be 10% higher than continuous corn yields in Tennessee. Cotton is a commonly grown crop in the mid-South that is rotated with corn to use corn herbicides to help control weeds in future cotton crops (Reddy et al., 2006). Reddy et al. (2006) compared 5 years of continuous corn to corn after cotton in Mississippi and discovered that corn grown after cotton had 1–13% higher yields than continuous corn. The aforementioned studies provide insight into the rotation effects on corn yields in the mid-South, but they did not evaluate the changes in the optimal N fertilization rates and net returns for each corn rotation. We are not aware of any studies that have evaluated corn yield response to N fertilizer, the profit-maximizing N fertilization rate, and expected net returns in the mid-South for corn grown after soybeans and cotton.

Yield gains for corn rotations have been attributed to reductions in insects, weeds, and diseases when pest control was suboptimal in previous years (Bullock, 1992; Varvel and Wilhelm, 2003). Reductions in optimal N rates were usually the result of residual soil N, especially if a legume such as soybeans was grown before corn (Hennessy, 2006; Varvel and Wilhelm, 2003). Along with the agronomic benefits, crop rotations can enhance water quality (Wu et al., 2004) and ecosystem diversity (Batra, 1982) by reducing chemical and fertilizer use. Consequently, the agronomic and environmental benefits from rotating corn with soybeans, oats, wheat, and alfalfa can result in economic gains for producers (Hennessy, 2006; Singer, Chase, and Karlen, 2003; Stanger, Lauer, and Chavas, 2008). Hennessy (2006) found that N fertilizer decreased and yield increased in the first year of corn after soybeans in Iowa. However, the corn yield gains and reduced N fertilization effects did not carry over more than one year in rotations using soybeans followed by two or more years of corn. Hennessy (2006) recommended using stochastic response functions in future research to capture weather-dependent nonmarket inputs in determining changes in yield and profit-maximizing N fertilizer rates from crop rotations.

Researchers have found the quadratic response plateau (QRP, also called quadratic-plus-plateau in the literature) to be the most suitable response function for modeling corn yield response to N (Bullock and Bullock, 1994; Cerrato and Blackmer, 1990; Roberts et al., 2002). Bullock and Bullock (1994) compared a QRP and a quadratic function and determined the QRP to be the more appropriate response function. Cerrato and Blackmer (1990) also found the QRP to describe corn yield response to N fertilizer better than the linear response plateau (LRP), quadratic, exponential, and square root functions. Recently, crop yield response functions to N such as the LRP have been modified to allow certain parameter estimates to be stochastic (Tembo et al., 2008). Tembo et al. (2008) extended the conventional LRP developed by Berck and Helfand (1990) and Paris (1992) by incorporating a normally distributed year random effect in the plateau. Tembo et al.'s (2008) linear response stochastic plateau (LRSP) function assumes yield responds linearly to N fertilizer until yield reaches a plateau (or where N is no longer a limiting input). They emphasize the effect of stochastic events such as insects, disease, and weather on crop yield response to N by including a plateau year random effect. Tembo et al.'s (2008) function was found more suitable than deterministic response functions for wheat (Biermacher et al., 2009; Boyer et al., 2012a; Roberts et al., 2011), wheat forage (Taylor et al., 2010), ryegrass forage (Tumusiime et al., 2011a, 2011b), and switchgrass (Boyer et al., 2012b). However, the LRSP function has not been estimated for corn yield response to N fertilizer and compared with the deterministic LRP.

Given that researchers have found the QRP best describes corn yield response to N, we expanded the QRP by developing a quadratic response stochastic plateau (QRSP) function. A plateau random effect was included in the deterministic QRP model to capture stochastic weather and other random events that affect corn yields, similar to how Tembo et al. (2008) modified the conventional LRP. We compare the QRSP results with the deterministic QRP, and by estimating the QRSP and LRSP functions, we make a unique comparison of stochastic plateau response functions for corn response to N fertilizer. Additionally, we fill a gap in the literature on changes in yield, profit-maximizing N fertilization rates, and net returns for corn grown after corn, cotton, and soybeans in the mid-South.

Our research objective was to determine the rotation effects for corn grown after soybeans and cotton in the mid-South while considering stochastic events. The specific objectives were 1) to compare the stochastic plateau response functions with their deterministic counterparts to determine the most suitable response function to describe the data for continuous corn, corn grown after cotton, and corn grown after soybeans; 2) to calculate the expected profit-maximizing N fertilization rates and yields for continuous corn, corn grown after soybeans, and corn grown after cotton using the most suitable response function; and 3) to analyze expected net returns for corn grown after corn, cotton, and soybeans using the profit-maximizing N fertilization rates and yields. The data were collected from a six-year (2006–2011) no-tillage corn rotation and N fertilization experiment in Tennessee. For each of the six years, corn was grown on plots where corn, cotton, and soybeans had been planted the previous year. The six years of corn data allow us to evaluate the first-year rotation effects on optimal N fertilization rates and corn yields for corn after corn, cotton, and soybeans.

