

Robo-Advisors: A Portfolio Management Perspective

Jonathan Walter Lam
Advised by David F. Swensen

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*Yale College
New Haven, Connecticut*

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To my parents

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INTRODUCTION

In many respects, financial advice is an enabler of risk-taking. Individuals who have little knowledge of or experience with the financial markets may not feel confident in their ability to design well-structured investment portfolios.¹ Hence, in giving individuals the confidence to take risk, financial advisors help individuals overcome their fears and act rationally. Robo-advisors, automated investment platforms that provide investment advice without the intervention of a human advisor, have emerged as an alternative to traditional sources of advice. While this paper does not study whether humans trust computers to provide sound investment advice, it conducts an examination of the robo-advisor model. As such, the paper may enable individuals to employ computer models to obtain sound investment advice.

This paper examines the robo-advisor model from the ground up. The first chapter discusses the benefits and limitations of mean-variance analysis, the primary asset allocation framework employed by robo-advisors, concluding that mean-variance analysis is a compelling framework for asset allocation that allows investors to construct efficiently diversified portfolios. While the model suffers from several limitations, such as the assumption of normally distributed returns and the sensitivity of optimized portfolios to estimation error, such limitations can be overcome through relatively straightforward techniques.

In the second chapter, the paper describes how robo-advisors work, emphasizing areas of commonality between robo-advisors and discussing the rationale for passive indexing, which is the investment strategy that most robo-advisors have adopted. It then describes robo-advisors' general investment methodology, showing that robo-advisors perform asset allocation with mean-variance analysis; implement portfolios in a low-cost, tax-efficient manner; and monitor and rebalance portfolios with the aid of automation.

The third chapter, which conducts an in-depth examination of three leading robo-advisors, discusses how robo-advisors differ from one another and concludes that the quality of investment advice is not consistent throughout the robo-advisory industry. Schwab Intelligent Portfolios, whose advice is compromised by material conflicts of interest, is an inferior robo-advisor compared to Wealthfront and Betterment. While both Wealthfront and Betterment possess well-grounded approaches to portfolio selection, they differ in some important respects. Wealthfront has created a general long-term investing platform, while Betterment has focused on goals-based investing. Wealthfront gauges an investor's subjective risk tolerance, while Betterment appears not to.

The fourth chapter assesses to what extent robo-advice could serve as an alternative to traditional sources of investment advice and as such has the greatest policy implications. The chapter makes the case that robo-advisors provide low-cost, transparent, well-grounded, and systematic investment advice, arguing that human advisors may fail on any of these counts. Critics of robo-advisors cite their provision of canned, non-personalized investment advice. At their current stage of development, robo-advisors do not consider an investor's entire financial profile. Yet empirical evidence suggests that human advisors also may not provide tailored

¹ Nicola Gennaioli, Andrei Shleifer, and Robert Vishny. Money Doctors. *The Journal of Finance*. February 2015.

advice; their biases may not only affect the data gathering process that is so essential to portfolio construction, but also the eventual recommendations that they make.

Critics of robo-advisors stress that these automated platforms cannot prevent investors from timing the markets and that the damage from such poor market timing behavior swamps all the benefits robo-advisors may provide. This paper argues that such claims are overblown and that the benefit of having a human advisor to “hold one’s hand” during times of market stress may be overstated. The paper presents qualitative and quantitative evidence supporting the view that robo-advisors can coach investors into better investing behaviors. It also presents evidence on the actual behavior of robo-advisor clients. To date, such evidence has lent support to the view that robo-advisors suppress clients’ inclination to time the markets.

This paper focuses on what robo-advice is, not what it will be. In principle, robo-advice could become infinitely customizable, as the design of ever more complex algorithms could allow robo-advisors to tailor portfolios to individuals with even the most unusual of financial circumstances. Data on clients’ income and career trajectory, saving and spending behavior, and assets and liabilities – coupled with artificial intelligence, machine learning, and other data science technologies – could be harnessed to make better investment recommendations. Robo-advisors will also become more adept at managing clients’ behavior. Data on clients’ trading, withdrawal, and rebalancing activity in robo-advisor and external accounts could improve risk measurement processes. Insights from behavioral economics and related fields could help robo-advisors re-design platforms to promote better investment behaviors. Robo-advice could one day become the norm for passive investing. Future indexers might look back on today’s market for financial advice, wondering why we ever trusted humans to provide sound and un-biased investment advice.

Yet we are not in the future. Robo-advice is still in its early days and it is the current state of robo-advice that policymakers and researchers seek to understand. Robo-advisors have become topical due to the Department of Labor’s proposed fiduciary rule, which critics argue would price small retirement savers out of the market for traditional investment advice, leaving them to invest on their own or through a robo-advisor.² To date, the regulatory debate has largely ignored the benefits of robo-advisors stemming from their sound investment philosophy and methodology. Robo-advisors espouse a strategy of passive indexing, which abundant empirical evidence has shown to be the best strategy for individual investors who do not have access to institutional quality active managers. Wealthfront and Betterment have selected a reasonable and diverse set of asset classes and use mean-variance optimization to construct efficient portfolios. These robo-advisors pay attention to tax efficiency, developing separate efficient frontiers for taxable and tax-deferred accounts. They provide unbiased, systematic advice, taking into account the investor’s time horizon in all cases and other investor attributes in some cases.

Robo-advisors may be sufficiently developed to provide advice to some, but not all, retirement investors. Betterment, in particular, has made a promising first attempt at a retirement investing product (see Chapter 3), dynamically adjusting individuals’ asset allocation in response

² Robert Litan and Hal Singer. Good Intentions Gone Wrong: The Yet-To-Be-Recognized Costs of the Department of Labor’s Proposed Fiduciary Rule. Economists Incorporated. July 2015.

to their spending needs. However, the robo-advisor does not appear to measure investors' subjective risk tolerance. Robo-advisor Wealthfront may provide adequate advice for some retirement savers, as investors' time horizon and risk tolerance, arguably the two most important factors for advisors to consider when making recommendations, are taken into account. Regardless of product quality, whether less tech-savvy investors will trust robo-advisors, however, remains an open question.

Taken as a whole, the findings of this paper suggest that investors who switch to robo-advisors may be better off than they were before. Robo-advisors are superior to many sources of traditional advice and will only become more sophisticated over time.

CHAPTER 1: BENEFITS AND LIMITATIONS OF MEAN-VARIANCE OPTIMIZATION

The mean-variance approach to portfolio selection, developed by Nobel laureates Harry Markowitz and James Tobin, is the most widely accepted model for asset allocation. Investors ranging from university endowments to Internet-based investment advisors (“robo-advisors”) employ mean-variance optimization to structure efficient portfolios. This chapter discusses the benefits and limitations of the mean-variance framework, often drawing examples from the Yale Investments Office.³

Benefits of Mean-Variance Optimization

Economists often say there is no such thing as a “free lunch.” Yet portfolio diversification, which one can achieve through mean-variance analysis, is perhaps the one exception to this adage, as diversification allows investors to reduce portfolio risk without sacrificing expected return or to increase expected return without accepting more risk.

Mean-variance optimization, introduced by Nobel laureate Harry Markowitz in his 1952 paper “Portfolio Selection,” was the first mathematical formalization of the idea of diversification of investments. The framework considers a set of risky assets and calculates portfolios for which the expected return is maximized for a given level of portfolio risk, where risk is measured as variance; an alternative formulation of the optimization minimizes portfolio risk for a given level of expected return.⁴ These optimized portfolios compose the “efficient frontier,” a band of portfolios that dominate all other feasible portfolios in terms of their risk-return tradeoff (Figure 1).

In a 1958 article entitled “Liquidity Preference as Behavior Toward Risk,” Nobel laureate James Tobin expanded upon Markowitz’s mean-variance framework, showing that the introduction of a riskless asset implies that there is an optimal risky portfolio on the efficient frontier whose selection is independent of the investor’s risk aversion. The capital market line, which passes through the riskless return and the optimal risky (“tangency”) portfolio, delineates the new set of efficient portfolios. Tobin’s work led to the famous “separation theorem,” the idea that portfolio selection is divided into two stages: first, an optimal sub-portfolio of risky assets is selected solely on the basis of the joint distribution of the returns of the risky and riskless assets; second, the investor divides wealth between the risky sub-portfolio and the riskless asset, choosing a portfolio from the capital market line on the basis of risk aversion or other factors.⁵

The primary benefit of employing mean-variance optimization is portfolio diversification, which is most easily explained through William Sharpe’s simplified model of portfolio theory, the so-called “one-factor model.”⁶ While the Sharpe model is usually applied to individual securities, the same logic extends to asset classes. Under the Sharpe model, the return on all securities is correlated to the market return through a constant called beta, but each security’s

³ The Yale Investments Office manages Yale's endowment and certain related assets.

⁴ The section on portfolio selection and investor objectives discusses how risk and volatility are not equivalent.

⁵ Mark Rubinstein. *A History of the Theory of Investments: My Annotated Bibliography*. John Wiley & Sons. 2006.

⁶ Harry M. Markowitz, Mark T. Hebner, Mary E. Brunson. *Does Portfolio Theory Work During Financial Crises?* www.ifaarchive.com

return is also subject to an idiosyncratic term that is independent of the market return and the idiosyncratic terms of all other securities. A portfolio's beta, the weighted average of the betas of the securities in the portfolio, measures the portfolio's correlation to the market. Similarly, the portfolio's idiosyncratic term is the weighted average of the idiosyncratic terms for each of the securities.

However, since the idiosyncratic term of each security is assumed to be independent of that of all other securities, the variance of the idiosyncratic term of the portfolio is not the weighted sum of the constituent securities' idiosyncratic variances.⁷ It is, in fact, less than the weighted sum, since the idiosyncratic terms tend to diversify – some are positive while others are negative, cancelling each other out.⁸ With a sufficiently large number of securities, idiosyncratic risk can be completely eliminated. However, risk from correlation to the market – the systematic risk – cannot be diversified away.

In contrast to the Sharpe model, mean-variance optimization takes into account the overall risk of securities (or asset classes), without separating out their systematic and idiosyncratic (unsystematic) components.⁹ Also, while in the Sharpe model securities correlate with one another through their relationship with the market return, in the Markowitz framework securities relate to one another more generally through a specified pattern of correlation – a correlation matrix. Despite their differences, both models of portfolio theory capture the basic insight that imperfect co-movement of returns – either through independent idiosyncratic risk components in the case of the Sharpe model or less than perfect correlation in the Markowitz framework – reduces portfolio risk. More specifically, as long as the correlation between asset classes is less than one, the variance of portfolio returns will be less than the weighted average of the variances of its constituent assets.

Limitations of Mean-Variance Optimization

Investors intending to employ the mean-variance asset allocation framework should possess a thorough understanding of its limitations. This section highlights many of the limitations of mean-variance optimization and presents solutions when applicable.

Normality Assumptions

Mean-variance optimization assumes that asset class returns are normally distributed, but real-world returns possess significant nonnormal characteristics. Perhaps the greatest limitation of the normality assumption is that it inadequately accounts for the possibility of extreme market moves.¹⁰ Yale economist William Nordhaus shows that for the 140-year period from 1871 to

⁷ Ibid.

⁸ Diversification is related to the Central Limit Theorem. If the idiosyncratic terms in the one-factor model are identical and independent random variables, the Central Limit Theorem implies that the variance of the average of the idiosyncratic terms goes to zero when the number of asset classes is sufficiently large. Thus, if the mean of the idiosyncratic terms is zero, the inclusion of more asset classes effectively diversifies away idiosyncratic risk.

⁹ Harry Markowitz. Crisis Mode: Portfolio Theory Under Pressure. *The Financial Professionals' Post*. June 8, 2010.

¹⁰ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 105; Ashvin B. Chhabra. *The Aspirational Investor*. HarperCollins. 2015. 90.

2010, the actual maximum and minimum monthly returns on the U.S. stock market were much larger than would be found with a normal distribution.¹¹ A study by Morningstar provides further evidence of “fat-tailed” asset class distributions, finding that between January 1926 and May 2011 there were 10 months when monthly returns were more than three standard deviations below the mean; the normal distribution implies that there should have only been 1.3 months with such returns.¹² The 2007-2008 global financial meltdown, during which U.S. stocks dropped 57 percent from peak to trough, and the 1987 stock market crash, during which U.S. stock prices fell by 23 percent on Black Monday, are two examples of tail events.

Another problem associated with the assumption of normally distributed returns is that variance is a symmetrical risk measure, one that does not distinguish between upside and downside moves.¹³ Investment returns with positive skew will appear riskier than they really are, leading to under-allocation of the asset class; similarly, returns with negative skew will appear less risky than they really are, leading to over-allocation of the asset class. These concerns are not merely academic musings. Some investors actively seek out returns with favorable asymmetry characteristics; for instance, the Yale Investments Office seeks to hire investment managers whose return distributions exhibit positive skew. Markowitz himself in his 1959 book on portfolio theory acknowledged that using the semi-variance, rather than the variance, as a measure of risk tends to produce better portfolios, as the former does not consider extremely high returns undesirable.¹⁴ However, Markowitz qualifies his critique of using variance as a risk measure, arguing that the variance and semi-variance produce the same efficient portfolios if return distributions are in fact symmetric or possess the same degree of asymmetry. Moreover, a portfolio with low variance must also have low semi-variance, though such a portfolio may sacrifice too much expected return in eliminating both upside and downside volatility.

Evidence suggests that failing to incorporate information about fat tails and skewness may lead to suboptimal portfolio decisions. Using a version of Conditional Value at Risk (CVaR) as their risk measure, Xiong and Idzorek (2011) show that incorporating skewness and kurtosis (fat tails) into portfolio optimization can have a significant impact on optimal allocations.¹⁵ The authors compared portfolio allocations from mean-variance optimization and the CVaR optimization by holding the expected return constant across both optimizations. Xiong and Idzorek show that the combination of skewness and kurtosis with mixed tails (meaning asset classes do not have uniformly fat tails) leads to the largest effect on optimal allocations; zero skewness and uniform tails, zero skewness and mixed tails, and non-zero skewness and uniformly fat tails lead to smaller effects.

¹¹ William Nordhaus. Elementary Statistics of Tail Events. *Review of Environmental and Economic Policy*. April 8, 2011.

¹² Morningstar. Asset Allocation Optimization Methodology. December 12, 2011.

¹³ Ashvin B. Chhabra. *The Aspirational Investor*. HarperCollins. 2015. 88.

¹⁴ Harry Markowitz. Portfolio Selection. Cowles Foundation for Research in Economics at Yale University. 1959. 194. The semi-variance is the average of the squared deviations of values that are less than the mean.

¹⁵ James X. Xiong and Thomas M. Idzorek. The Impact of Skewness and Fat Tails on the Asset Allocation Decision. *Financial Analysts Journal*. March/April 2011.

Value at Risk (VaR) is a statistical measure of the amount of money a portfolio, strategy, or firm might expect to lose over a specified time horizon with a given probability. Conditional Value at Risk (CVaR) is an extension of VaR that gives the total amount of loss given a loss event. For more on VaR and CVaR, please consult <http://www.cfapubs.org/doi/pdf/10.2469/irpn.v2012.n1.6>

By conducting “stress tests” of efficient portfolios, investors can overcome the inability of mean-variance optimization to account for extreme market events. The Yale Investments Office has been a leader in this arena, stress testing its portfolio across a range of human-generated return scenarios that would be unlikely to occur under a normal distribution.¹⁶ These scenarios are based on qualitative and quantitative analysis of particular stress scenarios and consideration of fundamental asset class attributes.

For example, the Yale Investments Office has modeled a “market shock” scenario comparable to the 2008 financial crisis.¹⁷ Under this scenario, U.S. equities fall by 35 percent, and Treasuries appreciate due to their safe-haven status. Since foreign equities, particularly in emerging markets, are typically more volatile than domestic equities during bear markets, they are projected to fall by more than domestic equities. The private equity portfolio, which is invested in smaller companies than its public market counterparts, might be expected to fall by more than domestic equities; however, the ability of Yale’s investment managers to implement aggressive cost-cuts, reposition businesses, and work with lenders to avoid portfolio company defaults exerts a countervailing force, mitigating the potential impact to private equity. Volatile commodity prices and higher risk premia due to investor deleveraging might leave natural resources particularly exposed during such a market shock, leading to a considerable drawdown in the price of natural resources equities. The Yale Investments Office also considers potential recovery paths from the initial shocks, extending the time horizon of the stress tests. Stress scenarios range from “market shocks” to “inflation induced collapses” to “deflationary recessions.” In each case, the impact of the shocks to each asset class is assessed with a high level of conservatism.

Unfortunately, no straightforward solution exists for correcting optimal allocations based on asymmetrical return distributions. Inasmuch as using mean-variance optimization is both an art and a science, investors may find it reasonable to make adjustments to optimal allocations to account for skewness of returns.

Static Inputs

Mean-variance optimization takes static inputs, but real-world correlations between asset class returns are time-varying.¹⁸ In particular, during periods of acute market stress, cross-asset correlations increase markedly, temporarily diverging from long-run correlation levels. As Figure 2 shows, the correlations of foreign developed equity markets, foreign emerging equity markets, commodities, and the price of oil to the S&P 500 increased during the 2007-2008 financial crisis.¹⁹ Commodities in particular experienced a sharp increase in their correlation with U.S. equities, as a negative demand shock and investor deleveraging pushed commodity

¹⁶ This discussion about stress tests relies on a conversation the author had with Alex Hetherington, a Director of the Yale Investments Office.

¹⁷ Ibid.

¹⁸ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 105.

¹⁹ Jeremy Siegel. *Stocks for the Long Run*. McGraw Hill. 2014. 49.

prices lower.²⁰ Before the crisis, numerous studies had touted commodities as the silver bullet for asset allocation due to their low correlation to other asset classes.²¹

That correlations among asset class returns approach one during financial crises is often cited as a major limitation of modern portfolio theory. But as Harry Markowitz has argued, this is exactly what portfolio theory predicts.²² As was discussed in the previous section, portfolio theory allows one to diversify away unsystematic risk, but systematic risk, due to beta, does not diversify away. Under the Sharpe model, a financial crisis is by definition a period of time during which the systematic risk swamps the unsystematic risk.²³ Users of mean-variance optimization should heed the lessons of the Sharpe model. Since mean-variance optimization does not separate risk into its systematic and unsystematic parts, care must be taken to limit beta exposure to reasonable levels.

Fortunately for investors, long-term correlations between asset class returns are significantly lower than short-term correlations.²⁴ By extending their time horizon, investors employing mean-variance optimization enjoy the benefits of diversification and stand a better chance of making accurate capital market assumptions.

Estimation Error

Estimation error invariably leads to inefficient portfolios. This can be explained by considering estimation error in the expected returns and three sets of portfolios: the *true efficient frontier*, the *estimated frontier*, and the *actual frontier*.²⁵ The *true efficient frontier* is the efficient frontier computed using the true (but unknown) parameters, while the *estimated frontier* is the frontier computed using estimated (and hence incorrect) parameters. The *actual frontier* is the frontier computed using the true expected returns but the weights of the portfolios from the *estimated frontier*. It should be quite clear from these definitions that the actual frontier, which is the frontier that determines actual investment outcomes, always lies below the *true efficient frontier*.

The forward-looking nature of capital market assumptions practically guarantees that the inputs for mean-variance analysis will be tainted by some degree of estimation error. Unfortunately, solutions to the mean-variance optimization process are highly unstable, as even small errors in input parameters can result in large changes in portfolio contents.²⁶

Unconstrained mean-variance optimization may also lead to unintuitive, nonsensical portfolios. As Richard Michaud has written in his critique of mean-variance optimization:

²⁰ Ibid. 49-50; J.P. Morgan. Rise of Cross Asset Correlations. Global Equity Derivatives & Delta One Strategy. May 2011.

²¹ J.P. Morgan. Rise of Cross Asset Correlations. Global Equity Derivatives & Delta One Strategy. May 2011.

²² Harry M. Markowitz, Mark T. Hebner, Mary E. Brunson. Does Portfolio Theory Work During Financial Crises? www.ifaarchive.com

²³ Ibid.

²⁴ Jeremy Siegel. Stocks for the Long Run. McGraw Hill. 2014. 50.

²⁵ Mark Broadie. Computing Efficient Frontiers Using Estimated Parameters. *Annals of Operations Research*. 1993.

²⁶ Taming Your Optimizer: A Guide Through the Pitfalls of Mean-Variance Optimization. Ibbotson Associates; Vijay Chopra. Improving Optimization. *The Journal of Investing*. Fall 1993.

The unintuitive character of many “optimized” portfolios can be traced to the fact that MV optimizers are, in a fundamental sense, ‘estimation-error maximizers.’ Risk and return estimates are inevitably subject to estimation error. MV optimization significantly overweights (underweights) those securities that have large (small) estimated returns, negative (positive) correlations and small (large) variances. These securities are, of course, the ones most likely to have large estimation errors.²⁷

Moreover, as Professor of Business at Columbia University Mark Broadie has shown through simulations, the error maximization property of mean-variance analysis becomes more pronounced as the number of asset classes increases.²⁸ With more asset classes in the analysis, the likelihood that some asset class has either a large positive error in the estimation of its expected return or a large negative error in the estimation of its standard deviation of return increases. Hence, as the number of asset classes increases, the *estimated frontier* tends to increasingly overstate actual portfolio performance. Figure 3 provides an example of the unintuitive portfolio weights that can result from unconstrained mean-variance optimization.

Assumptions about expected returns exert the largest effect in determining portfolio contents, while variances and covariances exert secondary and tertiary effects, respectively. Calculating the average portfolio turnover resulting from switching from a base portfolio to one based on error-tainted inputs, Vijay Chopra shows that for an investor with a moderate risk tolerance, the average turnover due to estimation errors in means is two to four times the average turnover from estimation errors in variances and about five to thirteen times the average turnover from errors in covariances.²⁹ A separate study by Chopra and his collaborator William Ziemba corroborates these results. By examining the cash equivalent loss from optimizing portfolios based on estimated, rather than true, input parameters, Chopra and Ziemba show that for an investor with a moderate risk tolerance, errors in means are eleven times as damaging as errors in variances.³⁰ Errors in variances are twice as damaging as errors in covariances. Moreover, they find that the relative importance of errors in means, variances, and covariances depends upon the risk tolerance of the investor. Since an investor with higher risk tolerance focuses on raising the expected return of the portfolio while deemphasizing the variance, errors in expected return exert a larger effect on investment results. Conversely, the investor with a low risk tolerance focuses on reducing portfolio risk and hence is less affected by errors in means than the investor with higher risk tolerance.

Investors may employ several tools to counteract the problems associated with estimation error. Setting reasonable constraints on asset class weights serves as a first defense against unintuitive, highly concentrated portfolios. Constraints on minimum allocations ensure that asset

²⁷ Richard Michaud. The Markowitz Optimization Enigma: Is ‘Optimized’ Optimal? *Financial Analysts Journal*. January-February 1989.

²⁸ Mark Broadie. Computing Efficient Frontiers Using Estimated Parameters. *Annals of Operations Research*. 1993.

²⁹ Vijay Chopra. Improving Optimization. *The Journal of Investing*. Fall 1993.

³⁰ Vijay Chopra and William Ziemba. The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice. *Journal of Portfolio Management*. Winter 1993. The cash equivalence of a portfolio is the amount of cash that provides the same utility as the risky portfolio. Cash equivalent loss is the difference in cash equivalence for optimal portfolios based on true and error-tainted inputs.

classes with low expected returns but desirable diversification qualities are not ignored in the optimization process. David Swensen, Chief Investment Officer of Yale University, advocates committing at least five percent to each asset class, as smaller commitments make little difference to overall portfolio performance.³¹ On the other hand, constraints on maximum allocations protect portfolios from overconcentration. Swensen suggests a maximum allocation constraint of 25 or 30 percent.

Applying constraints on asset class weights should not be taken to an extreme, however. As Swensen has written, “placing too many constraints on the optimization process causes the model to do nothing other than to reflect the investor’s original biases, resulting in the GIGO (garbage-in/garbage-out) phenomenon well known to computer scientists.”³²

Investors perform sensitivity analysis to reduce the effects of estimation error. The goal of sensitivity analysis is to identify a set of asset allocation weights that is close to efficient under several different sets of plausible capital market assumptions.³³ Sensitivity analysis might involve first choosing a portfolio from the efficient frontier and then altering the mean-variance optimization inputs to create a new efficient frontier.³⁴ The original portfolio, whose risk and return profile has changed due to the updated optimization inputs, could then be compared to portfolios on the new efficient frontier in terms of risk, return, and portfolio composition.

Rather than treating the portfolio optimization as a deterministic problem, investors could choose to incorporate uncertainty of input assumptions into the optimization process itself. Such techniques – commonly referred to as “robust optimization” – could help investors identify portfolios that perform well under a number of different scenarios.³⁵

Investors and economists have proposed the Black-Litterman model as a solution to the problems of unintuitive, highly concentrated portfolios, input-sensitivity, and estimation error maximization.³⁶ The Black-Litterman model, developed by economists Fischer Black and Robert Litterman at Goldman Sachs, provides investors with a systematic approach for combining their own views about asset class returns with the market equilibrium implied returns. Using portfolio weights from the market portfolio, which is assumed to lie on the efficient frontier, the Black-Litterman model uses “reverse optimization” to compute the Capital Asset Pricing Model equilibrium returns for each asset class in the market portfolio.³⁷ The investor then expresses views on asset class expected returns; these are allowed to be partial or complete and can be expressed in both absolute and relative terms.

³¹ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 101; David F. Swensen. *Unconventional Success*. Free Press. 2005. 83.

³² David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 107.

³³ Frank Fabozzi. *Robust Portfolio Optimization and Management*. John Wiley & Sons. 2007. 213

³⁴ *Taming Your Optimizer: A Guide Through the Pitfalls of Mean-Variance Optimization*. Ibbotson Associates.

³⁵ Frank Fabozzi. *Robust Portfolio Optimization and Management*. John Wiley & Sons. 2007. 214; Dimitris Bertsimas, David B. Brown, and Constantine Caramanis. *Theory and Applications of Robust Optimization*. https://faculty.fuqua.duke.edu/~dbbrown/bio/papers/bertsimas_brown_caramanis_11.pdf

³⁶ Thomas M. Idzorek. *A Step-By-Step Guide to the Black-Litterman Model*. Ibbotson Associates. April 26, 2005; Frank Fabozzi. *Robust Portfolio Optimization and Management*. John Wiley & Sons. 2007. 233-239; Jay Walters. *The Black-Litterman Model in Detail*. June 20, 2014.

³⁷ Jay Walters. *The Black-Litterman Model in Detail*. June 20, 2014.

The Black-Litterman model “blends” the market equilibrium implied returns with the investor’s views, producing a new vector of expected returns. Note that in the absence of investor views, the blended returns are those implied by the market equilibrium, meaning that the investor should hold the market portfolio. The degree to which the blended return estimates deviate from the market equilibrium depends on the magnitude of the expressed views and the investor’s confidence in both the equilibrium estimates and the investor views on expected returns.

The primary benefit of using the Black-Litterman model is that the vector of asset class returns that the model produces leads to reasonable portfolio weights without additional constraints on the portfolio optimization process.³⁸ In fact, the optimal portfolio resulting from the Black-Litterman process is the market equilibrium portfolio plus a weighted sum of the investor’s “view portfolios,” implying that views only affect portfolio weights when they have returns that differ from those implied by a combination of the equilibrium portfolio and all other views.³⁹

Despite its theoretical benefits, the Black-Litterman model suffers from several limitations. First, it may be difficult to define the market portfolio.⁴⁰ The public markets may not fully represent the universe of risky assets. For instance, since the vast majority of real estate is privately held, the market capitalization of publicly traded real estate (through REITs) is only a small fraction of the total real estate asset value. An investor using the market capitalization of publicly traded securities to determine the market portfolio may thus start with a baseline allocation to real estate that is too low. Moreover, due to data constraints, it may be difficult, if not impossible, to estimate accurately the market capitalization of illiquid assets, as coming to such estimates requires that investors both identify all private assets and assign a value to them.⁴¹ For instance, if an institution invests in natural resources, should state-owned oil and gas assets be included in the calculation of the asset class weights of the market portfolio? Even for publicly traded securities, the answers are not always easy – should investors only consider the free-float market capitalization?

In sum, investors using mean-variance optimization may reduce the effects of estimation error by applying reasonable constraints, conducting sensitivity analysis, performing robust optimization, or using the Black-Litterman model. Some of these solutions are not mutually exclusive.

³⁸ This is the case for unconstrained portfolio optimization. In the case of constraints, such as constraints on beta exposure or leverage, the results are less intuitive. However, as Rob Litterman has written, “the same trade-off of risk and return – which leads to intuitive results that match the manager’s intended views in the unconstrained case – remains operative when there are constraints or other considerations.”

Bob Litterman. *Beyond Equilibrium, the Black-Litterman Approach*. Modern Investment Management: An Equilibrium Approach. John Wiley & Sons, Inc. 2003. 81. 87.

³⁹ *Ibid.* 85.

⁴⁰ This discussion of the limitations of the Black-Litterman model relies heavily on a conversation the author had with Alex Hetherington, a Director of the Yale Investments Office.

⁴¹ *Ibid.*; Jay Walters. *The Black-Litterman Model in Detail*. June 20, 2014.

Time Horizon

Markowitz mean-variance optimization is a single-period model of investment. Disconnects between investor time horizon and the length of the mean-variance investment period may lead to suboptimal investment outcomes.

As David Swensen has written, investors may possess multiple objectives that span different time horizons.⁴² In such cases, a single-period model of investment might serve one objective at the expense of others, or simply serve none of them. Swensen highlights the dilemma facing university endowments: with the conflicting objectives of providing stable intermediate-term cash flows to the university's operating budget and preserving long-term endowment purchasing power, single-period mean-variance analysis sheds little light on how to achieve both objectives.⁴³

Making matters worse is the fact that the standard implementation of mean-variance optimization considers a one-year time horizon.⁴⁴ As Jeremy Siegel, Professor of Finance at the Wharton School, has shown in his book *Stocks for the Long Run*, the relative risk of different asset classes depends on the holding period.⁴⁵ This is due to the fact that stock and bond returns do not follow a random walk, a process whereby future returns are completely independent of past returns.⁴⁶ Rather, Siegel shows that stock returns exhibit mean-reverting behavior, while bond returns exhibit mean-averting behavior.⁴⁷ The mean-reverting behavior of stock returns means that periods of stock underperformance relative to the long-term trend are more likely to be followed by periods of outperformance, and vice versa. The mean-averting behavior of bond returns, on the other hand, means that once bond returns have deviated from their long-run average, there is an increased chance that they will deviate further.⁴⁸ The mathematical consequence of this behavior is that the relative risk of stocks compared to asset classes such as bonds declines as the holding period increases.⁴⁹

Clearly, then, the efficient frontier is a function of the holding period.⁵⁰ Siegel demonstrates this fact rather dramatically. As shown in Figure 4, the minimum variance portfolio for a one-year time horizon is 13 percent in stocks, while the minimum variance portfolios for 20-year and 30-year time horizons are 58 percent and 68 percent in stocks, respectively.⁵¹ However, Laura Spierdijk and Jacob Bikker of the Dutch Central Bank find that mean reversion

⁴² David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 106.

⁴³ *Ibid.*

⁴⁴ *Ibid.*

⁴⁵ Jeremy Siegel. *Stocks for the Long Run*. McGraw Hill. 2014. 102.

⁴⁶ *Ibid.* 97-98.

⁴⁷ *Ibid.* 98-99. The autocorrelation structure of asset class returns not only influences asset class expected returns but also the variances of and covariances between asset class returns.

⁴⁸ *Ibid.* 99.

