# An ABox Revision Algorithm for the Description Logic $\mathcal{EL}_\perp$

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**Abstract.** Revision of knowledge bases (KBs) expressed in description logics (DLs) has gained a lot of attention lately. Existing revision algorithms can be divided into two groups: model-based approaches (MBAs) and formula-based approaches (FBAs). MBAs are fine-grained and independent of the syntactical forms of KBs; however, they only work for some restricted forms of the DL-Lite family. FBAs can deal with more expressive DLs such as $\mathcal{SHOIN}$, but they are syntax-dependent and not fine-grained. In this paper, we present a new method for instance-level revision of KBs. In our algorithm, a non-redundant depth-bounded model is firstly constructed for the KB to be revised; then a revision process based on justifications is carried out on this model by treating a model as a set of assertions; finally the resulting model is mapped back to a KB which will be returned by the algorithm. Our algorithm is syntax-independent and fine-grained, and works for the DL $\mathcal{EL}_\perp$.

## 1 Introduction

Description logics (DLs) are playing a central role in the Semantic Web, serving as the basis of the W3C-recommended Web ontology language OWL [11]. The main strength of DLs is that they offer considerable expressive power going far beyond propositional logic, while reasoning is still decidable.

Traditionally DLs have been used for representing and reasoning about knowledge of static application domains [2]. Recently, however, there are some research works focused on dynamic aspects of knowledge bases (KBs) based on DL. One typical application background for these research works is ontology evolution [9], where the goal is to incorporate new information $\mathcal{N}$ into an original ontology $\mathcal{K}$ and guarantee the consistency of resulting ontology $\mathcal{K}'$. This problem is also called KB revision in artificial intelligence [1]. Another typical application background is reasoning about actions [3, 6], where the purpose is to bring the KB up to date when the world is changed by the execution of actions which are described by DL assertions. This problem is called KB update [10, 12, 15]. In this paper we investigate the revision problem.

A KB $\mathcal{K}$ based on DL is generally composed of two parts: a TBox $\mathcal{T}$ for representing intensional knowledge, and an ABox $\mathcal{A}$ for representing extensional knowledge. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an original KB and $\mathcal{N}$ a set of new information, then there are three types of revision problems based on DLs: (1) TBox revision, where $\mathcal{N}$ consists of TBox assertions only and $\mathcal{A}$ is empty [16, 17]; (2) ABox revision, where $\mathcal{N}$ consists of ABox assertions only and $\mathcal{T}$ is assumed to be fixed [13, 14]; (3) KB revision, where $\mathcal{N}$ consists of both TBox assertions and ABox assertions [5, 18, 19]. In this paper we focus on the ABox revision problem.

In order to capture the basic ideas and properties of revision, some postulates were proposed and well-studied in the literature [1, 12, 8]. According to these postulates, a revision operator or algorithm should hold the following properties:

**R1:** must preserve the consistency of KBs;
**R2:** must entail the new information and preserve the protected part;
**R3:** should not change the original KB if there is no conflict;
**R4:** should be independent of the syntactical forms of KBs; and
**R5:** should guarantee a minimal change.

Among these properties the last one is not precisely specified, since there are different approaches to define minimality for different applications; it is well-accepted that there is no general notion of minimality that will do the right thing under all circumstances [5].

According to the semantics adopted for defining minimality, existing revision operators and algorithms can be divided into two groups: *model-based approaches* (MBAs) and *formula-based approaches* (FBAs).

In model-based approaches, the semantics of minimal change is defined by measuring the distance between the models of new information $\mathcal{N}$ and the models of original KB $\mathcal{K}$ [13]. An advantage of MBAs is that they are independent of the syntactical forms of the KB. However, although they work well for propositional logic, it is difficult to adapt them to DLs. Until now, they are only applied in the DL-Lite family for revision problems [13, 17, 18].

In formula-based approaches, the semantics of minimal change is reflected in the minimality of formulas which will be changed. One group of FBAs is based on the deductive closure of KBs; they are powerful enough to guarantee syntax-independent but only works for restricted forms of DL-Lite [5, 14]. Another group of FBAs is based on justifications; they and capable for processing DLs such as $\mathcal{SHOIN}$, but are syntax-dependent and not fine-grained [16, 19].

In this paper we present a new method for ABox revision problem. Compared to MBAs and the FBAs based on deductive closures, our method is capable to support the DL $\mathcal{EL}_\bot$. Compared to the FBAs based on justifications, our method is syntax-independent and fine-grained for the minimal change principle. The proofs of all of our technical results are given in the technical report [7].

## 2 The Description Logic $\mathcal{EL}_\bot$

The DL $\mathcal{EL}_\bot$ extends $\mathcal{EL}$ with bottom concept (and consequently disjointness statements) [4]. Let $N_C$, $N_R$ and $N_I$ be disjoint sets of *concept names*, *role* *names* and *individual names*, respectively. $\mathcal{EL}_\bot$-*concepts* are built according to the syntax rule $C ::= \top \mid \bot \mid A \mid C \sqcap D \mid \exists r.C$, where $A \in N_C$, $r \in N_R$, and $C, D$ range over $\mathcal{EL}_\bot$-concepts.

The semantics is defined by means of an *interpretation* $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$, where the interpretation domain $\Delta^\mathcal{I}$ is a non-empty set composed of individuals, and $\cdot^\mathcal{I}$ is a function which maps each concept name $A \in N_C$ to a set $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$, maps each role name $r \in N_R$ to a binary relation $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$, and maps each individual name $a \in N_I$ to an individual $a^\mathcal{I} \in \Delta^\mathcal{I}$. The function $\cdot^\mathcal{I}$ is inductively extended to arbitrary concepts by setting $\top^\mathcal{I} := \Delta^\mathcal{I}$, $\bot^\mathcal{I} := \emptyset$, $(C \sqcap D)^\mathcal{I} := C^\mathcal{I} \cap D^\mathcal{I}$, and $(\exists r.C)^\mathcal{I} := \{x \in \Delta^\mathcal{I} \mid \text{there exists } y \in \Delta^\mathcal{I} \text{ such that } (x, y) \in r^\mathcal{I} \text{ and } y \in C^\mathcal{I}\}$.