## Profit-Maximizing Nitrogen Rate

Partial budgets are used to calculate the expected net returns for corn after corn, corn grown after cotton, and corn grown after soybean. Following profit-maximization theory, the producer is risk-neutral with the objective of maximizing expected corn net returns (Nicholson, 2005). The producer's objective is expressed as

$$(1) \qquad \max_{x_{itj}} E(\pi_{itj}) = pE(y_{itj}) - rx_{itj}$$

$$s.t.\ \ y_{itj} = F(x_{itj}),\ x_{itj} \geq 0$$

where $E(\pi_{itj})$ is the producer's expected corn net returns in \$/acre for corn grown after crop $i$ in time $t$ for the $j$th N fertilizer rate in lb N/acre; $p$ is expected corn price in \$/bu; $E(y_{itj})$ is the expected corn yield in bu/acre; $r$ is the price of N fertilizer in \$/lb of N; and $x_{itj}$ is the quantity of N fertilizer applied in lb/acre. Yield expectations are calculated using the production function $y_{itj} = F(x_{itj})$. The LRP, LRSP, QRP, and QRSP functions are used to estimate $E(y_{itj})$ for corn grown after corn, corn grown after cotton, and corn grown after soybeans.

The LRP function assumes corn yield increases linearly with additional N until a yield plateau (or knot point) is reached. At the plateau, N is no longer a limiting input, so additional N does not increase yield. The LRP is

$$(2) \qquad y_{itj} = \min(\beta_0 + \beta_1 x_{itj}, \mu) + \varepsilon_{itj},$$

where $y_{itj}$ is the corn yield in bu/acre; $\beta_0$ and $\beta_1$ are the yield response parameters; $x_{itj}$ is the quantity of N applied in lb/acre; $\mu$ is the expected plateau yield parameter in bu/acre; and $\varepsilon_{itj} \sim N(0, \sigma_\varepsilon^2)$ is the random error term. The response function in equation (2) is estimated using the NLMIXED procedure in SAS 9.1 (SAS Institute Inc., 2003). The profit-maximizing N rate is the N rate required to reach the plateau if the marginal value product of N is greater than the marginal factor cost of N (Tembo et al., 2008). Conversely, if the marginal value product of N is less than the marginal factor cost of N, the profit-maximizing N rate is zero (Tembo et al., 2008).

The LRSP function assumes corn yield responds linearly to additional units of N fertilizer until yield reaches its plateau. A random effect variable is included in the plateau to capture stochastic events such as insects, disease, and weather. This response function is specified as

$$(3) \qquad y_{itj} = \min(\beta_0 + \beta_1 x_{itj}, \mu + u_t) + \varepsilon_{itj}$$

where $y_{itj}$ is the corn yield in bu/acre; $\beta_0$ and $\beta_1$ are the yield response parameters; $x_{itj}$ is the quantity of N applied in lb/acre; $\mu$ is the expected plateau yield parameter in bu/acre; $\varepsilon_{itj} \sim N(0, \sigma_\varepsilon^2)$ is the random error term; and $u_t \sim N(0, \sigma_u^2)$ is the year plateau random effect. Independence is assumed across the two stochastic components. Tembo et al. (2008) included an intercept random effect in their LRP and LRSP response functions, which we do not do because enough years of data are not available for the yield response models to converge with a third random effect. Additional years of data are needed for the models to converge with the intercept random effect. Equation (3) is estimated using the NLMIXED procedure in SAS 9.1 (SAS Institute Inc., 2003).

To solve for the expected profit-maximizing N fertilization rate, the estimated response function is substituted into the net returns partial budget (equation [1]), and the first-order condition (first derivative) is solved for the expected profit-maximizing N fertilization rate. Tembo et al. (2008) solved this condition and found the expected profit-maximizing N fertilization rate as

$$(4) \qquad x^* = \frac{1}{\beta_1}(\mu + Z_\alpha \sigma_u - \beta_0)$$

where $Z_\alpha$ is the standard normal probability of $r/(p\beta_1)$ at the $\alpha$ significance level and the expected profit-maximizing yield is calculated by Tembo et al. (2008) as

$$(5) \qquad E(y_{ti}) = (1 - \Phi)a + \Phi(\mu - \frac{\sigma_u \varphi}{\Phi})$$

where $a = \beta_0 + \beta_1 x$; $\Phi = \Phi[a - \mu/\sigma_u]$ is the cumulative normal distribution function; and $\varphi = \varphi[a - \mu/\sigma_u]$ is the standard normal density function.