⁴⁹ *Ibid.* Under the random walk hypothesis, the standard deviation of each asset class's average real annual returns (defined as the arithmetic mean of real annual returns) will fall by the square root of the holding period because of the Central Limit Theorem. However, with mean reversion, the standard deviation of these returns falls faster than predicted by the random walk hypothesis.

⁵⁰ *Ibid.* 101.

⁵¹ *Ibid.* 102.

of stock returns has a more muted effect on portfolio weights.⁵² For instance, the first column of Table 1 shows that the difference in stock allocations for the minimum variance portfolio with and without mean reversion is less than 2.5 percentage points over a 20-year investment horizon. Moreover, in stark contrast to the minimum variance portfolios on Siegel's efficient frontiers, the difference in stock allocations due to mean reversion between the one-year and 20-year minimum variance portfolios in Spierdijk and Bikker (2012) is less than two percentage points.

The differences between the two studies can be attributed to the fact that Siegel's estimates are based on 210 years of historical data, while Spierdijk and Bikker's are based on approximately 30 years of data. It should be noted, however, that Spierdijk and Bikker's results also hinge on an assumption regarding the variance ratio, which is a key parameter in their mean reversion model. Spierdijk and Bikker use the mean reversion model introduced by Poterba and Summers (1987), which defines a mean-reverting log price process as the sum of a permanent and transitory component. The variance ratio is the return variance of the permanent component of the log price process divided by the return variance of the transitory component. Due to difficulties in estimating the variance ratio, Spierdijk and Bikker based their choice of the parameter on the existing literature. They show that a lower variance ratio would lead to a larger effect due to mean reversion, though these effects are still much smaller than those in Jeremy Siegel's study.

In "Short-Horizon Inputs and Long-Horizon Portfolio Choice," William Goetzmann and Franklin Edwards propose a solution to the mismatch between one-year mean-variance inputs and investor time horizon: simulating long-term returns.⁵³ Specifically, they estimate the parameters of a vector autoregression (VAR) model, which explicitly incorporates the autocorrelation (correlation of past and future returns, in contrast with the random walk assumption) structures of short-term asset class returns. They then use the estimated model to simulate long-term returns. Simulating long-term returns thousands of times results in a joint distribution of long-term asset class returns that can be used as inputs in the mean-variance framework.

Goetzmann and Edwards show that the short-horizon and simulated long-horizon returns lead to different efficient frontiers. In their study, the minimum variance portfolio exhibits the largest difference in portfolio composition; the long-horizon inputs lead to a minimum variance portfolio composed of 50 percent bonds and 50 percent bills, while the short-horizon inputs led to a minimum variance portfolio of 10 percent bonds and 90 percent bills. The simulated inputs not only increase the minimum achievable risk, but also reduce the curvature of the frontier due to slightly higher correlation across asset classes. While Goetzmann and Edwards find that return autocorrelations have relatively little impact on the high-risk, high-return portion of the efficient frontier, other research has shown that stocks are more attractive to long-term investors when the time structure of returns is taken into account.⁵⁴

⁵² Laura Spierdijk and Jacob A. Bikker. Mean Reversion in Stock Prices: Implications for Long-Term Investors. Dutch Central Bank. April 5, 2012.

⁵³ William N. Goetzmann and Franklin R. Edwards. Short-Horizon Inputs and Long-Horizon Portfolio Choice. *The Journal of Portfolio Management*. Summer 1994.

⁵⁴ *Ibid.* 78-80.

Explicitly considering longer time horizons is an example of how investors could incorporate return autocorrelations into their estimate of mean-variance parameters. As Goetzmann and Edwards write, “Investors wishing to use this technique should consider further simulations that perturb the underlying parameters: mean, standard deviations, correlations, and VAR coefficients.” They further write that their approach is “predicated on the assumption that investors can accurately identify both their investment horizon and the timing of future cash needs.” While using long-horizon capital market assumptions would bring the greatest benefit to investors whose time horizon is known with a high degree of certainty, investors with less well-defined holding periods could still benefit from a reduction of the mismatch between their investment horizon and the most commonly used one-year mean-variance inputs.

While the degree to which autocorrelation affects the relative risk of asset class returns is unclear, autocorrelation nonetheless affects portfolio allocations and highlights the important issue of time horizon. Investors must take care that mean-variance analysis corresponds to the appropriate time horizon. Forward-looking simulations of portfolios from a one-year mean-variance model could effectively extend the time horizon of the mean-variance analysis, allowing investors to assess portfolios over the relevant holding period.⁵⁵ Such simulations allow investors to translate portfolio risk and return characteristics into metrics quantifying the ability of portfolios to meet investor objectives over various time horizons.⁵⁶ This last point is elaborated upon in the section on portfolio selection and investor objectives.

Other Investment Attributes

Mean-variance optimization fails to consider important investment attributes such as liquidity and marketability. The standard implementation of mean-variance optimization, which is based on a one-year time horizon, implicitly assumes rebalancing of portfolio allocations.⁵⁷ However, the lack of marketability of illiquid assets such as real estate and private equity limits the ability of investors to rebalance portfolios in a low cost, efficient manner.⁵⁸ Even reasonable rebalancing methods – such as offsetting private asset shortfalls with investments in cash, bonds, and absolute return investments, and offsetting private asset surpluses through reductions in risky public investments – invariably lead to portfolios whose risk-return profiles differ from that of the target portfolio.⁵⁹

Uncertainties in both asset values and the rate and timing of cash flows for alternative investment vehicles limit the ability of investment managers to achieve the target allocation determined from the mean-variance portfolio selection process. Institutions invest in illiquid assets predominantly through commingled limited partnerships.⁶⁰ As Dean Takahashi, Senior Director of the Yale Investments Office, and Seth Alexander, a former Associate Director of the Yale Investments Office and current Chief Investment Officer at MIT, wrote in a 2001 paper

⁵⁵ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 128.

⁵⁶ *Ibid.*

⁵⁷ *Ibid.* 106.

⁵⁸ *Ibid.*

⁵⁹ *Ibid.* 135-6.

⁶⁰ Dean Takahashi and Seth Alexander. Illiquid Alternative Asset Fund Modeling. *The Journal of Portfolio Management*. Winter 2002.

entitled “Illiquid Alternative Asset Fund Modeling,” “The uncertain schedule of drawdowns, unknowable changes in the valuation of the partnership’s investments, and unpredictable distributions of cash or securities to the limited partners combine to make it difficult to predict accurately the future value of partnership interests.”⁶¹ These challenges, coupled with the uncertainties associated with projecting overall endowment growth, hamper the ability of investment managers to achieve the target allocation determined through the mean-variance portfolio selection process. In the above-cited paper, Takahashi and Alexander present a financial model that enables institutional investors to project future asset values and cash flows for funds in illiquid alternative asset classes.⁶² The model, which allows investors to assess the impact of changes to fund commitment levels and assumptions regarding contributions, distributions, and underlying net returns, significantly improves the ability of investors to bring asset allocations to target levels.⁶³

Empirical evidence supports the view that rebalancing improves the risk-return tradeoff of actual investment results. For example, using a three-asset framework, Chopra shows that the optimal portfolio with constraints on portfolio drift dominates the optimal portfolio without such constraints, as the former has a higher mean return and lower risk.⁶⁴ Specifically, the constrained portfolio is not allowed to deviate far from a 60-40-0 stock-bond-cash allocation, while the unconstrained portfolio has no such constraints. Mean returns, variances, and covariances are calculated on a sixty-month rolling basis, and the optimal mean-variance allocation is held for the month following the sixty-month estimation period. The unconstrained and constrained portfolios are tested out-of-sample for a 72-month interval from January 1985 through December 1990. Chopra finds that the constrained portfolio realizes a higher mean return with lower risk. A separate study by Vanguard largely corroborates these findings.⁶⁵ The study, which is based on data from 1960 to 2013, compares two portfolios: a 60-40 stock-bond portfolio that is rebalanced annually and a 60-40 stock-bond portfolio that is not rebalanced. While the former provides a marginally lower return (9.12 percent versus 9.36 percent), it does so with significantly lower risk (11.41 percent vs. 14.15 percent).

Mean-variance optimization fails to consider other costs associated with illiquidity, such as investors’ restricted ability to respond to unforeseen cash flow requirements.⁶⁶ In fact, naïve implementations of mean-variance optimization may lead to portfolios with unreasonable illiquidity levels, as mean-variance optimizers favor asset classes such as private equity – from which investors reap an illiquidity premium – with high expected returns.⁶⁷ Investors may

⁶¹ Ibid.

⁶² Ibid.

⁶³ Ibid.

⁶⁴ Vijay Chopra. Improving Optimization. *The Journal of Investing*. Fall 1993; This result might seem to not make much sense, since the unconstrained efficient frontier always lies above the constrained efficient frontier. However, the point Chopra is making is not about the risk-return tradeoff of portfolios on the efficient frontier, but rather the actual investment results obtained from portfolios whose weights are constrained to lie within a band of the target allocation.

⁶⁵ Francis M. Kinniry Jr. et al. Putting a value on your value: Quantifying Vanguard Advisor’s Alpha. Vanguard Research. March 2014.

⁶⁶ Sameer Jain. Investment Considerations in Illiquid Asset Classes. *Alternative Investment Analyst Review*.

⁶⁷ The author spoke with Alex Hetherington, a Director of the Yale Investments Office; 2010 Yale Endowment Report.

employ additional modeling to establish reasonable illiquid assets targets. For example, the Yale Investments Office has performed extensive modeling of different market scenarios to stress test its liquidity profile.⁶⁸ Once an illiquid assets target has been established, investors can continue to employ mean-variance optimization by setting an additional constraint on the total allocation to illiquid asset classes.

Lastly, it should be noted that target allocations obtained through the mean-variance portfolio selection process may not be achievable in the short-term, particularly for funds that pursue active strategies. It may take years to change the composition of institutional portfolios, as the pace of portfolio turnover is limited by the sourcing of high-quality investment managers.⁶⁹ Capacity constraints in funds with existing managers may also limit investors' ability to increase the allocation to certain asset classes.⁷⁰ On the other hand, investors who are over-allocated to a particular asset class may find it difficult to reduce the allocation due to lock-up periods, contractual fund commitments, and other factors. These are not issues for investors pursuing passive strategies.

Portfolio Selection and Investment Objectives

Perhaps the most obvious limitation of mean-variance optimization is that it delineates a *set* of efficient portfolios, but provides little guidance in choosing an optimal portfolio. Clearly, the investor must provide additional information to make mean-variance analysis a useful exercise.

Economists typically attempt to overcome this issue by introducing the idea of investor preferences, which they express in terms of a utility function. Utility in the context of mean-variance optimization is traditionally a function of the portfolio's expected return and variance, investor risk tolerance, and a scaling factor.⁷¹ The expected return enters positively into the function, while the variance enters negatively into the function. Variance discounts utility at a higher rate for lower levels of risk tolerance, and vice versa. The scaling factor is a constant coefficient on the variance term. By finding the point of tangency between the efficient frontier and an indifference curve, economists identify the optimal portfolio.

Unfortunately for economists, people are not mean-variance utility maximizers; that is, investor satisfaction cannot be expressed solely in terms of the portfolio's mean and variance.⁷² Other issues arise in the way expected return relates to variance in the utility model. Common sense dictates that investors with varying levels of risk tolerance should choose different optimal portfolios from a set of reasonable options. However, consider a scenario in which all indifference curves across the entire range of acceptable levels of risk tolerance choose the same portfolio (Figure 5). Adjusting the specification of the utility function (by changing the scaling factor) so that indifference curves with the acceptable levels of risk tolerance fall along the entire

⁶⁸ 2013 Yale Endowment Report.

⁶⁹ The author spoke with Alex Hetherington, a Director of the Yale Investments Office, and David Katzman, a Senior Associate of the Yale Investments Office.

⁷⁰ The author spoke with Daniel Otto, a Senior Financial Analyst of the Yale Investments Office.

⁷¹ Utility = (expected return) – (scaling factor)*(variance)/(risk tolerance)

⁷² David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 122.

span of the efficient frontier might seem to be a reasonable solution (Figure 6). Economists might refer to such a procedure as a scaling adjustment.⁷³

However, “scaling” the utility function changes the fundamental relationship between risk and return, as a larger (smaller) scaling factor causes the utility function to discount portfolio variance at a higher (lower) rate. In fact, for a given level of risk tolerance, the utility function could pick out any one of the efficient portfolios if the scaling parameter were varied sufficiently. Moreover, it seems completely arbitrary that indifference curves for the acceptable levels of risk tolerance should lie tangent to points along the entire span of the efficient frontier. Why should they not lie along the upper half of the frontier only, or the lower half? What is the point of calculating an investor’s risk tolerance if the way in which risk tolerance enters into the utility calculation is subject to such arbitrariness?

Granted, a case could be made that there exists a “true” specification of the utility function, a specification that most closely matches actual investor behavior. Yet portfolio selection based solely on utility maximization is still divorced from an assessment of tangible investment outcomes. In fact, risk and volatility are not equivalent. Variance, which is the measure of “risk” used in mean-variance analysis, is really a measure of volatility. As Ashvin Chhabra has written, “What matters is not the volatility of a security, but its price at the time you need to sell it to meet an obligation; risk is not simply ‘what happens’ in the abstract but rather the impact of what happens – the ‘event risk’ – on your ability to generate cash flow when you need it.”⁷⁴

Rather than relying on the mathematically appealing, but unintuitive approach of employing a mean-variance utility function to select an optimal portfolio from the efficient frontier, investors should articulate quantifiable investment goals and then evaluate efficient portfolios in terms of their ability to meet them. For example, the Yale Investments Office has articulated the two investment objectives of providing stable intermediate-term cash flows to the university’s operating budget and preserving long-term endowment purchasing power. To evaluate the ability of portfolios to meet its two objectives, the Yale Investments Office has defined two metrics. The first measures the average two-year spending decline in the worst 10 percent of years.⁷⁵ The second measure, purchasing power impairment risk, is defined as failure to preserve one-half of purchasing power over fifty years.⁷⁶

Unfortunately, little intuition about portfolios’ ability to meet Yale’s investment objectives can be gleaned from simply observing the risk and return characteristics of efficient portfolios. Will a lower-returning, lower risk portfolio necessarily lead to more stable spending?⁷⁷ Does the risk of purchasing power impairment increase or decrease with portfolio

⁷³ See page 166 of *Investments* by Bodie, Kane, and Marcus for a discussion of mean-variance utility.

⁷⁴ Ashvin B. Chhabra. *The Aspirational Investor*. HarperCollins. 2015. 89.

⁷⁵ This point relies on a conversation the author had with Alex Hetherington, a Director of the Yale Investments Office.

⁷⁶ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 122.

⁷⁷ The Yale endowment’s target spending rate currently stands at 5.25 percent. According to the current smoothing rule, endowment spending in a given year sums to 80 percent of the previous year’s spending and 20 percent of the targeted long-term spending rate applied to the fiscal year-end market value two years prior, adjusted for inflation (2013 Yale Endowment Report).

expected return and variance? Monte Carlo simulations of efficient portfolios provide some guidance, as thousands of simulation paths allow the Yale Investments Office to assign values to its spending decline and purchasing power impairment measures for each portfolio.⁷⁸ Some portfolios may be eliminated from consideration if they are dominated by others on the basis of both metrics.⁷⁹ In the end, however, Yale will need to exercise judgment to deal with the clear tradeoff between the two goals for the portfolios in contention.⁸⁰

In the case of personal investment, investors must specify quantifiable investment objectives. For instance, the investor's goals could be to maximize expected wealth-building above a certain threshold percentile return (e.g. the 50th percentile return would be the median outcome, while the 75th percentile return would be a more desirable outcome), given that the expected loss from return outcomes below the threshold is no less than a certain value. Specifying a wealth-building goal in this way protects against downside loss, while preserving the potential for wealth creation. An individual investing for retirement could design an investment program that minimizes the expected shortfall of wealth during retirement, where the shortfall is defined as the amount by which wealth falls short of what is needed.⁸¹

By clearly articulating quantifiable investment objectives, conducting the necessary tests to evaluate portfolios on the efficient frontier, and exercising sound judgment in the final portfolio selection process, investors employing mean-variance optimization stand a strong chance of achieving their investment goals.

Conclusion

Mean-variance optimization is a compelling framework for portfolio selection under uncertainty. It is no wonder that many investors, ranging from university endowments to Internet-based robo-advisors, have turned to mean-variance analysis as their primary asset allocation model.

As with any model, simplifying assumptions both increase the model's utility and detract from it. In the case of mean-variance optimization, the assumption that expected returns, variances, and covariances fully describe the behavior of asset class returns greatly simplifies the investment process, making mean-variance optimization an accessible tool for portfolio decision-making. Yet as was shown in this chapter, such assumptions also limit the ability of mean-variance analysis to model real-world asset class characteristics.

Fortunately, most of the limitations of mean-variance optimization can be overcome through relatively straightforward methods. Investors who cannot address these limitations, however, should think twice before employing mean-variance optimization.

⁷⁸ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 123.

⁷⁹ *Ibid.*

⁸⁰ *Ibid.*

⁸¹ Ben Inker and Martin Tarlie. *Investing for Retirement: The Defined Contribution Challenge*. GMO Whitepaper. April 2014.

CHAPTER 2: HOW ROBO-ADVISORS WORK

The investment methodology of all individual and institutional investors can be summarized as comprising three distinct steps: asset allocation, implementation, and monitoring and rebalancing. Robo-advisors, which generally adhere to a passive indexing strategy, are no exception to this methodology. This chapter begins by discussing the rationale for passive indexing. It then shows how robo-advisors execute each step of the investment methodology outlined above. While differences in investment process exist between robo-advisors, the general framework outlined in this chapter aims to give readers a foundational understanding of how robo-advisors work.

The Case for Passive Indexing

In his 1951 Princeton economics thesis, visionary John Bogle put forward an argument that would challenge the basic tenets of the mutual fund industry: “Mutual funds can make no claim to superiority over the market averages.”⁸² In the many decades since the writing of Bogle’s thesis, economists and investors have lent support to Bogle’s proposition that mutual fund managers in aggregate possess no stock selection skill, and that investors would be better served by investing in passive index funds. These arguments, much like Bogle’s, have emphasized the importance of giving mutual fund shareholders a “fair shake,” that is, a chance to succeed financially in an industry where the profit motives of mutual fund companies all too easily trump their fiduciary responsibility.⁸³

David Swensen, Burton Malkiel, and Charles Ellis are among the economists and investors who have championed the passive indexing approach to individual investment. In *Unconventional Success*, Swensen highlights the failure of the profit-seeking mutual fund industry to produce satisfactory results for individual investors through active management. He shows that most actively managed mutual funds fail to meet their goal of beating the market, citing an academic study placing the pre-tax and after-tax failure rates at 78 to 95 percent and 86 to 96 percent, respectively.⁸⁴ Such numbers understate the true underperformance of actively managed mutual funds due to survivorship bias, the omission of data on disappearing funds.⁸⁵ Moreover, the average margin of defeat for managers underperforming the index exceeded the average margin of victory for the few managers who outperformed the market, casting such numbers in an even dimmer light.⁸⁶ High fees and excessive portfolio turnover (which leads to greater commission costs, higher market impact costs, and the realization of greater taxable gains for taxable accounts) are among the obvious sources of mutual fund failure producing the performance deficit.⁸⁷ Yet, several hidden sources of mutual fund failure – including pay-to-play activity, stale-price market timing, and soft-dollar trading – further diminish the returns generated by mutual fund investors.⁸⁸ In contrast to actively managed funds, index funds exhibit

⁸² John C. Bogle. *John Bogle on Investing*. Mc-Graw Hill. 2001.

⁸³ *Ibid.*

⁸⁴ David F. Swensen. *Unconventional Success*. Free Press. 2005. 203.

⁸⁵ *Ibid.*

⁸⁶ *Ibid.* 213-217.

⁸⁷ *Ibid.* 204, 214.

⁸⁸ *Ibid.* 205, 219.

much lower fees (expense ratios) and lower portfolio turnover, the latter of which leads to better tax efficiency.⁸⁹ Unfortunately, winning the game of active management is a challenge, as identifying and monitoring high-quality managers is a difficult task.⁹⁰ Swensen encourages individuals to invest in passive instruments managed by not-for-profit money management firms.⁹¹

In *A Random Walk Down Wall Street*, Burton Malkiel argues that markets price stocks so efficiently that most professional investors cannot outperform the index.⁹² Specifically, he argues that while stock market returns do not conform perfectly to the random walk hypothesis, which posits that future returns are completely independent of past returns, past prices do not contain enough information to reliably inform predictions of future prices; hence, investing based on technical analysis of past returns is unlikely to generate better returns than a simple buy-and-hold strategy, which has the added benefit of postponing or avoiding capital gains taxes.⁹³ Using data on the historically poor performance of actively managed mutual funds relative to the market index, Malkiel also argues that very few investors are able to consistently beat the market through fundamental analysis.⁹⁴ Mutual fund performance is even worse than the data suggest, as the data do not include the performance of some failed firms.⁹⁵ Malkiel then dismisses several “market-beating” strategies based on the predictability of stock markets, arguing that critics of the efficient market hypothesis have overstated the extent to which the stock market is usefully predictable.⁹⁶ Such strategies may also result in investors accepting above-average risks.⁹⁷ Malkiel concludes that individuals would be best served by adopting a market-matching strategy of investing in index funds.⁹⁸

In *Winning the Loser’s Game*, Charles Ellis makes a compelling case in favor of passive indexing. He writes that in recent decades active management has evolved into a loser’s game, a game in which “winning” is determined by making fewer mistakes than one’s opponent, rather than beating one’s opponent outright.⁹⁹ In a kind of prisoner’s dilemma, institutional investors, in seeking to generate market-beating returns, have collectively made the markets so efficient that it is difficult for any one of them to stay ahead of the market.¹⁰⁰ In markets increasingly dominated by institutions, individual investors stand little chance of outperforming the benchmark index, especially once the costs of active management are taken into account.¹⁰¹ Ellis urges individuals to adopt a program of passive indexing, the winner’s game that every investor can enjoy.¹⁰²

⁸⁹ Ibid. 257-263.

⁹⁰ Ibid. 312.

⁹¹ Ibid. Chapter 11.

⁹² Burton G. Malkiel. *A Random Walk Down Wall Street*. W. W. Norton & Company. 2012. 19.

⁹³ Ibid. 144, 161-162.

⁹⁴ Ibid. Chapters 7 and 11.

⁹⁵ Ibid. Chapter 11.

⁹⁶ Ibid.

⁹⁷ Ibid.

⁹⁸ Ibid.

⁹⁹ Charles D. Ellis. *Winning the Loser’s Game*. McGraw Hill. 2013. 5.

¹⁰⁰ Ibid. 5-10.

¹⁰¹ Ibid. 6-7.

¹⁰² Ibid. 9-10.

Asset Allocation

Robo-advisors generally perform asset allocation with mean-variance analysis or a variant of mean-variance analysis, the benefits and limitations of which were discussed in the first chapter.¹⁰³ While the determination of asset classes and their portfolio weights constitute parts of the same asset allocation process, the following discussion of asset allocation is divided into several parts for clarity. Readers should note that robo-advisors' asset allocation process may be more fluid than the structure of this section suggests.

Determination of Asset Classes

Clients of robo-advisors may withdraw assets at any time, limiting robo-advisors' investable universe to liquid assets. Thus, asset classes such as private equity and private real estate are excluded from consideration from the outset, as funds in such asset classes typically employ lock-ups or other restrictions on redemptions. Robo-advisors' focus on passive investing also excludes actively managed but liquid strategies such as actively managed domestic or foreign equity mutual funds.

Since robo-advisors generally help individuals invest across different goal types, they may develop different sets of asset classes for taxable and tax-deferred accounts.¹⁰⁴ Asset classes may be chosen on the basis of the specific roles they are expected to play in a portfolio.¹⁰⁵ For example, U.S. stocks may be included in a portfolio due to their capital growth, long-run inflation protection, and tax efficiency attributes. Inflation-protected bonds may be chosen due to their income, low historical volatility, diversification, and inflation hedging attributes. Municipal bonds may be included in a portfolio due to their income, low historical volatility, diversification, and tax efficiency attributes.

Estimation of Mean-Variance Inputs

Having determined the ideal set of asset classes for portfolio construction, robo-advisors then estimate the capital market assumptions for each asset class. Since robo-advisors use different methods to estimate expected returns, which as shown in the first chapter exert the largest effect in determining portfolio contents, their methods are compared in the next chapter ("How Robo-Advisors Differ From One Another"). Unfortunately, some robo-advisors do not disclose information on how they estimate variances and correlations, but it is most likely that they primarily rely on historical data to form these estimates.¹⁰⁶ In some cases, however,

¹⁰³ Betterment, one of the robo-advisors studied in this paper, does not use a mean-variance optimizer in the strictest sense. This subject is discussed in the next chapter.

¹⁰⁴ For example, the asset classes Wealthfront and Betterment have chosen for taxable and tax-deferred accounts can be viewed here: Wealthfront Investment Methodology Whitepaper, Betterment Website. Portfolio.

¹⁰⁵ For example, see Wealthfront's Investment Methodology Whitepaper and Schwab Intelligent Portfolios' Guide to Asset Classes Whitepaper.

¹⁰⁶ Schwab Intelligent Portfolios does not provide information on how it estimates its variance-covariance matrix; Wealthfront Investment Methodology Whitepaper. Wealthfront generates standard deviation estimates by considering each asset class's long-term and short-term historical standard deviation and the expected volatility of each asset class as implied by pricing in options markets. "Long-term historical estimates benefit from a larger sample size, short-term estimates capture market evolution, and the option markets imply forward-looking

forward-looking measures of volatility as implied by options markets may influence capital market estimates.¹⁰⁷

Mean-Variance Analysis

With a full set of capital market assumptions for each asset class, robo-advisors then use mean-variance optimization or a variant of mean-variance optimization to generate the efficient frontier. In the optimization process, constraints are imposed on asset class weights to ensure proper diversification.¹⁰⁸ Although finance theory shows that investors may find “super-efficient” portfolios by choosing a portfolio on the capital market line (combinations of the risk-free asset with a portfolio on the efficient frontier), it appears that some robo-advisors do not use the capital market line to identify such portfolios.

As mentioned in the previous chapter, mean-variance optimization delineates a set of efficient portfolios, but provides little guidance in choosing the optimal portfolio. Robo-advisors have adopted different approaches to identifying and measuring the level of portfolio risk that is most appropriate for each client, and hence the bulk of the discussion about the selection of a portfolio from the efficient frontier is deferred to the next chapter. Suffice to say, robo-advisors use information from short questionnaires and/or clients’ stated investment objectives to determine the level of risk the client should take.

Implementation

Indexing

Once robo-advisors have selected a portfolio from the efficient frontier, they choose exchange-traded funds to represent each asset class, focusing on how ETFs contribute to net-of-fee, after-tax, risk-adjusted portfolio returns.¹⁰⁹ As mentioned previously, most robo-advisors have adopted a passive indexing strategy and hence select ETFs that passively track broad-market benchmarks.

Index funds work well for rebalancing, as they correct portfolio drift without causing slippage in returns. By contrast, rebalancing with actively managed funds conflicts with the

volatility.” To estimate correlations, Wealthfront considers long-term historical correlation and short-term correlation; Betterment Website. Support Center. 2013 Portfolio Optimization.

<http://support.betterment.com/customer/portal/articles/1295723-why-is-betterment-changing-the-portfolio-> To estimate expected returns, Betterment uses the Black-Litterman model, which requires users to specify a variance-covariance matrix for all asset classes. According to Dan Egan, Betterment uses historical data to generate a sample variance-covariance matrix and then performs Ledoit-Wolf shrinkage to reduce estimation error.

¹⁰⁷ Ibid.

¹⁰⁸ Wealthfront Investment Methodology Whitepaper. In their online materials, Schwab Intelligent Portfolios and Betterment do not disclose information on the use of constraints, but without constraints their suggested asset allocations would appear nonsensical, which they are not. For Schwab and Betterment, it is unclear whether constraints are set at the asset class level or for groups of assets, such as the overall equity or bond portfolios.

¹⁰⁹ Wealthfront Investment Methodology Whitepaper; Schwab Intelligent Portfolios Selecting Exchange-Traded Funds Whitepaper; Betterment ETF Portfolio Selection Methodology; Wealthfront Website. FAQ. <https://pages.wealthfront.com/faqs/what-etfs-does-wealthfront-use-to-implement-tax-loss-harvesting/>

investor's conviction in the active manager. For example, rebalancing with active managers may result in return slippage when the market index has performed well relative to other asset classes but a manager has not kept pace with the index. Some top-tier managers lag the index during bull markets and outperform during bear markets. Hence, rebalancing away from the outperforming asset class by reducing the position in the active manager would lead to poorly timed distributions from the manager, leading to return slippage. Conflicts could also arise between the time horizon of a manager's investment thesis and the timing of the rebalancing trade. For instance, making rebalancing trades away from overweight asset classes might constitute taking assets from managers whose investment theses are partially, but not fully, realized.

Robo-advisors generally select ETFs that minimize costs, provide ample market liquidity, and minimize tracking error. Robo-advisors consider ETF costs, because fund expenses impose definite, negative costs on the ETF investor. Sufficient liquidity allows for withdrawals at any time and also reduces bid-ask spreads and market impact. While trading costs impose a larger burden for active traders than long-term investors, minimizing these costs makes a difference for clients creating new portfolios or rebalancing existing ones. Lastly, while tracking error can be either positive or negative, the goal of passive indexing is to match the market return, and hence robo-advisors try to minimize this error.

The Silicon Valley robo-advisor Wealthfront has moved beyond ETFs for large accounts, using a strategy it calls "direct indexing" for the domestic equity asset class.¹¹⁰ With direct indexing, investors hold a combination of individual securities and one or two "completion ETFs" to track an index, rather than a single index fund or ETF. Hence, investors avoid some of the fees associated with index funds and ETFs under this strategy.

Tax-Loss Harvesting (For Taxable Accounts)

Many robo-advisors use algorithms to harvest tax losses on a daily basis.¹¹¹ Tax-loss harvesting is the process of selling securities for a loss and using the proceeds to buy highly correlated substitutable investments. By realizing capital losses and taking advantage of differences in tax rates between short-term and long-term capital gains, portfolios reap additional returns through both the compounding of tax savings (which come with tax filings) and tax rate arbitrage. Since robo-advisors replace investments that have been sold with highly correlated substitutes, the risk-return profile of the portfolio is largely maintained.

Most robo-advisors harvest tax losses at the ETF level, but through direct indexing, tax losses can be harvested at the individual security level. Thus, even when an overall index trades up, tax losses can be harvested on the individual securities that fell in value.¹¹² Robo-advisors that harvest tax losses avoid wash sales. A wash sale occurs when an investor sells a security that is "substantially identical" to another security purchased within 30 days after or before the

¹¹⁰ Wealthfront Tax-Optimized Direct Indexing Whitepaper.

¹¹¹ Ibid.; Betterment Tax-Loss Harvesting Whitepaper; Schwab Intelligent Portfolios Rebalancing and Tax-Loss Harvesting Whitepaper.

¹¹² Wealthfront Tax-Optimized Direct Indexing Whitepaper. The Investment Company Act of 1940 prohibits index funds and ETFs from passing on tax losses to investors.

sale.¹¹³ Robo-advisors that harvest tax losses at the ETF level avoid wash sales by selecting primary and secondary ETFs that track different, but highly correlated indexes. Robo-advisors that use direct indexing employ highly correlated primary and secondary stocks, such as Coca-Cola and PepsiCo, to avoid wash sales.