A *TBox* $\mathcal{T}$ is a finite set of *general concept inclusions* (GCIs) of the form $C \sqsubseteq D$, where $C$ and $D$ are concepts. An *ABox* $\mathcal{A}$ is a finite set of *concept assertions* of the form $C(a)$ and *role assertions* of the form $r(a, b)$, where $a, b \in N_I$, $r \in N_R$ and $C$ is a concept. A *knowledge base* (KB) is a pair $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$.

The *satisfaction relation* "$\models$" is defined inductively as follows: $\mathcal{I} \models C \sqsubseteq D$ iff $C^\mathcal{I} \subseteq D^\mathcal{I}$; $\mathcal{I} \models \mathcal{T}$ iff $\mathcal{I} \models X$ for every $X \in \mathcal{T}$; $\mathcal{I} \models C(a)$ iff $a^\mathcal{I} \in C^\mathcal{I}$; $\mathcal{I} \models r(a, b)$ iff $(a^\mathcal{I}, b^\mathcal{I}) \in r^\mathcal{I}$; and $\mathcal{I} \models \mathcal{A}$ iff $\mathcal{I} \models X$ for every $X \in \mathcal{A}$.

$\mathcal{I}$ is a *model* of a KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ if $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$. We use $mod(\mathcal{K})$ to denote the set of models of a KB $\mathcal{K}$. Two KBs $\mathcal{K}_1$ and $\mathcal{K}_2$ are *equivalent* (written $\mathcal{K}_1 \equiv \mathcal{K}_2$) iff $mod(\mathcal{K}_1) = mod(\mathcal{K}_2)$.

A KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ is *consistent* (or $\mathcal{A}$ is *consistent* w.r.t. $\mathcal{T}$) if $mod(\mathcal{K}) \neq \emptyset$. A KB $\mathcal{K}$ *entails* a GCI, assertion or ABox $X$ (written $\mathcal{K} \models X$) if $\mathcal{I} \models X$ for every $\mathcal{I} \in mod(\mathcal{K})$. It is obvious that $\mathcal{K}$ is inconsistent iff $\mathcal{K} \models \top \sqsubseteq \bot$ iff $\mathcal{K} \models \bot(a)$ for some individual name $a$ occurring in $\mathcal{K}$. We say that $\mathcal{K}$ *entails a clash* if $\mathcal{K}$ is inconsistent.

Let $X$ be a concept, GCI, ABox assertion, TBox, ABox or KB, then $N_C^X$ (resp., $N_R^X$ and $N_I^X$) is the set of concept names (resp., role names and individual names) occurring in $X$, and $sig(X) = N_C^X \cup N_R^X \cup N_I^X$.

For any concept $C$, the *role depth* $rd(C)$ is the maximal nesting depth of "$\exists$" in $C$. Let $X$ be an ABox or TBox, $sub(X) = \{C \mid C(a) \in X\}$ if $X$ is an ABox, and $sub(X) = \bigcup_{C \sqsubseteq D \in X} \{C, D\}$ if $X$ is a TBox, then the *depth* of $X$ is defined as $depth(X) = max\{rd(C) \mid C \in sub(X)\}$.

## 3 ABox Revision for $\mathcal{EL}_\bot$

Firstly we define the ABox revision problem in $\mathcal{EL}_\bot$ as follows.

**Definition 1.** *An $\mathcal{EL}_\bot$ KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and an ABox $\mathcal{N}$ is a* revision setting *if $\mathcal{N}$ is consistent w.r.t. $\mathcal{T}$.*

*A KB $\mathcal{K}' = \langle \mathcal{T}, \mathcal{A}' \rangle$ is a* solution *for a revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and $\mathcal{N}$ if $\mathcal{K}'$ is consistent and $\mathcal{K}' \models \mathcal{N}$.*

In a revision setting, $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ is called *original KB* and $\mathcal{N}$ is *new information*. The task is to incorporate new information into original KB while preserving the TBox $\mathcal{T}$ and guaranteeing the consistency of KB.

If we only consider the requirements on solutions specified in the above definition, then a straightforward solution for a revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and $\mathcal{N}$ is the KB $\mathcal{K}'' = \langle \mathcal{T}, \mathcal{N} \rangle$. However, obviously it is not a "good" solution, since information contained in the ABox $\mathcal{A}$ is completely lost. Therefore, besides the two necessary requirements, we hope that a solution has the properties specified by **R3-R5** in Section 1 of the paper. In the following subsections, we will show that existing revision approaches not work well for $\mathcal{EL}_\bot$ with respect to these properties, and then illustrate our method by an example.

### 3.1 Model-based Approaches

MBAs define revision operators over the distance between interpretations [13]. Let $\mathcal{K} \diamond \mathcal{N}$ be the set of models for the solution of revision setting, then $\mathcal{K} \diamond \mathcal{N}$ is generated by choosing models of $\langle \mathcal{T}, \mathcal{N} \rangle$ that are minimally distant from the models of $\mathcal{K}$, i.e.,

$$\mathcal{K} \diamond \mathcal{N} = \{\mathcal{J} \in mod(\langle \mathcal{T}, \mathcal{N} \rangle) \mid there\ exists\ \mathcal{I} \in mod(\mathcal{K})\ such\ that\ dist(\mathcal{I}, \mathcal{J}) = min\{dist(\mathcal{I}', \mathcal{J}') \mid \mathcal{I}' \in mod(\mathcal{K}), \mathcal{J}' \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)\}\ \}.$$

Let $\Sigma$ be the set of concept names and role names occurring in $\mathcal{K}$ and $\mathcal{N}$. There are four different approaches for measuring the distance $dist(\mathcal{I}, \mathcal{J})$:

- $dist^s_\sharp(\mathcal{I}, \mathcal{J}) = \sharp\{X \in \Sigma \mid X^\mathcal{I} \neq X^\mathcal{J}\}$,
- $dist^s_\subseteq(\mathcal{I}, \mathcal{J}) = \{X \in \Sigma \mid X^\mathcal{I} \neq X^\mathcal{J}\}$,
- $dist^a_\sharp(\mathcal{I}, \mathcal{J}) = \underset{X \in \Sigma}{sum}\ \sharp(X^\mathcal{I} \ominus X^\mathcal{J})$,
- $dist^a_\subseteq(\mathcal{I}, \mathcal{J}, X) = X^\mathcal{I} \ominus X^\mathcal{J}$ for every $X \in \Sigma$,

where $X^\mathcal{I} \ominus X^\mathcal{J} = (X^\mathcal{I} \setminus X^\mathcal{J}) \cup (X^\mathcal{J} \setminus X^\mathcal{I})$. Distances under $dist^s_\sharp$ and $dist^a_\sharp$ are natural numbers and are compared in the standard way. Distances under $dist^s_\subseteq$ are sets and are compared by set inclusion. Distances under $dist^a_\subseteq$ are compared as follows: $dist^a_\subseteq(\mathcal{I}_1, \mathcal{J}_1) \leq dist^a_\subseteq(\mathcal{I}_2, \mathcal{J}_2)$ iff $dist^a_\subseteq(\mathcal{I}_1, \mathcal{J}_1, X) \subseteq dist^a_\subseteq(\mathcal{I}_2, \mathcal{J}_2, X)$ for every $X \in \Sigma$. It is assumed that all models have the same interpretation domain and the same interpretation on individual names. In [13], the above four different semantics are denoted as $\mathcal{G}^s_\sharp$, $\mathcal{G}^s_\subseteq$, $\mathcal{G}^a_\sharp$, and $\mathcal{G}^a_\subseteq$ respectively.

It was shown that, under these semantics, the ABox revision problem in DL-Lite suffers from inexpressibility (i.e., the result of revision cannot be expressed in DL-Lite) [5, 13]. The reason is that the authors hope to compute a KB $\mathcal{K}' = \langle \mathcal{T}, \mathcal{A}' \rangle$ such that $mod(\mathcal{K}') = \mathcal{K} \diamond \mathcal{N}$, and the minimality principle of change intrinsically introduces implicit disjunction which is not supported by DL-Lite.

Since $\mathcal{EL}_\bot$ does not support disjunction, it is obvious that it suffers from the same problem of inexpressibility. However, in this paper, we do not require the solution to be a single KB. In the case that implicit disjunction is introduced by the revision process, we will generate more than one KBs $\mathcal{K}'_i$ $(1 \leq i \leq n)$ such that $\bigcup_{1 \leq i \leq n} mod(\mathcal{K}'_i) = \mathcal{K} \diamond \mathcal{N}$. As a result, the cause for inexpressibility studied in [5, 13] is avoided here.

There are two reasons for us to permit multi solutions. Firstly, in some applications, what we care about is to check whether an assertion $\varphi$ holds or not after the revision process. In this case, we can represent all the possible solutions as a set of KBs, and check whether $\varphi$ holds or not in each KB. Secondly, the implicit disjunction introduced by revision process means that there are different possible results for the revision process. We can compute and represent all the possible results as a set of KBs and let the user or experts to select the best one.

However, although the inexpressibility problem studied in [5, 13] can be solved by permitting multi solutions (or disjunctive KB), if we adopt MBAs, then there exists another cause of inexpressibility for ABox revision in $\mathcal{EL}_\bot$.

*Example 1.* Consider the revision setting $\mathcal{K}_1 = \langle \mathcal{T}, \mathcal{A}_1 \rangle$ and $\mathcal{N} = \{E(a)\}$, where

$$\mathcal{T} = \{A \sqsubseteq \exists R.A,\ A \sqsubseteq C,\ E \sqcap \exists R.A \sqsubseteq \bot\}, \quad \mathcal{A}_1 = \{A(a)\}.$$

Firstly, we apply the semantics $\mathcal{G}^s_\subseteq$ and $\mathcal{G}^s_\sharp$. Let $\Sigma = \{A, C, E, R\}$. It is obvious that, for any interpretations $\mathcal{I} \in mod(\mathcal{K}_1)$ and $\mathcal{J} \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)$, it must be $A^\mathcal{I} \neq A^\mathcal{J}$ and $E^\mathcal{I} \neq E^\mathcal{J}$. Therefore, $\{A, E\}$ is the minimal set of signatures whose interpretations must be changed. So, under both $\mathcal{G}^s_\subseteq$ and $\mathcal{G}^s_\sharp$, we have that

$$\begin{aligned}\mathcal{K}_1 \diamond \mathcal{N} = \{\mathcal{J} \in mod(\langle \mathcal{T}, \mathcal{N} \rangle) \mid\ & \textit{there exists } \mathcal{I} \in mod(\mathcal{K}_1) \textit{ such that} \\ & X^\mathcal{I} = X^\mathcal{J} \textit{ for any } X \in \Sigma \setminus \{A, E\}\}.\end{aligned}$$

For every positive integer $k$, construct an interpretation $\mathcal{J}_k = (\Delta^{\mathcal{J}_k}, \cdot^{\mathcal{J}_k})$ as follows: $\Delta^{\mathcal{J}_k} = \{x_1, ..., x_k\}$, $a^{\mathcal{J}_k} = x_1$, $A^{\mathcal{J}_k} = \emptyset$, $E^{\mathcal{J}_k} = \{x_1\}$, $C^{\mathcal{J}_k} = \{x_1, ..., x_k\}$, and $R^{\mathcal{J}_k} = \{(x_1, x_2), ..., (x_{k-1}, x_k), (x_k, x_k)\}$. It is obvious that $\mathcal{J}_k \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)$. Furthermore, let $\mathcal{I}_k = (\Delta^{\mathcal{I}_k}, \cdot^{\mathcal{I}_k})$ be an interpretation with $\Delta^{\mathcal{I}_k} = \Delta^{\mathcal{J}_k}$, $a^{\mathcal{I}_k} = a^{\mathcal{J}_k}$, $A^{\mathcal{I}_k} = \{x_1, ..., x_k\}$, $E^{\mathcal{I}_k} = \emptyset$, $C^{\mathcal{I}_k} = C^{\mathcal{J}_k}$, and $R^{\mathcal{I}_k} = R^{\mathcal{J}_k}$. Then it is obvious that $\mathcal{I}_k \in mod(\mathcal{K}_1)$ and consequently $\mathcal{J}_k \in \mathcal{K}_1 \diamond \mathcal{N}$.