The QRP yield response function is similar to the conventional quadratic, but a plateau is imposed. The QRP function assumes diminishing marginal physical productivity of yields with increasing N fertilizer until yield reaches a plateau where N is no longer a limiting input. This assumption is different from the LRP that assumes corn yield responds linearly to N fertilizer, which can be a limitation of the LRP model. This function is specified as

$$(6) \qquad \begin{aligned} y_{itj} &= \beta_0 + \beta_1 x_{itj} + \beta_2 x_{itj}^2 && \text{if} \quad x_{itj} < x^k \\ y_{itj} &= \mu && \text{if} \quad x_{itj} \geq x^k \end{aligned}$$

where $y_{itj}$ is the corn yield in bu/acre; $\beta_0$, $\beta_1$, and $\beta_2$ are the yield response parameters; $x_{itj}$ is the quantity of N applied in lb/acre; $x^k$ is the critical value of N (or the amount of N required to reach the plateau); and $\mu$ is the expected plateau yield parameter in bu/acre. This response function is estimated using the NLMIXED procedure in SAS 9.1 (SAS Institute Inc., 2003). Following Bullock and Bullock (1994), Monte Carlo integration is used to numerically solve for the expected profit-maximizing N fertilization rate $x^*$. The NLP procedure in SAS 9.1 (SAS Institute Inc., 2003) is used to solve for $x^*$. Because the QRP has a quadratic term, $x^*$ might occur at a lower yield than the plateau or yield maximum, $x^k$. The expected profit-maximizing N rate is substituted into equation (6) to calculate the expected profit-maximizing yield.

The QRSP is similar to the QRP, but a random effect is included for the plateau to capture annual variability as a result of weather and other stochastic events. This response function is

$$(7) \qquad y_{itj} = \min(\beta_0 + \beta_1 x_{itj} + \beta_2 x_{itj}^2, \mu + u_t) + \varepsilon_{itj}$$

where $y_{itj}$ is the corn yield in bu/acre; $\beta_0$, $\beta_1$, and $\beta_2$ are the yield response parameters; $x_{itj}$ is the quantity of N applied in lb/acre; $\mu$ is the expected plateau yield parameter in bu/acre; $\varepsilon_{itj} \sim N(0, \sigma_\varepsilon^2)$ is the random error term; and $u_t \sim N(0, \sigma_u^2)$ is the year plateau random effect. Independence is assumed across the two stochastic components. Again, an intercept random effect is not included as a result of limited years of data. Equation (7) is estimated using the NLMIXED procedure in SAS 9.1 (SAS Institute Inc., 2003).

Substituting the estimated QRSP into the partial budget (equation [1]), the first-order condition for the expected profit-maximizing N fertilizer rate is

$$(8)\qquad \frac{\partial E(\pi)}{\partial x} = p(\beta_1 + 2\beta_2 x)[1 - \Phi] - r = 0$$

where $\Phi = \Phi(\beta_0 + \beta_1 x + \beta_2 x^2)$is the cumulative normal distribution function. The quadratic formula must be used to solve for $x^*$. Thus, the QRSP function does not have an explicit analytical solution for $x^*$, so Monte Carlo integration is used to numerically solve $x^*$. We randomly draw 10,000 plateau yields using the expected plateau estimate and the standard error of the year plateau random effect and use the NLP procedure in SAS 9.1 (SAS Institute Inc., 2003) to solve for $x^*$. The expected profit-maximizing yield is calculated by substituting $x^*$ into the expected yield function. This function is similar to equation (5), except $a = \beta_0 + \beta_1 x^* + \beta_2 x^{*2}$.

## Empirical Tests

Likelihood ratio (LR) tests were used (Greene, 2008) to select between the deterministic (restricted) and stochastic (unrestricted) response functions with LR = −2 [log-likelihood (restricted) − log-likelihood (unrestricted)]. The critical value of the LR test was distributed chi-squared with one degree of freedom ($\chi_1^2$). If the LR statistic was greater than the critical value, the stochastic response function fit the data better and was chosen as the more suitable response function.

Given that the LRSP (restricted) model is nested in the QRSP (unrestricted) model, the LR test was used to determine the more suitable stochastic response function between the LRSP and QRSP models. The LR statistic was compared with the critical value of $\chi_1^2$. If the LR statistic was greater than the critical value, the QRSP (unrestricted) function fit the data better and was chosen as the more suitable response function.

