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Lanchester's Square Law in Theory and Practice

A Monograph
by

Major Ronald L. Johnson

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School of Advanced Military Studies
United States Army Command and General Staff College
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This monograph conducts a statistical analysis of the National Training Center engagement data to determine to what extent that training adheres to the square law. The monograph discusses the theory of the square law, summarizes the findings of previous verification attempts, discusses the National Training Center as a laboratory for this analysis, presents the results of the statistical analysis and suggests that an exponential law is more appropriate. A discussion follows centered around the reason for this gap between theory and practice. The monograph discusses the problem of suppression as the missing link in the square law. The conclusion drawn is that whenever combat simulations are used, those simulations should account for the problem of suppression or else the results will be distorted. Furthermore, it appears that if combat follows an exponential law then the National Training Center training may come closer to combat than the US Army thinks. (U)

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Major Ronald L. Johnson

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Approved by:

_____ Monograph Director
James J. Schneider, MA

_____ Director, School of
Colonel William H. Janes, MA, MMAS   Advanced Military
                                       Studies

_____ Director, Graduate
Philip J. Brookes, Ph.D.   Degree Program

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ABSTRACT

LANCHESTER'S SQUARE LAW IN THEORY AND PRACTICE by
MAJ Ronald L. Johnson, USA, 50 pages.

Military analysts have employed simple mathematical
models to obtain insights into the dynamics of armed combat
at least since 1914. In that same year, Frederick William
Lanchester proposed his own model of combat dynamics to
illustrate the principle of concentration. Lanchester's famous
square law states that the casualty ratio should vary inversely
as the force ratio. Hence a force which could concentrate to
achieve the "proper" force ratio could inflict a certain amount
of casualties on another force.

Several analyses using historical data have been
conducted. Attempts to verify Lanchester's square law using
historical data have had mixed results. In most cases, the
Lanchester square law has not stood up to empirical scrutiny
and an exponential law has appeared as being more
appropriate. Even though attempts at validation have
repeatedly failed, the modelling communtiy continues to rely
upon the model as a base for other models.

This monograph conducts a statistical analysis of the
National Training Center engagement data to determine to
what extent that training adheres to the square law. The
monograph discusses the theory of the square law, summarizes
the findings of previous verification attempts, discusses the
National Traing Center as the laboratory for this analysis,
presents the results of the statistical analysis and suggests that
an exponential law is more appropriate.

A discussion follows centered around the reason for this
gap between theory and practice. This monograph discusses the
problems of suppression and fire control as missing links in the
square law. The monograph concludes with a proposal that
whenever combat simulations are used, they should account
for the problem of suppression or else the results will be
distorted. Furthermore it appears that if combat follows an
exponential law then the NTC may come closer to combat than
the US Army thinks.

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I. Introduction

> Theory will have fulfilled its main task when it is used
> to analyze the constituent elements of war, to distinguish
> precisely what at first sight seems fused, to explain in
> full the properties of the means employed and to show
> their probable effects, to define clearly the nature of the
> ends in view and to illuminate all phases of warfare in a
> thorough critical inquiry. Theory then becomes a guide ...
> to learn about war... [1]

This passage from Clausewitz expresses the essential role of modelling combat. Military analysts have employed simple mathematical models to obtain insights into the dynamics of armed combat at least since 1914.[2] The model which serves as the standard for others is the one which was proposed by Frederick William Lanchester in 1914. During World War I, Lanchester's original work received little attention. During World War II, the U.S. Army Air Corps gave some consideration to Lanchester's model, however this consideration had no operational impact. Without capabilities to solve more complicated models, Lanchester's model tended to be universally accepted. His models lent themselves to ease of solution.

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[1] Carl von Clausewitz, On War, trans. by Michael Howard and Peter Paret, (Princeton, NJ: University of Princeton Press, 1976), p 141.

[2] Stephen P. Stuk, "Multivariable Systems Theory for Lanchester Type Models", Dissertation Georgia Institute of Technology, 1987, p 1.

With the invention of the digital computer, whole new generations of Lanchester-type differential equation models were developed. These models were developed because Lanchester's model fell short in its ability to capture the many unquantifiable aspects of combat. Even today these models provide the mathematical foundations for analytical investigations and serve as the primary basis for calculating combat attrition in large scale computer simulations.

The utility of these models to military operators and planners in modelling combat attrition is great. Every tactician when planning a military operation must plan so that he has allocated the proper number of resources to accomplish the mission. In combat, this relates to the allocation of men and materiel so that the potential is there to generate the right amount of combat power. Since combat power is a relative concept, the tactician normally looks at a force ratio to determine if he has the potential to accomplish the mission.

At a higher tactical level or at the operational level, the planners may look at a sequence of tactical operations. Sometimes this sequence of tactical operations is accomplished by the same tactical unit and, as a result, the potential combat power throughout the entire sequence of operations becomes important. Thus, the attrition of this unit becomes critical to the operational planner. The planner would like to be able to ascertain whether the unit can accomplish subsequent missions with current means or whether the unit will need

reinforcements. In essence, it would be convenient for the planners to be able to estimate the attrition that will ensue.

Lanchester's model allows such an estimate. The Lanchester square law theoretically estimates attrition under what Lanchester called "modern combat conditions"[3]. There are many examples of attempts to use Lanchester's square law to estimate theoretical attrition so that it can be compared to actual attrition. The attempts made have been a function of data availability and hence, limited. These verifications have been attempted for air, land and sea battles. The success of the verifications have been mixed.

A literature search shows that Lanchester's law serves as a basic foundation for attrition modelling in combat simulations. Therefore, when actual results do not mesh with theory, the implication is that our simulations are not duplicating reality and hence are of little value. Although attempts at verification have generally failed, the model continues to be used in one form or another.

The purpose of this paper is to examine the Lanchester square law's performance in the environment of a contemporary combat-like situation at the U.S. Army's National Training Center. The research question that will be answered is: to what extent does the experience of U.S. Army battalions at the National Training Center confirm or deny the

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[3] The concept of modern combat is explained in Section II of this monograph.

validity of Lanchester's square law? By validity, it is meant the degree which the model and the actual data agree. This agreement between theory and reality gives some experimental confirmation that the square law model is the right model for combat. Likewise disagreement between theory and reality gives some experimental confirmation that the square law model is not the right model for combat.

This monograph will proceed to answer the question by first examining the theory of the square law. Then an analysis of the findings of previous studies is conducted to demonstrate the importance of the model to the modelling community. A discussion of the National Training Center as the data source and an analysis of that data follows. Finally, implications and conclusions are drawn based upon the cumulative analysis of this and other studies. Of particular interest is the problem of suppression.

II. The Square Law and Its Theory

In 1914, Frederick William Lanchester proposed a simple model of combat dynamics to illustrate the principle of concentration. Lanchester's equations of combat, generally referred to as Lanchester's laws, provide algorithms for predicting the dynamics of attrition in a model of combat. Lanchester hypothesized that under conditions of modern warfare combat between two homogeneous forces could be described by the following diagram:

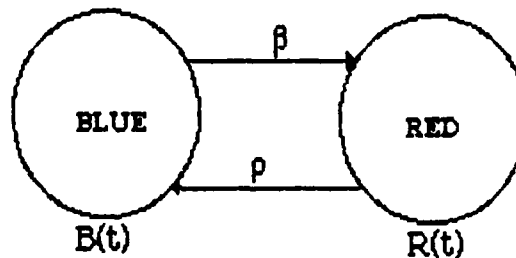Figure 1 - Lanchester's Model of Combat[4]

Before describing this model of combat which Lanchester proposed, it is appropriate to discuss what was meant by homogeneous forces and modern combat.

Forces are said to be homogeneous when they are similar in terms of weapons systems. The weapons systems are

4   James G. Taylor, Lanchester Models of Warfare, Vol I, (Arlington, VA: Ketron Inc, 1983), p.58.

essentially similar if they can accomplish the same effects. In the context of the National Training Center, that presupposes battle between like systems such as tank on tank or TOW on SAGGER, etc. In 1914, Lanchester used these equations to discuss a force-on-force battle with fighter aircraft against fighter aircraft.