For every positive integer $k$, construct another interpretation $\mathcal{J}'_k = (\Delta^{\mathcal{J}'_k}, \cdot^{\mathcal{J}'_k})$ as follows: $\Delta^{\mathcal{J}'_k} = \{x_1, ..., x_k\}$, $a^{\mathcal{J}'_k} = x_1$, $A^{\mathcal{J}'_k} = \emptyset$, $E^{\mathcal{J}'_k} = \{x_1\}$, $C^{\mathcal{J}'_k} = \{x_1, ..., x_k\}$, and $R^{\mathcal{J}'_k} = \{(x_1, x_2), ..., (x_{k-1}, x_k)\}$. Then, although $\mathcal{J}'_k \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)$, there is no interpretation $\mathcal{I}'_k \in mod(\mathcal{K}_1)$ with $R^{\mathcal{I}'_k} = R^{\mathcal{J}'_k}$. So, we have $\mathcal{J}'_k \notin \mathcal{K}_1 \diamond \mathcal{N}$.

Now, suppose solution for the revision setting under $\mathcal{G}^s_\subseteq$ and $\mathcal{G}^s_\sharp$ is a finite number of KBs $\mathcal{K}'_i = \langle \mathcal{T}, \mathcal{A}'_i \rangle$ $(1 \leq i \leq n)$ such that $\bigcup_{1 \leq i \leq n} mod(\mathcal{K}'_i) = \mathcal{K}_1 \diamond \mathcal{N}$. Consider the case that $k \geq 2$. From $\mathcal{J}_k \in \mathcal{K}_1 \diamond \mathcal{N}$, there must be some solution $\mathcal{K}'_i = \langle \mathcal{T}, \mathcal{A}'_i \rangle$ $(1 \leq i \leq n)$ such that $\mathcal{J}_k \in mod(\mathcal{K}'_i)$. At the same time, from $\mathcal{J}'_k \notin \mathcal{K}_1 \diamond \mathcal{N}$, we have $\mathcal{J}'_k \notin mod(\mathcal{K}'_i)$. Therefore, the ABox $\mathcal{A}'_i$ must contain some concept assertion $X(a)$ such that the role depth $rd(X)$ equals $k$. Since the value of $k$ can be infinitely big, such an ABox does not exist. In other words, solutions under the semantics $\mathcal{G}^s_\subseteq$ and $\mathcal{G}^s_\sharp$ can not be expressed.

Secondly, we apply the semantics $\mathcal{G}^a_\subseteq$ and $\mathcal{G}^a_\sharp$. It is obvious that, for any interpretations $\mathcal{I} \in mod(\mathcal{K}_1)$ and $\mathcal{J} \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)$, it must be $a^\mathcal{I} \in A^\mathcal{I}$, $a^\mathcal{J} \in E^\mathcal{J}$, $a^\mathcal{J} \notin A^\mathcal{J}$, and $a^\mathcal{I} \notin E^\mathcal{I}$. Therefore, under $\mathcal{G}^a_\sharp$, we have $min\{dist^a_\sharp(\mathcal{I}', \mathcal{J}') \mid \mathcal{I}' \in mod(\mathcal{K}_1), \mathcal{J}' \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)\} = 2$; under $\mathcal{G}^a_\subseteq$, we have $min\{dist^a_\subseteq(\mathcal{I}', \mathcal{J}', X) \mid \mathcal{I}' \in mod(\mathcal{K}_1), \mathcal{J}' \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)\} = \{a\}$ for $X \in \{A, E\}$, and $min\{dist^a_\subseteq(\mathcal{I}', \mathcal{J}',$

$X) \mid \mathcal{I}' \in mod(\mathcal{K}_1), \mathcal{J}' \in mod(\langle \mathcal{T}, \mathcal{N} \rangle)\} = \emptyset$ for $X \in \Sigma \setminus \{A, E\}$. So, under both $\mathcal{G}^a_{\subseteq}$ and $\mathcal{G}^a_{\sharp}$, we have

$$\begin{aligned} \mathcal{K}_1 \diamond \mathcal{N} = \{\mathcal{J} \in mod(\langle \mathcal{T}, \mathcal{N} \rangle) \quad & \mid \textit{there exists } \mathcal{I} \in mod(\mathcal{K}_1) \textit{ such that} \\ & A^{\mathcal{I}} \ominus A^{\mathcal{J}} = E^{\mathcal{I}} \ominus E^{\mathcal{J}} = \{a^{\mathcal{I}}\}, \textit{ and} \\ & X^{\mathcal{I}} = X^{\mathcal{J}} \textit{ for any } X \in \Sigma \setminus \{A, E\}\}. \end{aligned}$$

We can construct a KB $\mathcal{K}_1' = \langle \mathcal{T}, \{E(a), C(a), R(a,a)\} \rangle$ that satisfies $mod(\mathcal{K}_1')$ $= \mathcal{K}_1 \diamond \mathcal{N}$. Therefore, under both $\mathcal{G}^a_{\subseteq}$ and $\mathcal{G}^a_{\sharp}$, $\mathcal{K}_1'$ is a solution for the revision setting. However, this solution is very strange, since there seems to be no "good" reason to enforce the assertion $R(a,a)$ to hold. □

To sum up, there are four notions of computing models in existing MBAs. For the ABox revision problem in $\mathcal{EL}_{\bot}$, two notions suffer from inexpressibility and the other two notions are semantically questionable.

### 3.2 Formula-based Approaches

There are two typical formula-based approaches. The first one is based on deductive closures [5, 14]. Given an ABox revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and $\mathcal{N}$, it firstly calculates the deductive closure of $\mathcal{A}$ w.r.t. $\mathcal{T}$ (denoted $cl_{\mathcal{T}}(\mathcal{A})$), and then computes a maximal subset of $cl_{\mathcal{T}}(\mathcal{A})$ that does not conflict with $\mathcal{N}$ and $\mathcal{T}$. This method works for restricted forms of DL-Lite, where the ABox $\mathcal{A}$ only contains assertions of the form $A(a)$, $\exists R(a)$ and $R(a,b)$, with $A$ and $R$ concept names or role names. With such an assumption, $cl_{\mathcal{T}}(\mathcal{A})$ is finite and can be calculated effectively. However, it does not work for $\mathcal{EL}_{\bot}$, since in our revision setting we allow any assertion constructed on $\mathcal{EL}_{\bot}$ concepts occurring in the ABox $\mathcal{A}$.