Once the response function that best describes the data was determined, a joint LR test (McGuirk, Driscoll, and Alwang, 1993) was used to determine if the expected yield plateau, yield response, and intercept parameters were statistically different for corn after corn, corn after cotton, and corn after soybeans. The unrestricted model was a jointly estimated response function using the data for two corn rotations, allowing the parameter estimates to be different between the rotations. The restricted model jointly estimated the response function restricting the parameter estimates to be equal between the rotations. For example, to test if the expected plateaus for corn grown after soybeans ($\mu_{cs}$) and corn grown after corn ($\mu_{cc}$) were statistically different, the restricted model was estimated with the expected plateaus set equal and the null hypothesis was $\mu_{cs} = \mu_{cc}$. If the LR statistic was greater than the critical value of $\chi_1^2$, the null hypothesis was rejected.

## Data

Corn yield data were obtained from a series of crop rotation and N fertilization experiments conducted at the University of Tennessee Milan Research and Education Center at Milan, TN (latitude 35°56′ N, longitude 88°43′ W) from 2006 to 2011. The field used for this experiment was under no-tillage production for over a decade (Yin et al., 2011). The soil type was predominantly Grenada silt loam, which is well suited for corn production in Tennessee. Corn (cultivar Pioneer 33N58) was planted in a 30-inch row spacing under no-tillage in the month of April (Yin et al., 2011). Each plot was 15.09 feet wide, which is equivalent to six rows of corn, and 29.86 feet long. The experimental design was a randomized complete block with four replications. N fertilization treatments were randomly selected into corn after corn, corn after cotton, and corn after soybeans. The annual N fertilizer rates were 0, 55, 110, 165, and 220 lb N/acre in 2006 and 2007. In 2008, an additional N fertilizer rate of 275 lb/acre was added. N fertilizer was uniformly broadcast to the soil surface as ammonium nitrate (34N–0P–0K). All N applications occurred within a week after planting. For each year of the experiment, no-tillage corn was planted into plots that grew corn, cotton, or soybeans in the previous year. Yields for corn grown after cotton and corn grown after soybeans were collected from 2006 to 2011, and yields for corn after corn were collected from 2007 to 2011. Therefore, we analyze the first-year rotation effects on profit-maximizing N fertilization rates and corn yields for corn after corn, cotton, and soybeans. A visual representation of the average corn yield data at various N fertilization rates is shown in Figure 1.

Prices for N and corn were assumed in calculating net returns. Three prices of N were used: \$0.45, \$0.60, and \$0.75/lb. These prices of N were selected from average ammonium nitrate prices for 2006–2011 (United States Department of Agricultural National Agricultural Statistics Service [USDA NASS], 2011b). The price of corn in Tennessee increased from \$2.95/bu to \$6.50/bu between 2006 and 2011 and the average nominal corn price over this time period was \$4.36/bu (USDA NASS, 2011a). Three corn prices were selected to calculate net returns: \$2.95, \$4.36, and \$6.50/bu.

## Results

### *Yield Response Functions*

Parameter estimates for each of the corn rotations using the LRP and the LRSP functions are presented in Table 1, and parameter estimates for each of the corn rotations using the QRP and the QRSP functions are presented in Table 2. For the LRP and the LRSP functions, the parameter estimates were all significant at the 5% probability level. For the QRP and QRSP functions, the parameter estimates were significant at the 5% probability level with the exception of the quadratic parameter estimates for all corn rotations.

For the linear response plateau, the LR statistics comparing the deterministic and stochastic functions were 172.6, 69.2, and 131.8 for corn after corn, corn after cotton, and corn after soybeans, respectively. The LR statistics comparing the QRP and QRSP functions were 171.1, 67.1, and 132.8 for corn after corn, corn after cotton, and corn after soybeans, respectively. These LR statistics were greater than the critical value $\chi^2_{1,0.05} = 3.84$, suggesting that the LRSP functions describe yield response to N better than the LRP functions, and the QRSP functions describe the data better than the QRP functions.

The LR statistics comparing the LRSP and QRSP models were 0.3, 2.1, and 0.8, respectively, for corn after corn, corn after cotton, and corn after soybeans. The test statistics were less than the chi-squared critical value $\chi^2_{1,0.05} = 3.84$, indicating no statistical difference between the two stochastic plateau models at the 5% probability level. This result was expected given the quadratic N fertilizer parameter estimates were not different from zero.