When Lanchester wrote about modern combat, he was writing about combat which involved aimed fire. It was envisioned that firing units on both sides knew the locations of their opponents and could shift their fires to a new target when a kill was achieved. Additionally, this shifting was to occur immediately. A fundamental assumption in this process was predicated upon perfect command and control. In the desert environment at the National Training Center, it is quite feasible to achieve this aimed fire. With the use of the training devices, it also is quite clear when a kill is achieved. Hence, it is reasonable to achieve immediate shifting of fires which the model refers to. This shows that the conditions of modern combat and homogeneity of forces are achievable at the National Training Center. Hence, application of the training situation to the model is logical.

When examining Lanchester's combat model as shown in Figure 1 above, one sees a force-on-force interaction. In this diagram, a "red" force, R, acts upon a "blue" force, B in accordance with some effect, call it ρ. Likewise, B acts upon R in accordance with some effect, call it β. These effects are the attrition between forces. The variables which represent these

effects, ρ and β, are called attrition rates as they represent the rates at which reds kill blues and blues kill reds, respectively.[5]

Combination of these concepts and the diagram in Figure 1 allows one to appreciate the equations which Lanchester developed to model these attrition processes. The attrition process for the "blue" force is[6]:

Eqn 1	$dB/dt = -\rho R$ for $B > 0$
	where $dB/dt = 0$ for $B = 0$

Likewise, the attrition process for the "red" force is[7]:

Eqn 2	$dR/dt = -\beta B$ for $R > 0$
	where $dR/dt = 0$ for $R = 0$

Furthermore, the initial conditions which define the state of the forces at the beginning of a battle (i.e at time equal to zero) is given by[8]:

Eqn 3	$B(0) = B_0$

Eqn 4	$R(0) = R_0$

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[5] ibid, p.57.

[6] ibid, p.55.

[7] ibid, p.55.

[8] ibid, p.55.

The numerical strengths of the blue and red forces are defined by the variables B and R, respectively. In addition the constraints[9]:

$$dB/dt = 0 \text{ for } B \le 0 \quad \text{and} \quad dR/dt = 0 \text{ for } R \le 0$$

are established so that no more casualties occur after that force has been annihilated (i.e. when R = 0 or B = 0). It is also clear that R and B can assume no negative values since strength is never a negative number.

The solution to this model, found by integrating equations 1 and 2 with respect to time and imposing the initial conditions, is given by[10]:

$$\text{Eqn 5} \qquad \beta (B_0^2 - B^2) = \rho (R_0^2 - R^2)$$

Since the strengths of the opposing forces appear with exponents of two, the name "square law" is given to that law which the set of equations describes. The square law essentially states that the casualty ratio varies inversely as the force ratio, that is, a force outnumbering an opponent can expect to incur fewer casualties than the weaker opponent.[11]

Based upon Lanchester's notion of aimed fire, the concept for equations 1 and 2 is that each firer on the red side will pick

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[9] ibid, p.55.

[10] ibid, p.61.

[11] ibid, p.61.

targets on the blue side and try to kill it. As long as there are targets on the blue side then the rate of attrition will depend on the number of firers on the red side and that side's attrition rate or effectiveness.

In addition to the homogeneity of forces and aimed fire properties of the model, there are other assumptions as well. Those assumptions are[12]:

1. The forces are within weapons range of one another.
2. The effects of weapons rounds are independent.
3. Fire is uniformly distributed over the enemy targets.
4. Attrition coefficients are constant and known.[13]
5. All of the forces are committed at the beginning and there are no reinforcements.

The direct fire kills achieved at the National Training Center appear to satisfy the conditions of assumptions one and five. The attrition coefficients of the forces may be attributed to the training proficiency. Therefore, one could argue that those coefficients are known. Conditions may be such that assumptions two and three are not met. One would need to assess the combined effects firing multiple training devices at a particular target in order to support or refute the independence assumption. Assessment of unit fire control for

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12 ibid, pp. 159-164.
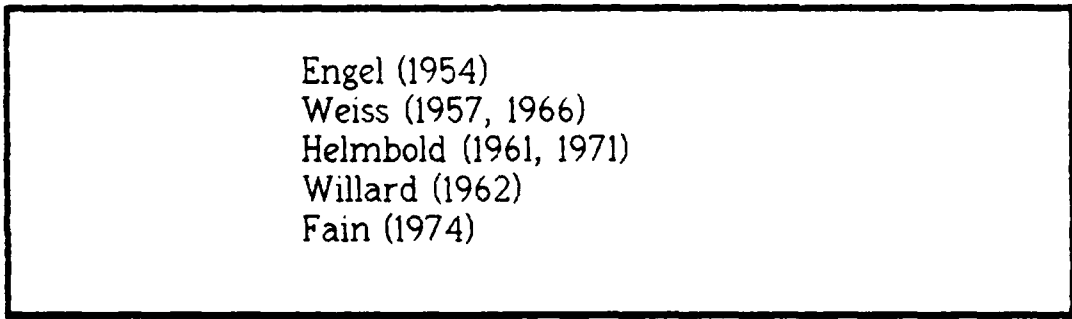
13 This assumption was hypothesized later by Weiss and Dolansky.

each engagement is necessary to support or refute the uniform distribution assumption. Both of these assessments are beyond the scope of this paper.

These assumptions appear to restrict the applicability of Lanchester's model to the National Training Center environment as well as to combat. Combat on today's battlefield is very complex and very different from the type proposed by Lanchester's models. Some of these assumptions no longer seem appropriate in modelling what we term combat under modern conditions. However, it is not necessary that all of the assumptions fit the experimental model perfectly; some deviation is quite acceptable.

III. History and the Application of the Theory of the Square Law

Since Lanchester's original hypothesis, military operations research analysts have employed Lanchester-like equations to model the dynamics of armed combat. In the history of the Lanchester model, seven[14] empirical attempts at verification have been made (see Figure 2) which are appropriate to this study. An assessment of these studies in terms of their impact is now in order.

Engel (1954)
Weiss (1957, 1966)
Helmbold (1961, 1971)
Willard (1962)
Fain (1974)

Figure 2 - Empirical Verification of Lanchester Theory of Combat

Iwo Jima Study by Engel

In 1954, J.H. Engel showed that an attrition model, of the form of Lanchester's, reasonably fit the data from the Battle of Iwo Jima which apparently verified the square law.[15] Engel's

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14 Taylor, pp 115-122.

15 J.H. Engel, "A Verification of Lanchester's Law", Operations Research 2 (1954), pp 163-171.

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work, however, has been a topic of considerable controversy. R Samz points out in his doctoral dissertation that Engel had not demonstrated an adequate statistical relationship between observed and theoretical data.[16] Engel's analysis was weakened by using a set of data to test a model after using that same set of data to estimate the parameters.[17]

Engel's basic approach involved graphing the theoretical data and the empirical data and then superimposing the two plots to determine "goodness of fit". This is not the typical manner in which statisticians show goodness of fit.

Civil War Study by H.K. Weiss

In 1966, when Weiss wrote his article, there was still a great deal of skepticism about the applicability of mathematical models to real combat. Weiss stated in that article that recent disbelief in the utility of the square law was discouraging especially since analysts had been attempting to better define the fog of war. The greatest challenge which he saw was just finding data in the right form so that some analysis could be done. Since the data on the Civil War was readily available, he decided to look at that war. Weiss conducted an analysis of the empirical data to determine the extent to which combat could be explained by Lanchester's model of combat. Additionally

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[16] Robert W. Samz, "Toward A Science of War Through Some Mathematical Concepts of Macrocombat", Dissertation Arizona State University, 1970.

[17] James G. Taylor, Force-on-Force Attrition Modelling, Arlington, VA : Ketron Inc, 1980), p 41.

Weiss wanted to determine the extent to which the equations were empirically valid. Weiss appreciated the criticism of his study as he stated that the Civil War did not fall in the domain of modern battle and so the analysis might not be intrinsically valuable with regard to its application to future wars.