The second FBA is based on justifications [19]. Given an ABox revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and $\mathcal{N}$, it firstly constructs a KB $\mathcal{K}_0 = \langle \mathcal{T}, \mathcal{A} \cup \mathcal{N} \rangle$ and finds all the minimal subsets of $\mathcal{K}_0$ that entail a clash (i.e., all justifications for clashes); then it computes a minimal set $\mathcal{R} \subseteq \mathcal{A}$ which contains at least one element from each justification; finally it returns $\mathcal{K}' = \langle \mathcal{T}, (\mathcal{A} \cup \mathcal{N}) \setminus \mathcal{R} \rangle$ as a solution. Obviously this approach can deal with $\mathcal{EL}_{\bot}$. However, as shown by the following examples, it is neither fine-grained nor syntax-independent.

*Example 2.* Consider the revision setting $\mathcal{K}_1 = \langle \mathcal{T}, \mathcal{A}_1 \rangle$ and $\mathcal{N}$ described in the previous example. It is obvious that $\langle \mathcal{T}, \mathcal{A}_1 \cup \mathcal{N} \rangle \models \bot(a)$ and for which there is only one justification $\mathcal{J} = \{A \sqsubseteq \exists R.A, E \sqcap \exists R.A \sqsubseteq \bot, A(a), E(a)\}$. Since $\mathcal{T}$ is protected and $E(a) \in \mathcal{N}$, our only choice is to remove $A(a)$ from $\mathcal{A}_1 \cup \mathcal{N}$ and get a solution $\mathcal{K}_1' = \langle \mathcal{T}, \{E(a)\} \rangle$.

This solution is not so good, since it loses many information which is entailed by $\mathcal{K}_1$ and not conflicted with $\mathcal{N}$, such as the assertions $C(a)$ and $\exists R.C(a)$. □

*Example 3.* Consider another revision setting $\mathcal{K}_2 = \langle \mathcal{T}, \mathcal{A}_2 \rangle$ and $\mathcal{N}$, where $\mathcal{A}_2 = \{A(a), C(a), \exists R.C(a)\}$. Apply the FBA based on justifications again, we will get a solution $\mathcal{K}_2' = \langle \mathcal{T}, \{E(a), C(a), \exists R.C(a)\} \rangle$.

Since the KBs $\mathcal{K}_1$ and $\mathcal{K}_2$ are logically equivalent, it is unhelpful that we get two different solutions w.r.t. the same new information $\mathcal{N}$. □

To sum up, for ABox revision in $\mathcal{EL}_\perp$, existing FBAs either can not be applied directly, or can be applied but is syntax-dependent and not fine-grained.

### 3.3 Our Approach

Our method is composed of three steps. Given an ABox revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and $\mathcal{N}$, we firstly construct a non-redundant depth-bounded model $\mathcal{G}$ (called $k$-MW in this paper) for the initial KB $\mathcal{K}$, and treat $\mathcal{G}$ as an ABox $\mathcal{A}_\mathcal{G}$ which contains some auxiliary variables. Then we execute a justification-based revision process on the KB $\mathcal{K}_0 = \langle \mathcal{T}, \mathcal{A}_\mathcal{G} \rangle$ plus the new information $\mathcal{N}$, and get a consistent KB $\mathcal{K}_0 = \langle \mathcal{T}, (\mathcal{A}_\mathcal{G} \setminus \mathcal{R}) \cup \mathcal{N} \rangle$. Finally we map $\mathcal{A}_\mathcal{G} \setminus \mathcal{R}$ back into an ABox $\mathcal{A}'$ by "rolling up" auxiliary variables, and get a solution $\mathcal{K}' = \langle \mathcal{T}, \mathcal{A}' \cup \mathcal{N} \rangle$.

*Example 4.* Consider the revision setting $\mathcal{K}_1 = \langle \mathcal{T}, \mathcal{A}_1 \rangle$ and $\mathcal{N}$ described in Example 1. Firstly, since $max\{depth(\mathcal{K}_1), depth(\mathcal{N})\} = 1$, we will introduce two variables $x_1, x_2$, and construct a 1-MW $\mathcal{G}$ for the KB $\mathcal{K}_1$ (details will be given in Section 4.1 and 4.2). By treating $\mathcal{G}$ as an ABox, we get $\mathcal{A}_\mathcal{G} = \{A(a), C(a), R(a, x_1), A(x_1), C(x_1), R(x_1, x_2), A(x_2), C(x_2)\}$.

Secondly, for the clash $\langle \mathcal{T}, \mathcal{A}_\mathcal{G} \cup \mathcal{N} \rangle \models \perp(a)$, there are two justifications: $\mathcal{J}_1 = \{A \sqsubseteq \exists R.A, E \sqcap \exists R.A \sqsubseteq \perp, E(a), A(a)\}$ and $\mathcal{J}_2 = \{E \sqcap \exists R.A \sqsubseteq \perp, E(a), R(a, x_1), A(x_1)\}$. Since $\mathcal{T}$ is protected and $E(a) \in \mathcal{N}$, we can get two possible minimal repairs for removing the clash: $\mathcal{R}_1 = \{A(a), A(x_1)\}$ and $\mathcal{R}_2 = \{A(a), R(a, x_1)\}$. However, since $\mathcal{R}_2$ contains a role assertion $R(a, x_1)$ where $x_1$ a variable, all the information related to $x_1$ will be lost if we apply $\mathcal{R}_2$ as a repair. Therefore, we choose $\mathcal{R}_1$ as the repair and get a consistent KB $\mathcal{K}_0 = \langle \mathcal{T}, (\mathcal{A}_\mathcal{G} \setminus \mathcal{R}_1) \cup \mathcal{N} \rangle = \langle \mathcal{T}, \{C(a), R(a, x_1), C(x_1), R(x_1, x_2), A(x_2), C(x_2)\} \cup \mathcal{N} \rangle$.

Finally, by rolling up variables occurring in the ABox of $\mathcal{K}_0$, we get a solution $\mathcal{K}' = \langle \mathcal{T}, \{C(a), \exists R.(C \sqcap \exists R.(A \sqcap C))(a), E(a)\} \rangle$. □

In our solution, information contained in $\mathcal{K}_1$ is inherited as more as possible. Furthermore, as we will see later, since the depth-bounded model $\mathcal{G}$ constructed in the first step is non-redundant, our solution is syntax-independent.

## 4 ABox Revision Algorithm for $\mathcal{EL}_\perp$

Before presenting our revision algorithm, we introduce three procedures which will be used in the revision algorithm.