Because the quadratic parameters were not significant for the QRSP functions, the LRSP functions appear to describe yield response best. The LRSP functions were tested for differences in the intercept, yield response, and plateau across the corn rotations. Table 3 presents the LR statistic for the joint tests between the corn rotations. The intercepts, which are the expected yields when zero N is applied, for the three corn rotations were statistically

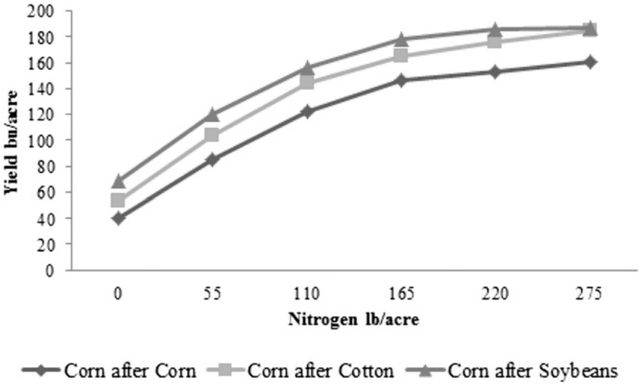

**Figure 1.** Average Yields (bu/acre) for Corn after Corn, Corn after Cotton, and Corn after Soybean by Nitrogen Rate (lb/acre) from 2006 to 2011 at Milan, Tennessee

**Table 1.** Estimated Corn Yield (bu/acre) Response to Nitrogen (N) (lb/acre) after Corn, Cotton, and Soybean Using Deterministic and Stochastic Linear Response Functions

| | Corn after Corn | | Corn after Cotton | | Corn after Soybean | |
|---|---|---|---|---|---|---|
| Parameter | Deterministic | Stochastic | Deterministic | Stochastic | Deterministic | Stochastic |
| Intercept $\beta_0$ | 41.08*** | 39.84*** | 55.08*** | 54.12*** | 71.47*** | 70.08*** |
| | (7.245) | (3.356) | (4.141) | (2.999) | (5.246) | (3.121) |
| N $\beta_1$ | 0.75*** | 0.86*** | 0.83*** | 0.86*** | 0.79*** | 0.87*** |
| | (0.102) | (0.043) | (0.059) | (0.039) | (0.075) | (0.042) |
| Plateau $\mu$ | 150.37*** | 157.93*** | 172.19*** | 177.13*** | 180.32*** | 182.79*** |
| | (4.336) | (2.186) | (2.531) | (2.062) | (3.070) | (1.974) |
| Plateau random | | 961.13*** | | 669.47*** | | 1078.50*** |
| effect $\sigma_u^2$ | | (85.903) | | (116.710) | | (136.41) |
| Random error $\sigma_\varepsilon^2$ | 1259.67*** | 280.21*** | 439.07*** | 276.36*** | 791.51*** | 297.56*** |
| | (158.07) | (35.210) | (57.318) | (32.176) | (89.910) | (33.884) |
| −2 Log-likelihood | 1267.0 | 1094.4 | 1337.7 | 1268.5 | 1474.3 | 1342.5 |

*** Significant at $p = 0.01$; ** significant at $p = 0.05$; * significant at $p = 0.10$.
Note: Standard errors are in parentheses.

different at the 1% probability level ($\chi^2_{1,0.01} = 6.64$). Corn after soybeans had the highest intercept followed by corn after cotton. Corn after corn had the lowest intercept.

Because we do not have soil N uptake for each corn rotation, we cannot directly calculate N use efficiency for the corn rotations. However, we could consider the linear slope parameters as proxies for changes in N use efficiency across the corn rotations. The slope parameters are the ratios of lb N/acre to bu corn/acre or the marginal physical productivity of N. The LR tests indicate no statistical differences in yield response to N for corn planted after corn, soybeans, or cotton (Table 3). Thus, corn response to N does not change when corn, cotton, or soybeans are grown in the previous year, indicating that N use efficiency may not be different among rotations.

The expected plateau yield for corn after cotton was significantly different from the expected plateau for continuous corn (Table 3),

**Table 2.** Estimated Corn Yield (bu/acre) Response to Nitrogen (N) (lb/acre) after Corn, Cotton, and Soybean Using Deterministic and Stochastic Quadratic Response Functions

| | Corn after Corn | | Corn after Cotton | | Corn after Soybean | |
|---|---|---|---|---|---|---|
| Parameter | Deterministic | Stochastic | Deterministic | Stochastic | Deterministic | Stochastic |
| Intercept $\beta_0$ | 39.40*** | 40.07*** | 53.47*** | 53.34*** | 70.24*** | 69.37*** |
| | (7.695) | (3.598) | (4.521) | (3.343) | (6.278) | (3.509) |
| N $\beta_1$ | 0.96*** | 0.90*** | 1.00*** | 0.97*** | 0.99*** | 0.92*** |
| | (0.223) | (0.118) | (0.210) | (0.120) | (0.183) | (0.120) |
| N squared $\beta_2$ | −0.0019 | −0.0006 | −0.0016 | −0.0009 | −0.0021* | −0.0004 |
| | (0.001) | (0.001) | (0.002) | (0.001) | (0.001) | (0.001) |
| Plateau $\mu$ | 153.98*** | 158.77*** | 172.19*** | 167.66*** | 182.67*** | 177.99*** |
| | (5.450) | (2.299) | (2.531) | (2.822) | (4.463) | (1.874) |
| Plateau random | | 936.90*** | | 607.91*** | | 1055.31*** |
| effect $\sigma_u^2$ | | (83.49) | | (139.41) | | (132.54) |
| Random error $\sigma_\varepsilon^2$ | 1247.33*** | 268.76 | 490.54*** | 272.70*** | 994.34*** | 305.57*** |
| | (156.53) | (83.49) | (57.024) | (32.526) | (150.68) | (34.717) |
| −2 Log-likelihood | 1265.8 | 1094.7 | 1337.7 | 1270.6 | 1476.1 | 1343.3 |