Based upon his analysis, Weiss discovered that Lanchester's model did appear to explain that combat casualties were significantly related to force ratios. In particular, Weiss found that in small battles, high force ratios appeared to be associated with low casualty ratios which was consistent with Lanchester's square law. But in large battles, those with greater than 15,000 troops on both sides, high force ratios tended to be associated with high casualty ratios. In other words, a side tends to lose men in proportion to the number committed to battle and independent of the other side's strength.[18] This latter phenomenon was noticed in earlier studies by other analysts. Weiss noted that the individual units of an apparently homogeneous force are not exactly equal in their capabilities. He also suggested that a force becomes a more vulnerable target as its size increases and simultaneously as its firepower producing ability tapers off. This effect, coined

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[18] Herbert K. Weiss, "Combat Models and Historical Data : The US Civil War", Operations Research 14 (1966), p 759-788.

the exponential decay by Schneider[19] or the logarithmic law by Peterson[20] was not explained by Lanchester's model.

Weiss' study also suggests that combat follows a model that relates attrition to decisionmaking. He states that after the battle begins and as casualties mount on both sides, each side continuously evaluates its ability to continue to fight, using as a single criterion its own cumulative fractional losses.[21] Supposedly, this quantitative tool facilitates decisionmaking concerning when an operational pause might be in order. Unfortunately it is not clear how a tactical commander knows his actual strength or casualty rate at any specific point in time during a battle.

Of particular interest was Weiss' foresight into a problem that would continue to be looked at up to the present day. He states that analysts should seek to examine real data from World War II, Korea, Vietnam and other contemporary conflicts and that such studies should be directed to deriving more fundamental combat relationships than those that deal merely with the probability of hitting or destroying a target. By doing so, we should be able to "substantiate the analytic representation of Principles of War and indicate the prospects

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[19] James J. Schneider, "The Exponential Decay of Armies", Theoretical Paper No. 1, School of Advanced Military Studies, 1985.

[20] Richard H. Peterson, "On the Logarithmic Law of Attrition and Its Application to Tank Combat", Operations Research 15 (1967), p 557-558

[21] Weiss, p 786.

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and uncertainties of such tools" .[22] I hope to contribute to this field of study with this present monograph.

Willard's Study

In 1962, Willard published "Lanchester as A Force in History". This study was an analysis of land battles covering the period 1618 through 1905. Dr. Willard's objective was to determine, by an examination of historical military data, the extent to which Lanchester's equations are an expression of a general property of battle.[23]

Willard conducted an analysis of battles which he separated into two distinct categories - Category I : meeting engagement-type combat and Category II: siege/attack on fortified positions-type combat. Willard's rationale for this categorization is attributed to the form in which the historical data was available.[24] Based upon a generalized form of Lanchester's equations and some mathematical manipulation, Willard devised the equation:

Eqn 6	$\log n_c/m_c = \log E + \gamma \log m_o/n_o$[25]

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[22] Weiss, p 789.

[23] Daniel A. Willard, Lanchester as a Force in History : An Analysis of Land Battles of the Years 1618 - 1905, (McLean, VA : Research Analysis Corporation, 1962), p 1.

[24] ibid, p 9.

[25] ibid, p 11.

where n_c = casualties to BLUFOR
m_c = casualties to REDFOR
n_0 = no of blues at start
m_0 = no of reds at start
E = casualty ratio between forces
γ = a parameter to determine the appropriate law

Willard's equation demonstrates the relationship between force ratios and casualty ratios. The analysis conducted to find gamma and the exchange ratio should give some measure of the extent to which the aforementioned ratios are related.

Dr. Willard hypothesized that a value of $\gamma = 0$ or 1 would be obtained depending upon whether the linear law or the square law, respectively applied to equation 1. Furthermore, he argued that real battle was a series of small frays[26], which may display the characteristics of one of Lanchester's laws. Then he argued that logically, one would suspect that the collective character of these frays would lie somewhere between following a linear and a square law. Thus Willard hypothesized that γ should lie somewhere between the values of 0 and 1.

The result of Willard's analysis was that gamma was found not to lie between 0 and 1, but between -0.27 and -0.87,

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[26] ibid, p 10.

with typical values centered around $\gamma = -0.5$.[27] Willard
expressed some dismay about this result, especially since his
interpretation was that for $\gamma < 0$, and for Lanchester's
formulation being correct, then the casualty producing power
of troops increases as they suffer casualties. Said another way
a smaller force incurs fewer casualties than a larger force.
Willard's conclusions were:

(1) Force ratios have little to do with predicting the
outcome of battle.

(2) Lanchester's square law does a poor job as a
deterministic law to model combat.

Willard's work was also significant in another respect. It
provided a useful methodology with which to evaluate
Lanchester's equations empirically.

Helmbold's Studies

When searching the literature pertaining to Lanchester
models, the name that appears the most is Robert L. Helmbold.
Helmbold's 1961 study approached the subject by assuming, as
Willard did, that the square law was valid. His findings dealt
with the notion that a defender becomes more effective as the
force ratios increase in favor of the attacker.[28] Based upon the
apparent absurdity of this finding, Helmbold suggested

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[27] ibid, p 20.

[28] Robert L. Helmbold, <u>Historical Data and Lanchester's Theory of
Combat</u>, (Fort Belvoir, VA: Combat Operations Research Group, 1961), p
7.

explanations for this mystifying result. His explanations centered around concepts involving redundancy of hits and suppression. More about the suppression problem will be discussed in this monograph.

Between 1961 and 1971, Helmbold seemed to be indecisive about whether the square law was valid or not. In a 1964 article, he used data from ninety-two historical battles and methods of linear regression and obtained a γ value of -0.367, which was consistent with Willard's 1962 findings.[29] Based upon that analysis, he concluded that victory in battle was primarily determined by factors other than numerical superiority and so implied that he distrusted Lanchester's square law as being adequate in describing combat. Even after that, Helmbold continued to evaluate, modify and reevaluate the validity of the square law as it applied to modelling combat.

Fain Study

In 1974, Dr. Janice Fain conducted an analysis of sixty World War II land engagements from four major Italian campaigns. Dr. Fain selected short duration engagements (typically three days long).[30] The rationale was that such a selection would be more appropriate for use with Lanchester's

[29] Robert L. Helmbold, "Some Observations on the Use of Lanchester's Theory for Prediction", Operations Research 12 (1964), p 779-780.

[30] Janice B. Fain, "Lanchester Equations and Historical Warfare", History, Numbers, and War (Spring 1977), p 34.

equations since short duration engagements did not involve lapses in combat intensity. A portion of Fain's work involved repeating Willard's analysis with a different set of data as Fain thought Willard's data was faulty.[31] That portion of Fain's work which we are concerned with considered the question: Are casualties during combat related to the numbers of men in the opposing forces? That is just another way of wording Lanchester's square law.

Using the Historical Evaluation and Research Organization's (HERO) data set, Willard's equation, [see Equation 6] and linear regression analysis, Fain calculated a value of γ to be equal to -0.594 [32] which was strikingly similar to Willard's value. Fain's numerical analysis would have also led to the conclusion that either Lanchester's model does not represent those engagements or that the casualty-producing power of a force increases as the force decreases. The first conclusion was unsatisfactory since Lanchester equations were generally accepted. The second conclusion made little sense at the time.[33]

Fain recognized that in Willard's simplification, he may have been incorrect in formulating the correct regression equation since he retained only the leading terms. Fain

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[31] Trevor N. DuPuy, "The Lanchester Equations: Lanchester's Original Article with a Commentary by Trevor N. DuPuy", History, Numbers, and War (Fall 1977), p 149.

[32] Fain, p 38.

[33] ibid, p 38.

formulated the following equation[34] and conducted another regression analysis:

$$\text{Eqn 7} \qquad \log n_c/m_c = \log E + \log (m_0 - m_c / n_0 - n_c)$$

The variables in this equation have the same meanings as in Equation 1. Note the difference in the second term on the right hand side. Also note that that ratio of differences is what is left as opposed to what we started with. Fain's analysis resulted in a γ value equal to -0.413, not any substantial improvement over the previous analysis.