### 4.1 Calculating Witness for Knowledge Base

The first procedure will construct a possibly redundant depth-bounded model for the initial KB based on a structure named revision graph.

A *revision graph* for $\mathcal{EL}_\perp$ is a directed graph $\mathcal{G} = (V, E, \mathcal{L})$, where

- $V$ is a finite set of nodes composed of individual names and variables;
- $E \subseteq V \times V$ is a set of edges satisfying:

  - there is no edge from variables to individual names, and
  - for each variable $y \in V$, there is at most one node $x$ with $\langle x, y \rangle \in E$;
- each node $x \in V$ is labelled with a set of concepts $\mathcal{L}(x)$; and
- each edge $\langle x, y \rangle \in E$ is labelled with a set of role names $\mathcal{L}(\langle x, y \rangle)$; furthermore, if $y$ is a variable then $\sharp\mathcal{L}(\langle x, y \rangle) = 1$.

For each edge $\langle x, y \rangle \in E$, we call $y$ a *successor* of $x$ and $x$ a *predecessor* of $y$. *Descendant* is the transitive closure of successor.

For any node $x \in V$, we use $level(x)$ to denote the *level* of $x$ in the graph, and define it inductively as follows: $level(x) = 0$ if $x$ is an individual name, $level(x) = level(y) + 1$ if $x$ is a variable with a predecessor $y$, and $level(x) = +\infty$ if $x$ is a variable without predecessor.

**Procedure 1** *Given a KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and a non-negative integer $k$, construct a revision graph $\mathcal{G} = (V, E, \mathcal{L})$ by the following steps:*

**Step 1.** *Initialize the graph $\mathcal{G}$ as:*
- $V = N_I^{\mathcal{K}}$,
- $\mathcal{L}(a) = \{C \mid C(a) \in \mathcal{A}\}$ *for each node* $a \in V$,
- $E = \{\langle a, b \rangle \mid$ *there is some* $R$ *with* $R(a, b) \in \mathcal{A}\}$,
- $\mathcal{L}(\langle a, b \rangle) = \{R \mid R(a, b) \in \mathcal{A}\}$ *for each edge* $\langle a, b \rangle \in E$.

**Step 2.** *Expand the graph by applying the following rules, until none of these rules is applicable:*

**GCI$_{Ind}$-rule:** *if* $x \in N_I^{\mathcal{K}}$, $C \sqsubseteq D \in \mathcal{T}$, $D \notin \mathcal{L}(x)$, *and* $\langle \mathcal{T}, \mathcal{A} \rangle \models C(x)$, *then set* $\mathcal{L}(x) = \mathcal{L}(x) \cup \{D\}$.

**GCI$_{Var}$-rule:** *if* $x \notin N_I^{\mathcal{K}}$, $C \sqsubseteq D \in \mathcal{T}$, $D \notin \mathcal{L}(x)$, *and* $\langle \mathcal{T}, \{E(x) \mid E \in \mathcal{L}(x)\} \rangle \models C(x)$, *then set* $\mathcal{L}(x) = \mathcal{L}(x) \cup \{D\}$.

**$\sqcap$-rule:** *if* $C_1 \sqcap C_2 \in \mathcal{L}(x)$, *and* $\{C_1, C_2\} \nsubseteq \mathcal{L}(x)$, *then set* $\mathcal{L}(x) = \mathcal{L}(x) \cup \{C_1, C_2\}$.

**$\exists$-rule:** *if* $\exists R.C \in \mathcal{L}(x)$, $x$ *has no successor* $z$ *with* $C \in \mathcal{L}(z)$, *and* $level(x) \leq k$, *then introduce a new variable* $z$, *set* $V = V \cup \{z\}$, $E = E \cup \{\langle x, z \rangle\}$, $\mathcal{L}(z) = \{C\}$, *and* $\mathcal{L}(\langle x, z \rangle) = \{R\}$.

**Step 3.** *Return the graph* $\mathcal{G} = (V, E, \mathcal{L})$.

Given any KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and non-negative integer $k$, we call the revision graph $\mathcal{G}$ returned by Procedure 1 a *$k$-role-depth-bounded witness* ($k$-W) for $\mathcal{K}$.

Revision graphs can be seen as ABoxes with variables. Given a revision graph $\mathcal{G} = (V, E, \mathcal{L})$, we call $\mathcal{A}_{\mathcal{G}} = \bigcup_{x \in V} \{C(x) \mid C \in \mathcal{L}(x)\} \cup \bigcup_{\langle x, y \rangle \in E} \{R(x, y) \mid R \in \mathcal{L}(\langle x, y \rangle)\}$ as the *ABox representation* of $\mathcal{G}$, and call $\mathcal{G}$ as the *revision-graph representation* of $\mathcal{A}_{\mathcal{G}}$.

**Proposition 1.** *Let $\mathcal{G} = (V, E, \mathcal{L})$ be a $k$-W for a KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, and let $\mathcal{A}_{\mathcal{G}}$ be the ABox representation of $\mathcal{G}$. Then, for any ABox assertion $\alpha$ with $sig(\alpha) \subseteq sig(\mathcal{K})$ and $depth(\alpha) \leq k$, $\mathcal{K} \models \alpha$ iff $\langle \emptyset, \mathcal{A}_{\mathcal{G}} \rangle \models \alpha$.*

### 4.2 Calculating Minimal Witness for Knowledge Base

A $k$-W of KB possibly contains some redundant information which will make two logically equivalent KBs have different witnesses. In this subsection, we introduce a procedure to remove these redundant information.

A graph $\mathcal{B} = (V', E', \mathcal{L}')$ is a *branch* of a revision graph $\mathcal{G}$ if $\mathcal{B}$ is a tree and a subgraph of $\mathcal{G}$.

A branch $\mathcal{B}_1 = (V_1, E_1, \mathcal{L}_1)$ is *subsumed* by another branch $\mathcal{B}_2 = (V_2, E_2, \mathcal{L}_2)$ if $\mathcal{B}_1$ and $\mathcal{B}_2$ have the same root node, $\sharp(V_1 \cap V_2) = 1$, and there is a function $f : V_1 \to V_2$ such that: $f(x) = x$ if $x$ is the root node, $\mathcal{L}_1(x) \subseteq \mathcal{L}_2(f(x))$ for every node $x \in V_1$, $\langle f(x), f(y)\rangle \in E_2$ for every edge $\langle x, y\rangle \in E_1$, and $\mathcal{L}_1(\langle x, y\rangle) \subseteq \mathcal{L}_2(\langle f(x), f(y)\rangle)$ for every edge $\langle x, y\rangle \in E_1$.