*** Significant at $p = 0.01$; ** significant at $p = 0.05$; * significant at $p = 0.10$.
Note: Standard errors are in parentheses.

**Table 3.** Likelihood Ratio Test Statistics for the Estimated Intercept, Slope, and Plateau Parameters of the Linear Response Stochastic Plateau Functions for Corn after Corn, Cotton, and Soybeans

| | Likelihood Ratio Statistic | | |
|---|---|---|---|
| Parameter Estimate | Corn after Corn versus Corn after Cotton | Corn after Corn versus Corn after Soybean | Corn after Cotton versus Corn after Soybean |
| Intercept $\beta_0$ | 10.7*** | 36.7*** | 12.1*** |
| Slope $\beta_1$ | 0.0 | 0.0 | 0.1 |
| Plateau $\mu$ | 9.2*** | 8.5*** | 2.9* |

*** Significant at $p = 0.01$; ** significant at $p = 0.05$; * significant at $p = 0.10$.
Note: The critical value of the test statistic are $\chi^2_{1,0.1} = 2.71$, $\chi^2_{1,0.05} = 3.84$, $\chi^2_{1,0.01} = 6.64$.

producing 19 bu/acre (or 12%) more yield than continuous corn. The yield increase for corn grown after cotton was within the range found by Reddy et al. (2006) in Mississippi at a fixed N fertilizer rate. The expected plateau yield for corn grown after soybeans was significantly different from the expected plateau for continuous corn (Table 3). The expected plateau yield for corn after soybeans increased by 25 bu/acre (or 16%) relative to continuous corn. Howard, Chambers, and Lessman (1998) found corn grown after soybeans to have 6% higher yields than continuous corn yields in Tennessee at a fixed N fertilizer rate. Improved corn varieties might explain why our yield boost for this rotation is larger than what they observed. The expected plateaus for corn after soybeans and corn after cotton were not statistically different at the 5% probability level, but they were different at the 10% probability level ($\chi^2_{1,0.1} = 2.71$) (Table 3). At the latter probability level, the expected plateau corn yield was six bu/acre (or 3%) higher for corn grown after soybeans than for corn grown after cotton. The first-year yield gains for corn grown after soybeans and corn grown after cotton have never been directly compared in the mid-South, although these are common rotations among mid-South crop producers. We find the expected plateau is higher for corn after soybeans than continuous corn and the expected plateau for corn after cotton is higher than continuous corn. These results show the agronomic benefits of corn rotations with cotton and soybeans, but they do not consider how corn and N prices affect expected yields. The plateau random effect variable represents the variance in the plateau; thus, the greater the plateau random effect, the more plateau variability is measured.

### *Profit-Maximizing Nitrogen Rates, Yields, and Net Returns*

The results in Table 4 are conditional on the LRSP parameter estimates and show the sensitivity of the expected profit-maximizing N fertilization rates, expected yields, and expected net returns to price changes for each corn rotation. The expected profit-maximizing N fertilization rates for corn grown after corn, corn grown after cotton, and corn grown after soybeans ranged from 156 to 189 lb/acre, 159 to 186 lb/acre, and 150 to 184 lb/acre, respectively. Standard errors for the expected profit-maximizing N fertilization rates were calculated using the delta method (Greene, 2008, p. 69) and used to build confidence intervals. The 95% confidence intervals for the expected profit-maximizing N rates overlap, indicating no difference in economically optimal N rates across rotations. On average, the reduction in profit-maximizing N fertilization rate for corn rotated after soybeans is five to six pounds per acre relative to corn after corn and two to nine pounds per acre relative to corn after cotton, depending on the price of N and corn. Results from studies in the Corn Belt found a reduction in optimal N rates from rotating corn with soybeans to range from zero to 60 lb/acre (Bullock and Bullock, 1994; Chase and Duffy, 1991; Hennessy, 2006; Kanwar, Colvin, and Karlen, 1997; Paulson and Babcock, 2010). The warmer climate conditions and different soil types in the mid-South likely result in