Fain then made another attempt to "fix" Willard's approach by reevaluating what data to use in the model. Fain's logic was that the force ratio was not descriptive enough and did not account for other operational factors involving elements of the cybernetic, physical and moral domains. As a result, the combat power ratio devised by HERO was used in lieu of the force ratio m_0/n_0. Using that approach, a value of γ equal to 0.466 [35] was computed. This was the type of result expected by Willard's analysis. The problem with this "substitution" was that it distorted the Lanchester concept. It has been argued that this was Lanchester's intent in the first

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[34] ibid, p 38.

[35] ibid, p 39.

place.[36] The general results from Fain's study are shown in Figure 3.[37]

Results using Eqn #	γ	log E	correlation coeff.
6	-0.594	.00115	-0.262
7	-0.413	.0539	-0.184
6 with HERO	0.466	-0.638	0.303

Figure 3 - Results of Fain's Analysis using various
Regression Equations

Summary

When these studies are viewed together, one immediately appreciates the challenge posed by the research question. Previous studies, after Engel's, have generally suggested, that Lanchester equations may be poor descriptors of large battles extending over periods during which forces are not constantly engaged in combat. Perhaps Lanchester equations may be adequate for predicting losses while forces are actually engaged in fighting. So now the question becomes one of : how to find some engagement data that "fits" the conditions described so that the model can be tested ? The best answer might be to look at the engagement data from the National Training Center.

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[36] DuPuy, p 150.

[37] Fain, p 42.

IV. The National Training Center as a Data Source

The mission of the National Training Center, as stated in
the information guide [38], is to provide tough, realistic combined
arms and services joint training in accordance with air land
battle doctrine for brigades and regiments in a medium to high
intensity environment, while retaining the training feedback
and analysis focus at battalion/task force level. Part of the
explicitly stated mission is to provide a data source for
training, doctrine, organization and equipment improvement.
To what extent that data is used is unknown. It is clear
however that the trends which the data projects receives
attention at the highest Army echelons. This was made clear
during the direct fire briefing during Forces Command Leaders'
Training Program (FLTP) rotation number 89-13 at the
National Training Center.

The National Training Center provides an unequalled
opportunity for battalions to exercise under conditions that
approach those of actual combat. The terrain and climate are
harsh and serve to intensify the stress and fatigue for men
and materiel.

As the brochure reiterates, the unique training
experience offered by the National Training Center cannot be
overstated. No other training exercises approach the realism

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[38] This brochure is issued to FORSCOM Troop Leaders' Training
Program attendees. The title is "National Training Center"; no other
publication information exists.

routinely achieved at the National Training Center. No other training presents, in combination, the scope, scale and intensity of effort that is captured and portrayed. Its uniquely instrumented battlefield provides instant feedback and heightens learning at all levels. The National Training Center enables commanders to train as they will fight. Moreover, it develops a level of stress unequalled outside of actual combat. In addition, this training is conducted with a well trained opposing force.

This force-on-force training allows us to operate against a live opposing force and approaches that type characteristic of the Lanchester model. Riflemen, gunners and armored vehicle crews engage live enemy targets in the form of individuals, crew-served weapons, armored vehicles and helicopters, all equipped with the multiple integrated laser engagement system (MILES). Thus, the National Training Center provides the military scientist with the unique environment of an experimental laboratory.

Combat is portrayed by means of five basic scenarios that include movement to contact, hasty attack, deliberate attack, defense in sector and defense from a battle position.[39] The force ratios between the opposing force and the task forces replicate in numbers and types of equipment that which might be expected in a European scenario.

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[39] As described on the 'Force On Force' page of the brochure.

To capture the specifics of the battle dynamics, special instrumentation is used. This data collection is transparent to the training unit. Throughout the entire training complex are some forty-four solar-powered "interrogator" relay stations. These stations receive signals from special transponders which are linked to the MILES systems on combat vehicles. These stations allow analysts at the National Training Center to capture data such as locations, types of vehicles, and types of weapons fired. Synthesis of this information yields data useable for this study.

Recalling the seven assumptions of Lanchester's model and previous comparison with the model, one might investigate the extent to which the National Training Center lends itself to this analysis. Previous analyses have demonstrated difficulties. Recall that those three difficulties were that:

(1) Previous analyses were not intrinsically applicable to conditions of modern warfare - e.g the Civil War.
(2) The data was not available in the format desired or at least some interpretation of the data was required.
(3) The data itself was considered unreliable in some instances.

The form of the data available from the National Training Center, about which more will be said later, is exactly what Lanchester's model calls for and so no interpretation is required. Because of the highly realistic nature of the exercises conducted at the National Training Center, it was decided that the Center would provide an ideal vehicle to conduct a validation of Lanchester's square law. As far as is known, no

attempt to use National Training Center data in this fashion has ever been considered.

V. Statistical Analysis of National Training Center Data

Data was available from National Training Center rotations from fiscal year 1987 and 1988. The data used in this study was obtained from both the Center for Army Lessons Learned and the Concepts Analysis Agency.[40] The following information is available from this database:

(1) battle number - this is a reference number used by the analysts

(2) date - self explanatory

(3) rotation - this is a year and month designator keyed to the actual National Training Center rotation number

(4) mission type - this is a two or three letter alphabetic code that defines the type of mission. The different type missions were explained in the previous section. The code is developed by taking the first letter in each word of the type mission (e.g defense in sector = DIS).

(5) Mission Kind - a code that defines whether the mission is an offensive or a defensive one.

(6) Location - self explanatory

(7) Unit TF - M for mech units and A for armor units.

(8) Unit component - either active component or reserve component

(9) MACOM - by division designation

(10) Type - type of division

(11) Remarks - shows H/L for a heavy/light rotation

(12) Weapon Types - shows major systems such as M1, M2 or M113 and the particular division of the light infantry forces if H/L rotation

(13) Initial Forces - shows quantities of both blue forces and OPFOR major weapons systems along with the force ratio(OPFOR to blue)

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[40] The Concepts Analysis Agency data was obtained from MAJ Forrest Crain in the form of a LOTUS 123 spreadsheet data disk.

(14) Loss Rates - shows quantities of blue forces and OPFOR major systems losses and the ratio(OPFOR to blue)

(15) Exchange ratio - ratio of loss ratios to initial force ratios

(16) Exchange ratios by type weapons system - for direct fire systems, fire support systems, air defense systems, and fratricide

(17) Loss Table: blue - shows the type of each blue major system and the type of OPFOR system which caused the loss.

(18) Loss Table: red - shows the type of each red major system and the type of blue system which caused the loss

The method used to determine the statistical validity of the square law was linear regression analysis. The linear regression equation for the square law model that was used is:

$$\log [B_C/R_C] = e + \gamma \log [R_O/B_O]$$

where B_C = number of blue casualties

R_C = number of red casualties

R_O = number of red forces at start of battle

B_O = number of blue forces at start of battle

e = the x-intercept for the regression line

γ = the slope of the regression line

The first indication of the validity of the square law to this data is through the value of the parameter γ. This is the same methodology as first used by Willard. In this analysis the parameter γ is expected to take on the values of +1, 0 or -1 depending on whether the square, linear, or logarithmic laws are appropriate. Those are ideal values for the parameter.

Practically, the sign and magnitude of γ in the vicinity of these values will support the validity of one law or another.

In order to conduct the regression analysis the data for one hundred and fifty seven battles from fields 13 and 14, as explained previously, was extracted and verified. The logarithms of the appropriate ratios were calculated and then were regressed using the methods of linear regression. The method of least squares was used to compute the parameters e and γ. The specific data and calculations used are attached in Appendix A.

The least squares estimates for the parameters for the regression equation are e = -0.0217 and γ = -0.9111. An interpretation of these results are that the average change in the logarithm of the casualty ratio -0.9111 is associated with a unit change in the logarithm of the force ratio. The negative value for e may appear to be nonsense since no casualties can result when no forces are fighting. On the other hand, a review of the loss tables suggests that those losses could occur as a result of mines or fratricide. The utility of conducting the analysis was to predict the casualty ratio given the force ratio and assuming a constant value for e. It is important however to ascertain first whether there is a cause-effect relationship.