A branch $\mathcal{B}$ is *redundant* in $\mathcal{G}$ if every node in $\mathcal{B}$ except the root is a variable, and $\mathcal{B}$ is subsumed by another branch in $\mathcal{G}$.

**Procedure 2** *Given a KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}\rangle$ and a non-negative integer $k$, let $\mathcal{G} = (V, E, \mathcal{L})$ be a $k$-W for $\mathcal{K}$, construct a revision graph $\mathcal{G}' = (V', E', \mathcal{L}')$ by the following steps:*

**Step 1.** *Initialize the graph $\mathcal{G}' = (V', E', \mathcal{L}')$ as*
- *$V' = V$, $E' = E$, $\mathcal{L}'(\langle x, y\rangle) = \mathcal{L}(\langle x, y\rangle)$ for every $\langle x, y\rangle \in E'$, and*
- *$\mathcal{L}'(x) = \{C \in \mathcal{L}(x) \mid C$ is a concept name$\}$ for every $x \in V'$.*

**Step 2.** *Prune $\mathcal{G}'$ by the following rule until the rule is not applicable:*
**R-rule:** *if there is a redundant branch $\mathcal{B} = (V_\mathcal{B}, E_\mathcal{B}, \mathcal{L}_\mathcal{B})$ in $\mathcal{G}'$, then set $E' = E' \setminus E_\mathcal{B}$ and $V' = V' \setminus (V_\mathcal{B} \setminus \{x_\mathcal{B}\})$, where $x_\mathcal{B}$ is the root of $\mathcal{B}$.*

**Step 3.** *Return the graph $\mathcal{G}'$.*

For any KB $\mathcal{K}$ and non-negative integer $k$, we call the graph $\mathcal{G}'$ returned by Procedure 2 a *$k$-role-depth-bounded minimal witness* ($k$-MW) for $\mathcal{K}$.

**Proposition 2.** *Let $\mathcal{G}'$ be a $k$-MW for a KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}\rangle$, and let $\mathcal{A}_{\mathcal{G}'}$ be the ABox representation of $\mathcal{G}'$. Then, for any ABox assertion $\alpha$ with $sig(\alpha) \subseteq sig(\mathcal{K})$ and $depth(\alpha) \le k$, $\mathcal{K} \models \alpha$ iff $\langle \emptyset, \mathcal{A}_{\mathcal{G}'}\rangle \models \alpha$.*

The following proposition states that, for any logically equivalent KBs $\mathcal{K}$ and $\mathcal{K}'$, their $k$-MW are identical up to variable renaming in the case that $k$ is sufficiently large.

**Proposition 3.** *Let $\mathcal{K}_i = \langle \mathcal{T}, \mathcal{A}_i\rangle$ $(i = 1, 2)$ be two consistent KBs, let $k$ be an integer with $k \ge depth(\mathcal{T})$, $k \ge depth(\mathcal{A}_1)$ and $k \ge depth(\mathcal{A}_2)$, and let $\mathcal{G}_i = (V_i, E_i, \mathcal{L}_i)$ be a $k$-MW for $\mathcal{K}_i$ $(i = 1, 2)$. If $mod(\mathcal{K}_1) = mod(\mathcal{K}_2)$, then there is a bijection $f : V_1 \to V_2$ such that*

- *$f(x) = x$ if $x$ is an individual name,*
- *$\mathcal{L}_1(x) = \mathcal{L}_2(f(x))$ for every node $x \in V_1$,*
- *$\langle x, y\rangle \in E_1$ iff $\langle f(x), f(y)\rangle \in E_2$, and*
- *$\mathcal{L}_1(\langle x, y\rangle) = \mathcal{L}_2(\langle f(x), f(y)\rangle)$ for every edge $\langle x, y\rangle \in E_1$.*

### 4.3 Transforming a Witness Back into a Knowledge Base

In this subsection we introduce a procedure to transform a revision graph into an ABox without variables.

**Procedure 3** *Given a TBox $\mathcal{T}$ and a revision graph $\mathcal{G} = (V, E, \mathcal{L})$, construct an ABox by the following steps:*

**Step 1.** *Delete from $V$ all the variables which are not descendants of any individual names in $\mathcal{G}$.*

**Step 2.** *Roll up the graph $\mathcal{G}$ by applying the following substeps repeatedly, until there is no variable contained in $V$:*

1. *Select a variable $y \in V$ that has no successor.*
2. *Let $x$ be the predecessor of $y$.*
3. *If $\mathcal{L}(y) \neq \emptyset$, then let $C_y$ be the concept $\bigsqcap_{C \in \mathcal{L}(y)} C$, else let $C_y$ be $\top$.*
4. *If $\langle \mathcal{T}, \{D(x) \mid D \in \mathcal{L}(x)\}\rangle \not\models (\exists R.C_y)(x)$, then set $\mathcal{L}(x) = \mathcal{L}(x) \cup \{\exists R.C_y\}$, where $R$ is the role name contained in $\mathcal{L}(\langle x, y\rangle)$.*
5. *Set $E = E \setminus \{\langle x, y\rangle\}$ and $V = V \setminus \{y\}$.*

**Step 3.** *Return $\mathcal{A} = \bigcup_{x \in V} \{C(x) \mid C \in \mathcal{L}(x)\} \cup \bigcup_{\langle x,y\rangle \in E} \{R(x, y) \mid R \in \mathcal{L}(\langle x, y\rangle)\}$.*

**Proposition 4.** *For any revision graph $\mathcal{G}$ and TBox $\mathcal{T}$, let $\mathcal{A}$ be the ABox returned by Procedure 3, and let $\mathcal{A}_\mathcal{G}$ be the ABox representation of $\mathcal{G}$. Then, $\langle \mathcal{T}, \mathcal{A}_\mathcal{G}\rangle \models \mathcal{A}$.*

### 4.4 The Revision Algorithm

Given a TBox $\mathcal{T}$ and two ABoxes $\mathcal{A}$ and $\mathcal{N}$, if $\langle \mathcal{T}, \mathcal{A} \cup \mathcal{N}\rangle \models \top \sqsubseteq \bot$, then:

– a set $\mathcal{J} \subseteq \mathcal{A}$ is a $(\mathcal{A},\mathcal{N})$-*justification for clash* w.r.t. $\mathcal{T}$ if $\langle \mathcal{T}, \mathcal{J} \cup \mathcal{N}\rangle \models \top \sqsubseteq \bot$ and $\langle \mathcal{T}, \mathcal{J}' \cup \mathcal{N}\rangle \not\models \top \sqsubseteq \bot$ for every $\mathcal{J}' \subset \mathcal{J}$;
– a set $\mathcal{R} \subseteq \mathcal{A}$ is a $(\mathcal{A},\mathcal{N})$-*repair for clash* w.r.t. $\mathcal{T}$ if $\sharp(\mathcal{R} \cap \mathcal{J}) = 1$ for every $\mathcal{J}$ which is an $(\mathcal{A},\mathcal{N})$-justification for clash w.r.t. $\mathcal{T}$.