**Table 4.** Expected Profit-maximizing Nitrogen (N) Rates (lb/acre), Corn Yields (bu/acre), and Net Returns ($/acre) for Corn after Corn, Cotton, and Soybean Using the Linear Response Stochastic Plateau Functions

| | Corn Price | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Corn after Corn | | | Corn after Cotton | | | Corn after Soybeans | | |
| Nitrogen Price | $2.95 | $4.36 | $6.50 | $2.95 | $4.36 | $6.50 | $2.95 | $4.36 | $6.50 |
| $0.45 | | | | | | | | | |
| Profit-maximizing N rate (lb/acre) | 170 | 180 | 189 | 170 | 179 | 186 | 164 | 175 | 184 |
| Profit-maximizing yield (bu/acre) | 155 | 156 | 157 | 174 | 175 | 176 | 180 | 181 | 182 |
| Net returns ($/acre) | $379 | $599 | $933 | $438 | $685 | $1060 | $455 | $709 | $1097 |
| $0.60 | | | | | | | | | |
| Profit-maximizing N rate (lb/acre) | 163 | 173 | 182 | 164 | 172 | 180 | 156 | 167 | 177 |
| Profit-maximizing yield (bu/acre) | 154 | 155 | 156 | 174 | 175 | 176 | 178 | 180 | 181 |
| Net returns ($/acre) | $355 | $574 | $905 | $413 | $659 | $1033 | $431 | $685 | $1070 |
| $0.75 | | | | | | | | | |
| Profit-maximizing N rate (lb/acre) | 156 | 167 | 177 | 159 | 168 | 176 | 150 | 161 | 172 |
| Profit-maximizing yield (bu/acre) | 152 | 154 | 156 | 173 | 174 | 175 | 176 | 178 | 180 |
| Net returns ($/acre) | $330 | $547 | $878 | $389 | $633 | $1007 | $408 | $659 | $1043 |

less residual N remaining in the soil from year to year; therefore, optimal N rates for corn do not decrease when grown after soybeans or cotton. Furthermore, some differences in results might be explained by our use of different production functions than in previous literature.

The expected yields for the corn after corn, corn after cotton, and corn after soybeans ranged from 152 to 157 bu/acre, 173 to 176 bu/acre, and 176 to 182 bu/acre, respectively (Table 4). Substituting the expected yields and expected profit-maximizing N fertilizer rates in Table 4 from the LRSP into the profit equation (equation [1]) gives expected net returns ranging from $330–933/acre for corn after corn, $389–1060/acre for corn after cotton, and $408–1097/acre for corn after soybeans (Table 4). Relative to continuous corn, a producer's expected net returns increased by rotating corn with soybeans and cotton under all the price scenarios (Table 4). Economic gains from corn after cotton and soybeans were mainly the result of yield gains given that expected profit-maximizing N fertilization rates were similar across rotations and price scenarios. Growing corn after cotton increased expected net returns for corn by $85/acre relative to corn after corn at the average corn price ($4.36/bu) and the average N price ($0.60/lb). At the average corn price ($4.36/bu) and the average N price ($0.60/lb), expected net returns for corn increased by $111/acre when grown after soybeans relative to corn grown after corn. Several studies in the Corn Belt found corn grown after soybeans to increase profitability relative to continuous corn (Hennessy, 2006; Singer, Chase, and Karlen, 2003; Stanger, Lauer, and Chavas, 2008).

The expected profit-maximizing N fertilization rates, expected yield, and net returns were also calculated for the QRSP functions and compared with the results from the LRSP functions (Table 5). The same three N fertilizer prices and three corn prices show the sensitivity of the expected profit-maximizing N fertilization rate, expected yields, and expected net returns across the corn rotations using the QRSP. The results in Table 5 are conditional on parameter estimates of the QRSP, price of N, and price of corn. The expected profit-maximizing N fertilization rates were 161–202 lb/acre for corn after corn, 146–179 lb/acre for corn after cotton, and 143–180 lb/acre for corn after soybeans. Again, the expected profit-maximizing N fertilization rates estimated using the QRSP were not statistically different across rotations, similar to the LRSP results.

Expected yields using the QRSP model results at the expected profit-maximizing N

**Table 5.** Expected Profit-maximizing Nitrogen (N) Rates (lb/acre), Corn Yields (bu/acre), and Net Returns ($/acre) for Corn after Corn, Cotton, and Soybeans Using the Quadratic Response Stochastic Plateau Functions