To determine the degree of this cause-effect relationship, the coefficient of determination is computed. The coefficient of determination, called R^2, is a formal measure of the explanatory power of the model. R^2 records the proportion of variation in the logarithm of the casualty ratio that is

explained by the logarithm of the force ratio. An R^2 value of 0 means that the variables are independent and one tells nothing about the other. An R^2 value of 1 means that the one value accounts completely for the other. The value of R^2 for this analysis was 0.4828. Based upon that value one can say that the logarithm of the force ratio accounts for an estimated 48% of the variation in the logarithm of the casualty ratio and that there is evidence of a relationship between those two variables.

Such an analysis suggests that the attrition process tends toward a logarithmic law, which states that the attrition rate of a force is proportional to the size of that force. Therefore, the experience of U.S Army battalions at the National Training Center does not support the Lanchester square law theory. A logical question is: why does the theory fail? This question is important because the infatuation with Lanchester's square law by the bulk of the ORSA community continues despite its repeated invalidation. Why is this the case? In the first place the ORSA community has never been able to postulate a model that would adequately supplant Lanchester's square law. In the second place no one has been able to explain adequately why the model fails to stand up to empirical scrutiny. The next section will attempt to explain some reasons for the failure of the model to stand up to empirical scrutiny.

VI. The Problems of Suppression and Fire Control

In a nutshell, there are two specific reasons why the theory fails in this case. The primary reason is the phenomenon of suppression. A related reason which often manifests itself as suppression is the problem of fire control in battle. If the blue force is suppressed will attrition occur in accordance with the square law? If fire control is not possible will attrition occur in accordance with the square law? These problems are pivotal in explaining the inadequacy of the square law.

Suppression as an effect has not been well defined in our doctrine. Generally, the effects of suppression are such that, if effective, it hinders one's ability to observe, fire at, or maneuver against the force doing the suppressing. In a broader sense, suppression degrades unit effectiveness and more specifically influences the attrition coefficients of Lanchester's equations. Factors in the moral and cybernetic domain figure as prominently in s' ppressive behavior. A confidential study[41] conducted by Kushnik and Duffy in 1972 establishes that suppression is a function of:

(1) weapon characteristics (type, munition, rate of fire, lethality)
(2) weapon employment (accuracy, volume, pattern of fire, proximity, weapons mixes)

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[41] Stephen A. Kushnik, John O. Duffy, The Identification of Objective Relationships Between Small Arms Fire Characteristics (U), (Sunnyvale,CA: Litton Systems, Inc., 1972), p 2-24 - 2-26.

(3) situational variables (characteristics of forces, terrain, type and results of engagements)
(4) psychological factors (prior combat experience and individual factors)

A cursory view of this list shows the influences of the moral and cybernetic, as well as the physical domains of battle. A study conducted by the RAND Corporation suggests that there is a relationship between casualty production, troop suppression and combat effectiveness.[42] Figures 4 and 5 show the quantification of this relationship.

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[42] S.G. Spring and S.H. Miller Relationships Among Casualties, Suppression, and the Performance of Company-Sized Units, (Santa Monica, CA: The RAND Corporation, 1972) pp 39-40.

Figure 4 Attacking Infantry Company Casualties versus effectiveness - from page 12 of Spring and Miller FAST-VAL Report
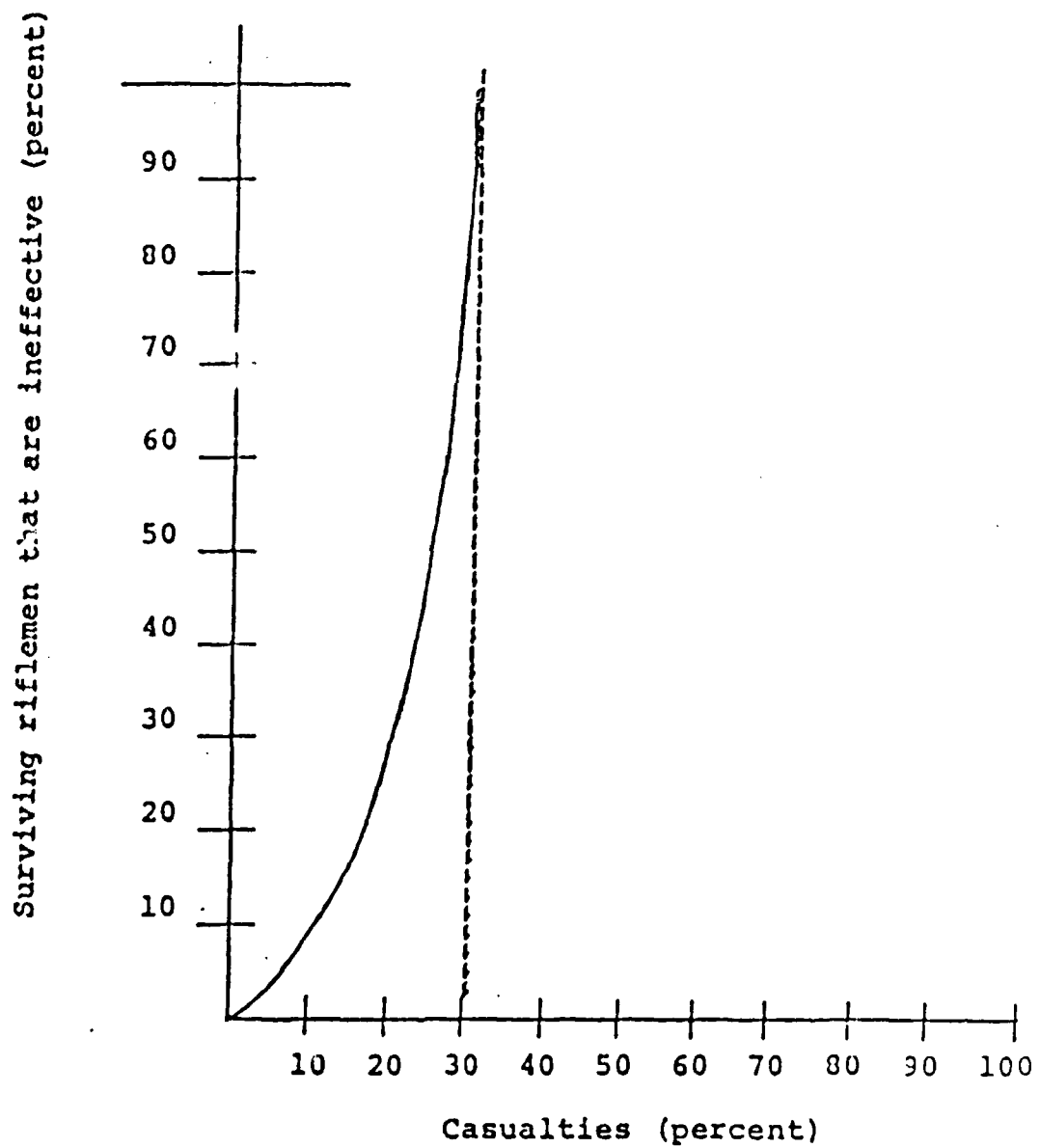
Figure 5 Defending Infantry Company Casualties versus
effectiveness - from page 16 of Spring and Miller FAST-VAL
Report
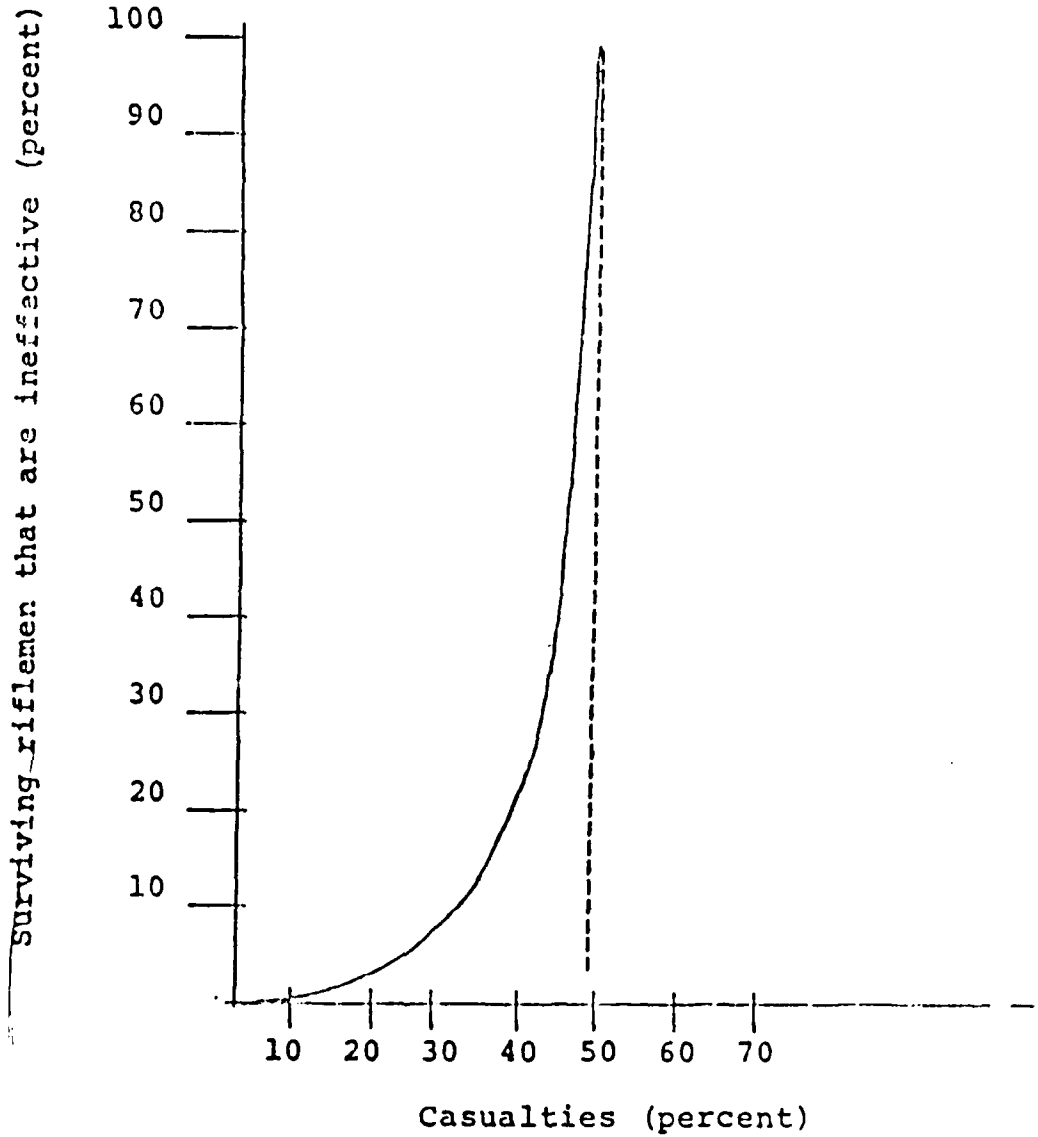
Most combat simulation models attempt to account for suppression. Suppression is often accounted for by applying a factor of degradation. This degradation typically manifests itself as a restriction on normal engagement rules. An example is when a unit is suppressed by smoke and is not allowed to engage any targets until the next game turn. By assuming this total degradation, a hiatus in the activity of the suppressed force is assumed. The entire suppressed force is assumed to be ineffective. As a result, the ability of a firer to detect a target is degraded to zero. The logical question is: what are the consequences of doing this if suppression is not, in fact, total. Suppressive fires may slow down activity, but they won't stop activity. Suppressive fires directed at a unit does not, in reality, affect all components of the unit equally. The vulnerabilities of the various individual components are different. Lanchester's square law assumes constant attrition and hence does not account for these differences.