Monitoring and Rebalancing

Robo-advisors generally employ threshold-based rebalancing (rather than time-based rebalancing) to maintain investment discipline.¹¹⁴ That is, once asset class weights have drifted away from the target allocation by a certain amount, an algorithm automatically conducts the trades necessary to bring the asset allocation back to target. For instance, in the absence of cash flows into or out of the investment account, overweight asset classes are sold to buy underweight asset classes, reducing overall portfolio drift.

The investor's target allocation may also change over time. For instance, the portfolio risk an investor is able to assume is usually a positive function of time horizon. With each passing year, the investor's time horizon decreases, leading the robo-advisor to adjust portfolio risk downward. An investor's risk tolerance and investment goals may also change over time.¹¹⁵ Investors can usually indicate these changes through the robo-advisor's online platform, and target allocations are adjusted accordingly.

¹¹³ Betterment Tax-Loss Harvesting Whitepaper.

¹¹⁴ Schwab Intelligent Portfolios Rebalancing and Tax-Loss Harvesting Whitepaper; Wealthfront Investment Methodology Whitepaper; How and When My Portfolio is Rebalanced. Betterment Support Center. <http://support.betterment.com/customer/portal/articles/987453-how-and-when-is-my-portfolio-rebalanced->

¹¹⁵ Wealthfront Investment Methodology Whitepaper.

CHAPTER 3: HOW ROBO-ADVISORS DIFFER FROM ONE ANOTHER

Although most robo-advisors adhere to the general investment methodology outlined in the previous chapter, significant differences still exist. These differences relate to issues ranging from the definition of asset classes to the measurement of investment risk to conflicts of interest between robo-advisors and their affiliate companies. This chapter highlights these differences, focusing on matters that affect the first two steps of the general investment framework outlined in the previous chapter: asset allocation and implementation.

Asset Classes

Robo-advisors possess different attitudes toward defining asset classes. While most of these automated platforms invest mainly across stocks and bonds, the extent to which they divide asset classes into smaller sub asset classes varies greatly. For example, Schwab Intelligent Portfolios splits the broad U.S. stock asset class into U.S. small- and large-capitalization stocks, reflecting its belief that size is an important differentiating characteristic. Betterment differentiates between value and growth stocks, favoring value stocks due to their historically higher returns in both domestic and foreign markets.

The division of asset classes on the basis of such characteristics contrasts sharply with the inclusion of a fundamentally different asset class to mean-variance analysis. While both activities may lead to an improvement of the efficient frontier, the former leads to a false sense of improved diversification based on estimation error, while the latter meaningfully introduces a new asset class with different fundamental attributes.¹¹⁶ As there is no technical limit to splitting asset classes into sub asset classes, defining asset classes on the basis of characteristics such as industry or country, or, in the extreme case, specifying capital market inputs for individual stocks, could raise the efficient frontier even further.¹¹⁷ Yet as discussed in the first chapter, the error maximization property of mean-variance optimization becomes more pronounced as the number of asset classes increases. With more asset classes in the analysis, the likelihood that some asset class has either a large positive error in the estimation of its expected return or a large negative error in the estimation of its variance increases.

In *Pioneering Portfolio Management*, David Swensen provides some guidance toward specifying a reasonable number of asset classes. He writes:

While market participants disagree on the appropriate number of asset classes, the number should be small enough so that portfolio commitments make a difference, yet large enough so that portfolio commitments do not make too much of a difference. Committing less than 5 percent or 10 percent of a fund to a particular type of investment makes little sense; the small allocation holds no potential to influence overall portfolio

¹¹⁶ The following discussion assumes that robo-advisors optimize portfolio allocations with all asset classes. It is possible that they optimize with groups of assets such as U.S. stocks, even if U.S. small-, mid-, and large-capitalization stocks are separate sub asset classes in the final asset allocation. In such a case, the error maximization property of mean-variance optimization would be less of an issue, though small portfolio commitments (e.g. less than five percent of total portfolio assets) hardly affect overall portfolio results.

¹¹⁷ This interesting observation was made by David Katzman, a Senior Associate at the Yale Investments Office.

results. Committing more than 25 or 30 percent to an asset class poses the danger of overconcentration. Most portfolios work well with around a half a dozen asset classes.¹¹⁸

Asset Classes Across Different Account Types

Schwab Intelligent Portfolios (28)	Wealthfront (11)	Betterment (13)
U.S. Large Company	U.S. Stocks	U.S. Total Stock Market
U.S. Large Company - Fundamental	Foreign Developed Stocks	U.S. Large-Cap Value Stocks
U.S. Small Company	Emerging Market Stocks	U.S. Mid-Cap Value Stocks
U.S. Small Company - Fundamental	Dividend Growth Stocks	U.S. Small-Cap Value Stocks
International Developed Large Company	U.S. Government Bonds	International Developed Stocks
International Developed Large Company - Fundamental	Corporate Bonds	Emerging Market Stocks
International Developed - Small Company	Emerging Market Bonds	Short-Term Treasuries
International Developed Small Company - Fundamental	Municipal Bonds	Inflation Protected Bonds
International Emerging Markets	TIPS	U.S. High Quality Bonds
International Emerging Markets - Fundamental	Real Estate	U.S. Municipal Bonds
U.S. Exchange-Traded REITS	Natural Resources	U.S. Corporate Bonds
International Exchange - Traded REITS		International Developed Bonds
U.S. High Dividend		Emerging Market Bonds
International High Dividend		
Master Limited Partnerships		
U.S. Treasuries		
U.S. Investment Grade Corporate Bonds		
U.S. Securitized Bonds		
U.S. Inflation Protected Bonds		
U.S. Corporate High Yield Bonds		
International Developed Country Bonds		
International Emerging Markets Bonds		
Preferred Securities		
Bank Loans		
Investment Grade Municipal Bonds		
Investment Grade California Municipal Bonds		
Gold and Other Precious Metals		
Cash		

Sources: Schwab Intelligent Portfolios website, Wealthfront website, Betterment website

With 28 asset classes across its different account types, Schwab Intelligent Portfolios has almost certainly over-specified its asset class mix. For some investment goals, Schwab recommends investing across 20 or more asset classes.¹¹⁹ The large number of asset classes and the inevitable estimation error in their capital market assumptions practically guarantee that mean-variance optimization without minimum and maximum constraints on each asset class will produce nonsensical portfolio allocations. Without minimum constraints, some asset classes may not receive any allocation at all. Unfortunately, with a large number of asset classes, setting minimum and maximum constraints on portfolio allocations is also problematic. For example,

¹¹⁸ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 101.

¹¹⁹ Schwab Intelligent Portfolios Website. FAQ. <https://intelligent.schwab.com/public/intelligent/about-intelligent-portfolio>. See the question “Can you give me an example of what these asset allocations look like?” Investor 2 invests in 20 asset classes. Filling out the questionnaire on the Schwab Intelligent Portfolios website leads to asset allocation recommendations, some of which have more than 20 asset classes.

setting a minimum constraint of five percent on each of 20 asset classes leads to a perfectly balanced portfolio reflecting the investor's original bias that all 20 asset classes should be part of the portfolio. Setting lower minimum constraints might seem to mitigate this problem. However, asset classes constituting a paltry two or three percent of the overall portfolio hardly affect investment results. In such a case, one might wonder why the broader asset classes were divided into sub asset classes in the first place.

Robo-advisors Schwab Intelligent Portfolios, Wealthfront, and Betterment invest in foreign bonds, which may sacrifice expected return while providing little diversification benefits. Since international bond markets might be subject to interest rate fluctuations, inflation and economic cycles, and other monetary conditions that differ from those in the domestic bond market, the advisors might argue that the diversifying power of foreign bonds merits their inclusion in a portfolio. However, the functional attributes of an unhedged foreign currency bond are equivalent to those of a U.S. dollar bond plus foreign exchange exposure, the latter of which cannot be relied upon to produce positive expected returns.¹²⁰ This result follows from the fact that the foreign exchange risk of foreign bonds can be hedged away by selling foreign currency forward contracts whose timing and magnitude coincide with the interest and principal payments from the foreign bond; with such a hedge, the U.S. dollar cash flows from the foreign-currency-denominated bond would match the U.S. dollar cash flows from the dollar-denominated bond.¹²¹ Hence, unhedged foreign bonds provide similar returns to U.S. bonds; however, they do not afford the same protection against financial crisis or deflation as U.S. bonds, since it is unclear how exchange rates would change in either situation.¹²² Investors seeking the diversification benefits of foreign exchange exposure might invest in foreign equities rather than foreign bonds, as the former provides foreign exchange exposure without sacrificing expected return.¹²³

In spite of the concerns outlined above, robo-advisors might make a reasonable case in favor of foreign bonds based on the efficiency of the market portfolio. As measured by market capitalization, foreign bonds have become a more important asset class to investors in recent years; foreign bonds' share of the global investable market (global public equities and fixed income) rose from approximately 19 percent in 2000 to 32 percent in 2013.¹²⁴ Excluding an asset class that constitutes a large proportion of the market portfolio may philosophically be at odds with robo-advisors' reliance on mean-variance optimization, which – together with the capital market line – implies the existence of an efficient market portfolio.

While robo-advisors generally invest in domestic equities, foreign developed equities, emerging market equities, and a range of fixed income investments, they possess different attitudes toward investing in real assets such as real estate and natural resources. For instance, Schwab invests in ETFs tracking the price of gold and other precious metals. Wealthfront invests in both REITs and natural resources ETFs, while Betterment does not invest in any real assets. The primary ETF Schwab uses for its investment in gold is the iShares Gold Trust (ticker: IAU),

¹²⁰ David F. Swensen. *Unconventional Success*. Free Press. 2005. 123.

¹²¹ *Ibid.*

¹²² *Ibid.*

¹²³ *Ibid.*

¹²⁴ *Global Fixed Income: Considerations for U.S. Investors*. Vanguard Research. February 2014.

the assets of which “consist primarily of gold held by a custodian on behalf of the Trust.”¹²⁵ According to the ETF’s prospectus, “The Trust seeks to reflect generally the performance of the price of gold.”¹²⁶ Investments in commodities such as gold may provide some degree of diversification, but such diversification comes at the expense of expected return. As Jeremy Siegel has shown in *Stocks for the Long Run*, the annualized real return of gold for the period 1802-2012 was 0.7 percent, lagging the 6.6 percent and 3.6 percent return of stocks and bonds, respectively.¹²⁷ By contrast, investments in asset classes such as real estate and natural resources (e.g. oil and gas, timber) provide price exposure in addition to an intrinsic rate of return.¹²⁸

Investment in international real estate involves a tradeoff between diversification and expected return. Real estate assets exhibit characteristics of both fixed income and equity. The fixed income attributes of real estate result from the regular, contractual lease payments made by tenant to landlord.¹²⁹ Equity attributes stem from the residual value of the property, as the uncertainty associated with existing lease agreements, vacancies, and the terms of future leases combine to increase the risk and potential reward of owning real estate assets.¹³⁰ Hence, the income derived from regular lease payments on foreign real estate exhibits similar characteristics to those of a foreign bond, which as argued above is equivalent to a U.S. dollar bond plus foreign exchange exposure. Since foreign equities are typically higher returning than foreign real estate assets, which display both bond-like and equity-like characteristics, investing in international real estate presents an opportunity cost; both foreign equities and international real estate provide foreign exchange exposure, but foreign equities dominate international real estate with respect to expected return. However, since international real estate likely responds to different fundamental drivers than foreign equities, the lower expected return of foreign real estate might be offset by its additional diversification benefits. Investors choosing to invest in international real estate might reasonably exclude emerging markets, as generally weaker legal systems and more unstable political regimes threaten the safety of such investments. Of Schwab Intelligent Portfolios, Wealthfront, and Betterment, Schwab is the only robo-advisor invested in international real estate assets.

Schwab Intelligent Portfolios, Wealthfront, and Betterment invest in U.S. corporate bonds, which may not provide adequate risk-adjusted returns. The undesirability of corporate bonds stems mainly from three risk factors – credit risk, illiquidity, and callability – and the lack of adequate compensation for undertaking such risk.¹³¹ First, unlike the U.S. government, whose full faith and credit guarantee full and timely payments on its debt, U.S. corporations are at risk of not meeting their debt obligations. Second, corporate bonds trade in much shallower markets than U.S. Treasury bonds. While illiquidity is less of an issue for long-term investors than short-term traders, illiquidity nonetheless poses a risk that investors should be compensated for. Third, the callability of corporate bonds creates an undesirable asymmetry for investors in corporate bonds. When rates fall, the corporation is more likely to call the bond, preventing the investor

¹²⁵ iShares Gold Trust Prospectus. <https://www.ishares.com/us/products/239561/ishares-gold-trust-fund>

¹²⁶ Ibid.

¹²⁷ Jeremy Siegel. *Stocks for the Long Run*. McGraw Hill. 2014. 5-6.

¹²⁸ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 199-200.

¹²⁹ David F. Swensen. *Unconventional Success*. Free Press. 2005. 203.

¹³⁰ Ibid.

¹³¹ Ibid. 93-104. The entire paragraph relies on this source.

from enjoying the benefits of a now high-coupon bond; when rates rise, the bond becomes less valuable, leading to mark-to-market losses.

In addition to these risk factors, a misalignment of interests between shareholders and bondholders further skews the return distribution for corporate bonds.¹³² Since a firm's value is independent of its capital structure, and since enterprise value – the sum of all equity and debt – is a measure of total firm value, actions that increase the value of equity decrease the value of debt. Corporate management, which typically has an equity interest in the corporation, generally acts in the interests of stockholders versus bondholders. Data from Ibbotson Associates show that from 1926-2009, long-term U.S. government bonds generated a compound annual return of 5.4 percent, marginally trailing the 5.9 percent return of long-term corporate bonds.¹³³ Investors in corporate bonds undertake considerable risk for incremental reward.

Misalignment of interests between shareholders and owners of high-yield bonds may be even more acute than for investment-grade bonds.¹³⁴ Since management usually focuses on improving or preserving the value of equity during distressed situations, cost cutting measures – of which reducing interest expenses and otherwise minimizing debt obligations are one such strategy – may act against the interests of bondholders.¹³⁵

Like investment-grade bonds, high-yield bonds suffer from illiquidity, credit risk, and callability concerns.¹³⁶ Illiquidity is a concern for investors seeking to diversify into high-yield bonds, as the cost of transacting in the high-yield market is significantly higher than in the investment-grade market, especially during times of market stress.¹³⁷ The weighted liquidity cost spread – the weighted average cost of immediately executing a round-trip transaction for a standard institutional trade for the securities in an index – is about two basis points for high-yield bonds (as measured by the Barclays U.S. Corporate High Yield Bond Index) and one basis point for corporate bonds (as measured by the Barclays U.S. Corporate Bond Index) in “normal” economic times. However, during the 2007-2008 financial crisis, the weighted liquidity cost spread increased to over six basis points for high-yield bonds, while the cost spread for corporate bonds rose only slightly to just above one basis point.

While investors may reduce the liquidity costs associated with high-yield bonds by investing in bonds with greater trading volume, doing so limits the investment opportunity set for high-yield bonds.¹³⁸ The Barclays U.S. Very Liquid High Yield Corporate Bond Index contained only 211 issues with a market capitalization of \$226 billion as of June 30, 2012, compared to the 1,915 issues with a capitalization of \$1 trillion for the broader Barclays U.S. High Yield Corporate Bond Index. In a study, Vanguard showed that adding the Barclays U.S. Very Liquid High Yield Corporate Bond Index or the Barclays Ba/B High Yield Corporate Bond Index

¹³² Ibid. Most of the paragraph relies on this source.

¹³³ Burton G. Malkiel. *A Random Walk Down Wall Street*. W. W. Norton & Company. 2012. 201.

¹³⁴ David F. Swensen. *Unconventional Success*. Free Press. 2005. 109.

¹³⁵ Ibid.

¹³⁶ Ibid. 109.

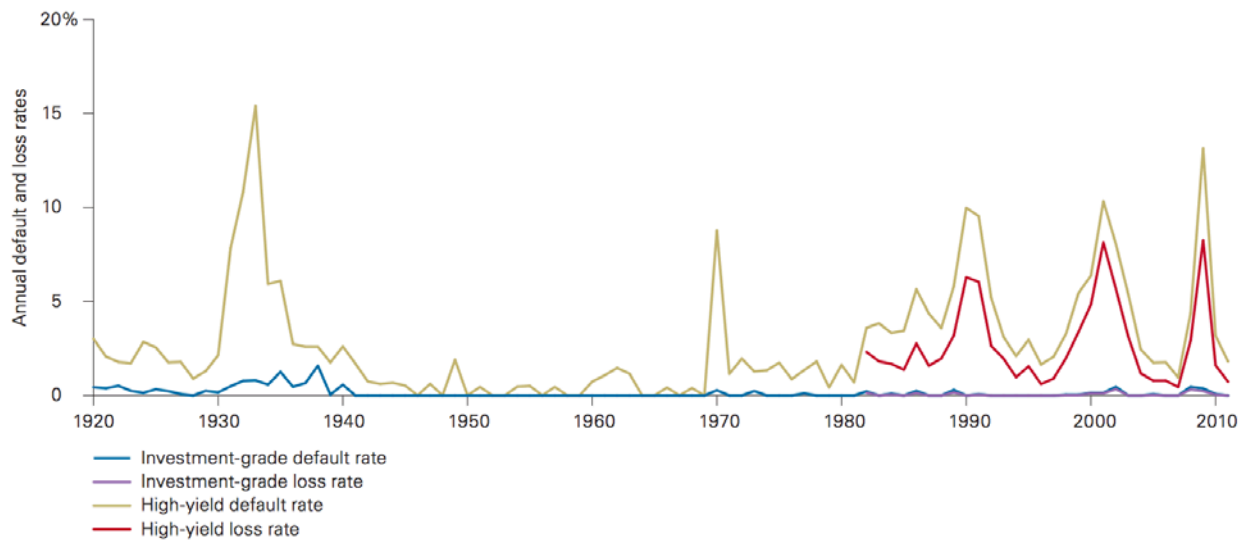
¹³⁷ Christopher B. Philips. *Worth the risk? The appeal and challenges of high-yield bonds*. Vanguard Research. December 2012. Same source for the entire paragraph.

¹³⁸ Ibid. Same source for the entire paragraph.

(another index that excludes less liquid issues) to its mean-variance analysis did not lead to a material improvement of the efficient frontier.

Investors in high-yield bonds undertake considerable credit risk. As shown in the graph below, the default rate of high-yield bonds has not only exceeded that of investment-grade bonds but also exhibited significant volatility since 1920.¹³⁹ Moreover, the average annual return realized by investors of high-yield bonds between 1987 and 2012 trailed the bonds' average yield. During the same period, the average annual return of investment-grade bonds exceeded the bonds' average yield. Since one could reasonably expect total returns to be on par or even exceed the average yield during a period of generally declining interest rates such as 1987-2012, defaults most likely led to the negative difference between high-yield bonds' total returns and average yield.

Annual Default and Loss Rates for High-Yield and Investment-Grade Bonds¹⁴⁰

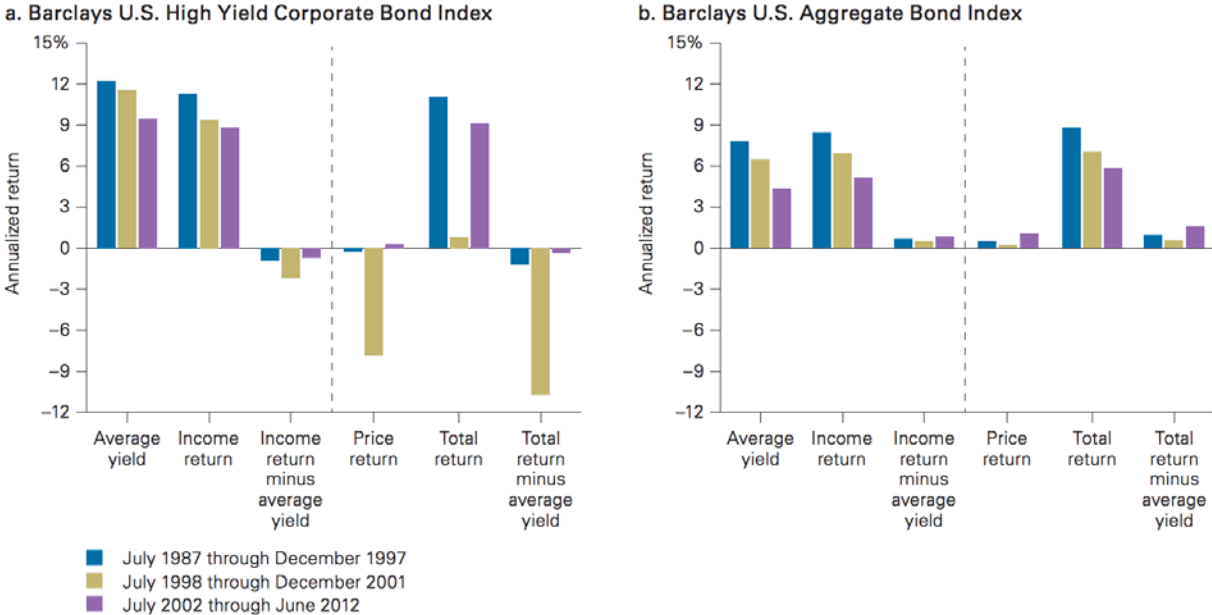


Source: Christopher B. Philips. Worth the risk? The appeal and challenges of high-yield bonds. Vanguard Research. December 2012.

¹³⁹ Ibid. Same source for the entire paragraph.

¹⁴⁰ The loss rate is the value of a given default that is not recovered during bankruptcy proceedings.

Average Return Trails Average Yield for High-Yield Bonds But Not Investment-Grade Bonds



Source: Christopher B. Philips. Worth the risk? The appeal and challenges of high-yield bonds. Vanguard Research. December 2012.

High-yield bonds do not provide fair compensation for the credit risk investors assume. Data from September 30, 1983 to December 31, 2015 show that high-yield bonds, U.S. corporate bonds, and U.S. Treasury bonds delivered annualized returns of 8.8 percent, 8.09 percent, and 7.13 percent as measured by the Barclays U.S. Corporate High Yield Index, Barclays Aggregate Bond Index, and Barclays U.S. Treasury Index, respectively.¹⁴¹ Data were drawn from this period because the Barclays U.S. Corporate High Yield Index began tracking high-yield bonds on September 30, 1983. High-yield bonds' meager spread over both U.S. corporate bonds and U.S. Treasuries calls for their exclusion from a portfolio.

As with all fixed income investments, the return distributions of high-yield bonds are negatively skewed, as the best outcome for bond investments consists of full and timely payments of interest and principal.¹⁴² Foreign, corporate, and high-yield fixed income investments do not exhibit the same upside potential as equity investments yet offer little downside protection. Rather than investing in high-yield bonds, whose negative attributes have been reviewed in this section, less risk-averse individuals might instead choose to increase portfolio risk and expected return by increasing their equity orientation. Schwab Intelligent Portfolios might reconsider its allocation to high-yield bonds in light of the risks outlined in this section.

Unlike Wealthfront and Betterment, Schwab Intelligent Portfolios invests in securitized bonds, which may not adequately compensate investors for the risk they take on. Securitized bonds are securities whose interest and principal payments are backed by underlying assets such

¹⁴¹ Data from Bloomberg.

¹⁴² David F. Swensen. *Unconventional Success*. Free Press. 2005. 100.

as home mortgages, automobile loans, and credit card debt. As the robo-advisor concedes in its online documents, mortgage-backed securities in particular might not perform well in environments with falling interest rates, as declining rates might lead the homeowner to prepay and refinance the mortgage, shortening the stream of now high-interest payments.¹⁴³ Mortgage-backed securities might also not perform well in environments with rising rates, as homeowners are less inclined to refinance, leading to low-returning payments being drawn out in a high-rate environment.¹⁴⁴ Whether investors in securitized bonds are adequately compensated for such optionality is a difficult question to answer.¹⁴⁵ In addition to these risks, investors may unknowingly assume considerable credit risk. The subprime mortgage crisis was perhaps the most dramatic display of credit risk, as credit rating agencies indiscriminately assigned AAA ratings to mortgage-backed securities backed by subprime loans.

Bank loans, perhaps one of the most obscure asset classes employed by Schwab Intelligent Portfolios, also may not provide fair compensation to investors. According to the robo-advisor, the loans typically have floating rates and are generally rated below investment grade.¹⁴⁶ However, the loans are usually secured and senior to other corporate debt. Such loans perform well in environments with rising interest rates, as the floating interest rate is typically a fixed spread over a floating reference rate such as LIBOR. It is unclear whether the spread is large enough to constitute fair compensation for the credit risk embedded in the loans, however.

Estimation of Mean-Variance Inputs

As discussed in the chapter on the benefits and limitations of mean-variance optimization, asset class expected returns exert a much larger influence on portfolio composition than variances and covariances. Hence, estimation error in expected returns is much more damaging to investment results than estimation error in the other mean-variance inputs.

The degree to which the Black-Litterman model enters into the expected return estimation process varies greatly across robo-advisors. Schwab Intelligent Portfolios seems to disregard completely the Black-Litterman model, while Betterment exclusively relies on the reverse optimization of the market portfolio to generate expected return estimates, refusing to blend its own views with the market equilibrium implied returns. In fact, Betterment believes blending views on expected returns is a form of “short-term market timing.”¹⁴⁷ Wealthfront takes the middle ground, combining its own views with those of the market.¹⁴⁸ Such differences may reflect the philosophical views of robo-advisors, as investors with greater confidence in the efficiency of the market portfolio may feel more comfortable relying on the market equilibrium implied returns.¹⁴⁹

¹⁴³ Ibid. 118-119.; Schwab Intelligent Portfolios Guide to Asset Classes Whitepaper.

¹⁴⁴ Ibid.

¹⁴⁵ David F. Swensen. *Unconventional Success*. Free Press. 2005. 119.

¹⁴⁶ Schwab Intelligent Portfolios Guide to Asset Classes Whitepaper.

¹⁴⁷ Betterment Website. Support Center. 2013 Portfolio Optimization.

[http://support.betterment.com/customer/portal/articles/1295723-why-is-betterment-changing-the-portfolio-](http://support.betterment.com/customer/portal/articles/1295723-why-is-betterment-changing-the-portfolio)

¹⁴⁸ Wealthfront Investment Methodology Whitepaper; Schwab Intelligent Portfolios uses the long-term return estimates of Charles Schwab Investment Advisory, which makes no mention of the Black-Litterman model in its online materials.

¹⁴⁹ The problems associated with defining the market portfolio were discussed in the first chapter.

Robo-advisors that do not rely solely on the market equilibrium implied returns generally use data on historical returns, interest rates, credit spreads, dividend yields, GDP growth, and other macroeconomic variables to form long-term expected return views for each asset class. For equities, robo-advisors may use the Gordon growth model to generate their own expected return estimates. The Gordon growth model is a special case of the dividend discount model, which posits that the value of a company's stock is the net present value of all future dividends. These dividends (or earnings) are assumed to grow at a constant rate in the Gordon growth model, leading to a simple equation: asset class expected returns equal the sum of the current dividend yield, dividend or earnings growth, and change of the price-to-earnings ratio.¹⁵⁰

Schwab Intelligent Portfolios and Wealthfront generate their own views on expected returns for equities by using the Gordon growth model. For example, Schwab first estimates the equity risk premium for U.S. large-capitalization stocks.¹⁵¹ It does this by calculating the difference between the historical average return on U.S. large-capitalization stocks and the historical income return provided by the risk-free asset, for which Schwab uses the Ibbotson Long-Term Government Bond Index as a proxy. The historical average return on U.S. large-capitalization stocks is calculated by using the Gordon growth model with data beginning in 1926. Schwab assumes that the price-to-earnings expansion in the historical data will not repeat in the future. Schwab then adds the historical equity risk premium to the current risk-free rate, which is measured as the yield on a 20-year U.S. Treasury bond, to generate the long-term expected return estimate for U.S. large-capitalization stocks. (Schwab estimates expected returns over a 20-year time horizon.)

To measure the asset class premium for mid- and small-capitalization stocks, Schwab again uses data beginning in 1926 to find the historical premium of mid- and small-capitalization stocks relative to large-capitalization stocks. Schwab then adds this premium to the return estimate for U.S. large-capitalization stocks to come to its return estimate for mid- and small-capitalization stocks. For international stocks, Schwab measures the beta of international market returns to U.S. large-capitalization stock returns; the beta is then multiplied by the equity risk premium of U.S. large-capitalization stocks, resulting in the international asset premium. This international asset premium is then added to the current risk-free rate to generate the long-term return estimate for international equities.

The expected return estimation process for bonds is somewhat less complicated. Wealthfront estimates the expected return of a fixed income asset by its yield to maturity at the time of its purchase.¹⁵² Over the long term, the yield to maturity has been an accurate forward-

¹⁵⁰ One could also include an inflation term but it has been omitted for simplicity.

¹⁵¹ Schwab Intelligent Portfolios Goal Tracker Whitepaper; Michael E. Lind. Q and A: Estimating Long-Term Market Returns. April 24, 2015. <http://www.schwab.com/public/schwab/nn/articles/Q-and-A-Estimating-Long-Term-Market>Returns>

¹⁵² This detail relies on a conversation the author had with Qian Liu, former Director of Research at Wealthfront. It is also the approach advocated by Wealthfront Chief Investment Officer Burton Malkiel in his book *A Random Walk Down Wall Street* (243).

looking indicator of the total compensation received by fixed income investors.¹⁵³ Schwab takes the same approach as Wealthfront, using the yield to maturity on the Barclays U.S. Aggregate Bond Index as the foundation for its expected return estimate for U.S. bonds.¹⁵⁴ However, since the average maturity of bonds in the index is shorter than the time horizon of 20 years over which Schwab estimates expected returns, Schwab adds a horizon premium to account for this additional maturity risk.

Of the companies studied in this paper, Schwab Intelligent Portfolios seems to be the only robo-advisor that estimates capital market inputs for time horizons other than one year.¹⁵⁵ Since asset returns do not follow a random walk, the efficient frontier is a function of the holding period. Specifically, the mean-reverting behavior of stock returns and the mean-averting behavior of bond returns lower stocks' risk relative to bonds as the holding period increases. Thus, using one-year capital market assumptions may be misleading for making asset allocation decisions when the investment horizon is not one year. Explicitly considering longer time horizons is an example of how robo-advisors could incorporate return autocorrelations into their estimate of mean-variance parameters.

Portfolio Optimization

While the spirit of mean-variance analysis undergirds the portfolio optimization processes of Schwab Intelligent Portfolios, Wealthfront, and Betterment, Schwab complements mean-variance analysis with another optimization technique, while Betterment does not use a plain vanilla mean-variance optimizer. Only Wealthfront uses mean-variance optimization in its purest form, which was described in the first chapter. Mean-variance optimization and the capital market line are together the theoretical foundation of the Capital Asset Pricing Model and the Black-Litterman model. As will be shown below, Betterment relies heavily on the Black-Litterman model to optimize portfolios.¹⁵⁶

Schwab Intelligent Portfolios complements mean-variance analysis with full-scale optimization, an approach that can incorporate an investor's preference for loss aversion and considers all features of return distributions, such as skewness and kurtosis.¹⁵⁷ Full-scale optimization considers these higher moments by using historical return data.¹⁵⁸ In Schwab's full-scale optimization, the pain of losses is twice as strong as the benefit of an equal-sized gain,

¹⁵³ Burton G. Malkiel. *A Random Walk Down Wall Street*. W. W. Norton & Company. 2012. 343; Michael E. Lind. Q and A: Estimating Long-Term Market Returns. April 24, 2015.