Now we are ready to present our ABox revision algorithm.

**Algorithm 1** *Given a revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}\rangle$ and $\mathcal{N}$, given a non-negative integer $k$, construct a finite number of KBs by the following steps:*

**Step 1.** *If $\mathcal{A} \cup \mathcal{N}$ is consistent w.r.t. $\mathcal{T}$, then return $\mathcal{K}' = \langle \mathcal{T}, \mathcal{A} \cup \mathcal{N}\rangle$, else continue the following steps.*

**Step 2.** *Construct a $k$-MW $\mathcal{G}$ for $\mathcal{K}$.*

**Step 3.** *Let $\mathcal{A}_\mathcal{G}$ be the ABox representation of $\mathcal{G}$. Calculate all the $(\mathcal{A}_\mathcal{G},\mathcal{N})$-repairs $\mathcal{R}_1, ..., \mathcal{R}_n$ for clash w.r.t. $\mathcal{T}$.*

**Step 4.** *For each $\mathcal{R}_i$ $(1 \leq i \leq n)$ do the following operations:*

1. *Construct an ABox $\mathcal{A}_i = \mathcal{A}_\mathcal{G} \setminus \mathcal{R}_i$.*
2. *Let $\mathcal{G}_i$ be the revision-graph representation of $\mathcal{A}_i$. Call Procedure 3 to generate an ABox $\mathcal{A}'_i$ by taking $\mathcal{T}$ and $\mathcal{G}_i$ as inputs.*

*3. Construct a KB $\mathcal{K}'_i = \langle \mathcal{T}, \mathcal{A}'_i \cup \mathcal{N} \rangle$.*

**Step 5.** *Let $S = \{\mathcal{K}'_1, ..., \mathcal{K}'_n\}$. For each $\mathcal{K}'_i = \langle \mathcal{T}, \mathcal{A}'_i \cup \mathcal{N} \rangle \in S$, if there is another $\mathcal{K}'_j \in S$ such that $\mathcal{K}'_j \models \mathcal{A}'_i \cup \mathcal{N}$, then remove $\mathcal{K}'_i$ from $S$.*

**Step 6.** *Return all the KBs contained in $S$.*

The following theorems state that our algorithm satisfies the properties specified by **R1-R4** in Section 1 of the paper.

**Theorem 1.** *For any revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and $\mathcal{N}$, let $\mathcal{K}' = \langle \mathcal{T}', \mathcal{A}' \rangle$ be any KB returned by Algorithm 1. Then, (1) $\mathcal{K}'$ is consistent, (2) $\mathcal{K}' \models \mathcal{N}$ and $\mathcal{T}' = \mathcal{T}$, and (3) $\mathcal{A}' = \mathcal{A} \cup \mathcal{N}$ if $\mathcal{A} \cup \mathcal{N}$ is consistent w.r.t. $\mathcal{T}$.*

**Theorem 2.** *Given any two revision settings $\mathcal{K}_i = \langle \mathcal{T}, \mathcal{A}_i \rangle$ and $\mathcal{N}_i$ $(i = 1, 2)$, and any integer $k$ such that $k \geq depth(X)$ for $X \in \{\mathcal{T}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{N}_1, \mathcal{N}_2\}$. If $\mathcal{K}_1 \equiv \mathcal{K}_2$ and $\langle \mathcal{T}, \mathcal{N}_1 \rangle \equiv \langle \mathcal{T}, \mathcal{N}_2 \rangle$, then for any KB $\mathcal{K}'_1$ returned by Algorithm 1 for the revision setting $\mathcal{K}_1$ and $\mathcal{N}_1$, there must be a KB $\mathcal{K}'_2$ returned by the algorithm for the revision setting $\mathcal{K}_2$ and $\mathcal{N}_2$ such that $\mathcal{K}'_1 \equiv \mathcal{K}'_2$.*

Theorem 2 is based on Proposition 3 where $k$ is required to be sufficiently large. Theorem 1 has no requirement on $k$; the first result of it is based on Proposition 4, and the other two results are obvious from the algorithm.

In our algorithm, the $k$-MW $\mathcal{G}$ for the KB $\mathcal{K}$ is in fact a non-redundant $k$-depth-bounded model for $\mathcal{K}$. Therefore, our revision process works on fine-grained representation of models and guarantees the minimal change principle in a fine-grained level. So, our algorithm satisfies the property specified by **R5**.

**Theorem 3.** *For any revision setting $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and $\mathcal{N}$, assume the role depth of every concept occurring in $\mathcal{K}$ and $\mathcal{N}$ is bounded by some integer $k$, then Algorithm 1 runs in time exponential with respect to the size of $\mathcal{K}$ and $\mathcal{N}$.*

## 5 Conclusion

We studied instance level KB revision in $\mathcal{EL}_\bot$. There are two main families of approaches to revision: model-based ones and formula-based ones. We illustrated that they both have disadvantages and are inappropriate for $\mathcal{EL}_\bot$. We presented a new method for the revision problem. Our method is closer in spirit to the formula-based approaches, but it also inherits some ideas of model-based ones. We showed that our algorithm behaves well for $\mathcal{EL}_\bot$ in that it satisfies the postulates proposed in the literature for revision operators.

For future work, we will extend our method to support ABox revision in $\mathcal{EL}^{++}$. Another work is to formalize the notion of minimality of change. Finally, we will make an optimization to the algorithm and test the feasibility of it in practice.

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