A study[43] conducted by Horrigan Analytics, Inc states that representing suppression as a hiatus leads to major misrepresentation of an important combat factor. In fact, the impact is large since small errors in representing suppression result in large errors in representing effects of detection and hence attrition results.

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[43] Timothy J. Horrigan, <u>Detection, Suppression, and Fractional Suppression</u>, (Bethesda, Md: Horrigan Analytics, 1973), pp 1-10.

Mathematically speaking, it can be argued that casualty production and suppression are interrelated through some probability statement which suggests that not all targets are suppressed. Direct and indirect fires are often massed in the conduct of engagements. The effects of these fires are catastrophic only to the degree with which the massing is adequate. When the fires are directed at a target or region, only a portion of the region is susceptible to casualty-producing fires. Even if the target region is saturated with fires, there is still some positive probability that not all targets will become casualties. The Soviets indirectly acknowledge this phenomenon in their field artillery norms. By assuming only a certain percentage of kills based upon a given concentration of fires, the Soviets make such an acknowledgement.[44]

Another shortcoming of the model has to do with a dynamic of suppression - fire control. The Lanchester model assumes that all firers are actively engaged in battle and that the control of the firing is perfect. In essence, the dynamics of suppression are ignored. At the National Training Center, as well as in battle itself, it has been proved time and time again that all firers do not engage actively. According to Marshall, only about twenty-five percent of men engaged in combat will ever fire their weapons. SLA Marshall states:

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[44] Evidence is seen on pages 119 through 121 of the 1982 version of Tactical Calculations by A. Bayner. Other evidence from P118 discussions.

> Now I do not think I have seen stated in the
> military manuals of this age, or in any of the
> writings... that a commander of infantry will be
> well advised that when he engages the enemy not
> more than one quarter of his men will ever strike
> a real blow ...[45]

Marshall goes on to tell us that that this is even true of well-trained and seasoned campaign troops. During World War II while serving as a combat historian, Marshall interviewed approximately four hundred infantry companies in the Pacific and European theaters. During those post combat interviews, Marshall found that not a single battalion, company or platoon commander made any attempts to determine how many of their soldiers had actually fired their weapons against an enemy.[46] This passive firer phenomenon is hard to refute without that information.

The specific data was not available at the time of this study but the "killer tank" concept is a well known phenomenon at the National Training Center. This phenomenon exists when you have an "ace" who is knocking out all of the targets. The specific example which comes to mind is the one in which the operations officer of a task force was credited with several BMP kills during one mission.[47] Schneider cited a similar phenomenon that was present during the Western

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[45] S.L.A. Marshall, Men Against Fire, (Gloucester, MA: Peter Smith, Inc., 1978), p 50.

[46] Ibid, p 53

[47] This story was related to me during a seminar discussion about tactical dynamics.

Desert aerial engagements during World War II. He stated that generally ten percent of the pilots accounted for fifty to seventy-five percent of the kills. The other significant example was that of the 1300 kills claimed by the Germans , a total of 647 were claimed by fifteen pilots, with the top 53 aces claiming an aggregate 1042 kills. His analysis of this dynamic of combat is quite correct. If the square law adequately modelled the engagements, one would expect the kills to be more evenly distributed among all pilots.[48] The only place one can expect better fire control is on a firing range. The situation at the National Training Center is oftentimes similar and so the majority of the force is not actively engaged. Although this effect is not directly related to suppression, the effects are the same as if the nonfirers were suppressed.

As previously stated, suppression degrades the suppressed force's ability to see and hence engage its target. If a force is suppressed then it is clear that not all weapons are actively engaged at the critical moment. History, as well as current day tactical exercises are replete with examples of forces attacking after the preparatory fires which are designated to suppress the defender. If the defender is suppressed then he cannot fire because he has lost fire control.

Fire control is also influenced by terrain. Lanchester hypothesized that naval warfare adhered to the square law. He stated that the open sea is essentially without factors that

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[48] Schneider, p 85.

may influence the perfect model. When ships come into battle on the sea, they cannot use the "terrain" to cover or conceal themselves and and thus exposed. Erwin Rommel once inferred that desert battle was a lot like naval battle. Rommel's view of the desert is revealed in notes by General Bayerlein, the chief of staff of Afrika Corps in World War II. The notes describe the desert:

> ... there are no obstructions, no lines, water or woods for cover; everything is open and incalculable; the commander must adapt and reorient himself daily, even hourly...[49]

So why doesn't this desert warfare at the National Training Center adhere to the model?