<http://www.schwab.com/public/schwab/nn/articles/Q-and-A-Estimating-Long-Term-Market>Returns>

¹⁵⁴ Michael E. Lind. Q and A: Estimating Long-Term Market Returns. April 24, 2015.

<http://www.schwab.com/public/schwab/nn/articles/Q-and-A-Estimating-Long-Term-Market>Returns>

¹⁵⁵ In an article describing its expected return estimation process, Schwab Intelligent Portfolios calculated returns for a 20-year time horizon. It is unclear whether Schwab customizes capital market estimates for each client based on the client's time horizon.

¹⁵⁶ Since the focus of this paper is mean-variance optimization-based investment advisory services, a discussion of robo-advisors using other asset allocation models is omitted.

¹⁵⁷ Schwab Intelligent Portfolios Asset Allocation Whitepaper; Bjorn Hagstromer et al. Mean-Variance vs. Full-Scale Optimization: Broad Evidence for the UK. Federal Reserve Bank of St. Louis. April 2007.

¹⁵⁸ Ibid. For more information on full-scale optimization, see: Timothy Adler and Mark Kritzman. Mean-Variance Optimization versus Full-Scale Optimization: In and Out of Sample. Revere Street Working Paper Series. April 27, 2006.

building in a measure of loss aversion. Schwab adopted this approach because “Studies suggest that the psychological pain investors feel from a loss is twice as strong as the joy they receive from a similar size gain.”¹⁵⁹ Schwab averages the portfolio weights from full-scale optimization with those from mean-variance optimization, resulting in the robo-advisor’s optimal portfolio weights. By averaging the weights in this manner, half of the portfolio is mean-variance efficient, while the other half is efficient with respect to full-scale optimization. The portfolio as a whole, however, most likely will not be efficient with respect to either optimization technique.

While research has shown that individuals psychologically weight losses more than gains, it may be unwise to optimize portfolios based on such factors. For example, portfolio losses relative to a reference point may produce twice the psychological pain as the joy of an equal-size gain for an investor in the short-term. However, it would be irrational to optimize an asset allocation based on the investor’s short-term behavioral preferences when the investor has long-term investment objectives.¹⁶⁰ As Peter Brooks of Barclays Bank and Director of Investments and Behavioral Finance at Betterment Dan Egan have written, “if an investor wants to have the best possible long-term portfolio solution, the utility function used to optimize the portfolio should eliminate short-term behavioral biases, not replicate them.”¹⁶¹ Moreover, the investor’s reference point is inherently unstable and may be very different in 20 years from what it is today.¹⁶² Hence, optimizing portfolios on the basis of a momentary reference point may lead to suboptimal long-term portfolios. Nevertheless, it is still important to consider investors’ short-term behavioral preferences as investors who cannot stomach market volatility may have extreme reactions – such as withdrawing all assets – during times of market stress.¹⁶³

As discussed in the previous section, Betterment uses the Black-Litterman model to generate capital market assumptions. The proportion of stocks and bonds in Betterment’s global market portfolio is approximately 41 percent and 59 percent, respectively.¹⁶⁴ Representing U.S. bonds, foreign bonds, U.S. stocks, and foreign stocks by the Barclays U.S. Aggregate Bond Index, Barclays Global Aggregate ex-US Bond Index, MSCI USA Index, and MSCI All Country World Index ex-USA Index, respectively, a 2014 Vanguard study arrives at a similar stock-bond split.¹⁶⁵ To construct efficient portfolios that are less risky than the market portfolio, Betterment combines the market portfolio with cash (short-term Treasuries) and short-term inflation-protected securities; these portfolios delineate the portion of the capital market line without leverage. The minimum variance portfolio, which might be held by an individual who is about to liquidate an investment account, consists entirely of cash. Betterment uses the market equilibrium implied capital market inputs to generate optimized portfolios that are riskier than the market portfolio. Specifically, Betterment maximizes the Sharpe ratio at each stock allocation.¹⁶⁶ For example, Betterment determines its 70 percent stock portfolio by maximizing

¹⁵⁹ Schwab Intelligent Portfolios Asset Allocation Whitepaper.

¹⁶⁰ *Ibid.* 58-9.

¹⁶¹ *Ibid.* 59.

¹⁶² *Ibid.* 145.

¹⁶³ *Ibid.*

¹⁶⁴ This paragraph relies on a phone conversation the author had with Dan Egan, Director of Behavioral Finance and Investments at Betterment. The stock-bond split of the market portfolio: What is the financial model Betterment used to determine these changes? 2013 Portfolio Optimization. Betterment Support Center.

¹⁶⁵ Global Fixed Income: Considerations for U.S. Investors. Vanguard Research. February 2014.

¹⁶⁶ The Sharpe ratio is defined as: $(\text{mean portfolio return} - \text{risk-free rate}) / (\text{standard deviation of portfolio return})$

the Sharpe ratio subject to the constraint that equity investments sum to 70 percent of the portfolio.

Risk and Investment Objectives

As discussed in the first chapter, one of the major limitations of mean-variance optimization is that it delineates a set of efficient portfolios, but provides little guidance in selecting a particular portfolio from the efficient frontier.¹⁶⁷ Some robo-advisors have adopted a goals-based approach to portfolio selection while others have focused on choosing the optimal portfolio for general investing purposes. Schwab, Wealthfront, and Betterment each possess different methods of measuring risk.

Goals-Based Investing

Schwab and Betterment sub-optimize portfolios for each of an investor's goals, while Wealthfront optimizes more general investment portfolios. Investment goals on the Schwab and Betterment platforms range from generating retirement income to building a rainy day fund to saving for an anticipated future expenditure. Proponents of the goals-based approach argue that the division of assets into sub-portfolios improves mental accounting and allows investors to specify a different level of risk – and effectively, a different attitude toward risk – for each sub-portfolio.¹⁶⁸ (For example, an investor with a luxury goal of buying a Tesla might be less risk averse for this goal than for a retirement savings goal.) They argue that it may be difficult to correctly specify the proper risk level of the aggregate portfolio without determining the proper level of risk for each goal and the proportion of the aggregate portfolio dedicated to each goal.¹⁶⁹ Moreover, specifying risk aversion in terms of variance or the tradeoff between expected return and variance – which is how risk tolerance enters into the Markowitz utility function – may be unintuitive for the investor.¹⁷⁰ Goals-based investing also allows investors to customize asset allocations. For instance, if an investor were saving to buy a home, it might be reasonable to assign a higher weight to REITs relative to an efficient mean-variance portfolio of the same risk. That is, it might make sense to accept modest amounts of inefficiency in order to align the portfolio with a certain goal. The opposing view is that it is difficult for investors to articulate goals. Investors may not know when and in what quantity they will spend invested assets.¹⁷¹ For example, if an investor is saving to make a down payment on a home, the exact value of the down payment will not be known until the home is purchased. This is particularly true for long-term investors, as optimizing portfolios based on goals becomes more imprecise as the time

¹⁶⁷ As discussed in the last section, Schwab Intelligent Portfolios complements mean-variance optimization with full-scale optimization. Betterment uses the capital market line for portfolios with risk less than or equal to that of the market portfolio. For simplicity, this discussion assumes that all robo-advisors only use mean-variance analysis. Hence, all portfolios are assumed to be chosen from the efficient frontier.

¹⁶⁸ These details rely on a phone conversation the author had with Dan Egan, Director of Behavioral Finance and Investments at Betterment; Sanjiv Das, Harry Markowitz, Jonathan Scheid, and Meir Statman. Portfolio Optimization with Mental Accounts. *Journal of Financial and Quantitative Finance*, April 2010.

¹⁶⁹ Sanjiv Das, Harry Markowitz, Jonathan Scheid, and Meir Statman. Portfolio Optimization with Mental Accounts. *Journal of Financial and Quantitative Finance*, April 2010. 325.

¹⁷⁰ Ibid. 325.

¹⁷¹ These details rely on a conversation the author had with Qian Liu, former Director of Research at Wealthfront.

horizon increases.¹⁷² Hence, it is not obvious whether optimizing based on goals would influence the optimal allocation and if it did whether portfolios would stand a better chance of helping investors achieve their goals.

Optimizing portfolios on the basis of goals produces locally optimal portfolios that in combination may not be globally optimal.¹⁷³ Das, Markowitz, Scheid, and Statman (2010) show that when short-selling is permitted, combinations of mean-variance efficient portfolios are also mean-variance efficient.¹⁷⁴ This is due to the famous two-fund theorem, which states that combinations of mean-variance efficient portfolios are mean-variance efficient. When short sales are not allowed, however, sub-portfolio mean-variance optimization may lead to minor reductions in efficiency relative to optimizing a single aggregate portfolio.¹⁷⁵ The authors show that maximizing sub-portfolios' expected return subject to an intuitive Value-at-Risk constraint is mathematically equivalent to mean-variance analysis. They argue that the loss of efficiency due to such sub-portfolio optimization is small relative to the damage investors may incur from mis-specifying the risk aversion parameter of the aggregate portfolio in traditional applications of mean-variance analysis.¹⁷⁶

Risk Measurement

Schwab uses its questionnaire (see Figure 7), the Investor Profile Questionnaire (IPQ), to gain insight into an investor's objective capacity and subjective willingness to take risk, or in other words, their objective and subjective risk tolerance. Schwab gains insight into an individual's risk capacity by asking specific objective questions, such as the length of time to retirement and investment goals.¹⁷⁷ Schwab learns about the investor's willingness to take risk by asking questions related to behavioral tendencies, such as the action the investor may take after experiencing significant investment loss.¹⁷⁸ The IPQ assigns to each individual a Risk Capacity Score and a Risk Willingness Score and weights the two scores equally in determining the appropriate level of risk the individual should take.¹⁷⁹ This approach of equally weighting objective and subjective risk scores contrasts with the approach advocated by the CFA Institute:

When ability to take risk and willingness to take risk are consistent, the investment adviser's task is the simplest. When ability to take risk is below average and willingness to take risk is above average, the investor's risk tolerance should be assessed as below average overall. When ability to take risk is above average but willingness is below average, the portfolio manager or adviser may seek to counsel the client and explain the conflict and its implications. For example, the adviser could outline the reasons why the client is considered to have a high ability to take risk and explain the likely consequences, in terms of reduced expected return, of not taking risk. The investment adviser, however,

¹⁷² This point relies on a conversation the author had with Duncan Gilchrest, a Data Scientist at Wealthfront.

¹⁷³ *Ibid.*

¹⁷⁴ Sanjiv Das, Harry Markowitz, Jonathan Scheid, and Meir Statman. Portfolio Optimization with Mental Accounts. *Journal of Financial and Quantitative Finance*, April 2010. 320-321.

¹⁷⁵ *Ibid.* 315, 326-330.

¹⁷⁶ *Ibid.* 315, 325-326.

¹⁷⁷ Schwab Intelligent Portfolios Investor Profile Questionnaire Whitepaper.

¹⁷⁸ *Ibid.*

¹⁷⁹ Schwab Intelligent Portfolios Asset Allocation Whitepaper.

should not aim to change a client's willingness to take risk that is not a result of a miscalculation or misperception. Modification of elements of personality is not within the purview of the investment adviser's role. The prudent approach is to reach a conclusion about risk tolerance consistent with the lower of the two factors (ability and willingness) and to document the decisions made.¹⁸⁰

While Schwab does not ask clients about the total value of their liquid assets or their annual pre-tax income, it does ask investors when they intend to use the monies for each goal. Such information complements the question on age, as the answers to both questions may help the robo-advisor determine the investor's time horizon.

Wealthfront also assigns objective and subjective risk scores to each individual. The objective risk score is determined by estimating whether the client is likely to have enough savings at retirement to support projected spending needs (see Figure 8, which shows Wealthfront's questionnaire).¹⁸¹ The main metric Wealthfront uses to gauge an individual's objective risk capacity is the annual after-tax income to expense ratio in retirement.¹⁸² The greater the individual's excess income, the greater is the individuals' capacity for risk. By using information on the client's current portfolio size and estimating an average rate of return and savings rate until retirement, Wealthfront approximates the size of the client's portfolio at retirement. Retirement income (the numerator of the ratio) is simply the yield on the client's portfolio at retirement. Wealthfront assumes an income replacement ratio of 80 percent (the denominator of the ratio); that is, the income the investor will need in retirement is 80 percent of pre-retirement income. Wealthfront estimates the investor's pre-retirement income by applying a growth rate to the investor's current annual after-tax income. This ratio then undergoes a transformation process, leading to the assignment of an objective risk score. This process of estimating an objective risk score carries over to taxable accounts for general savings purposes.

Wealthfront estimates investors' subjective risk tolerance by asking clients whether they are focused on maximizing gains, minimizing losses, or both equally. It also asks a hypothetical question gauging investors' response to a market decline. Wealthfront's overall risk metric is a weighted combination of the subjective and objective risk measures, with a higher weight assigned to the component indicating higher risk aversion.¹⁸³ Wealthfront adopted this approach because behavioral economics research has shown that individuals consistently overstate their true risk tolerance.¹⁸⁴

The robo-advisor finds the optimal portfolio for each individual by maximizing the Markowitz utility function over all portfolios on the efficient frontier.¹⁸⁵ The classic utility function assumes that an investor's utility is a function of expected return and risk, with the former entering positively and the latter negatively into the calculation of utility. The reduction

¹⁸⁰ Alistair Byrne and Frank E. Smudde. *Basics of Portfolio Planning and Construction*. CFA Institute.

¹⁸¹ Wealthfront Investment Methodology Whitepaper.

¹⁸² The information in this paragraph relies heavily on a conversation the author had with Qian Liu, former Director of Research at Wealthfront.

¹⁸³ Wealthfront Investment Methodology Whitepaper.

¹⁸⁴ *Ibid.*

¹⁸⁵ Some of the limitations of using a utility function for portfolio selection were discussed in the first chapter.

in utility due to portfolio volatility is discounted by a larger factor for investors with higher levels of risk tolerance, and vice versa. Thus, less risk-averse individuals maximize utility by selecting portfolios with higher risk and higher expected return than more risk-averse investors.

It is unclear whether the behavioral tests of Schwab and Wealthfront accurately gauge investor risk tolerance. Hypothetical questions, such as how one would react to a market decline, almost certainly do not elicit emotional responses commensurate to those experienced by investors with actual portfolio losses. Moreover, as Betterment Director of Investments and Behavioral Finance Dan Egan has argued, investors who monitor their portfolios frequently have shorter emotional time horizons and will feel like their investments are riskier.¹⁸⁶ More frequent monitoring does not necessarily lead to poorer market timing behavior, however.¹⁸⁷ While more frequent monitoring could increase the chances of an investor logging into an account when the markets are down, potentially precipitating an extreme reaction from the investor, a sophisticated investor might learn how to psychologically handle market volatility.

Robo-advisors might experiment with alternative behavioral tests that examine investors' actual trading activity. For instance, robo-advisors that transfer assets from clients' brokerage accounts could use data on investors' past rebalancing and trading activity to assign subjective risk scores. A more controversial approach might allow clients of robo-advisors to make small market timing bets on a small portion of their assets managed by robo-advisors. Robo-advisors could then directly assess investors' subjective risk tolerance.

Betterment does not appear to incorporate measures of subjective risk tolerance into portfolio selection.¹⁸⁸ For each goal, Betterment constructs a "glide path," a function that determines the recommended asset allocation.¹⁸⁹ In most cases, the recommended allocation is purely a function of investor time horizon, with Betterment's retirement goal glide path being the one exception. In other words, the glide paths for each goal (other than the retirement investing goal) are the same for each investor. The investor can change the risk of the portfolio but the recommended portfolio adheres to the glide path. Subjective risk tolerance is an important consideration in portfolio selection. Investors taking more risk than they can stomach are more likely to lose conviction in their investment program, increasing the chances of selling low and buying high.

¹⁸⁶ Greg B. Davies and Arnaud de Servigny. *Behavioral Investment Management*. McGraw-Hill. 2012. 62; This point relies on a phone conversation the author had with Dan Egan, Director of Behavioral Finance and Investments at Betterment.

¹⁸⁷ This point relies on a conversation the author had with Duncan Gilchrest, a Data Scientist at Wealthfront.

¹⁸⁸ Dunleavy, MP. Inside Betterment's Investment Advice. <https://www.betterment.com/resources/inside-betterment/investment-advice/>. "When you invest with Betterment you'll notice that we don't give you a risk-tolerance questionnaire. Instead, we ask about time as it pertains to your investments: your age in relation to your goal (with retirement, say) or the time horizon to reach your goal (e.g. three years to save up a Safety Net fund). That's because a goal-specific time horizon is an objective measure of the potential range of outcomes which you should be exposed to. If you are investing for a longer period of time, stocks actually have less potential for loss than bonds, and vice versa for the short-term (bonds are typically less volatile than stocks). So our advice suggests an allocation that exposes you to the optimal level of objective risk, without reference to any personal self-assessment."

¹⁸⁹ Ibid.; Our Goals and Advice Explained. Betterment Website. <https://www.betterment.com/resources/research/goals-advice-explained/>

Betterment assumes that all investors possess a downside risk bias, which runs counter to the spirit of allowing individuals to express different levels of risk aversion for each goal type. In determining the glide paths for each goal, Betterment focuses on the 5th to 50th return outcomes.¹⁹⁰ (Imagine simulating each portfolio on the efficient frontier 100,000 times for the time horizon in question. Then for each portfolio, order the 100,000 outcomes in ascending order. The 5,000th outcome represents the 5th percentile performance of the particular portfolio. Now compare the 5th percentile performance of each portfolio on the efficient frontier. The portfolio with the best 5th percentile performance is the best portfolio for the 5th percentile return outcome.) By imposing a downside risk focus on all investors in this way, Betterment may recommend overly conservative portfolios.

Betterment has implemented a dynamic asset allocation strategy for retirement accounts that considers the investor's unique financial situation.¹⁹¹ In the accumulation phase of retirement investing, the investor adheres to a static glide path (i.e. the allocation is purely a function of time horizon) that gradually reduces portfolio risk until retirement. However, in the decumulation phase, stock allocation advice is tailored to the individual's specific financial circumstances, considering the investor's current balance, desired monthly income amount, minimum acceptable income level, desired certainty about not falling below the minimum income level, and conditional life expectancy, which is based on projections used by the Social Security Administration.

Conflicts of Interest

Conflicts of interest influence both the asset allocation and implementation process of some robo-advisors. These conflicts of interest are particularly acute for Schwab Intelligent Portfolios. Evidence suggests that Schwab's relatively large cash allocation and high ETF expense ratios are linked to compensation flows to Schwab affiliates.

The Role of Cash in a Portfolio

Maintaining a significant cash position is an essential element of some investment strategies. Investing legend Warren Buffet and founder of The Baupost Group Seth Klarman are well known for holding considerable amounts of cash. While these liquid war chests can be a drag on investment returns, they also ensure that capital is available when unique investment opportunities arise. The willingness to devote a significant portion of assets to cash allows investors such as Buffet and Klarman to be selective, as the scope of their investment universe widens to encompass not only today's opportunity sets, but also those of the future.¹⁹²

¹⁹⁰ Our Stock Allocation Advice. Betterment Website. <https://www.betterment.com/resources/research/stock-allocation-advice/>; Betterment does not provide details on how it optimizes its glide paths. It only says that it focuses on unfavorable return outcomes.

¹⁹¹ Our Goals and Advice Explained. Betterment Website. <https://www.betterment.com/resources/research/goals-advice-explained/>

¹⁹² Michael Ide. Klarman Held 50 Percent Cash Amid Scarce Value. <http://www.valuewalk.com/2014/01/klarman-cash-letters-to-investors-2013/>

Unlike Buffet and Klarman, however, robo-advisors for the most part do not engage in security selection. In fact, most robo-advisors have adopted a passive investment strategy. With asset allocation, market timing, and security selection being the three drivers of investment results, adopting a passive strategy excludes security selection as a determinant of investment performance.¹⁹³ Hence, for most robo-advisors the inclusion of cash in a portfolio can only be justified on the basis of arguments pertaining to asset allocation or market timing.

Neither of these considerations supports the inclusion of cash in a long-term investment portfolio, however. Many investors believe cash is a risk-free asset. However, for an investment to truly be risk-free, it must have zero default risk and no reinvestment risk.¹⁹⁴ Default-free zero coupon bonds (zero coupon U.S. Treasuries) whose duration matches the time horizon of the investor are the only assets meeting these criteria. Hence, for an investor with a time horizon greater than one year, cash – defined as either cash deposits or money market investments – clearly does not fit the bill.

Investors might also attempt to justify a cash position on the basis of market timing considerations. Consider a statement published by Schwab Intelligent Portfolios in defense of its cash allocation:

It's easy to question cash in the sixth year of a bull market and when the Federal Reserve is artificially suppressing interest rates, but we don't invest based on the last six years. We invest based on what we expect the future may hold. Bull markets end and interest rates rise. When they do, a little cash will feel pretty good.¹⁹⁵

In environments with dismal projections for equity returns, investing in cash might appear to constitute prudent investment policy. Yet, investing in cash is a drag on returns. U.S. Treasury bonds, which offer higher – albeit modest – returns, are a compelling alternative to cash investments.¹⁹⁶

Schwab Intelligent Portfolios stands apart from its robo-advisor peers due to its significant allocation to cash.¹⁹⁷ While Schwab Intelligent Portfolios disclosed in a SEC filing that its cash allocation could range from six to 30 percent of an account's value, the cash allocation of an investor with a medium- to long-term orientation could realistically range from six to 10 percent on the Schwab platform.¹⁹⁸ The SEC filing pertained to all investment programs on Schwab Intelligent Portfolios, including investments with short time horizons.¹⁹⁹

¹⁹³ David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 50. "Investment returns stem from decisions regarding three tools of portfolio management: asset allocation, market timing, and security selection."

¹⁹⁴ Aswath Damodaran. Estimating Risk Free Rates.

<http://people.stern.nyu.edu/adamodar/pdfiles/papers/riskfree.pdf>

¹⁹⁵ Response to Blog by Wealthfront CEO Adam Nash. <https://aboutschwab.com/press/statements/response-to-blog-by-wealthfront-ceo-adam-nash>

¹⁹⁶ As David Swensen wrote in *Pioneering Portfolio Management*, "Based on delivery of poor real returns and failure to serve as a riskless asset for long-term investors, cash plays no significant role in a well-constructed endowment portfolio." The same might be said of a long-term portfolio for an individual investor.

¹⁹⁷ In the case of Schwab Intelligent Portfolios, cash refers to cash deposits.

¹⁹⁸ Response to Blog by Wealthfront CEO Adam Nash. <https://aboutschwab.com/press/statements/response-to-blog-by-wealthfront-ceo-adam-nash>; Schwab Intelligent Portfolios Disclosure Brochure. Securities and Exchange

Schwab's competitors have levied justifiable criticism of the robo-advisor's cash policy, emphasizing not only the potential damage cash might bring to a long-term oriented portfolio, but also the conflicts of interest underlying Schwab's cash allocation. For example, Betterment Director of Behavioral Finance and Investing Dan Egan brought attention to this part of Schwab Intelligent Portfolios' January 22, 2015 filing with the Securities and Exchange Commission:

In most of the investment strategies, the percentage of the Sweep Allocation [in cash deposits] is higher than the cash allocation would be in a similar strategy in a managed account program sponsored by a Schwab entity or third parties. This is because, as described below under "Fees," clients do not pay a Program fee [i.e. an advisory fee].²⁰⁰

Schwab essentially admits to offsetting part of the costs of the Intelligent Portfolios program by allocating more of client assets to cash than it would under different investment programs. Cash investments from Schwab Intelligent Portfolios are deposited at Schwab Bank, which profits from the spread between the interest rate it pays on deposits and the amount it earns on the investment of such deposits.²⁰¹

In another part of Schwab Intelligent Portfolios' disclosure brochure, the robo-advisor makes its conflict of interest with Schwab Bank more explicit. It also acknowledges that a cash allocation can hurt investment performance:

Because Schwab Bank earns income on the Sweep Allocation for each investment strategy, [Schwab Wealth Investment Advisory, Inc. (SWIA), which sponsors Schwab Intelligent Portfolios,] has a conflict of interest in setting the parameters for the Sweep Allocation. In most of the investment strategies, this results in a Sweep Allocation which is higher than the cash allocation would be in a similar strategy in a managed account program sponsored by a Schwab entity or third parties. A higher cash allocation can negatively impact performance for an investment strategy in a rising market.²⁰²

ETF Selection

Adding insult to injury, Schwab Intelligent Portfolios does not minimize ETF expenses. As shown in the figure below, Schwab's ETF expense ratios are significantly higher than those of its competitors. In its 70 percent stock allocation, 11 of the 15 primary ETFs used by Schwab Intelligent Portfolios are Schwab ETFs. By comparison, none of the primary ETFs used by Wealthfront and Betterment for their 70 percent stock portfolio are sponsored by Schwab.

Commission. January 22, 2015.

http://www.adviserinfo.sec.gov/Iapd/Content/Common/crd_iapd_Brochure.aspx?BRCHR_VRSN_ID=277224

¹⁹⁹ Ibid.

²⁰⁰ Dan Egan. The Real Cost of Cash Drag. Betterment Blog. March 13, 2015.

²⁰¹ Schwab Intelligent Portfolios Disclosure Brochure. Securities and Exchange Commission. January 22, 2015.

http://www.adviserinfo.sec.gov/Iapd/Content/Common/crd_iapd_Brochure.aspx?BRCHR_VRSN_ID=277224

²⁰² Ibid.

Comparison of ETF Expense Ratios

	All Asset Classes	70 Percent Stock Allocation	
	Average	Taxable	Retirement
Schwab Intelligent Portfolios	0.25%	0.25%	0.25%
Wealthfront	0.16%	0.13%	0.15%
Betterment	0.15%	0.13%	0.11%

Source: Schwab Intelligent Portfolios website, Wealthfront website, Betterment website. See footnote for more details.²⁰³ The 70 percent stock allocation is approximate. The exact allocations, given in the format (% Stock, % Bond, % Commodities, % Cash), are: Schwab/Taxable (69, 17, 5.8, 8.2), Wealthfront/Taxable (71, 29), Betterment/Taxable (70,30), Schwab/Retirement (69, 17, 5.8, 8.2), Wealthfront/Retirement (70, 30), Betterment/Retirement (70,30). The calculated expense ratios for the 70 percent stock allocations for Schwab understate the expense ratio of ETFs used by Schwab Intelligent Portfolios as a whole, since the cash allocation was assumed to have an expense ratio of zero. Computations were performed by the author of this paper.

Schwab Intelligent Portfolios also receives compensation from using third-party ETFs in its OneSource program, creating an additional conflict of interest.²⁰⁴ Schwab ETF OneSource provides investors with commission-free trading of select ETFs in Schwab accounts. As Schwab Intelligent Portfolios wrote in its disclosure brochure:

[Charles Schwab Investment Advisory, which provides portfolio management services for Schwab Intelligent Portfolios,] has a potential conflict in selecting ETFs, because Schwab ETFs pay compensation to [Charles Schwab Investment Management], and ETFs in ETF OneSource pay compensation to Schwab, but other ETFs that are eligible for the investment strategies do not.²⁰⁵

²⁰³ Portfolio Allocations: Wealthfront Website. <https://www.wealthfront.com/plan>; Schwab Intelligent Portfolios Website. <https://hg.schwab.com/client/#/planRecommendation>; Betterment Website. <https://www.betterment.com/app/#portfolio>

Primary ETFs: Schwab Intelligent Portfolios Guide to Asset Classes Whitepaper, Wealthfront Investment Methodology Whitepaper, Wealthfront Tax-Loss Harvesting Whitepaper, Betterment Website. Portfolio. <https://www.betterment.com/portfolio/>; Rukun Vaidya. Investment Selection: Building Portfolios, Fund by Fund. <https://www.betterment.com/resources/investment-strategy/etfs/good-investment-selection-science-art/>; Our Investment Selection Methodology. Betterment Website. <https://www.betterment.com/resources/research/etf-portfolio-selection-methodology/>

Expense Ratios: Schwab Intelligent Portfolios Guide to Asset Classes Whitepaper, Wealthfront Tax-Loss Harvesting Whitepaper, Wealthfront Website. FAQ. <https://pages.wealthfront.com/faqs/what-etfs-does-wealthfront-use-to-implement-tax-loss-harvesting/>; Our Investment Selection Methodology. Betterment Website. <https://www.betterment.com/resources/research/etf-portfolio-selection-methodology/>

²⁰⁴ Schwab Intelligent Portfolios Disclosure Brochure. Securities and Exchange Commission. January 22, 2015. http://www.adviserinfo.sec.gov/Iapd/Content/Common/crd_iapd_Brochure.aspx?BRCHR_VRSN_ID=277224; Schwab ETF OneSource Website.

http://www.schwab.com/public/schwab/investing/accounts_products/investment/etfs/schwab_etf_onesource

²⁰⁵ Schwab Intelligent Portfolios Disclosure Brochure. Securities and Exchange Commission. January 22, 2015. http://www.adviserinfo.sec.gov/Iapd/Content/Common/crd_iapd_Brochure.aspx?BRCHR_VRSN_ID=277224

Indexing

Robo-advisors have adopted different indexing strategies to implement asset allocations determined through mean-variance analysis and other portfolio optimization techniques. While Wealthfront and Betterment exclusively rely on traditional capitalization-weighted ETFs (or, in the case of Wealthfront, a combination of individual securities and capitalization-weighted ETFs for accounts with direct indexing) to represent each asset class, Schwab Intelligent Portfolios complements capitalization-weighted ETFs with fundamentally weighted ETFs. This section provides an overview of the debate surrounding the use of each weighting scheme.

Some investors and economists tout fundamental indexes as a superior alternative to capitalization-weighted indexes due to their historical outperformance. In contrast to capitalization-weighted indexes, which weight stocks based on their proportion of the overall market capitalization, fundamentally weighted indexes use fundamental measures of value such as dividends, earnings, cash flows, or book value to determine index weights.²⁰⁶ In a paper entitled “Fundamental Indexation,” Robert D. Arnott, Jason Hsu, and Philip Moore showed that fundamentally based indexing strategies outperformed the benchmark capitalization-weighted portfolio for the 43 years from 1962 to 2004.²⁰⁷

Proponents of fundamental indexes often emphasize that markets are noisy – i.e., stock price movements can be caused by factors unrelated to fundamental changes in firm value – and that fundamental indexes systematically arbitrage price excursions from fair value. By contrast, since capitalization-weighted indexes weight stocks according to their price, proponents of fundamental indexing argue that capitalization-weighted indexes overweight overvalued stocks, dampening future returns. In a paper entitled “Cap-Weighted Portfolios are Sub-Optimal Portfolios,” Jason Hsu, a co-founder of Research Affiliates, presented a mathematical proof showing that if stock prices are more volatile than warranted by changes in fundamentals, capitalization-weighted indexes are no longer mean-variance optimal because of their tendency to overweight stocks whose prices are high relative to fundamentals and underweight stocks whose prices are low relative to fundamentals.²⁰⁸

Fundamental indexes may not always be less prone to overweighting overvalued stocks or underweighting undervalued stocks than their capitalization-weighted counterparts, however. This is due to the fact that growth stocks – stocks whose prices are high relative to fundamentals – may not always be overvalued. As Derek Jun and Burton Malkiel argue in “New Paradigms in Stock Market Indexing,” stocks such as Google – which at one point was selling at \$100 per share with very low earnings, revenues, and other fundamental measures of value – may have been undervalued, rather than overpriced, at the time.²⁰⁹ In this way, fundamental indexing could actually discriminate against undervalued stocks with growth prospects. Conversely, one could imagine fundamental indexes overweighting companies with low share prices relative to

²⁰⁶ Jeremy Siegel. *Stocks for the Long Run*. McGraw Hill. 2014. 370.