| | Corn Price | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Corn after Corn | | | Corn after Cotton | | | Corn after Soybeans | | |
| Nitrogen Price | $2.95 | $4.36 | $6.50 | $2.95 | $4.36 | $6.50 | $2.95 | $4.36 | $6.50 |
| $0.45 | | | | | | | | | |
| Profit-maximizing N rate (lb/acre) | 179 | 190 | 202 | 160 | 170 | 179 | 159 | 169 | 180 |
| Profit-maximizing yield (bu/acre) | 155 | 156 | 157 | 164 | 165 | 166 | 174 | 176 | 177 |
| Net returns ($/acre) | $375 | $595 | $930 | $413 | $645 | $1000 | $442 | $689 | $1066 |
| $0.60 | | | | | | | | | |
| Profit-maximizing N rate (lb/acre) | 169 | 181 | 194 | 153 | 163 | 172 | 151 | 162 | 173 |
| Profit-maximizing yield (bu/acre) | 153 | 155 | 156 | 163 | 165 | 166 | 174 | 175 | 176 |
| Net returns ($/acre) | $349 | $559 | $900 | $389 | $621 | $974 | $419 | $665 | $1040 |
| $0.75 | | | | | | | | | |
| Profit-maximizing N rate (lb/acre) | 161 | 175 | 187 | 146 | 157 | 167 | 143 | 155 | 167 |
| Profit-maximizing yield (bu/acre) | 151 | 154 | 156 | 162 | 164 | 165 | 172 | 174 | 175 |
| Net returns ($/acre) | $325 | $540 | $872 | $367 | $596 | $948 | $397 | $641 | $1014 |

fertilization rates were 151–157 bu/acre for corn after corn, 162–166 bu/acre for corn after cotton, and 172–177 bu/acre for corn after soybeans (Table 5). Using the results from the QRSP, the range of profit-maximizing net returns were $325–$930/acre, $367–1000/acre, and $397–1066/acre for corn after corn, corn after cotton, and corn after soybeans, respectively. Expected net returns relative to corn after corn at the average corn price ($4.36/bu) and the average N price ($0.60/lb) increased $62/acre and $107/acre by growing corn after cotton and soybeans, respectively. The results for the QRSP were similar to the LRSP results.

## Conclusions

The objectives of this research were to compare stochastic plateau response functions with their deterministic counterparts to find the yield response function that described the data the best and to then use this response function to determine expected yields, profit-maximizing N rates, and net returns for corn grown after corn, corn grown after cotton, and corn grown after soybeans. Yield response to N was estimated using the LRP, LRSP, QRP, and QRSP functions. Expected net returns were calculated for the optimal N rates using the stochastic plateau response functions to show the economic benefits from growing corn after corn, cotton, and soybeans. The data were collected from a corn rotation and N fertilization experiment in Tennessee from 2006 to 2011.

The LRSP functions describe corn yield response to N better than the LRP for each corn rotation, and the QRSP functions describe yield response to N better than the QRP functions. The LRSP and the QRSP functions were not statistically different. The LRSP was chosen for the analysis because the quadratic parameters of the QRSP functions were not significant for any of the rotations, suggesting that the LRSP function described yield response to N fertilizer rates the best.

Using the LRSP functions, we found the expected plateau yield for corn after cotton increased by 19 bu/acre (12%) relative to continuous corn, and the expected plateau yield for corn after soybeans increased by 25 bu/acre (16%) relative to continuous corn. Yield response to N was not different across the corn rotations, and the profit-maximizing N rates were similar as well. Relative to continuous corn, a producer's expected net returns increased by growing corn after soybeans and corn after cotton using the LRSP function. Using the QRSP functions, expected net returns relative to corn grown after corn increased by rotating corn after soybeans and corn after cotton. Thus, expected net returns from corn can increase in the first year after shifting from corn grown after corn to growing corn after cotton or soybeans; however, we cannot conclude about the profitability of the continuous corn, corn and cotton, and corn and soybean rotations.

We extend the QRP similar to how Tembo et al. (2008) extended the conventional linear response plateau, making a unique contribution to the literature. Results show the LRSP function describes the corn rotations better than the QRSP function. Previous research found the deterministic QRP more suitable than the LRP to model corn yield response to N (Bullock and Bullock, 1994; Cerrato and Blackmer, 1990; Roberts et al., 2002), but we find the LRSP to be more suitable for our data than the QRSP. Additionally, we show how changes in yield gains and N fertilization rates for corn grown after corn, soybean, and cotton impact the expected net returns in the mid-South region. Changes in yields and optimal N fertilizer rates for corn rotated with soybeans and cotton are not as well understood in the mid-South as in the Corn Belt. We intend to extend these results to producers by updating the University of Tennessee Corn Nitrogen Rate Calculator to include corn rotation effects (University of Tennessee Agricultural and Resource Economics Department, 2013).

As Funk et al. (1999) and Paulson and Babcock (2010) highlighted, risk management is an important component to consider when discussing the benefits of corn rotations. For future research, comparing the different corn rotations using stochastic dominance would be interesting and useful for corn producers in the mid-South. Additional research could focus on applying these stochastic plateau functions to other crops common to the mid-South and analyzing the economics of other crop rotations.

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