The wadis present throughout the training area at the National Training Center provide units with masking from enemy fires. The open seas may be featureless, but the desert floor is not. Lanchester's model presupposes that the firers know the location of their targets, are within range and can see to fire at them. If you can't see the target, you cannot engage it. The earlier anecdote about the killer tank involved a tank well-positioned in a wadi on the desert floor. That tank could engage, but it could not be engaged.

Terrain serves also as a way for firers to avoid being engaged. The moral domain of battle tells us that soldiers try to hide when they become afraid. While this notion of fear of

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[49] B.H. Liddell Hart, The Rommel Papers, (New York,NY: Da Capo Press Inc., 1953), pp 184-185.

death does not manifest itself at the National Training Center, the notion of fear of failure does and so the ultimate reaction of engagement avoidance is the same. If moral disintegration sets in, soldiers tend to take evasive action to avoid enemy fires and often become separated. Ardant du Picq reminds us that once in action, the infantryman of today escapes the control of his officers due to the disorder inherent in battle with the control being relinquished to his comrades.[50] As casualties increase, fire control becomes looser as a function of two actions. One action is troops taking care of their friends and the other is troops fleeing to avoid being fired upon. In essence, the effort of the unit becomes disjointed and the principle of concentration, which Lanchester's model explicitly assumes, is violated. If that concentration is not achieved, then the square law does not apply. After all, the square law does assume simultaneous projection of force upon a target. Units that become separated as is characteristic of the modern empty battlefield, tend to take themselves out of the fight. They lose the motivating influence of leadership and become more willing to let those few self-motivated individuals, the "aces", carry on the fight without them.

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[50] Charles Ardant du Picq, Battle Studies, trans by J.N. Greely and R.C. Cotton, (Harrisburg, PA: Military Service Publishing Company, 1946), p 194.

Summary

It becomes apparent that suppression, both as a combat dynamic and as a manifestation of fire control, impacts upon Lanchester's square law. Studies conducted by professional organizations suggest that many factors play a part in suppression. Other studies suggest that the manner in which most simulations account for suppression in combat is inadequate. If this last suggestion is true,and there is clear evidence to support that, then it is no wonder that Lanchester's square law does not stand up to empirical scrutiny.

VII. Conclusions and Implications

Generally, Lanchester's square law does not do a good job of explaining National Training Center engagement data, just as it has had limited success with actual combat data. Quantifying all of those factors that affect combat is too complex. Lanchester realized this in his treatise Aircraft in Warfare. He suggested that it is difficult to quantify the differences in units because there are just too many variables. He said that those variables cannot be accounted for in any equation anymore than the quality of wine or steel from the weight of the wine or the steel.[51] If Lanchester's square law cannot represent historical combat, then its utility as a tool for analyzing future combat is questionable. The refutation of the square law implies that in mathematical terms, tactical combat and, by implication, operational art are qualitatively different forms of war than the classical variety.

The suppression problem is a big one when it comes to combat. It obviously impacts greatly on our combat model outputs. That fact is borne out by previous analyses and the analysis of the National Training Center data.

As that analysis shows, any tactical training exercise that attempts to capture the essence of real combat must use some method of simulating suppression. Whatever analog (such as BAI, etc) of suppression which exists at higher levels should

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[51] Frederick W. Lanchester, "Mathematics in Warfare", The World of Mathematics, 1956 edition, p2144.

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receive the same attention. Failure to account for suppression will result in a gross miscalculation of casualties and render the simulation to be of little utility.

Of all the previous analyses, the analysis of the National Training Center data adheres mathematically to the exponential law better. An interesting implication is that if actual combat really does adhere to an exponential law then the National Training Center may be a more realistic training exercise than is realized. If that is true then the lessons learned from the Center are quite valuable ones.

The gap that exists here between reality and theory is only dangerous to the extent that those who use analytical models fail to understand this phenomenon. Models are designed to help us explain or predict real world activity but they all accomplish those functions with varying degrees of success. The challenge here is to continue to study the problem and seek ways in which to improve the models. To dismiss the utility of mathematical models would be, in Lanchester's words, unintelligent and illogical.

Initial Red	Initial Blue	Red Casualties	Blue Casualties	Init Force Ratio (red/blue)	Casualty ratio blue/red	log of casualty ratio (Y value)	log of force ratio (X value)
59	108	57	76	0.546296963	1.333333333	0.12	-0.26
49	87	13	43	0.563218839	3.69230769	0.57	-0.25
53	122	15	55	0.43442623	3.666666667	0.56	-0.36
49	132	16	74	0.371212121	4.625	0.67	-0.43
78	89	69	67	0.37640449	0.9710144449	-0.01	-0.06
42	93	9	45	0.451612129	5.111111111	0.71	-0.35
75	112	25	30	0.5696428	1.2	0.08	-0.17
26	115	17	51	0.22608696	3	0.48	-0.65
40	139	19	59	0.287769788	3.105263316	0.49	-0.54
177	136	155	39	1.301470594	0.251616129	-0.60	0.11
152	113	135	45	1.345132743	0.33333333	-0.48	0.13
172	117	150	46	1.470085547	0.30666667	-0.51	0.17
182	120	110	42	1.51666667	0.381818181	-0.42	0.18
148	127	131	67	1.165354333	0.51145038	-0.29	0.07
11	109	8	12	0.1009743	1.5	0.18	-1.00
167	115	1	6	1.452173591	6	0.78	0.16
169	116	90	80	1.456894655	0.8888889	-0.05	0.16
158	86	101	51	1.8372093	0.5049505	-0.30	0.26
177	92	70	69	1.923913304	0.98571429	-0.01	0.28
163	123	63	80	1.325203325	1.269841127	0.10	0.12
151	96	45	69	1.572916667	1.40816327	0.15	0.20
67	126	45	63	0.53174603	1.4	0.15	-0.27
163	80	108	43	2.0375	0.44444444	-0.35	0.31
163	121	91	64	1.3842975	0.7032967	-0.15	0.14
58	77	40	27	0.75324675	0.675	-0.17	-0.12
138	100	79	22	1.38	0.27848101	-0.56	0.14
144	114	101	67	1.263157589	0.66336634	-0.18	0.10
138	107	71	52	1.2897196	0.7323943	-0.14	0.11
138	115	95	29	1.2	0.305263316	-0.52	0.08
157	115	69	56	1.365217391	0.8115942	-0.09	0.14
170	134	51	82	1.268656672	1.607843114	0.21	0.10
158	97	59	48	1.628866598	0.81355932	-0.09	0.21
154	121	99	20	1.272727727	0.202020202	-0.69	0.10
149	94	95	70	1.585106638	0.7568842211	-0.13	0.20

185	129	88	71	1.4453125	0.80681818	-0.09	0.16
50	101	24	62	0.4950495	2.58333333	0.41	-0.31
54	99	28	44	0.54545455	1.57142857	0.29	-0.26
68	126	39	34	0.53968254	0.87179487	0.06	-0.27
136	87	119	36	1.56321839	0.30252101	-0.52	0.19
68	108	31	66	0.62962963	2.12903226	0.33	-0.20
34	136	10	56	0.25	5.6	0.75	-0.60
64	140	24	25	0.45714286	1.04166667	0.02	-0.34
30	115	20	42	0.26086957	2.1	0.52	-0.58
46	135	13	37	0.34074074	2.84615385	0.45	-0.47
64	140	27	48	0.45714286	1.77777778	0.25	-0.34
26	115	5	6	0.22608696	1.2	0.03	-0.65
33	101	8	56	0.32673267	7	0.85	-0.49
75	86	42	51	0.87209302	1.21428571	0.08	-0.06
23	130	7	32	0.17692308	10.6666667	1.05	-0.59
30	134	7	44	0.2238806	6.28571429	0.66	-0.75
44	141	6	41	0.31205674	6.83333333	0.35	-0.51
23	88	5	66	0.26136364	13.2	1.12	-0.58
25	127	18	53	0.19685039	2.94444444	0.47	-0.71
22	111	7	25	0.1981982	3.57142857	0.55	-0.70
33	87	17	61	0.37931034	3.58823529	0.42	-0.42
61	129	51	32	0.47286822	0.62745098	-0.20	-0.33
45	75	32	35	0.6	1.09375	0.04	-0.22
163	123	63	80	1.3252032	1.26984127	0.13	0.12
182	120	110	42	1.51666667	0.38181818	-0.42	0.18
68	108	31	66	0.62962963	2.12903226	0.33	-0.20
57	85	36	44	0.67058824	1.22222222	0.09	-0.17
33	124	18	57	0.26612903	3.16666667	0.50	-0.57
61	123	36	45	0.49593496	1.25	0.10	-0.30
26	90	6	47	0.28888889	7.83333333	0.89	-0.54
27	95	9	39	0.28421053	4.33333333	0.64	-0.55
59	108	57	76	0.5462963	1.33333333	0.12	-0.26
25	127	18	53	0.19685039	2.94444444	0.47	-0.71
23	88	5	66	0.26136364	13.2	1.12	-0.58
11	109	8	12	0.10091743	1.5	0.13	-1.00