²⁰⁷ The Value Effect. NBIM Discussion Note. Norges Bank. April 12, 2012.

²⁰⁸ Jason Hsu. Cap-Weighted Portfolios are Sub-Optimal Portfolios. *Journal of Investment Management*. 2006.

²⁰⁹ Derek Jun and Burton G. Malkiel. New Paradigms in Stock Market Indexing. *European Financial Management*. 2007.

fundamentals and negative prospects (for instance, the company could be in a near-obsolete industry). Such companies might be overvalued, rather than cheap.

Even if fundamentally weighted indexes outperform their capitalization-weighted counterparts in the future (they very well may not – past performance is not indicative of future performance), investors employing fundamental indexes may inadvertently increase portfolio risk. Fundamental indexes may have derived their historical outperformance from their active tilt toward value and small-capitalization stocks, which empirically have been shown to produce positive alpha relative to the market. However, economists remain at odds about the reasons for the existence of the value and size effects. Theoretical explanations can be broadly classified into two categories: rational explanations and behavioral-bias explanations.²¹⁰ Proponents of rational explanations argue that smaller and more value-oriented companies are inherently more risky. Proponents of behavioral-bias explanations assert that mispricings result from the suboptimal behavior of investors. While explanations predicated on investor rationality imply that tilting a portfolio toward smaller, more value-oriented stocks is a strategy that can increase returns – and risk – in the long term, behavioral explanations imply that investors can exploit market inefficiencies, eventually arbitraging the size and value effects away.

By employing capitalization-weighted indexes, investors express their conviction in passive indexing. Fundamental indexing, which tilts portfolios toward value and small-capitalization stocks, is a form of active management, creating winners and losers relative to the market return. Employing capitalization-weighted indexes results in lower index turnover, as absent a reconstitution of the index, indexes automatically rebalance. By contrast, trades must be conducted to rebalance fundamental indexes when prices do not move in tandem with the fundamental measure(s) determining index weights. Such rebalancing trades may lead to the realization of capital gains, decreasing tax efficiency. As discussed in the previous chapter, capitalization-weighted indexes also work well from a portfolio rebalancing standpoint.

Capitalization-weighted indexes are the best alternative for individual investors who do not have access to top-tier investment managers. Capitalization-weighted indexes' low expense ratios relative to fundamental indexes, transparency, simplicity, and tax efficiency warrant their inclusion in a portfolio for the individual investor.

Conclusion

As this chapter has shown, robo-advisors have adopted different approaches to asset allocation and implementation. This chapter has focused on three robo-advisors – Schwab Intelligent Portfolios, Wealthfront, and Betterment – and revealed significant differences between them with respect to issues such as asset class definition, estimation of mean-variance parameters, and attitudes toward risk.

Schwab has adopted a series of questionable practices that are likely to damage investor returns. It has invested in several asset classes with unfavorable risk-return characteristics and

²¹⁰ The Value Effect. NBIM Discussion Note. Norges Bank. April 12, 2012; Derek Jun and Burton G. Malkiel. New Paradigms in Stock Market Indexing. *European Financial Management*. 2007.

has over-specified its asset class mix. It has also implemented allocations with expensive (and potentially riskier) fundamental indexes. Most importantly, Schwab is subject to material conflicts of interest that bias its investment recommendations. Wealthfront and Betterment, by contrast, have adopted a sound investment methodology that is free of such conflicts.

Wealthfront and Betterment differ in some important respects, however. Wealthfront blends its own views on asset class returns with those implied by the equilibrium market portfolio, while Betterment is more confident about market efficiency, relying exclusively on the market implied returns. Wealthfront has focused on general long-term investing while Betterment is more concerned with goals-based investing. Wealthfront gauges investors' subjective risk tolerance while Betterment appears not to.

While an assessment of robo-advisors and their approaches to portfolio selection is an interesting exercise, a more important avenue of research is how robo-advisors compare to their traditional human counterparts. As with any question in economics, the benefits and limitations of robo-advice should be evaluated not only in isolation, but also with respect to the next best alternative – the counterfactual scenario. To what extent is robo-advice better or worse than traditional investment advice? That is the subject of the next chapter.

CHAPTER 4: HOW ROBO-ADVISORS DIFFER FROM TRADITIONAL ADVISORS

A diverse set of professionals and institutions provide financial advice. Traditional sources of investment advice have been registered investment advisors and broker-dealers. Advisors, by which this paper means all professionals providing investment advice, may possess different beliefs about investment best practices, adhere to different legal standards, and respond differently to incentives and conflicts of interest. This section focuses on the robo-advisor model, showing where traditional advice may depart from the purely automated model. Such deviations from the robo-advisor model may not apply to all traditional advisors.

Investment Philosophy and Methodology

Robo-advisors generally adhere to an investment philosophy and methodology that is grounded in finance theory and economics. While some traditional advisors have also adopted a well-grounded investment methodology, many have not for reasons such as conflicts of interest or misguided beliefs. This section focuses on the basic robo-advisor model, establishing a benchmark against which traditional advisors might be evaluated. It also presents some evidence showing that some traditional advisors may give advice that is inconsistent with investment best practices.

Passive Indexing

Robo-advisors have generally adopted a strategy of passive indexing, the merits of which were reviewed in the chapter on how robo-advisors work. Traditional advisors may or may not (exclusively) recommend passive funds. For example, in an audit study of advisors in the Boston and Cambridge area, Mullainathan et al. (2012) found that advisors encouraged the client to invest in index funds in only 7.5 percent of advice sessions, and suggested investing in actively managed funds in 50 percent of the visits.²¹¹ In another paper studying Canadian financial advisory firms, the average client portfolio held more than 99 percent of total assets in actively managed funds.²¹²

Tax Location

During the asset allocation process, robo-advisors typically develop different sets of asset classes for taxable and tax-deferred accounts. Robo-advisors' attention to asset location, the placement of assets into either taxable or tax-deferred accounts, improves investors' tax efficiency. Some traditional advisors recommend actively managed funds for taxable accounts even though actively managed funds' generally higher portfolio turnover compared to index

²¹¹ Sendhil Mullainathan, Markus Noeth, and Antoinette Schoar. The Market for Financial Advice: An Audit Study. NBER. Working Paper 17929. March 2012. As the authors of the paper write, "The audit data of 284 client visits was collected between April and August 2008, i.e. after the problems of Bear Stearns surfaced but before the bankruptcy of Lehman Brothers in mid-September. We had initially planned for 480 observations but unfortunately had to stop our audit study prematurely, since in the ensuing financial contraction the market for financial advice in the Boston area was significantly restructured." However, the authors show that the randomization of visits to advisors remained intact despite the smaller sample size.

²¹² Juhani T. Linnainmaa, Brian T. Melzer, and Alessandro Previtero. Costly Financial Advice: Conflicts of Interest or Misguided Beliefs? December 2015.

funds leads to the realization of greater taxable gains.²¹³ Robo-advisors avoid actively managed funds and invest in tax-efficient municipal bonds in taxable accounts.

Asset Allocation

Robo-advisors perform asset allocation with mean-variance optimization or a variant of mean-variance analysis, which was shown in the first chapter to be a compelling framework for portfolio selection. Traditional advisors may not use mean-variance analysis or may use it improperly. Although differences exist in the way robo-advisors select a single portfolio from the efficient frontier, they have generally adopted systematic approaches to measuring investor risk, taking into account factors such as time horizon and risk tolerance.

Empirical evidence suggests that traditional advisors may not provide investment advice in a systematic manner because of biases that influence the information gathering process. Mullainathan et al. (2012) found that women were asked for their age less often than men and that the relationship was statistically significant at the 5 percent confidence interval. Women were also less likely to be asked about their annual income and whether they had a 401(k) plan. These relationships were statistically significant at 5 percent and 1 percent confidence intervals, respectively. The study also showed that clients were more likely to be asked about their age, current occupation, and financial situation – including their annual income and whether they had a 401(k) – if they were older. These results suggest that young or female clients may receive less personalized advice due to unsystematic data gathering by traditional advisors. Robo-advisors face no such biases.

Nevertheless, in some respects the way in which traditional advisors measure risk may be systematic and consistent with lifecycle models and portfolio theory. Mullainathan et al. show that in 75 percent of client visits, advisors assessed clients' demographic characteristics, asking for information about the client's income; other savings, such as 401(k) assets; occupation; and marital and parental status. Moreover, advisors recommended riskier, more equity-oriented portfolios to individuals with higher annual income. Since *ceteris paribus* higher annual income increases an individual's objective risk tolerance, such recommendations make sense.

However, in other cases, traditional advisors' recommendations did not seem to be consistent with portfolio theory. For instance, in the study conducted by Mullainathan et al., the recommended allocation to equities decreased with the amount invested.²¹⁴ Female clients were advised to hold more liquidity and less international exposure, and were advised less frequently to invest in actively managed funds.²¹⁵ Foerster et al. (2015) similarly found that the advised portfolios of female investors exhibited on average a 1.4 percentage point lower equity allocation than those of men after controlling for demographic characteristics and risk tolerance.²¹⁶ It is unclear why women would be more risk averse than men. Mullainathan et al. (2012) also found

²¹³ Colleen M. Jaconetti. Asset Location for Taxable Investors. Vanguard Investment Counseling & Research.

²¹⁴ Sendhil Mullainathan, Markus Noeth, and Antoinette Schoar. The Market for Financial Advice: An Audit Study. NBER. Working Paper 17929. March 2012.

²¹⁵ Ibid.

²¹⁶ Stephen Foerster, Juhani T. Linnainmaa, Brian T. Melzer, Alessandro Previtero. Retail Financial Advice: Does One Size Fit All? Chicago Booth Paper No. 14-38. *Journal of Finance*, forthcoming. 9.

that client age did not seem to affect the stock allocation of recommended portfolios.²¹⁷ Since older individuals, who generally have shorter time horizons than younger investors, cannot afford to take on as much risk, the lack of a negative relationship between age and portfolio risk is striking. Foerster et al. (2015) found that younger clients assumed less risk and older clients assumed more risk relative to a lifecycle fund benchmark.²¹⁸

Implementation, Monitoring, and Rebalancing

In their implementation of asset allocations, robo-advisors have generally focused on lowering costs for investors. Traditional advisors who do not minimize fees for clients may also indirectly hurt investors by selecting underperforming funds. Gil-Bazo and Ruiz-Verdu find evidence of a negative relationship between fees and before-fee performance of U.S. equity mutual funds.²¹⁹ These results are robust to multiple checks and support the view that funds strategically set fees as a function of their past or expected performance. Specifically, the results are consistent with the view that 1) investors in funds with poor historical performance exhibit inelastic demand for fund shares and that funds optimally increase fees; 2) funds with lower expected performance anticipate that they will not be able to compete for dollar flows from more sophisticated investors looking for strong risk-adjusted returns, and hence target performance-insensitive investors, optimally raising fees; and 3) funds with low expected performance are marketed to performance-insensitive investors and incur higher marketing costs that are passed on to investors.²²⁰

Robo-advisors have focused not only on selecting ETFs that minimize expense ratios but also on maximizing tax efficiency. A study by Betterment found that the industry average expense ratio for a 70 percent stock portfolio was 0.43 percent, compared to an average expense ratio of 0.15 percent for Betterment.²²¹ Through daily monitoring for tax-loss harvesting opportunities, robo-advisors can significantly increase after-tax returns. Traditional advisors may or may not harvest tax losses or may do so less frequently. Lastly, robo-advisors monitor for rebalancing opportunities on a daily basis, maintaining investment discipline.

Transparency of Investment Advice

Human advisors may initially cater to clients' beliefs to gain their trust, adding a layer of complexity to the provision of investment advice. Mullainathan et al. found evidence that advisors showed early support of clients' portfolios, but that their eventual recommendations

²¹⁷ Sendhil Mullainathan, Markus Noeth, and Antoinette Schoar. The Market for Financial Advice: An Audit Study. NBER. Working Paper 17929. March 2012.

²¹⁸ Stephen Foerster, Juhani T. Linnainmaa, Brian T. Melzer, Alessandro Previtero. Retail Financial Advice: Does One Size Fit All? Chicago Booth Paper No. 14-38. *Journal of Finance*, forthcoming. 9.

²¹⁹ Javier Gil-Bazo and Pablo Ruiz-Verdu. The Relation Between Price and Performance in the Mutual Fund Industry. *Journal of Finance*. 2009.

²²⁰ Ibid.

²²¹ Betterment Investment Selection Methodology Whitepaper. <https://www.betterment.com/resources/research/etf-portfolio-selection-methodology/>. Expense ratios were calculated for a 70 percent stock portfolio with 50 percent of assets in primary ETFs and 50 percent of assets in secondary ETFs. Robo-advisors select primary and secondary ETFs tracking different, but highly correlated, indexes to harvest tax losses and avoid wash sales.

varied significantly from their initial reaction to clients' existing strategies.²²² The authors write that their results "highlight that advisers have to be aware of the fact that they are facing a sales situation and they cannot bluntly criticize what clients might have done in the past."²²³ Robo-advisors, by contrast, do not cater to clients' beliefs. They provide unambiguous advice and tout their product on the basis of academic research.

Robo-advisors are transparent about the securities in which they intend to invest client assets. Before clients hire robo-advisors as their asset manager, robo-advisors show clients the exact asset allocation of their future portfolio. Traditional advisors may or not exhibit such transparency. In the study conducted by Mullainathan et al., roughly 30 percent of advisors refused to offer any specific advice as long as the advisee had not transferred assets to the advisor.²²⁴ Moreover, advisors were almost 40 percent more likely to impose such conditions on women than men. These data evince the biased nature of some sources of traditional investment advice and cast doubt on the ability of individual investors to identify high-quality human advisors.

Summary

Some traditional advisors may adhere to a sound investment methodology. However, it is unclear to what extent the traditional investment advisory profession adheres to such best practices. From an investment philosophy and methodology standpoint, the well-grounded, systematic, and low-cost investment advice provided by robo-advisors is a compelling alternative to traditional sources of advice. Robo-advisors generally adhere to the highest standards as determined by finance theory and economics.

Personalized Investment Advice

A common criticism of robo-advisors is that they provide "canned" investment recommendations that do not adequately take into account the individual's overall financial picture.²²⁵ Some critics have cast doubt on the ability of robo-advisors to provide comprehensive and personalized investment advice. As Melanie Fein, an attorney who recently served on the adjunct faculty of Yale Law School, wrote in a review of robo-advisors:

Rather than characterize robo-advisors as providing personal investment advice, it is more accurate to describe them providing online tools for a client to use in determining the client's own risk tolerance and investment preferences and then enabling the client to subscribe to an investment strategy based on asset allocation formulas recommended for investors with similar preferences...it would be a mistake for retail or retirement investors to view robo-advisors as providing comprehensive personal investment advice designed to meet their individual needs.²²⁶

²²² Sendhil Mullainathan, Markus Noeth, and Antoinette Schoar. The Market for Financial Advice: An Audit Study. NBER. Working Paper 17929. March 2012.

²²³ Ibid. 4.

²²⁴ Ibid.

²²⁵ Melanie L. Fein. Robo-Advisors: A Closer Look. Social Science Research Network. June 30, 2015. 4.

²²⁶ Ibid. 12.

This section examines to what extent traditional advisors provide customized investment advice and how such advice compares with robo-advice. As shown in the previous section on Investment Philosophy and Methodology, biases in the information gathering process affect the ability of some traditional advisors to provide consistent advice. Robo-advisors, by contrast, gather information from clients in systematic fashion, making investment recommendations with the aid of computer algorithms.

Evidence from a study of Canadian financial firms suggests that advisors do not tailor portfolio recommendations to their clients' financial situation. Using regression analysis, Foerster et al. (2015) showed that advised individuals' observable characteristics – including their risk tolerance, time horizon, salary, and other demographic characteristics – jointly explained only 12.2 percent of the cross-sectional variation in portfolio equity orientation and 4.1 percent of the cross-sectional variation in portfolio home bias, as measured by adjusted R-squared.²²⁷ The low explanatory power of clients' observable characteristics on portfolio equity orientation has been corroborated by other studies.²²⁸

Rather, unobservable advisor characteristics may better explain the cross-sectional variation in portfolio equity orientation and portfolio home bias. Foerster et al. showed that including advisor fixed effects to their regression model increased the explanatory power of the portfolio risk and home bias regression models from 12.2 percent to 30.2 percent and 4.1 percent to 27.9 percent, respectively (see Table 2).²²⁹ (Advisor fixed effects capture common variation in portfolios among investors of the same advisor.) In other words, advisor fixed effects were a much better predictor of the risk-orientation and home bias of client portfolios than observable client characteristics. These results were not due to endogeneity bias. Using data on clients who were forced to switch to a new advisor, Foerster et al. showed that advisor fixed effects continued to explain much of the cross-sectional variation in portfolio equity orientation and portfolio home bias.²³⁰ Specifically, upon switching advisors, investors' equity share and home bias tended to shift toward that of the average portfolio held by the new advisor's clients.²³¹

Advisors' influence over investor portfolios can be linked to the beliefs and preferences of advisors, suggesting that advisors do not cater recommendations to the specific needs of clients, but impose their own beliefs and preferences on their clients. To assess whether advisors' influence on clients' risk taking could be explained by advisors' beliefs, Foerster et al. regressed the advisor fixed effects from the second regression in Table 2 on advisor attributes such as age, gender, and risk tolerance, and the equity orientation of the advisor's personal portfolio. As shown in Table 3, the coefficient on advisor's risky share was positive and highly significant. Similarly, for the home bias regression, the authors found that the advisor's home bias positively and significantly related to the advisor fixed effect. The results suggest that some traditional advisors provide biased, one-size-fits-all advice to their clients.

²²⁷ Stephen Foerster, Juhani T. Linnainmaa, Brian T. Melzer, Alessandro Previtero. Retail Financial Advice: Does One Size Fit All? Chicago Booth Paper No. 14-38. *Journal of Finance*, forthcoming. 10.

²²⁸ Ibid.

²²⁹ Ibid. 13.

²³⁰ Ibid. 16-17.

²³¹ Ibid. 17.

In contrast to such traditional advisors, robo-advisors provide consistent and unbiased advice in a systematic fashion, generally responding to individuals' risk tolerance, time horizon, other personal attributes, and investment purposes. As the above study has shown, traditional advisors may not provide advice in accordance with the information they collect. It is also important to note that algorithmic advice does not necessarily lack customization. A complex algorithm that takes into account all factors that are relevant to the investor's financial situation could provide recommendations that are both customized and supported by quantitative tests.

Granted, robo-advisors may not provide personalized investment advice if one defines "personalized investment advice" as fully informed advice. For many robo-advisors, the answers to questionnaires provide the only inputs in the design of long-term portfolios. As such, the degree to which investment advice can be personalized is limited by the scope of robo-advisors' questionnaires. Considerations that may be relevant to investment decision-making but have yet to be considered by some robo-advisors include, but are not limited to, assets, liabilities, and timing and magnitude of anticipated withdrawals. For instance, if an investor owns real estate, the robo-advisor might reasonably reduce the investor's REIT exposure. Or, if the investor owns a small business, the investment portfolio might reasonably tilt toward safer assets, offsetting part of the equity risk of the private holding.²³²

Yet in other respects, robo-advice is highly personalized. Robo-advisors differentiate between taxable and tax-deferred accounts, choosing a set of asset classes on the basis of their tax efficiency, income and dividend payouts, and risk-return characteristics. They avoid wash sales for clients harvesting tax losses, taking into account securities that the investor may be holding in other accounts. In some cases, they may offer goals-based investment advice, allowing the investor to manage investments with a greater degree of precision (albeit at the expense of efficiency). Robo-advisors will provide more personalized advice as they continue to add new features to their online platforms, allowing for greater flexibility in investment recommendations.

Fiduciary Responsibility

Robo-advisors are registered as investment advisors under the Investment Advisers Act of 1940 (henceforth, "Advisers Act"), which generally requires any person who receives compensation for providing investment advice to register with the Securities and Exchange Commission or a State.²³³ A broker-dealer (henceforth, "broker") that provides investment advice is exempt from the Advisers Act as long as the performance of investment advisory services is "solely incidental" to the conduct of the broker's business and the broker receives no "special compensation" for its advisory services.²³⁴ As the SEC wrote in a 2011 report:

Generally, the "solely incidental" element amounts to a recognition that broker-dealers commonly give a certain amount of advice to their customers in the course of their

²³² David F. Swensen. *Unconventional Success*. Free Press. 2005. 86.

²³³ *Fiduciary Investment Advice*. Regulatory Impact Analysis. Department of Labor. April 14, 2015. 27.

²³⁴ *Study on Investment Advisers and Broker-Dealers*. Securities and Exchange Commission. January 2011. 15-16; *Fiduciary Investment Advice*. Regulatory Impact Analysis. Department of Labor. April 14, 2015. 27.

regular business as broker-dealers and that “it would be inappropriate to bring them within the scope of the [Advisers Act] merely because of this aspect of their business.” On the other hand, “special compensation” “amounts to an equally clear recognition that a broker or dealer who is specially compensated for the rendering of advice should be considered an investment adviser and not be excluded from the purview of the [Advisers Act] merely because he is also engaged in effecting market transactions in securities.”²³⁵

This paper refers to registered investment advisers (RIAs), brokers, and other professionals providing advice as “advisers.”

RIAs are held to a fiduciary standard, while brokers adhere to a “suitability” standard. The Advisers Act imposes on RIAs “an affirmative duty to their clients of utmost good faith, full and fair disclosure of all material facts, and an obligation to employ reasonable care to avoid misleading their clients.”²³⁶ RIAs must provide advice that satisfies duties of loyalty and care.²³⁷ The duty of loyalty requires an advisor to act in the best interests of clients, and includes an obligation to not subordinate clients’ interest to the advisor’s own.²³⁸ The duty of care requires an advisor to make a reasonable investigation to determine that investment recommendations are not based on materially inaccurate or incomplete information.²³⁹ According to a report by the Department of Labor:

RIAs must employ reasonable care to avoid misleading clients and must eliminate, or at least disclose, all conflicts of interest that might incline them to render advice that is not disinterested. If RIAs do not avoid a conflict of interest that could impact the partiality of their advice, they must provide full and frank disclosure of the conflict to their clients. They cannot use their clients’ assets for their own benefit or the benefit of other clients, except with the clients’ consent.²⁴⁰

Brokers, who are regulated under the Securities Exchange Act of 1934 and are generally required to become members of the Financial Industry Regulatory Authority (FINRA), adhere to a “suitability” standard.²⁴¹ Although they generally are not subject to a fiduciary standard, brokers may become subject to a duty to act in the client’s best interest under common law if the broker acts in a position of trust and confidence with its client.²⁴² Under certain circumstances, brokers are also required to disclose material conflicts of interest to clients, but in practice such disclosures are more limited with brokers than with registered investment advisers.²⁴³ For instance, a broker generally is not required to disclose that it receives compensation from a

²³⁵ Study on Investment Advisers and Broker-Dealers. Securities and Exchange Commission. January 2011. 16. In this block quote, the SEC report cites Opinion of the General Counsel Relating to Section 202(a)(11)(C) of the Investment Advisers Act of 1940, Investment Advisers Act Release No. 2 (Oct. 28, 1940).

²³⁶ Fiduciary Investment Advice. Regulatory Impact Analysis. Department of Labor. April 14, 2015. 28.

²³⁷ Study on Investment Advisers and Broker-Dealers. Securities and Exchange Commission. January 2011. 22.

²³⁸ *Ibid.*

²³⁹ *Ibid.*

²⁴⁰ *Ibid.*

²⁴¹ *Ibid.* 26.

²⁴² *Ibid.*; Melanie Fein. Brokers and Investment Advisers. Standards of Conduct: Suitability vs. Fiduciary Duty. Fein Law Offices. Social Science Research Network. August 31, 2010. 3, 38.

²⁴³ Study on Investment Advisers and Broker-Dealers. Securities and Exchange Commission. January 2011. iv, 106.

mutual fund it recommends to its clients, while a registered investment adviser is required to disclose such conflicts.²⁴⁴

The duties applicable to registered investment advisors are more principles-based than rules-based, while the duties applicable to brokers are more rules-based than principles-based. Attorney Melanie Fein has written that it is difficult to make generalizations about whether the regulations governing registered investment advisors or the regulations governing brokers afford greater investor protection.²⁴⁵ For instance, while registered investment advisors have a duty to act in their clients' best interest, there is no explicit "suitability" criteria for investment advisors specifying what information they must evaluate when making investment recommendations.²⁴⁶ The suitability standard for brokers, by contrast, requires that they make recommendations that take into account specified information such as the client's financial situation, investment experience, and investment objectives.²⁴⁷

Although robo-advisors are RIAs, the extent to which they embrace their fiduciary duty is a point of contention. In "Robo-Advisors: A Closer Look," Fein writes, "it cannot be said that robo-advisors act in the best interest of the client but rather leave it to the client to act in his or her own best interest."²⁴⁸ Among the many excerpts Fein cites from robo-advisor client agreements is one stating that the client is responsible for determining that investments are in the best interests of the client's financial needs.²⁴⁹ The excerpt does not say that the robo-advisor is (or is not) responsible for determining that investments are in the client's best interests. Another excerpt states that the robo-advisor is and will act as an independent contractor, but is not an employee of the client and has no other relationship with the client.²⁵⁰ Fein has interpreted this excerpt as an attempt by robo-advisors to limit their fiduciary duty. Other excerpts seek to limit robo-advisors' financial liability. Fein writes, "While a fiduciary generally is not responsible for losses in a client's account that are beyond its control, the extent to which robo-advisors seek to limit their liability suggests that they do not perceive themselves as under a fiduciary duty to act in the client's best interest."²⁵¹

Robo-advisors generally are not fiduciaries as defined under the Employee Retirement Income Security Act of 1974 (ERISA), which governs plan assets and in some respects imposes more stringent rules on advisors than the Advisers Act.²⁵² As Fein has written, robo-advisors generally exclude accounts that are subject to ERISA, thereby avoiding ERISA's strict fiduciary duties.²⁵³ Betterment for Business – Betterment's 401(k) advisory platform – may be the one

²⁴⁴ Melanie Fein. Brokers and Investment Advisers. Standards of Conduct: Suitability vs. Fiduciary Duty. Fein Law Offices. Social Science Research Network. August 31, 2010. 2, 4.

²⁴⁵ *Ibid.* 2.

²⁴⁶ *Ibid.* 3.

²⁴⁷ *Ibid.*; Fiduciary Investment Advice. Regulatory Impact Analysis. Department of Labor. April 14, 2015. 28.

²⁴⁸ Melanie L. Fein. Robo-Advisors: A Closer Look. Social Science Research Network. June 30, 2015. 26.

²⁴⁹ *Ibid.* 23-24.

²⁵⁰ *Ibid.* 24.

²⁵¹ *Ibid.* 25-26.

²⁵² The author is unsure whether robo-advisors are fiduciaries under the Internal Revenue Code (IRC). IRC rules govern both plan and retail IRA accounts. This is a potentially interesting avenue of future research.

²⁵³ Melanie L. Fein. Robo-Advisors: A Closer Look. Social Science Research Network. June 30, 2015. 27.

robo-advisor that acts as an ERISA fiduciary.²⁵⁴ Rules under ERISA are separate from federal securities laws such as the Securities Exchange Act of 1934 and Advisers Act.²⁵⁵ ERISA rules govern the conduct of RIAs and brokers who provide advice on employer-sponsored retirement plans, such as 401(k) plans.²⁵⁶ ERISA does not apply to retail IRAs.²⁵⁷ Under ERISA, any person paid directly or indirectly to provide plan officials or participants with advice on the investment of assets in retirement plans is a fiduciary.²⁵⁸

However, in 1975, the Department of Labor issued a rule that narrowly limited fiduciary status under ERISA.²⁵⁹ Before an adviser is held to such fiduciary standards, the advisor must (1) make recommendations on investing in, purchasing or selling securities or other property, or give advice as to their value (2) on a regular basis (3) pursuant to a mutual understanding that the advice (4) will serve as a primary basis for investment decisions, and (5) will be individualized to the particular needs of the plan. The advisor must meet each of these conditions for each instance advice is rendered to be classified as having acted as a fiduciary in rendering such advice.

Fiduciary status under ERISA is not identical to fiduciary status under federal securities laws. Fiduciaries under ERISA must act prudently and solely in the interest of clients when providing investment recommendations.²⁶⁰ ERISA generally requires fiduciaries to avoid certain prohibited transactions, which may involve conflicts of interest.²⁶¹ ERISA fiduciaries also generally may not self-deal, meaning they may not deal with retirement plan assets for their own interest or account, or receive compensation from a third party in connection with a transaction involving retirement plan assets.²⁶²

The Department of Labor has written that ERISA complements, rather than contradicts, federal securities laws. It has also written that ERISA is more stringent than the Advisers Act in some important respects:

The specific duties imposed on advisers by the SEC stem, in large part, from antifraud provisions. Accordingly, certain conflicts of interest are not themselves violations as long as they are disclosed in order to ensure that the implied representation of fairness is not misleading. In contrast, ERISA and the [Internal Revenue] Code place greater emphasis on the elimination or mitigation of conflicts of interest. Thus, under ERISA and the Code,

²⁵⁴ Betterment website. <https://www.bettermentforbusiness.com/for-plan-sponsors/>

²⁵⁵ Fiduciary Investment Advice. Regulatory Impact Analysis. Department of Labor. April 14, 2015. 2.

²⁵⁶ Ibid.

²⁵⁷ Ibid.

²⁵⁸ Ibid. 13. ERISA refers to persons who handle funds or other property of an employee benefit plan as “plan officials.”

²⁵⁹ Ibid. 19; Federal Register. Department of Labor. April 20, 2015.

<https://www.federalregister.gov/articles/2015/04/20/2015-08831/definition-of-the-term-fiduciary-conflict-of-interest-rule-retirement-investment-advice>. From the Executive Summary: “In 1975, the Department issued regulations that significantly narrowed the breadth of the statutory definition of fiduciary investment advice by creating a five-part test that must, in each instance, be satisfied before a person can be treated as a fiduciary adviser. This regulatory definition applies to both ERISA and the [Internal Revenue] Code.”

²⁶⁰ Ibid. 13.

²⁶¹ Ibid.

²⁶² Ibid.

fiduciary advisers are generally prohibited from making recommendations with respect to which they have a financial conflict of interest unless the Department of Labor first grants an exemption with conditions designed to protect the interests of plan participants and IRA owners. This is true regardless of whether the fiduciary has disclosed his or her conflicts of interest to their plan or IRA customer.²⁶³

The Department of Labor proceeded to write:

In particular, the Advisers Act generally permits self-dealing transactions that would largely be prohibited under ERISA, as long as the RIA fully discloses the conflict to the client. Further, because many of the Adviser Act standards are outgrowths of the antifraud provisions of federal securities law, a private action to establish a violation of those provisions generally requires proving that the adviser acted with the intent to deceive, manipulate, or defraud his or her customer. This is a much more difficult standard of proof than required under ERISA.²⁶⁴

It is unclear why robo-advisors have generally chosen to exclude ERISA accounts from their platforms. Their decision may be related to the narrow definition of a fiduciary under the 1975 rules, which requires advisors to provide “individualized” advice. The ambiguous nature of the term “individualized” could impart a certain amount of liability to robo-advisors. Robo-advisors may have chosen not to assume fiduciary status under ERISA due to its more stringent rules generally requiring that fiduciaries eliminate or mitigate, rather than disclose, conflicts of interest. It might be the case that some conflicts of interest are too difficult, if not impossible, for robo-advisors to avoid. For example, some robo-advisors route orders through Apex Clearing, from which they may receive monetary rebates.²⁶⁵ There may be few or no cost-effective alternatives to Apex.

The Costs of Conflicted Advice

Robo-advisors are subject to some conflicts of interest. As shown in the previous chapter, serious conflicts of interest have tainted the ETF selection process of Schwab Intelligent Portfolios. The robo-advisor’s cash allocation can also be traced back to such conflicts. Other robo-advisors may face milder conflicts of interest, such as engaging in agency cross transactions from which robo-advisors may receive commissions, or receiving payments for routing orders to clearing firms. These milder conflicts pale in comparison to those of some traditional advisors. This section focuses on three channels through which traditional advisors, but not robo-advisors, may harm investors due to conflicts of interest: biased recommendations and asset flows, the poor performance of funds sold through intermediaries, and poor market timing. In each case,

²⁶³ Ibid. 24.

²⁶⁴ Ibid. 28. The Department of Labor includes this footnote: “The SEC can enforce breaches of fiduciary duties under Advisers Act Section 206, however, without proving scienter. In addition, some states permit claims based on breach of fiduciary duty, negligence, or fraud.”

²⁶⁵ Melanie L. Fein. Robo-Advisors: A Closer Look. Social Science Research Network. June 30, 2015. 17.

conflicted advice and its consequences can be traced back to the misalignment of interests between clients and advisors or clients and the funds in which they invest.²⁶⁶

Biased Recommendations and Asset Flows

As shown in the section on Investment Philosophy and Methodology, human advisors may recommend that clients invest in actively managed funds. Mullainathan et al. (2012) also show that advisers may dissuade clients from pursuing a passive indexing strategy even when they are already holding an efficient, albeit home-biased, index portfolio. In Mullainathan et al. (2012), prospective clients who were recruited by the authors were assigned a return-chasing portfolio, “company stock” portfolio, or low-fee index portfolio representing their existing investment strategy; there was also a control group of clients who held all-cash portfolios and did not espouse any particular view on a preferred investment strategy.²⁶⁷ Return-chasing portfolios were invested in a sector that had recently performed well, and the corresponding client expressed interest to the advisor in identifying more industries that had also recently performed well. The client with the company stock portfolio held 30 percent of the portfolio in stock of the client’s employer. The client with a low-fee index portfolio was invested solely in U.S. stocks and bonds and held the most efficient portfolio. Using a regression analysis, Mullainathan et al. (2012) found that advisors were most likely to recommend actively managed funds to clients with either an index fund portfolio or an all-cash portfolio.²⁶⁸

Mullainathan et al. (2012) write that such recommendations are evidence of conflicts of interest:

Most strikingly, even if a client had a well-diversified index funds portfolio, the adviser encouraged investment in actively managed funds. The objective of the adviser in this behavior might have been to signal that they could add value to the client by suggesting something different from the existing portfolio. This behavior was particularly pronounced for wealthier clients where the fee income mattered more to the adviser. But advisers could also have achieved this goal by suggesting low-fee international diversification. In general, advisers did not proactively reach out to clients to rebalance the portfolio due to changing circumstances of the client, but only to sell them new funds and generate fees. The advice that we observe in our treatments are a good proxy for the different situations that an adviser might encounter with their clients throughout a longer term relationship. The evidence suggests that most of the interaction is driven by the need to generate fees rather than to respond to the clients’ rebalancing needs.²⁶⁹

²⁶⁶ For a more comprehensive discussion of conflicted advice, please see the Department of Labor’s Regulatory Impact Analysis of the Department’s proposed fiduciary rule and the Council of Economic Advisers’ report “The Effects of Conflicted Investment Advice on Retirement Savings.” The author of this paper relied heavily on these reports in framing the discussion on conflicts of interest. Robert Litan and Hal Singer’s rebuttal to these reports (“Good Intentions Gone Wrong”) presents an interesting opposing view.

²⁶⁷ Sendhil Mullainathan, Markus Noeth, and Antoinette Schoar. The Market for Financial Advice: An Audit Study. NBER. Working Paper 17929. March 2012.

²⁶⁸ Ibid. 16.

²⁶⁹ Ibid.

In a perverse twist, advisors were almost 20 percent less likely to recommend actively managed funds to clients holding the “company stock portfolio,” a less efficient portfolio than the diversified index portfolio.

Mullainathan et al. present additional evidence showing that advisors support strategies that result in greater advisor compensation.²⁷⁰ In doing so they may fail to correct investors’ biases and in some cases may exacerbate them. Specifically, using a regression analysis, Mullainathan et al. showed that advisors were least supportive of the efficient index portfolio, followed by the “company stock” portfolio. The advisors were significantly more likely to support the return-chasing strategy, which would allow the advisor to churn the portfolio more often and generate fees at the expense of the client. These results were robust to additional specifications of the model that controlled for client characteristics such as gender, marital status, and investment size. Mullainathan et al. also ran a complementary test showing that advisors were most likely to discourage the client from continuing an existing investment strategy when the client was invested solely in index funds.

A paper by Christoffersen et al. (2013) links advisor compensation to investment flows, illustrating how brokers’ incentives taint their recommendations.²⁷¹ The paper found that broker-intermediated asset flows to mutual funds increased with the load paid to the broker by the particular mutual fund. Specifically, a 50 basis point increase in the load paid to the broker increased monthly inflows to the fund by 0.0186 percent. For the median fund, this translated to \$1 in loads increasing flows by \$6.71. The effect was more pronounced for funds that were unaffiliated with the broker. Specifically, Christoffersen et al. found that a 50 basis point increase in load payment to unaffiliated brokers increases flows into the average fund by 0.0476 percent. For the median fund, this translated to \$1 in loads increasing flows by \$14.20, more than double the increase experienced by all brokers. Christoffersen et al. argue that since an unaffiliated broker may sell a larger number of funds for many fund families, in contrast with a captive broker who focuses on the funds of a single family, more funds compete for the unaffiliated broker’s influence in directing client dollars. Hence, an unaffiliated broker sees more offers of broker payments, leading to greater sensitivity of fund inflows to broker payments. The authors found similar results for revenue sharing. That is, flows to mutual funds increased with revenue sharing.

Conflicted payments may lead to higher costs for clients. The results from Christoffersen et al. (2013) provide convincing evidence that load payments and revenue sharing bias investment recommendations. Such biases narrow the set of mutual fund choices available to the client, directly harming the investor.²⁷²

Underperformance of Funds Sold Through Conflicted Intermediaries

Christoffersen et al. (2013) also show that higher conflicted payments lead to poorer investment performance, where investment performance is measured as a fund’s return net of

²⁷⁰ Ibid.

²⁷¹ Susan E.K. Christoffersen, Richard Evans, David K. Musto. What do Consumers’ Fund Flows Maximize? Evidence from Their Brokers’ Incentives. *The Journal of Finance*. February 2013.

²⁷² David F. Swensen. *Unconventional Success*. Free Press. 2005. 272-281.

expenses during the 12 months after the initial investment. Specifically, they show that the funds paying higher loads to brokers subsequently perform worse. The effect is stronger for unaffiliated brokers. As they write, “the average 2.3% payment to unaffiliated brokers corresponds to a 1.13% reduction in annual performance.” Critics of the study may note that Christoffersen et al. only study returns over a 12-month period. However, as the Council of Economic Advisors has noted, the authors control for cyclical fluctuations that might have made the study results time-dependent.²⁷³ Moreover, other studies have shown that “annual estimates of underperformance over time are consistent with the first-year effect,” i.e. the investment performance during the first 12 months.”²⁷⁴

However, when advisor compensation is tied to investment performance, funds exhibit milder underperformance, suggesting that advisors respond to incentives. As Christoffersen et al. show, revenue sharing is not significantly correlated with future performance. Specifically, revenue sharing is predictive of underperformance when loads paid to the broker are excluded from the regression, but do not enter significantly when loads are included. The authors write that their results are “consistent with brokers’ exposure to the future performance of the investment that revenue sharing, but not load sharing, imposes through ongoing asset-based payments.”²⁷⁵ In contrast to front-end loads, revenue sharing not only involves upfront payments upon investment, but also continuing payments until redemption that are proportional to the value of the investment.²⁷⁶ Hence, under revenue sharing agreements, brokers are exposed to clients’ realized returns.

Additional economic evidence suggests that fund performance is linked to the incentives of brokers and mutual fund companies. Del Guercio and Reuter (2014) cite evidence showing that the market for actively managed funds is segmented: funds are either sold directly to investors or are sold through brokers, but rarely are they sold to both groups.²⁷⁷ They then show that the after-fee alphas of actively managed funds sold directly to investors are economically and statistically indistinguishable from those of index funds. However, actively managed funds sold through brokers underperform index funds by between 112 and 132 basis points per year. They attribute the difference in mutual fund performance across direct-sold and broker-sold segments to mutual funds’ incentive (or disincentive) to generate alpha. They write:

Because experienced and knowledgeable investors are likely to self-select into direct-sold funds, flows in this segment are more likely to respond to risk-adjusted returns, giving direct-sold families a strong incentive to generate alpha. In contrast, the findings in Christoffersen, Evans, and Musto (2013) and Chalmers and Reuter (2012) suggest that competition in the broker-sold segment is likely to focus on characteristics other than alpha, such as the level of broker compensation. The weaker the sensitivity of investor

²⁷³ The Effects of Conflicted Investment Advice on Retirement Savings. The Council of Economic Advisors. February 2015. 15-16.

²⁷⁴ Ibid.

²⁷⁵ Susan E.K. Christoffersen, Richard Evans, David K. Musto. What do Consumers’ Fund Flows Maximize? Evidence from Their Brokers’ Incentives. *The Journal of Finance*. February 2013.

²⁷⁶ Ibid. 13.

²⁷⁷ Del Guercio, Diane, and Jonathan Reuter. Mutual Fund Performance and the Incentive to Generate Alpha. *The Journal of Finance*. August 2014. 1674.

flows to alpha, the weaker the incentive to generate alpha. Indeed, we find strong evidence that the underperformance of the average actively managed fund can be explained by variation across market segments in the incentive that funds face to generate alpha.²⁷⁸

The results suggest that payments from mutual fund companies to brokers not only bias broker recommendations, but also limit mutual funds' incentive to deliver strong risk-adjusted returns to investors.

Underperformance Through Poor Market Timing

Investors who purchase load-carrying funds through investment professionals exhibit poorer market timing performance than self-directed investors who purchase pure no-load index funds.²⁷⁹ This is the conclusion of Bullard, Friesen, and Sapp (2008), which was cited by the Department of Labor in its Regulatory Impact Analysis of the proposed fiduciary rule.²⁸⁰ Using data on domestic common stock funds that existed from 1991 to 2004, Bullard, Friesen, and Sapp found that investors' performance gap due to market timing – defined as the difference between investors' annual dollar-weighted returns and the time-weighted returns of the funds in which they were invested – was larger for load-carrying funds than for no-load funds. Specifically, the annual performance gap between investor and fund returns was 1.82 percent for load funds (share classes A, B, and C) and 0.78 percent for pure no-load funds, representing an economically and statistically significant difference.²⁸¹

The results of Bullard, Friesen, and Sapp (2008) are consistent with the view that conflicted advisors who recommend load-carrying funds may espouse a return-chasing strategy that allows the advisor to churn the portfolio more often and generate fees at the expense of the client. Granted, alternative explanations may stress the fact that investors who seek out professional guidance may be less knowledgeable about investing and more susceptible to short-term performance biases than self-directed investors. According to this view, the poorer market timing performance of investors purchasing load-carrying funds would not be due to conflicted advice, but to the investor's own lack of experience. Yet as will be shown in a study in the next section, advisors exert a large influence on clients' trading behavior, suggesting that the study results are evidence of poor market timing due to conflicts of interest.

²⁷⁸ Ibid. 1675.

²⁷⁹ Pure no-load funds charge neither loads nor 12b-1 fees.

²⁸⁰ Mercer Bullard, Geoff Friesen, and Travis Sapp. Investor Timing and Fund Distribution Channels. Social Science Research Network. June 1, 2008.

²⁸¹ Ibid. "Class A, B, and C shares are similar in that they use a load structure to compensate brokers for providing investment-related services to their customers...Class A shares typically impose: (1) a front-end sales load that is deducted from the price when the fund share are purchased and (2) an ongoing asset-based fee, known as a 12b-1 fee, of approximately 0.25 percent. Class B shares typically impose: (1) a contingent deferred sales load (CDSL) that declines the longer that the shares are held and (2) a 12b-1 fee of approximately 1.00 percent. After the CDSL declines to zero, Class B shares typically convert to Class A shares and thereafter pay a reduced 12b-1 fee. Class C shares typically charge a 12b-1 fee of approximately 1%, and often a 1% sales load on shares that are redeemed within one year."

Differences in cross-share market timing performance and advisors' compensation also suggest that the study results are evidence of conflicted advice, rather than the inexperience of investors transacting through intermediaries. Bullard, Friesen, and Sapp showed that investors in Class B shares, for which advisors can receive higher compensation upon sale compared to other share classes, exhibited a performance gap of 2.28 percent, which was 41 percent and 71 percent greater than the performance gaps of Class A and Class C shareholders, respectively; the differences in timing performance between share classes B and A and share classes B and C were statistically significant. One reason why Class B shares may provide higher compensation is that their sales loads do not decline with investment size.²⁸² The recommendation of Class B shares over other share classes could be an indicator of the extent to which advice is conflicted. More conflicted advisors may recommend Class B shares and may be more likely to chase returns, inflicting greater damage to client portfolios through poor market timing.

Poor Advice Due to Misguided Beliefs

A different, and relatively new, strain of economics research has suggested another reason for poor investment advice: the misguided beliefs of financial advisors. Explanations of poor advice predicated on misguided beliefs are not necessarily mutually exclusive from explanations based on conflicts of interest. In a study of advisers from three Canadian firms, Juhani Linnainmaa, Brian Melzer, and Alessandro Previtero found some evidence suggesting that advisors make poor recommendations in response to conflicts of interest and stronger evidence that advisors give poor investment advice due to misguided beliefs.²⁸³

Specifically, Linnainmaa et al. found that trades that were both costly and without apparent benefits to the client were significantly more frequent when the advisor gained financially from the trade, providing evidence of conflicted advice. Such trades – which the authors called “self-serving trades” – amounted to 5.4 percent of total trades. By contrast, trades that were both costly and without apparent benefits to the client, but did not increase the advisor's compensation, only occurred 2.9 percent of the time. The difference between these two types of trades was both statistically and economically significant. (Table 4 presents the data from the study.) In the study, a trade was defined to benefit the advisor if, *ceteris paribus*, the advisor 1) earned a new sales commission from the fund company; 2) charged the client a front-end load; or 3) increased the trailing commission.²⁸⁴ A trade was defined to cost the client financially if, *ceteris paribus*, the client 1) paid a front-end load to the advisor; 2) experienced an increased management expense ratio as a result of the trade; or 3) had to pay a deferred sales

²⁸² Ibid. As Bullard, Friesen, and Sapp write, “One reason that Class B shares can be more lucrative is that Class A share sales loads typically decline with the size of the investment, whereas Class B share deferred sales loads do not. When a client invests a large amount, his broker therefore can receive a much higher payment by purchasing Class B shares instead of Class A shares. Some fund firms have addressed this concern by capping the size of Class B share purchases. Even when the client does not sell the shares and pay the deferred sales load, the broker often receives a commission because many funds' principal underwriters pay the broker a flat commission at the time of the Class B share sale, which the underwriter then finances from the 12b-1 fee income stream.”

²⁸³ Juhani T. Linnainmaa, Brian T. Melzer, and Alessandro Previtero. *Costly Financial Advice: Conflicts of Interest or Misguided Beliefs?* December 2015.

²⁸⁴ Ibid. 12. “A trailing commission is a recurring payment from the mutual fund company to the advisor. The fund pays the trailing commission for as long as the client remains invested in the fund. Trailing commissions of 0.25% to 1% per year are standard on all funds sold by advisors.”

charge. A trade was defined to benefit the client if 1) the management expense ratio decreased or 2) the client obtained diversification benefits; the authors assumed that any trade into a fund category that changed the risk-return tradeoff of the client's portfolio was beneficial to the client. This is a rather large assumption. While the authors acknowledged that it is difficult to measure diversification benefits, assuming that all asset allocation changes are beneficial to the client almost certainly leads to an underestimate of the number of conflicted transactions in the sample and an overestimate of the number of transactions that benefit the client. It is not surprising that the authors made this assumption given that their argument is that most advisors give poor advice due to misguided beliefs, rather than conflicts of interest.

Interestingly, several other statistically and economically significant patterns emerged from the study, nuancing the view that advisors strictly respond to incentives without regard for client interests. These results, however, should be taken with a grain of salt due to the authors' assumption regarding diversification benefits. First, the paper found that advisors were more likely to benefit from a trade when the client also benefitted from the trade. Second, trades were more likely to cost the client when the client also appeared to obtain some benefit from the trade. Linnainmaa et al. also found that advisors who recommended self-serving trades gained substantially from doing so, earning commissions of more than 3 percent of assets per year, compared with commissions of between 1.5 to 2 percent of client assets for the typical advisor. However, the clients of self-serving advisors performed better than other advisors' clients; Linnainmaa et al. argued that this is possible because an advisor who maximizes commissions may collect such commissions from mutual funds and not necessarily from clients. The implication is that advisors may recommend trades that are mutually beneficial to the advisor and client. As Linnainmaa et al. write, "To the extent that self-serving trades raise costs for investors, they may do so in an indirect way. That is, mutual funds may respond to increased commissions [they pay to advisors] by charging investors higher management fees."²⁸⁵

Linnainmaa et al. found stronger evidence that poor investment advice is due to the misguided beliefs of investment advisors. They first showed that advisors trade similarly to their clients. In their sample, both advisors and their clients exhibited a high degree of portfolio turnover and invested almost exclusively in actively managed funds. Advisors chased returns to an even greater extent than clients and were more likely to sell losing mutual funds (a phenomenon called the reverse disposition effect). The portfolios of both advisors and their clients displayed pronounced home bias in both retirement and open accounts and were invested in expensive mutual funds. Table 5 in the Appendix presents the data from the study.

Linnainmaa et al. then used a series of regressions to show that an advisor's own investing behavior is predictive of the client's investing behavior. Using a panel regression, Linnainmaa et al. first showed that advisor fixed effects explain part of the cross-sectional variation in client behavior. The independent variables in the model included the year, advisor, and investor fixed effects in addition to a vector of investor attributes including age, risk tolerance, investment horizon, and income. As shown in Table 6 in the Appendix, the inclusion of advisor fixed effects increased the explanatory power of the model. For instance, in the return-

²⁸⁵ Juhani T. Linnainmaa, Brian T. Melzer, and Alessandro Previtero. Costly Financial Advice: Conflicts of Interest or Misguided Beliefs? December 2015. 3.

chasing regression, client attributes explained only 2.3 percent of the variation (as measured by adjusted R-squared) in the return-chasing estimates; including the advisor fixed effects increased the model's explanatory power to 8.3 percent. As shown in Table 6, the advisor fixed effects increased the explanatory power of the regressions for clients' portfolio turnover, share of active management, return chasing, disposition effect, home bias, growth bias, and percentile fee within fund type.

Such increases in explanatory power were not due to endogenous matching between advisors and their clients. Endogenous matching could occur if advisors tended to attract similar clients; hence, including advisor fixed effects would lead to an increase in adjusted R-squared, but might not be evidence that advisors' trading behavior predicts client trading behavior. To show that their results were not due to endogeneity bias, Linnainmaa et al. ran separate regressions using data on clients who were forced to switch advisors when their previous advisor died, retired, and left the industry. In these regressions, the unit of observation was the advisor-client pair. Both advisor and investor fixed effects were included to control for unobserved heterogeneity. As shown in Table 6, the inclusion of advisor fixed effects increased the explanatory power of the model, showing that clients forced to move from one advisor to another changed their trading patterns coincident with the switch. The results imply that advisors instigate trades, not their clients.

Regressing advisor fixed effects on advisor attributes and advisor behavior, Linnainmaa et al. then showed that advisors' influence on clients' investing behavior was linked to advisors' own investing behavior. Such advisor fixed effects were the same fixed effects used in the previous regressions, i.e. the regressions that included advisor fixed effects, not the regressions without advisor fixed effects that they were compared to. For every regression – including the regressions for portfolio turnover, active management, and return chasing – the relationship between the advisor's behavior and advisor fixed effects was statistically significant. As shown in Table 6, the slope estimates were also economically significant. The results imply that client behavior can be explained by advisor fixed effects, which can be explained by advisor behavior. In other words, an advisor's own investing behavior is predictive of the client's investing behavior. The results suggest that advisors give poor advice because they have misguided beliefs about turnover frequency, active management, return chasing, and other investing principles.

Linnainmaa et al. showed that advisor returns relate significantly to client returns, suggesting that advisors make recommendations to clients that are consistent with their beliefs. As the authors wrote, "An advisor's personal portfolio is a good indicator of how he thinks money should be invested. A comparison of clients' performance against their advisors therefore measures how much differences in advisors' investment beliefs affect their clients' returns."²⁸⁶ The regression results showed that an advisor's investment performance was highly predictive of the client's investment performance. The adjusted R-squared for the panel regressions was 70 percent, and the average R-squared across the advisor-specific regression was 78 percent. The slope coefficients on advisors' performance ranged from 0.65 to 0.73, showing that client performance varied significantly with advisor performance and that advisors tended to hold

²⁸⁶ Juhani T. Linnainmaa, Brian T. Melzer, and Alessandro Previtero. Costly Financial Advice: Conflicts of Interest or Misguided Beliefs? December 2015. 29.

similar but riskier versions of the portfolios held by their clients. Client portfolio returns were net of fund expense ratios, but unadjusted for front-end loads and sales charges. Returns on advisor portfolios were also net of fund expenses, but unadjusted for sales and trailing commissions.

The average advisor's portfolio performs just as poorly or worse than those of the advisor's clients. Specifically, the advisor's portfolio underperforms client portfolios before the advisor's sales and trailing commissions on their own purchases are taken into account. Advisor portfolios experience the same poor performance as client portfolios when such rebates are factored into advisor returns. Linnainmaa et al. attribute the performance gap between advisors and clients to advisors' preference for even more expensive mutual funds than those recommended to clients.

Lastly, Linnainmaa et al. show that advisor behavior is largely unchanged post-career, suggesting that their behavior reflects ingrained beliefs about investing. Specifically, advisors' portfolio turnover post-career is only slightly lower and advisors still predominantly invest in expensive, actively managed funds. Advisors continue to chase returns and exhibit home bias. The results suggest that advisors do not engage in "window dressing" during their careers, i.e. they do not chase returns and invest in expensive actively managed funds to convince their clients to do the same. Rather, they believe that active management, return-chasing, and other ill-founded strategies will lead to superior returns.

That advisors' poor investment behavior and investment recommendations could be due to misguided beliefs suggests that even well-intentioned advisors who are not subject to conflicts of interest might recommend investment programs that are not in their clients' best interest. Advisors' embrace of return chasing, active management, and other poor investment behaviors in the sample suggest that the advisory profession may be subject to adverse selection problems. As the authors of the paper wrote:

Those who believe that active management does not add value are probably less likely to pursue a career in the financial advisory industry; and those who believe the opposite may be drawn in. Financial advisors are financial advisors because they hold misguided beliefs.²⁸⁷

Advisors may espouse active management, market timing, and frequent trading because they believe they can generate alpha through these strategies. Robo-advisors Wealthfront and Betterment, by contrast, possess an investment methodology that is grounded in the academic literature. They eschew return-chasing strategies and limit portfolio turnover. They invest solely in low-cost passive funds and diversify investments across many different asset classes, both domestic and foreign. Wealthfront and Betterment clients can rest assured that their assets are invested according to a well-grounded investment methodology.

²⁸⁷ Ibid. 34.

Market Timing and Behavioral Coaching

Critics of robo-advisors liken their clients to self-directed investors who have received no guidance and no education on market timing and long-term investing. In an op-ed appearing in the *Wall Street Journal*, Robert Litan, previously a non-resident senior fellow at the Brookings Institution, and Hal Singer, a senior fellow at the Progressive Policy Institute, wrote:

As research from Vanguard has shown, brokers and advisers perform a vital service by keeping clients invested for the long-term, rather than trying to time the market. The decision to stay invested during times of market stress swamps all other factors affecting retirement savings. “Robo advice” is not a substitute. An email or text message in the fall of 2008 would not have sufficed to keep millions of panicked savers from selling, with devastating consequences for their nest eggs.²⁸⁸

“Good Intentions Gone Wrong: The Yet-To-Be Recognized Costs of the Department of Labor’s Fiduciary Rule,” the research paper upon which the op-ed was based, caused a kerfuffle that led to Litan’s resignation from his Brookings Institution position. In letters to the Department of Labor and the Brookings Institution, Senator Elizabeth Warren raised concerns about conflicts of interest that may have biased the study.²⁸⁹ Warren provided details on the financial industry’s editorial input into the study that, along with “the exact amount of and sole nature of the industry’s financial support for” the paper, had not been disclosed in Litan’s testimony before the Health, Education, Labor, and Pensions Committee on the proposed fiduciary rule.²⁹⁰

It is important to note that the Vanguard study cited by Litan and Singer cite is not specific to human advisors.²⁹¹ The phrasing of Litan and Singer’s op-ed is misleading, as it presents the Vanguard study as if it applies only to human advisors. The study (henceforth, the “Advisor Alpha study”) used the results of another study (henceforth, the “Benchmark study”) to quantify the benefits of advisors’ behavioral coaching value – i.e., the value advisors might add by preventing clients from timing the markets.²⁹² The Benchmark study analyzed the investment performance of 58,168 self-directed Vanguard IRA investors over the five years ending December 31, 2012. These investors’ internal rates of return (IRR) were compared to “personal rate-of-return benchmarks” (or equivalently, IRR benchmarks) over the same five-year period. These benchmarks were based on Vanguard “best practice” investing policy and incorporated the balances and cash flows of each individual account. One of these benchmarks was a Vanguard target-date fund mapped to each account based on the investor’s age at the beginning of the study period. The study quantified to what extent each investor in the study fell short of or exceeded his or her IRR benchmark.

²⁸⁸ Robert Litan and Hal Singer. Obama’s Big Idea for Small Savers. ‘Robo’ Financial Advice. *Wall Street Journal*. July 21, 2015. <http://www.wsj.com/articles/obamas-big-idea-for-small-savers-robo-financial-advice-1437521976>

²⁸⁹ Elizabeth Warren. Letter to The Honorable Thomas Perez re: Conflict of Interest Rule, RIN 1210-AB32. September 24, 2015; Elizabeth Warren. Letter to Strobe Talbott. September 24, 2015.

²⁹⁰ Elizabeth Warren. Letter to Strobe Talbott. September 24, 2015.

²⁹¹ For more details, see Litan and Singer’s report “Good Intentions Gone Wrong” on the Department of Labor’s proposed fiduciary rule.

²⁹² Francis M. Kinniry Jr. et al. Putting a value on your value: Quantifying Vanguard Advisor’s Alpha. Vanguard Research. March 2014; Stephen M. Weber. Most Vanguard IRA Investors shot par by staying the course: 2008-2012. Vanguard Research. May 2013.

The performance of the IRR benchmarks, which the Advisor Alpha study assumed were a reasonable proxy for the returns investors would generate with an advisor, may not be indicative of the market timing advice human advisors provide. As the authors of the Advisor Alpha study wrote, “For the purpose of our example, we are assuming that Vanguard target-date funds provide some of the structure and guidance that an advisor might have provided.” The key word is proxy. No advisor, human or robot, was involved in either study; hence, the results of the study are not specific to human advisors. As will be argued later in this section, human advisors are subject to the same emotional biases as their clients and may not reliably be counted upon to provide sound market timing advice. Moreover, as shown in the section on conflicts of interest, some advisors may be incentivized to recommend frequent, return-chasing trades to their clients. The Advisor Alpha study concluded that the behavioral coaching value-add of advisors could be estimated by the difference in returns of the IRR benchmarks and the portfolios of self-directed investors corresponding to such benchmarks. By writing that robo advice is “not a substitute” for human advice, Litan and Singer effectively argue that robo-advisors do not provide any of the “structure and guidance” that a human advisor can provide. This claim cannot be substantiated.

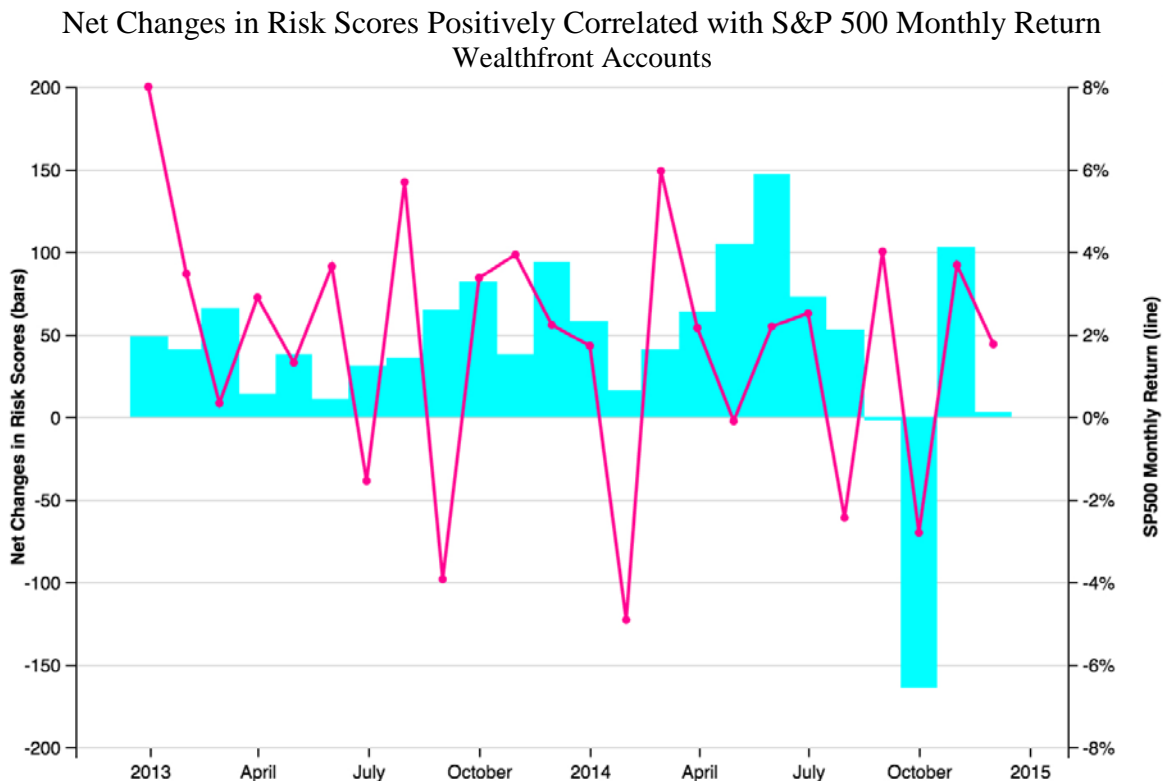
Litan and Singer’s critique of robo-advisors is predicated on the assumption that robo-advisors, unlike human advisors, cannot improve investor behavior. Both qualitative and quantitative data cast doubt on this assumption. Through blog posts, videos, and other media, robo-advisors have educated individual investors about the benefits of diversification and the dangers of market timing. Robo-advisors’ online platforms discourage clients from changing their asset allocation, often limiting the number of asset allocation changes clients can make. Through investor education and website modification, some robo-advisors shift investors’ focus away from history and performance, turning their attention to how their actions today can make them better off in the future.²⁹³ Robo-advisors’ future websites may include features allowing clients to stress test portfolios, psychologically preparing individuals for extreme market scenarios.