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42	95	9	46	0.4516129	5.111111111	0.71	-0.35
75	112	25	30	0.66964286	1.2	0.08	-0.17
27	94	32	32	0.2547698	1.0666667	1.05	-0.9?
171	106	3?	59	1.8191489	1.34090909	0.13	0.26
172	135	69	57	1.2666667	0.826086696	-0.08	-0.26
171	135	150	46	1.47008547	0.306666667	-0.51	-0.29
143	127	131	67	1.16535433	0.51145038	-0.29	-0.07
167	115	1	6	1.45217391	6	0.78	0.16
177	92	70	69	1.9239 304	0.98571429	-0.01	0.28
163	80	108	43	2.0375	0.44444444	0.31	0.31
58	77	40	24	0.7532675	0.6	-0.22	-0.12
33	101	8	65	0.32673267	8.125	0.91	-0.42
27	119	18	44	0.22689076	2.44444444	0.39	-0.64
49	86	34	47	0.59976744	1.38235294	0.14	-0.24
64	140	27	48	0.45714286	1.77777778	0.25	0.54
49	152	16	74	0.37121212	4625	0.6?	-0.43
46	135	13	37	0.34074074	2.84615385	0.45	-0.47
30	115	20	42	0.26086957	2.1	0.32	-0.58
64	140	24	25	0.45714286	1.04166667	0.02	-0.34
152	113	135	45	1.34513274	0.33333333	-0.43	-0.13
169	116	90	80	0.45442623	0.88888889	-0.05	-0.65
30	121	91	64	1.38842975	0.7032967	-0.15	0.14
26	122	15	55	0.22608696	3.66666667	0.56	-0.36
170	134	5	6	1.2986672	1.2	0.08	-0.65
53	115	90	44	0.2238806	6.2857 429	0.80	-0.10
168	134	91	80	1.45689655	0.88888889	-0.05	0.16
169	116	67	67	0.87640449	0.97101449	-0.01	-0.06
15	96	49	69	1.57291667	1.40816327	0.15	0.20
78	89	49	63	0.53174603	1.4	0.15	-0.65
34	136	10	56	0.25	5.6	0.75	-0.60
67	126	13	43	0.56321839	3.69230769	0.57	-0.25
15	96	101	51	1.8372093	0.5049505	-0.30	-0.26
49	85	90	51	0.52830189	1.91489362	0.28	-0.28
158	159	77	90	1.03614458	1.14925373	0.06	-0.23
84	166	67	77	1.05333333	1.86486486	0.27	0.02
172	150	31	69	0.55333333	1.9090909	0.26	-0.26
83	94	33	63	0.54255319	1.9090909	0.23	-0.2?
51							

32	134	19	30	0.23880597	4.210526632	0.62	-0.62
49	94	18	44	0.52127766	2.444444444	0.39	-0.28
32	82	9	40	0.39024390	4.444444444	0.65	-0.41
63	134	49	50	0.47014925	1.34693878	0.13	-0.33
55	117	23	76	0.47008547	3.30434783	0.52	-0.53
72	95	54	73	0.75789474	1.351351851	0.13	-0.12
34	118	21	55	0.28813559	2.619047762	0.42	-0.54
72	97	51	49	0.74226804	0.960784351	-0.02	-0.13
51	111	47	66	0.45945946	1.404255532	0.15	-0.34
45	136	5	32	0.33088235	6.4	0.81	-0.48
53	160	19	57	0.33125	0.9125	0.29	-0.48
43	94	7	16	0.45744681	2.285714329	0.36	-0.34
164	95	80	73	1.72631579	1.647368842	0.81	-0.48
200	129	139	29	1.55038760	0.20863309	-0.68	-0.19
177	129	120	81	1.37209302	0.675	-0.17	-0.14
192	132	88	81	1.45454545	0.920454555	-0.04	-0.16
180	92	130	50	1.95652174	0.384615538	-0.41	-0.29
139	96	136	76	1.44791667	0.558382353	-0.25	-0.16
199	114	135	13	1.74561404	0.0962963	-1.02	-0.24
207	130	151	68	1.59230769	0.450053113	-0.35	-0.20
173	123	52	82	1.40650407	1.576922308	-0.64	-0.15
223	85	157	36	2.62352941	0.252929336	0.20	0.42
173	95	77	65	1.82105263	0.844155584	0.42	0.26
78	158	53	16	0.49367089	0.5013866879	0.26	-0.51
178	159	80	68	1.11949686	0.85	-0.07	0.05
132	161	88	40	0.81987578	0.454545455	-0.34	-0.09
136	183	87	70	0.74316694	0.8045977	-0.09	-0.13
168	89	84	43	1.88764045	0.571428571	-0.24	0.28
168	139	113	74	1.20863309	0.65436726	-0.18	0.08
35	88	7	49	0.39772727	7	0.85	-0.40
34	96	10	55	0.35416667	5.5	0.74	-0.45
28	104	7	46	0.26923077	6.571428571	0.82	-0.57
59	116	33	43	0.50862069	1.454545455	0.16	-0.29
57	83	31	43	0.68674699	1.493571	0.19	-0.16
26	111	14	35	0.23423423	2.571428571	0.41	-0.53034960
81	106	58	50	0.76415094	0.862068897	-0.06	-0.12

81	129	58	65	0.62790698	1.1206390	0.05	-0.20
29	124	22	43	0.19727891	2.181919	0.34	-0.70
71	83	41	51	0.85542169	1.48790489	0.17	-0.07
60	120	25	43	0.42857143	1.72	0.24	-0.37
43	108	40	25	0.39814815	0.625	-0.29	-0.40
62	117	36	74	0.52991453	2.0555556	0.31	-0.28
74	95	39	50	0.78894737	1.282051	0.11	-0.11
70	118	29	64	0.59322034	2.2068965	0.34	-0.23
80	137	41	57	0.50995414	1.390243	0.14	-0.29
64	105	53	69	0.60377358	1.5018867	0.11	-0.22
165	111	91	36	1.48648649	0.3956044	-0.40	0.17
74	131	49	55	0.5648855	1.2448980	0.05	-0.25
169	127	95	97	1.33070866	1.02105263	0.01	0.12
75	116	31	13	0.62931034	0.41935484	-0.38	-0.20
18	111	14	17	0.16216216	1.21428571	0.08	-0.79

FOR $n = 157$, USING LEAST SQUARES

$\sum x = -29.81...$ $\sum xy = -19.0596$ REGR EQN: $\log\left(\frac{B_c}{R_c}\right) = \beta + \gamma \log\left(\frac{R_o}{B_o}\right)$

$\sum x^2 = 21.62...$ $\bar{x} = -0.1899$

$\sum y = 23.76$ $\bar{y} = 0.1513$ INTERCEPT: $\beta = -0.02...$

$\sum y^2 = 31.04...$ SLOPE: $\gamma = -0.911$

$\hat{r}^2 = .1828 = r^2$ $r = -.6949$

*USING HPISC w/ ABOVE 157 DATA POINTS

DEP. VAR = $\log\left(\frac{BLUE\ CASUALTIES}{RED\ CASUALTIES}\right) = \log\left(\frac{B_c}{R_c}\right)$

INDEP. VAR = $\log\left(\frac{RED\ FORCE\ @\ START}{BLUE\ FORCE\ @\ START}\right) = \log\left(\frac{R_o}{B_o}\right)$

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