Empirical evidence suggests that advisors exert a large influence on investor behavior. As shown in “Costly Financial Advice: Conflicts of Interest or Misguided Beliefs?” – a paper that was cited in the previous section – advisors’ investing behavior and beliefs are predictive of clients’ investing behavior.²⁹⁴ The authors of the paper showed that this effect was not due to endogeneity matching, the tendency of clients to select advisors with similar views. The authors did not investigate the mechanism by which advisors influence investor behavior, but it is most likely a combination of the advisor imparting certain beliefs to the client and the advisor instigating (or not instigating) certain investing behaviors. The implication of the study is that clients of robo-advisors will invest in a manner similar to how they are advised. That is, since robo-advisors espouse a strategy of long-term investing, clients of robo-advisors will likely maintain a long-term orientation and eschew market timing. It seems unreasonable to suggest that the study results only apply to human, but not robo-, advisors.

²⁹³ This point relies on a conversation the author had with Dan Egan, Director of Investments and Behavioral Finance at Betterment.

²⁹⁴ Juhani T. Linnainmaa, Brian T. Melzer, and Alessandro Previtero. Costly Financial Advice: Conflicts of Interest or Misguided Beliefs? December 2015.

Granted, not all robo-advisor clients will invest exactly as they are advised. This is true of clients of both human and robo-advisors. As Wealthfront Executive Chairman Andy Rachleff showed in a blog post, Wealthfront clients exhibit a mild tendency to time the markets through changes to their asset allocation.²⁹⁵ Specifically, he showed that net changes in risk scores were positively correlated with market performance as measured by the monthly return on the S&P 500; the relationship was statistically significant. The data spanned a two-year period beginning in early 2013.



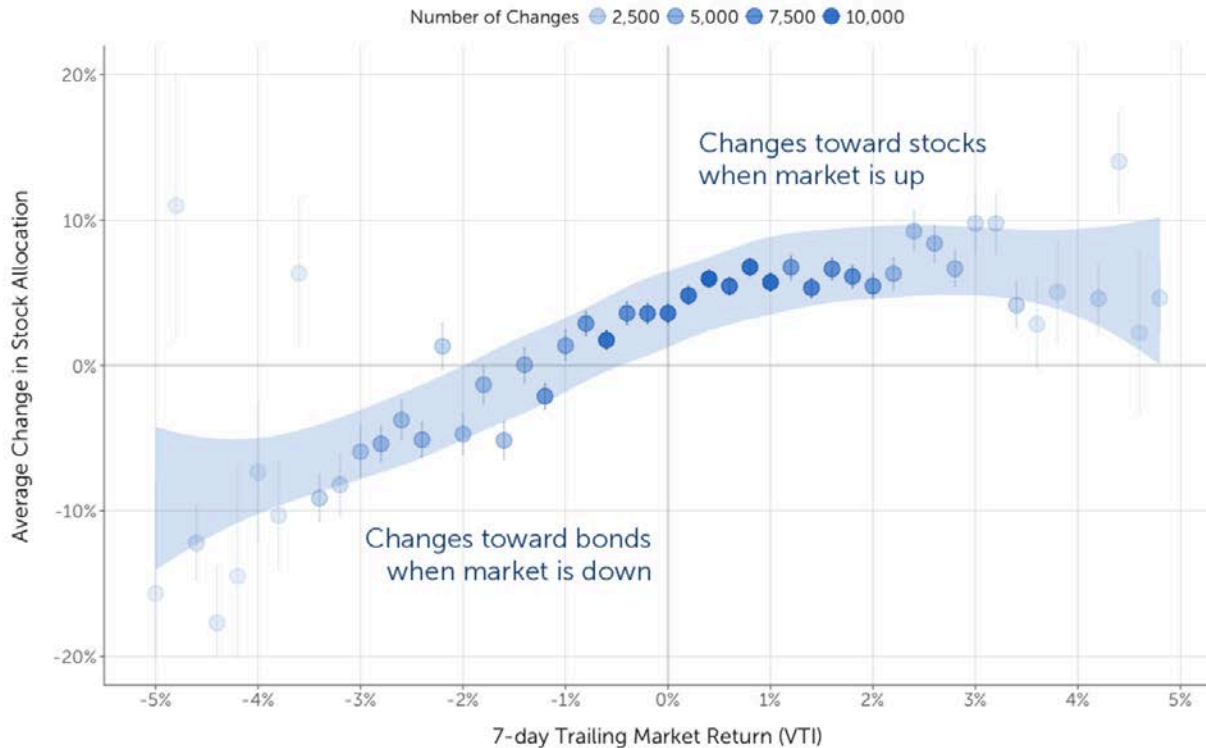
Source: Andy Rachleff. The Right and Wrong Reasons to Change Your Risk Tolerance. Wealthfront Blog. December 5, 2014.

Betterment has released similar data showing that clients tend to increase their portfolio risk following periods of strong market performance, and decrease their portfolio risk following periods of weak market performance.²⁹⁶ Nevertheless, more than 99 percent of clients do not change their asset allocation during a given 7-day period on the Betterment platform. The graph below, which shows the correlation between asset allocation changes and the 7-day trailing market return, pertains to the less than 1 percent of clients who do change their allocation.

²⁹⁵ Andy Rachleff. The Right and Wrong Reasons to Change Your Risk Tolerance. Wealthfront Blog. December 5, 2014.

²⁹⁶ Sam Swift. Betterment Customers Stay the Course, Stay Clear of Behavior Gap. Betterment Blog.

Average Changes in Stock Allocation Positively Correlated with Market Performance Betterment Accounts



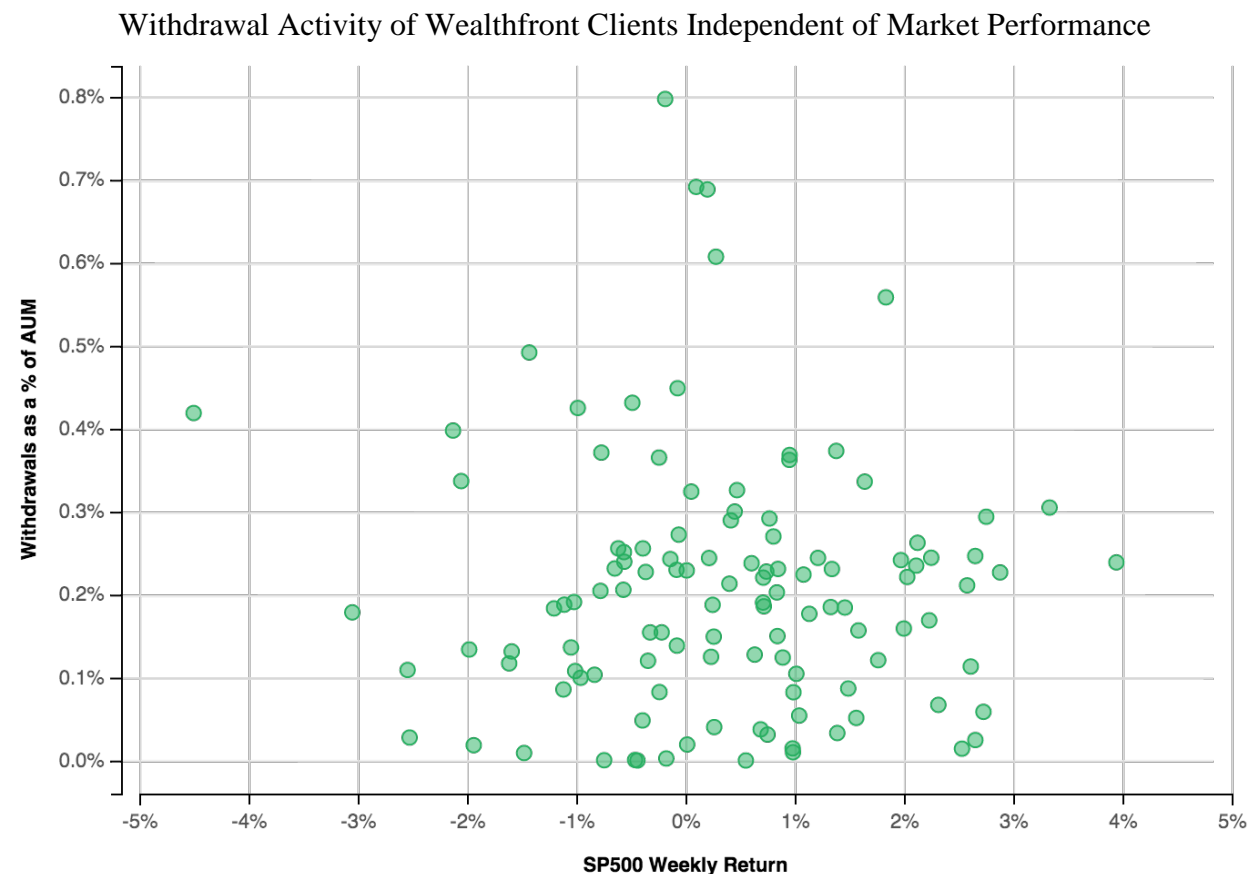
Source: Sam Swift. Betterment Customers Stay the Course, Stay Clear of Behavior Gap. Betterment Blog.

Comparing the time-weighted returns of clients' actual asset allocations to the time-weighted returns of their average time-weighted allocation, Betterment showed that making such asset allocation changes tended to reduce returns.²⁹⁷ Betterment found that across all accounts, including the accounts that made no allocation changes, the mean gap between clients' actual returns and the returns of their average time-weighted allocation was -22 basis points. Studying only the accounts that made at least one allocation change, the average gap was -41 basis points. Betterment showed that the average behavioral gap increased with the number of asset allocation changes. Overall, however, Betterment clients have largely stayed the course, steering clear of market timing tendencies. In 78 percent of accounts, clients have made less than one allocation change per year on average. Some clients may change their allocation due to changing financial circumstances rather than market timing. Granted, the promising results from the Wealthfront and Betterment studies could be due to sample bias; more sophisticated investors may have been more likely to become early adopters of robo-advice, suggesting that the average behavioral gap will increase in the future. Yet these forces might be counter-balanced by improvements to robo-advisor platforms that discourage market timing.

Data on client withdrawals suggest that robo-advisors suppress investors' inclination to time the markets. As Rachleff and his colleague Roberto Medri showed in a recent blog post, the

²⁹⁷ Ibid.

withdrawal activity of Wealthfront clients was independent of market performance.²⁹⁸ The R-squared from regressing withdrawals on market performance was only 0.002, and the p-value of the weekly S&P 500 return, the measure of market performance, was 0.662, statistically insignificant at any reasonable confidence interval. The data support the conclusion that Wealthfront clients do not attempt to time the markets through withdrawals. As the study notes, during the 128-week time period from which data were collected, there were a number of significant market declines: -4.5 percent (week of June 17, 2013), -3 percent (week of January 20, 2014), and -2.5 percent (week of August 12, 2013). It is unclear whether and to what extent robo-advisor withdrawal activity and market performance might become more correlated during times of more acute market stress.



Source: Andy Rachleff and Roberto Medri. Passive Investors Need Less Hand Holding. Wealthfront Blog. September 18, 2014.

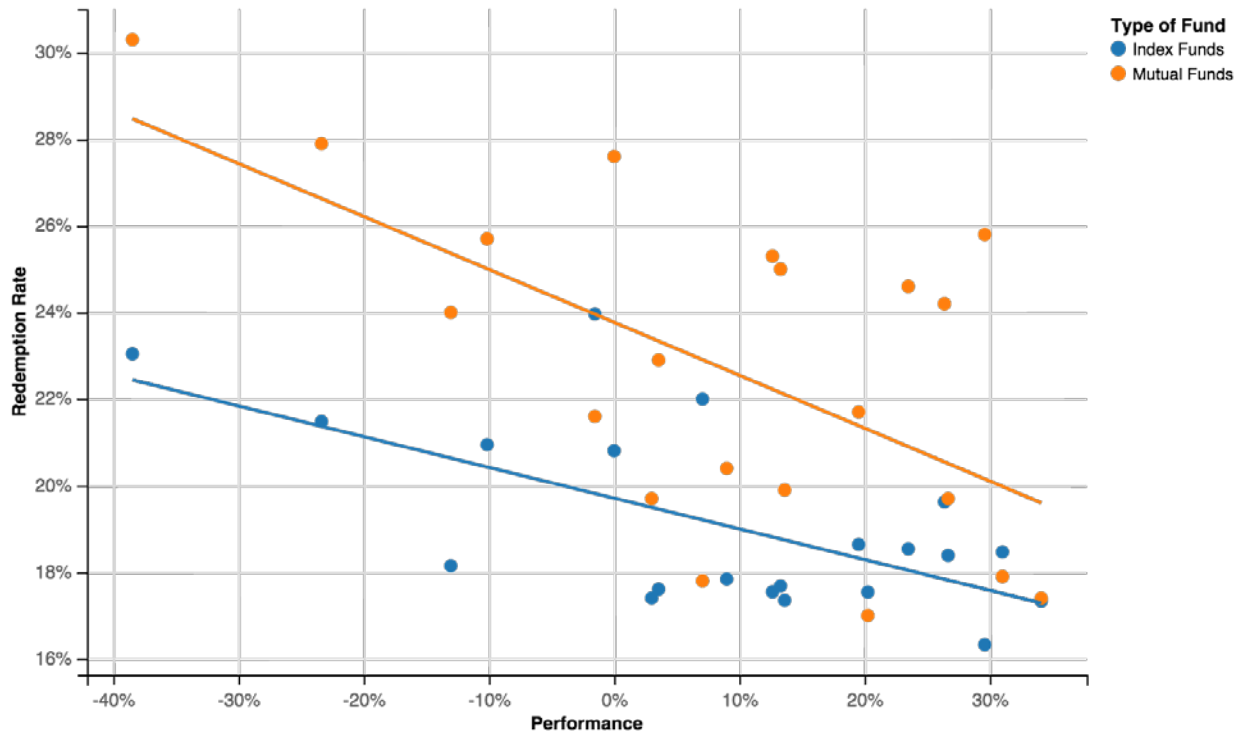
A comparison of the withdrawal activity of investors in index funds and investors in actively managed funds further suggests that market mis-timing may be a smaller problem for clients of robo-advisors than critics of robo-advisors claim. Rachleff and Medri present evidence showing that investors in index funds are less likely to withdraw assets in down markets than investors in actively managed funds.²⁹⁹ Regressing the aggregate redemption rates of all mutual

²⁹⁸ Andy Rachleff and Roberto Medri. Passive Investors Need Less Hand Holding. Wealthfront Blog. September 18, 2014.

²⁹⁹ Ibid.

funds and index funds from 1993 to 2013 on the performance of the U.S. stock market as measured by the annual return on the S&P 500, Rachleff and Medri show that index fund withdrawals are less sensitive to the U.S. stock market return than aggregate mutual fund (actively managed and index fund) withdrawals and that the difference is statistically significant at the 99 percent confidence interval. They show that a one percentage point decline in S&P performance is associated with a 0.12 percentage point increase in the overall mutual fund redemption rate and a 0.07 percentage point increase in the index fund redemption rate.³⁰⁰ Rachleff and Medri argue that the difference in withdrawals is likely more pronounced than their results suggest since the data for all mutual funds include the data for index funds, which represent approximately 20 percent of total mutual fund assets. The implication of Rachleff and Medri's study is that clients of robo-advisors – investors who have chosen a passive strategy by selecting a robo-advisor as their asset manager – are less likely to withdraw assets during a down market than investors pursuing active strategies.

Withdrawals from Index Funds Less Sensitive to Market Performance than Withdrawals from All Mutual Funds



Source: Andy Rachleff and Roberto Medri. Passive Investors Need Less Hand Holding. Wealthfront Blog. September 18, 2014.

³⁰⁰ In their blog post, Rachleff and Medri write that a “1% decline in S&P performance causes a 0.12% increase in withdrawals. For index funds, a 1% decline in S&P performance causes a 0.07% increase in withdrawals.” However, the graph they include in their blog post, which is shown in this section, clearly shows that they were referring to percentage point, not percent, changes.

Granted, a limitation of this study is that it does not control for the effect of advisors in influencing clients' decision to buy into or sell out of mutual funds.³⁰¹ If a greater proportion of index fund assets were advised than actively managed funds, a case might be made that it is the guidance of advisors – not passive investors' reduced inclination to withdraw assets, be it due to their greater investment acumen or fundamental belief that market timing is a losing strategy or other factors – that is the reason for the lower withdrawal rate for index funds. Conversely, if a lower proportion of index fund assets were advised than actively managed funds, this fact would bolster Rachleff and Medri's argument that passive investors need less hand holding than investors in actively managed funds; index investors refrain from market timing even without the aid of advisors.

Lastly, it is important to note that human advisors are subject to the same emotional biases as their clients. Critics of robo-advisors often claim that robo-advisors can do little to prevent poor market timing behavior and that clients would be better served by working with human advisors. Yet such recommendations are predicated on the assumption that human advisors will provide sound market timing advice during times of market stress. It is too easy to imagine that during times of extreme market volatility, human advisors – fearful of what is to come – might recommend asset allocation changes in the hopes of protecting portfolios from losses. These actions may not be limited to brokers; fiduciaries may believe that paring down portfolio risk is in the best interest of their clients. Robo-advisors, by contrast, are not subject to such behavioral biases. When the markets turn south, one can be confident that robo-advisors will maintain their composure.

Fees and Minimums

As shown in the chart below, robo-advisors charge much lower advisory fees than most traditional investment advisors. They also have much lower minimums. As a point of comparison, the chart below includes the pricing information for “hybrid” robo-advisor Vanguard Personal Advisor Services. Although hybrid robo-advisors combine technology with the guidance of a human advisor, they are much closer in nature to traditional advisors.

³⁰¹ The Investment Company Institute provides some information on the source of mutual fund purchases, but it does not separate the data between index mutual funds and actively managed mutual funds.

Fee Comparison

Advisor	Advisor Type	Annual Advisory Fee
Schwab Intelligent Portfolios	Robo-Advisor	N/A*
Wealthfront	Robo-Advisor	0.25% First \$10,000 managed for free \$500 minimum
Betterment	Robo-Advisor	For account balance of: - \$0-\$10,000: 0.35% with minimum of \$100/month auto- deposit OR \$3/month without auto- deposit - \$10,000-\$100,000: 0.25% - \$100,000+: 0.15%
Fee-Based Advisors in Aggregate**	Traditional Advisor	For account balance of: - Less than \$250,000: 1.43% - \$250,000-\$500,000: 1.37% - \$500,000- \$1M: 1.28% - \$1M-\$2M: 1.16% - \$2M+: 0.79%
Vanguard Personal Advisor Services	Hybrid Robo-Advisor	0.30% \$50,000 minimum

* As Schwab Intelligent Portfolios has written on its website, "Schwab Intelligent Portfolios charges no advisory fees. Schwab affiliates do earn revenue from the underlying assets in Schwab Intelligent Portfolios accounts. This revenue comes from managing Schwab ETFs™ and providing services relating to certain third-party ETFs that can be selected for the portfolio, and from the cash feature on the accounts. Revenue may also be received from the market centers where ETF trade orders are routed for execution."

** Numbers for fee-based advisors based on PriceMetrix data

Sources: Schwab Intelligent Portfolios Website; Wealthfront Website; Betterment Website; The State of Retail Wealth Management 5th Annual Report. PriceMetrix. Fees based on 2014 data; Vanguard Personal Advisor Services Website.

The Power of Automation

Monitoring and Rebalancing

Robo-advisors possess a much higher degree of technological sophistication than traditional advisors. They have built systems automating the monitoring and rebalancing process of client portfolios. As such, robo-advisors can easily monitor portfolios for rebalancing opportunities on a daily basis. By contrast, traditional advisors may monitor portfolios less frequently, as manually checking for rebalancing opportunities is a time-consuming task.

Frequent monitoring for rebalancing opportunities allows investors to control portfolio risk.³⁰² During the asset allocation process, investors select a target portfolio on the basis of investor attributes such as time horizon and risk tolerance. However, since financial assets are imperfectly correlated and experience price movements of different magnitudes, the portfolio will inevitably deviate from the target allocation.³⁰³ Disciplined investors limit portfolio drift and maintain the desired risk level, regularly rebalancing the portfolio to long-term policy targets.

Regular rebalancing may improve risk-adjusted returns. As rebalancing constitutes selling strong relative performers and purchasing poor relative performers, investors who rebalance regularly buy low and sell high, arbitraging markets' excess volatility.³⁰⁴ Yet the primary benefit of rebalancing is maintaining a portfolio risk level that is close to target.

Some studies conclude that daily monitoring for rebalancing opportunities is unnecessary, as weekly, monthly, or less frequent monitoring produces similar risk-adjusted returns without meaningfully increasing portfolio drift. However, the results of such studies may be time-dependent. For example, a study by Vanguard, whose results are shown below, examines threshold-only rebalancing strategies for the period 1989-2009. (Schwab Intelligent Portfolios, Wealthfront, and Betterment employ threshold-only rebalancing.³⁰⁵) It is strange that with a rebalancing threshold of 5 percent, weekly, monthly, and annual monitoring lead to an average equity allocation that is closer to target than with daily monitoring.

The more important point, however, is that rebalancing enables investors to manage risk. While more frequent monitoring for rebalancing opportunities may lead to greater portfolio turnover and a larger number of rebalancing events, monitoring for rebalancing opportunities on a daily basis with reasonable rebalancing thresholds helps investors achieve the risk-return profile that is best suited to their needs.

³⁰² David F. Swensen. *Pioneering Portfolio Management*. Free Press. 2009. 105; Ashvin B. Chhabra. *The Aspirational Investor*. HarperCollins. 2015. 132.

³⁰³ Ibid.

³⁰⁴ Ibid.

³⁰⁵ Schwab Intelligent Portfolios Rebalancing and Tax-Loss Harvesting Whitepaper. <https://intelligent.schwab.com/public/intelligent/insights/whitepapers/tax-loss-harvesting-rebalancing.html>; Wealthfront FAQ. <https://pages.wealthfront.com/faqs/how-often-do-you-rebalance-my-portfolio/>; Betterment Support. <http://support.betterment.com/customer/portal/articles/987453-how-and-when-is-my-portfolio-rebalanced->

Comparison of Threshold-Only Rebalancing for Different Monitoring Frequencies and Rebalancing Thresholds

Data from 1989 to 2009, 60-40 Stock-Bond Portfolio

Monitoring Frequency	Rebalancing Threshold	Average Annualized Return	Volatility	Average Equity Allocation	Annual Turnover	Number of Rebalancing Events
Daily	1%	8.9%	12.9%	60.1%	7.9%	270
Monthly	1%	8.8%	9.5%	60.1%	4.4%	79
Quarterly	1%	8.9%	9.5%	60.1%	4.0%	43
Annually	1%	9.0%	9.4%	60.2%	2.8%	16
Daily	5%	8.9%	13.0%	61.7%	3.1%	14
Monthly	5%	8.9%	9.6%	61.5%	2.8%	12
Quarterly	5%	9.0%	9.5%	61.4%	2.9%	11
Annually	5%	9.1%	9.4%	61.2%	2.8%	8
Never	N/A	8.6%	10.9%	69.2%	0	0

Source: Colleen M. Jaconetti et al. Best Practices for Portfolio Rebalancing. Vanguard Research, July 2010.

Tax-Loss Harvesting (For Taxable Accounts)

In contrast to traditional advisors, who typically only offer tax-loss harvesting services to clients with large accounts (e.g. Wealthfront writes that tax-loss harvesting is traditionally only available to accounts in excess of \$5 million), robo-advisors offer such services to all clients.³⁰⁶ As explained in the chapter on how robo-advisors work, automated investment platforms typically perform tax-loss harvesting by selling investments that have declined in value and using the proceeds to buy highly correlated substitutable investments. Due to their adoption of software and automation, robo-advisors can perform tax-loss harvesting on a daily basis and in many cases can achieve high levels of tax efficiency with advanced computer algorithms. By contrast, traditional advisors may perform tax-loss harvesting on an annual basis without the aid of software.³⁰⁷

Some robo-advisors have published white papers quantifying the value of their tax-loss harvesting services. Using Monte Carlo simulations, back-tests, and empirical tests based on actual client account data, Wealthfront found that the annual tax alpha – the additional performance benefit gained from tax-loss harvesting – is roughly one percent for an investor who withdraws half of the portfolio at the end of the assumed 20-year investment horizon. Using a back-test for the period 2000-2013, Betterment found comparable results under slightly different assumptions.³⁰⁸ The remainder of this section uses the results of Wealthfront’s Monte Carlo simulations as a basis for evaluating the value-add of tax-loss harvesting, as the results of back-tests and empirical tests are time-dependent and may not be the best indicator of future performance. While differences may exist between robo-advisors’ implementations of tax-loss harvesting, the Wealthfront study nonetheless provides a baseline estimate of the tax alpha one can expect to achieve through a robo-advisor.

In its Monte Carlo simulations, Wealthfront compared the returns of two portfolios: the Wealthfront portfolio with risk level 7 and daily tax-loss harvesting, and the Wealthfront portfolio with risk level 7 without tax-loss harvesting. It was assumed that any tax savings generated by tax-loss harvesting were reinvested into the portfolio at the beginning of the next tax year. As shown below, quarterly add-on deposits of \$10,000 were assumed to follow the

³⁰⁶ Wealthfront Tax-Loss Harvesting White Paper.

³⁰⁷ Ibid.

³⁰⁸ Betterment Tax-Loss Harvesting White Paper.

initial deposit of \$100,000. At the end of the investment period, three liquidation strategies – no liquidation, 50 percent liquidation, and full liquidation – were applied to the simulated portfolio. The taxes corresponding to each liquidation strategy were subtracted at this time, producing the after-tax values for all portfolios. The assumptions underpinning Wealthfront’s simulations, back-tests, and empirical tests have been reproduced below. The table displays the results of the Monte Carlo study.

Wealthfront Tax-Loss Harvesting Test Assumptions

- Client age: 37 (median age of Wealthfront tax-loss harvesting clients)
- Marital status: Married (the majority of Wealthfront clients are married)
- Annual income: \$260,000 (the average joint income reported by Wealthfront tax-loss harvesting clients)
- State of residence: California (the most popular state of residence of Wealthfront tax-loss harvesting clients)
- Combined federal and state short-term capital gain tax rate: 42.7%
- Combined federal and state long-term capital gain tax rate: 24.7%
- Portfolio risk level: 7 (the average risk score on a scale of 0 to 10 for Wealthfront tax-loss harvesting clients)
- Investment cash flows: An initial deposit of \$100,000 followed by add-on deposits of \$10,000 each quarter (the average Wealthfront tax-loss harvesting client actually adds an average of nearly 20% of the original deposit each quarter)

Annual Tax Alpha from Tax-Loss Harvesting Results Based on Monte Carlo Simulations

<u>Investment Length</u>	<u>No Liquidation</u>	<u>50% Liquidation</u>	<u>Full Liquidation</u>
10 Years	1.55%	1.31%	1.08%
20 Years	1.22%	1.06%	1.00%
30 Years	1.07%	1.00%	0.93%

Source: Wealthfront Tax-Loss Harvesting White Paper.

The results show that under more aggressive liquidation strategies, the benefit of tax-loss harvesting declines. This is due to the fact that tax-loss harvesting lowers the cost basis of the portfolio, and more aggressive withdrawals lead to the realization of more capital gains. Aggressive liquidations of short-term holdings may also push the investor into a higher tax bracket. The benefits of tax-loss harvesting similarly decline as the investment period increases. Realizing tax losses becomes more difficult over time because of the positive expected return of the portfolio.

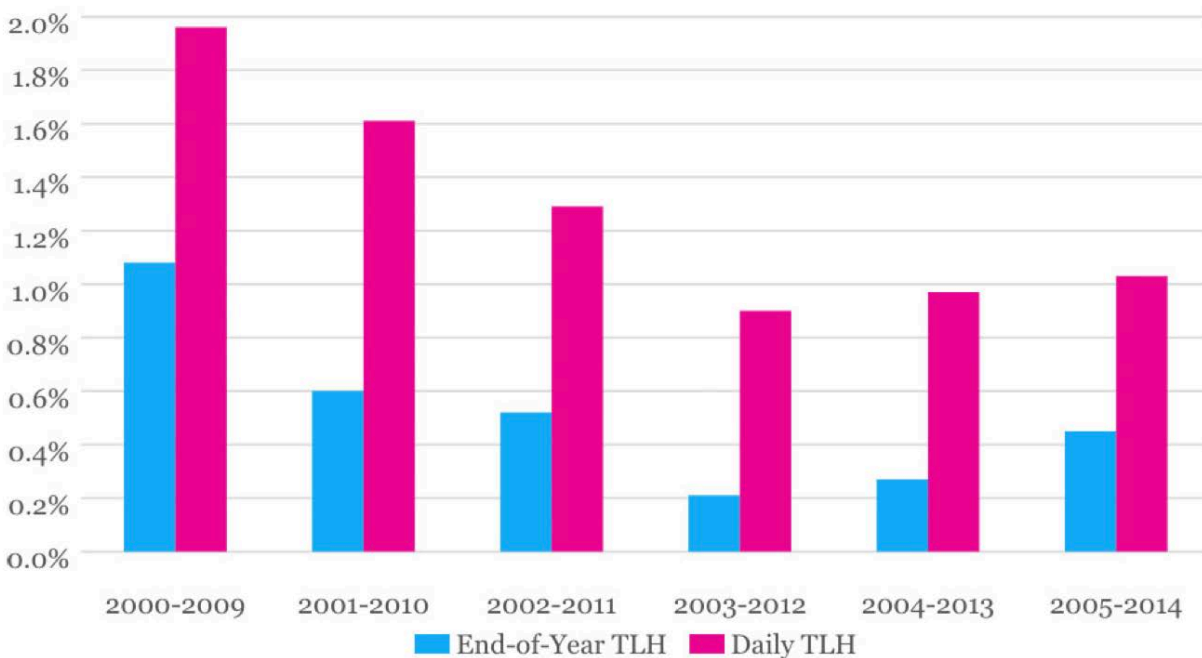
Wealthfront’s assumptions regarding the relevant short-term and long-term capital gains tax rates, portfolio risk, and cash flows also affect the simulation results. Tax-loss harvesting is both a tax deferral strategy and a tax arbitrage strategy. All else equal, the higher the tax rate, the greater the tax savings that are available for investment. The larger the spread between the short-term and long-term capital gains rates, the larger the economic benefit from tax-loss harvesting for long-term investors. Portfolio risk affects the potential benefit of tax-loss harvesting, as riskier, more volatile portfolios are more likely to fall in value below cost basis, creating tax-loss harvesting opportunities. Regular deposits similarly create more opportunities to harvest tax

losses, as securities are bought at multiple price points, leading to a more diverse set of cost bases for each asset class. Hence, Wealthfront's Monte Carlo results may overstate or understate the benefits of tax-loss harvesting for any individual investor.

In back-tests covering the period 2000 to 2014, Wealthfront showed that daily tax-loss harvesting generated more than double the annual tax alpha compared to end-of-year tax-loss harvesting. As mentioned previously, traditional advisors may harvest tax losses on an annual basis. In its white paper, Wealthfront showed that its tax-loss harvesting algorithm was able to achieve approximately 80 percent of the maximum tax alpha from 2000 to 2014. The maximum tax alpha was calculated assuming that all future prices were known, i.e. tax-loss harvesting trades were timed perfectly. The frequency with which robo-advisors can harvest tax losses and the efficiency of their algorithms suggest that robo-advisors generate at least double the tax alpha as traditional advisors.

Annual Average Tax Alpha for Daily Tax-Loss Harvesting and End-of-Year Tax-Loss Harvesting

Results Based on Back-Tests for Ten-Year Periods Between 2000 and 2014



Source: Wealthfront Tax-Loss Harvesting White Paper.

It should be noted that tax-loss harvesting generates the most benefits when there are capital gains to offset.³⁰⁹ When there are no such gains, or if losses remain after all gains have been offset, up to \$3,000 of losses can be used to offset ordinary income for the year. If any losses still remain, they can be carried forward indefinitely for future use; however, carrying forward losses would not generate tax deferral savings and would not create opportunities for

³⁰⁹ Betterment Tax-Loss Harvesting White Paper; Michael Kitces. Evaluating the Tax Deferral and Tax Bracket Arbitrage Benefits of Tax Loss Harvesting. December 3, 2014.

compounding growth or tax arbitrage until such losses were used to offset future capital gains or ordinary income.

Direct indexing increases the benefits of tax-loss harvesting, as even when an overall index trades up, tax losses can be harvested on the individual securities that fell in value. Very few advisors and asset managers use direct indexing, and Wealthfront is the only robo-advisor thus far to offer a direct indexing service (the robo-advisor offers direct indexing for accounts with at least \$100,000 in assets). Wealthfront's back-tests for the period 2000 to 2014 showed that its most basic version of direct indexing, which uses up to 100 individual stocks and several ETFs to represent the domestic equity asset class, generated an average 10-year differential IRR of 1.77 percent relative to the same portfolio without tax-loss harvesting.³¹⁰ In comparison, the same portfolio with tax-loss harvesting at the asset class level produced an average 10-year differential IRR of 1.55 percent. Portfolios that used up to 500 and 1000 stocks for direct indexing generated average 10-year differential IRRs of 1.88 percent and 2.03 percent, respectively, suggesting that the use of more individual securities in direct indexing increases the benefits of tax-loss harvesting. Hence, compared to individuals with small accounts, investors with large accounts may reap even greater benefits from robo-advisors' embrace of automation.

Conclusion

The picture that emerges from a review of robo-advisors, their human counterparts, and the relevant academic literature is clear: robo-advisors are a compelling alternative to many sources of traditional advice, and in many cases may dominate such sources of advice due to their lower costs, well-grounded investment methodology, and alignment with clients' interests.

Granted, robo-advisors are not perfect: their advice is not fully customizable and may not take into account important investor attributes such as assets and liabilities, anticipated spending, occupation and stability of income. Yet the advice they provide is systematic and unbiased, well-grounded in the finance and economics literature, and transparent. Technology has facilitated robo-advisors' implementation and rebalancing of client portfolios and has enabled robo-advisors to tap into sources of value-add such as tax-loss harvesting.

This chapter has focused much of its attention on the slimy underbelly of the traditional investment advisory profession. Conflicts of interest abound, biasing advisor recommendations and leading to underperformance of advisor-intermediated funds. Conflicted advisors churn investor portfolios and tout actively managed funds even when clients are already invested in efficient index funds. Yet not all bad behavior on the part of advisors is driven by conflicts of interest. Advisors' misguided beliefs lead to the provision of poor investment advice, potentially implicating a much wider array of advisors than those simply driven by misaligned incentives.

The investment advice robo-advisors provide will only become more sophisticated and more customizable over time. Improvements to client questionnaires and other on-boarding and monitoring processes will improve the ability of robo-advisors to assess individuals' risk

³¹⁰ Wealthfront Tax-Optimized Direct Indexing White Paper.

tolerance and behavioral tendencies, leading to superior portfolio optimization and management processes.

APPENDIX

Figure 1

The Efficient Frontier and the Capital Market Line

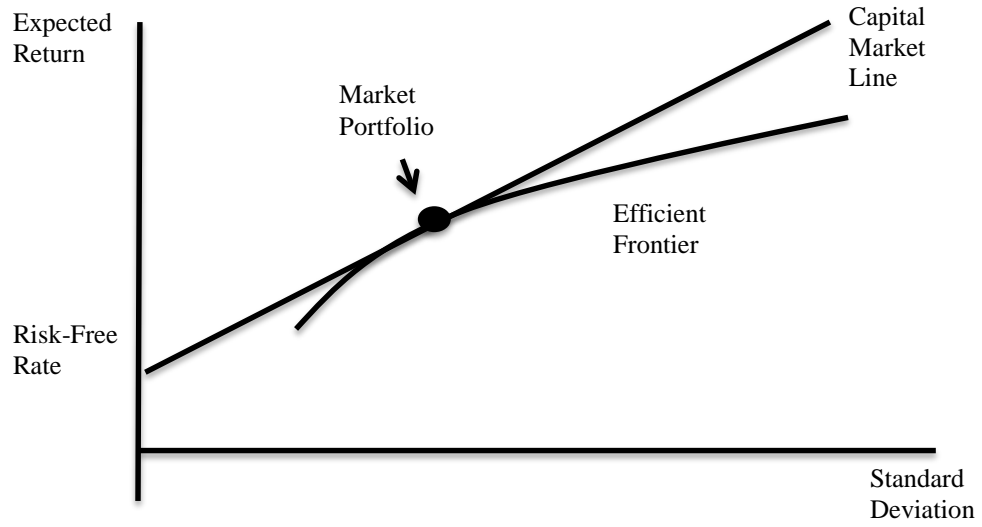
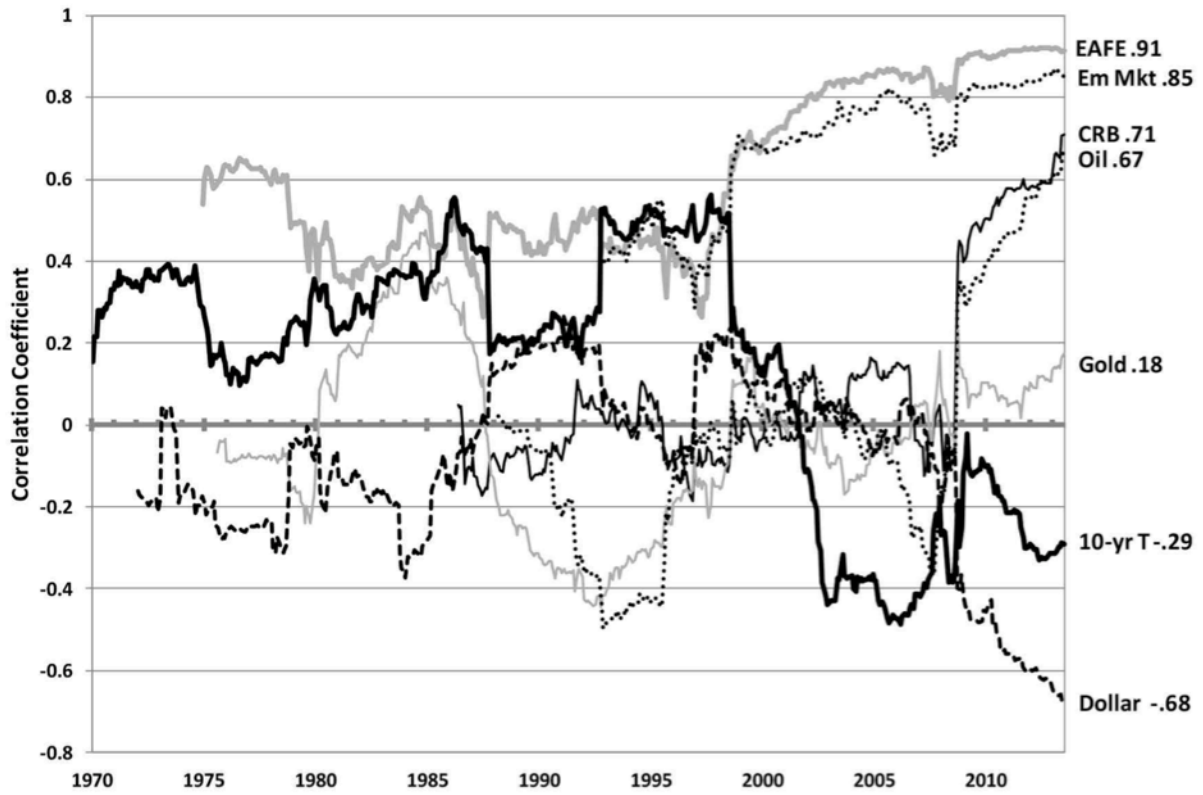


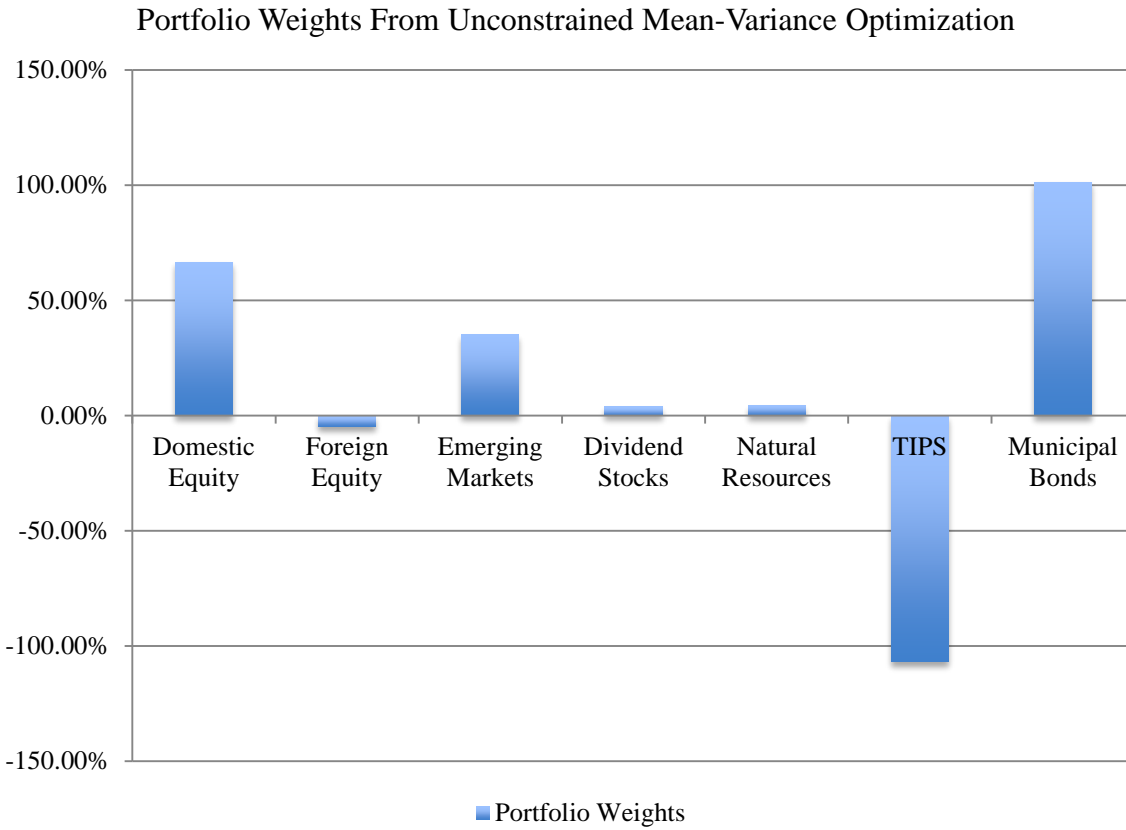
Figure 2

Monthly Correlations of S&P 500 and Various Asset Classes from 1970-2012



Source: Jeremy Siegel. Stocks for the Long Run.

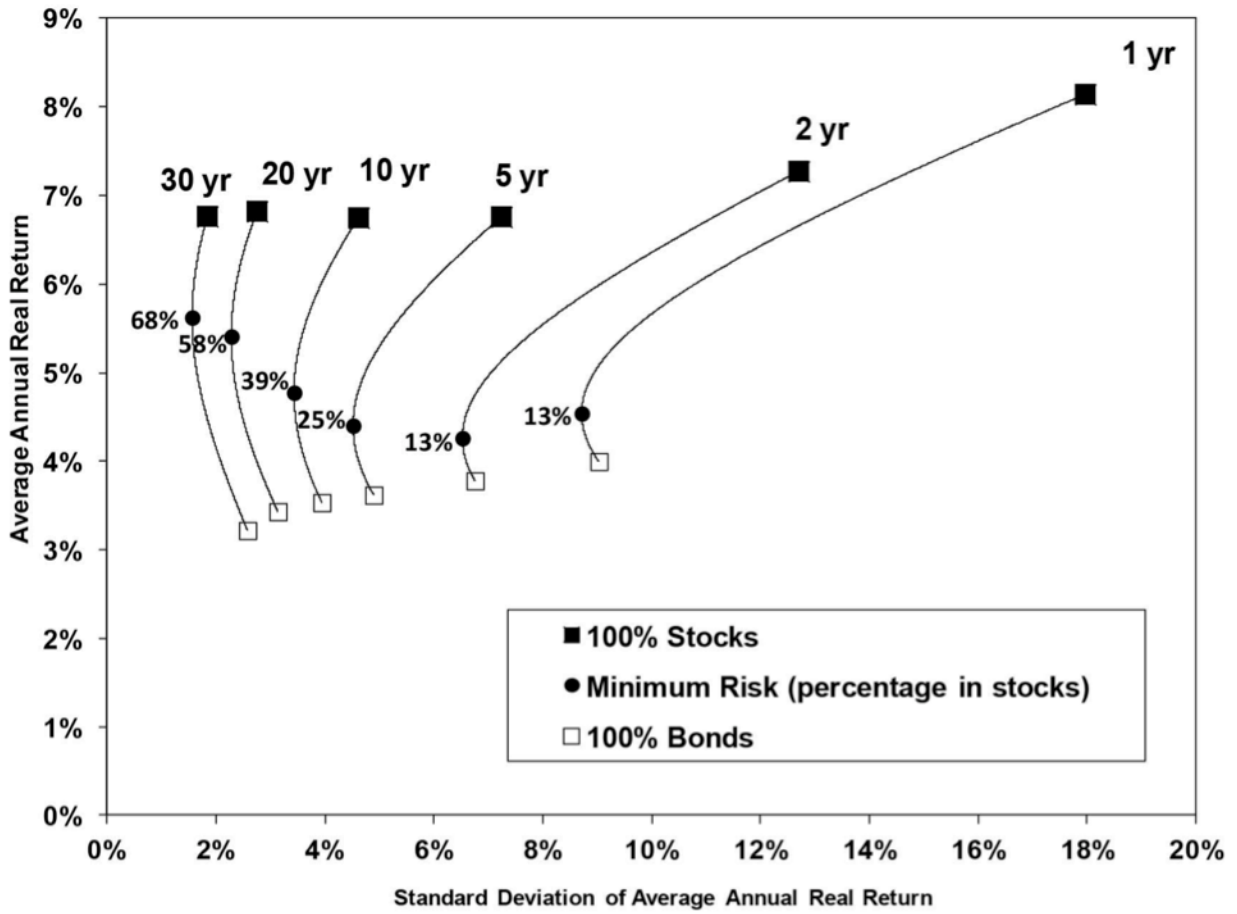
Figure 3



Source: Wealthfront Investment Methodology Whitepaper. This graph was created using Wealthfront’s capital market assumptions for taxable accounts. The portfolio weights correspond to a portfolio with an expected return of 5 percent and a standard deviation of 18.37 percent. Unconstrained mean-variance optimization was used to calculate the efficient frontier. Computations were performed by the author of this paper.

Figure 4

Risk-Return Tradeoffs (Efficient Frontiers) for Stocks and Bonds Over Various Holding Periods 1802-2012



Source: Jeremy Siegel. Stocks for the Long Run.

Table 1

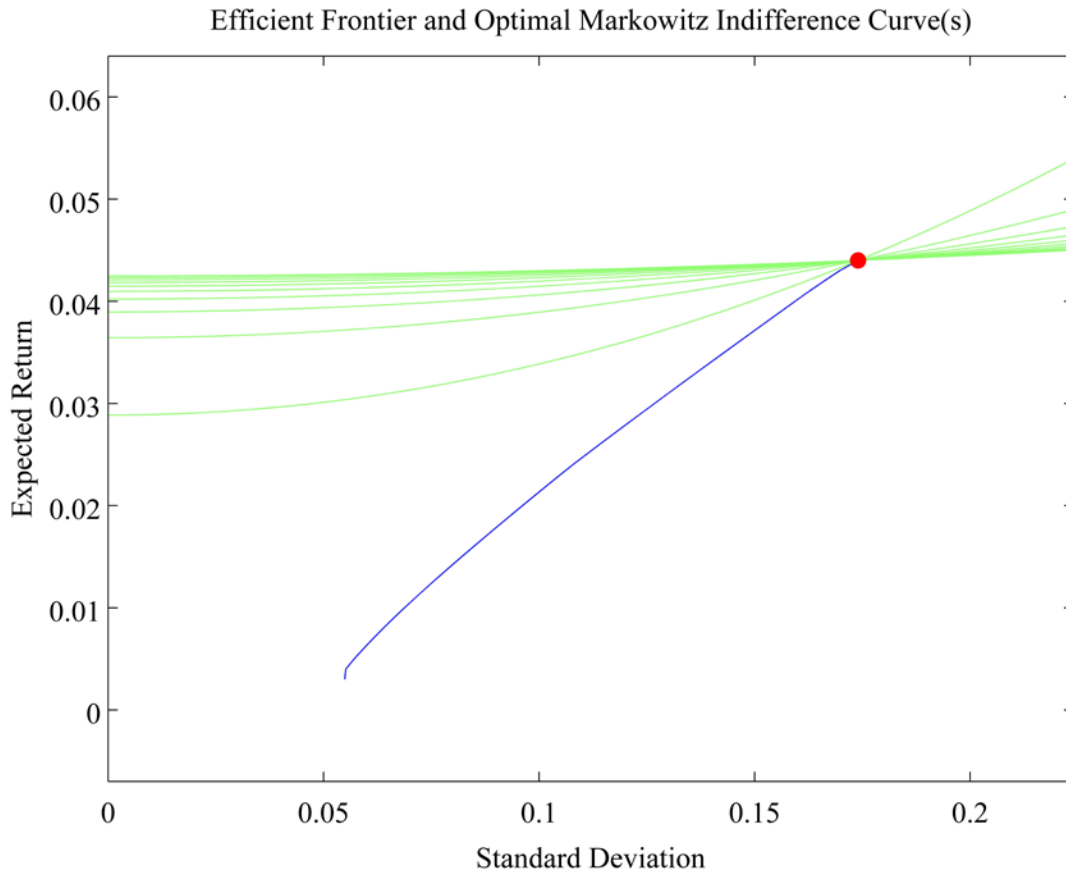
Optimal Portfolio Weights (In Percentage Terms) With and Without Mean Reversion in Stock Prices

This table reports the optimal portfolio weights for the global minimum variance portfolio (GMVP) and the tangency portfolio (TP). The investment categories are stocks (Datastream U.S. Aggregate Stock Market Index) and bonds (Citigroup U.S. Overall Bond Investment Grade Total Return Index) for different values of the variance ratio. The variance ratio is defined as the return variance of the permanent price component divided by the return variance of the transitory component. The risk-free rate is based on the nominal interest rate term-structure as compiled by the Dutch Central Bank. The last two columns in each panel display the expected portfolio return (μ_p) and the portfolio volatility (σ_p).

	variance ratio = 1:1				variance ratio = 1:2				variance ratio = 1:3			
	w_s	w_b	μ_p	σ_p	w_s	w_b	μ_p	σ_p	w_s	w_b	μ_p	σ_p
	<i>with mean reversion</i>				<i>with mean reversion</i>				<i>with mean reversion</i>			
GMV												
1 year	4.36	95.64	8.50	4.80	4.44	95.56	8.51	4.80	4.48	95.52	8.51	4.80
5 years	5.37	94.63	42.64	10.70	5.73	94.27	42.69	10.69	5.93	94.07	42.71	10.68
10 years	5.86	94.14	85.41	15.11	6.45	93.55	15.08	14.90	6.79	93.21	85.63	15.06
20 years	6.26	93.74	171.00	21.34	7.07	92.93	171.39	21.27	7.54	92.46	171.62	21.24
TP												
1 year	7.20	92.80	8.57	4.82	7.30	92.70	8.58	4.82	7.36	92.64	8.58	4.82
5 years	8.40	91.60	43.01	10.75	8.88	91.12	43.07	10.74	9.14	90.86	43.10	10.73
10 years	9.05	90.95	86.17	15.18	9.83	90.17	86.36	15.15	10.27	89.73	86.46	15.13
20 years	9.57	90.43	172.59	21.44	10.64	89.36	173.11	21.38	11.26	88.74	173.40	21.35
	<i>without mean reversion</i>				<i>without mean reversion</i>				<i>without mean reversion</i>			
GMVP												
1 year	4.14	95.86	8.50	4.81	4.14	95.86	8.50	4.81	4.14	95.86	8.50	4.81
5 years	4.43	95.57	42.53	10.73	4.43	95.57	42.53	10.73	4.43	95.57	42.53	10.73
10 years	4.43	95.57	85.06	15.18	4.43	95.57	85.06	15.18	4.43	95.57	85.06	15.18
20 years	4.43	95.57	170.13	21.47	4.43	95.57	170.13	21.47	4.43	95.57	170.13	21.47
TP												
1 year	6.90	93.10	8.57	4.83	6.90	93.10	8.57	4.83	6.90	93.10	8.57	4.83
5 years	7.14	92.86	42.86	10.78	7.14	92.86	42.86	10.78	7.14	92.86	42.86	10.78
10 years	7.14	92.86	85.71	15.24	7.14	92.86	85.71	15.24	7.14	92.86	85.71	15.24
20 years	7.13	92.87	171.42	21.55	7.13	92.87	171.42	21.55	7.13	92.87	171.42	21.55

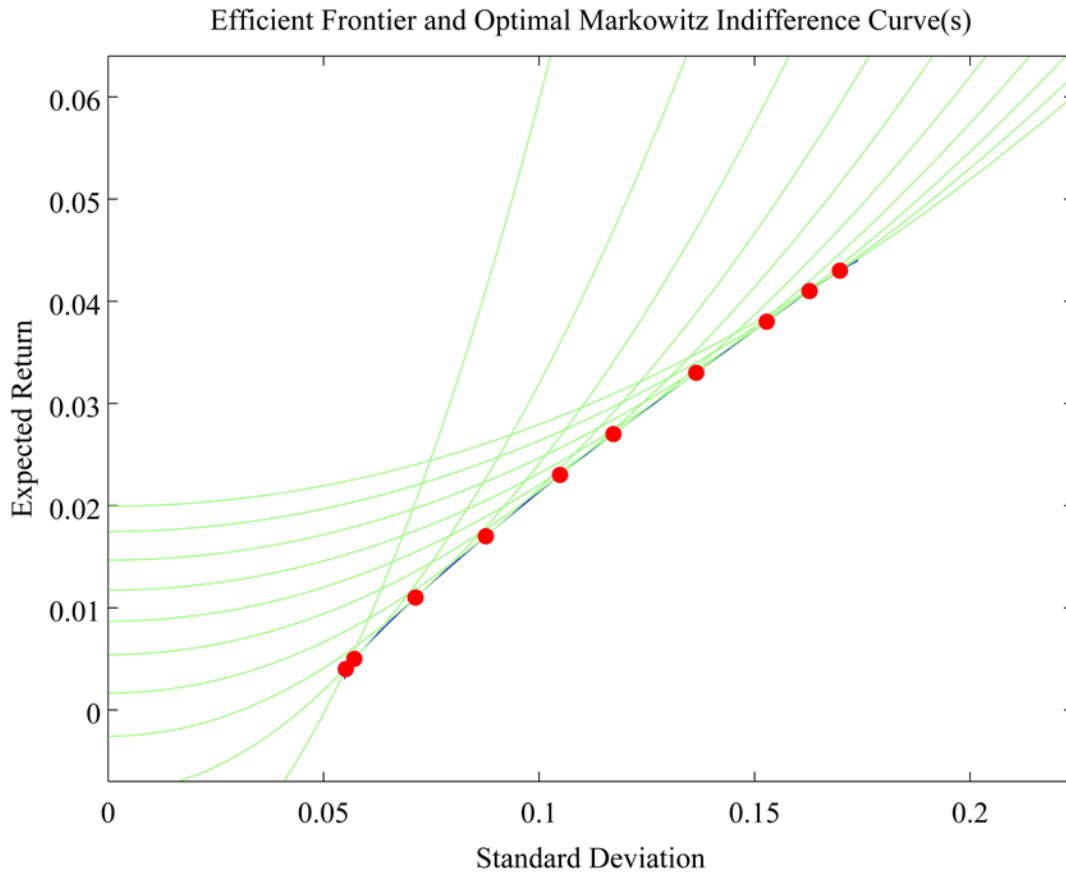
Source: Laura Spierdijk and Jacob Bikker. Mean Reversion in Stock Prices: Implications for Long-Term Investors. Dutch Central Bank. April 5, 2012.

Figure 5



Source: Wealthfront Investment Methodology Whitepaper. This graph was created in Matlab using Wealthfront's capital market assumptions for taxable accounts. The utility function in this graph assumes a scaling factor equal to $\frac{1}{2}$, which is the scaling factor Wealthfront has published in its investment methodology whitepaper. Indifference curves have been drawn for integer value risk tolerance levels 1, 2, ..., 10, which are within the range of values (0,10] Wealthfront considers acceptable. Indifference curves for lower risk tolerances bend upward at a faster rate, as compared to an investor with high risk tolerance, the investor with lower risk tolerance must be compensated by more expected return to accept the same amount of incremental portfolio risk. Computations were performed by the author of this paper.

Figure 6



Source: Wealthfront Investment Methodology Whitepaper. This graph was created in Matlab using Wealthfront's capital market assumptions for taxable accounts. In contrast to the previous figure, the utility function in this graph assumes a scaling factor equal to 8, meaning that portfolio variance reduces utility at a higher rate. Indifference curves have been drawn for integer value risk tolerance levels 1, 2, ..., 10, which are within the range of values (0,10] Wealthfront considers acceptable. Indifference curves for lower risk tolerances bend upward at a faster rate, as compared to an investor with high risk tolerance, the investor with lower risk tolerance must be compensated by more expected return to accept the same amount of incremental portfolio risk. Computations were performed by the author of this paper.

Figure 7Schwab Intelligent Portfolios Questionnaire³¹¹

1. My goal for this account is to
 - a. Prepare for retirement
 - b. Save for major upcoming expenses (education, health-bills, etc.)
 - c. Save for something special (vacation, new car, etc.)
 - d. Build a rainy day fund for emergencies
 - e. Generate income for expenses
 - f. Build long-term wealth
2. I have ___ understanding of stocks, bonds and ETFs.
 - a. No
 - b. Some
 - c. Good
 - d. Extensive
3. When I hear "risk" related to my finances,
 - a. I worry I could be left with nothing
 - b. I understand that it's an inherent part of the investing process
 - c. I see opportunity for great returns
 - d. I think of the thrill of investing
4. Have you ever lost 20% or more of your investments in one year?
 - a. Yes
 - b. No
5. In the year I lost 20% of my investments/If I ever were to lose 20% or more of my investments in one year, I would
 - a. Sell everything
 - b. Sell some
 - c. Do nothing
 - d. Reallocate my investments
 - e. Buy more
6. When it comes to making important financial decisions,
 - a. I try to avoid making decisions
 - b. I reluctantly make decisions
 - c. I confidently make decisions and don't look back
7. I am ___ years old.
8. My initial investment for this goal is ___.
9. One year from now, I'd be comfortable with my initial investment fluctuating between:
 - a. Indicate range around initial investment size (see figure below)
10. I plan to save an additional ___ per month for this goal.
11. I need the money for this goal starting in x years for y years. Specify x and y. OR I need income for x years ("Generate income for expenses" goal)

³¹¹ Schwab Intelligent Portfolios Website. There may be conditional questions that are not captured above. Please read the Schwab Intelligent Portfolios Investor Profile Questionnaire Whitepaper for more details.

Figure 8Wealthfront Questionnaire³¹²

1. What's your primary reason for investing?
 - a. General savings
 - b. Retirement
 - c. Other
2. What are you looking for in a financial advisor? Select all that apply
 - a. I'd like to create a diversified investment portfolio
 - b. I'd like to save money on my taxes
 - c. I'd like someone to completely manage my investments, so that I don't have to
 - d. I'd like to match or beat the performance of the markets
3. What is your current age?
4. What is your annual pre-tax income?
5. Which of the following best describes your household?
 - a. Single income, no dependents
 - b. Single income, at least one dependent
 - c. Dual income, no dependents
 - d. Dual income, at least one dependent
 - e. Retired or financially independent
6. What is the total value of your cash and liquid investments? (e.g. savings, CDs, mutual funds, IRAs, 401(k)s, public stocks)
7. When deciding how to invest your money, which do you care about more?
 - a. Maximizing gains
 - b. Minimizing losses
 - c. Both equally
8. The global stock market is often volatile. If your entire investment portfolio lost 10% of its value in a month during a market decline, what would you do?
 - a. Sell all of your investments
 - b. Sell some
 - c. Keep all
 - d. Buy more

³¹² Wealthfront Website.

Table 2**Regressions of Risky Share and Home Bias on Investor Attributes and Advisor Fixed Effects**

This table reports estimates from panel regressions of risky share (Panel A) and home bias (Panel B) on investor attributes, advisor fixed effects and year fixed effects. Risky share is the fraction of wealth in equity and home bias is the fraction of equity in Canadian funds. We measure risky share and home bias at year-ends 1999 through 2011. We omit the indicator variables for the lowest categories. The first two regressions are estimated using data on all advisors. The regressions in the low-dispersion and high-dispersion columns divide advisors each year into two groups of equal size based on client heterogeneity. The measure of heterogeneity is the within-advisor standard deviation of the fitted values from column (1)'s regression. The last row, "Adjusted R^2 w/o advisor FEs," reports the adjusted R^2 from an alternative model that does not include the advisor fixed effects. The adjusted R^2 that we report measures incremental explanatory power over a model with year fixed effects. Figure 1 Panel A reports the age-coefficient estimates from column (1)'s regression. Standard errors are clustered by advisor.

Panel A: Dependent variable = Risky share

Independent variable	(1)		(2)		(3)		(4)	
	All advisors		All advisors		Low-dispersion advisors		High-dispersion advisors	
	\hat{b}	t	\hat{b}	t	\hat{b}	t	\hat{b}	t
Constant	37.12	10.32	35.73	13.46	39.77	6.48	33.06	10.98
Risk tolerance								
Low	6.78	2.65	6.58	3.06	-4.51	-0.76	7.05	3.17
Low to Moderate	17.44	6.59	17.38	8.15	15.26	2.91	17.41	7.90
Moderate	30.52	11.45	28.90	13.16	27.36	5.23	28.90	12.69
Moderate to High	32.94	12.09	31.88	14.48	30.80	5.91	31.39	13.76
High	38.29	14.03	37.25	16.46	35.30	6.81	37.57	15.61
Fin. knowledge								
Moderate	2.87	7.58	1.46	10.16	1.23	5.73	1.61	9.07
High	3.99	7.09	2.76	9.68	1.94	5.45	3.30	8.54
Time horizon								
Short	3.98	4.90	3.81	5.75	3.58	3.26	3.75	4.75
Moderate	6.16	8.28	5.20	8.37	4.88	4.67	5.20	7.10
Long	6.57	8.07	5.62	8.91	4.84	4.52	6.03	8.10
Female	-1.37	-9.56	-1.34	-13.46	-1.28	-9.44	-1.40	-10.62
French speaking	-2.96	-2.37	-0.97	-2.01	-0.12	-0.14	-1.30	-2.33
Salary								
\$30-50k	0.42	2.39	0.69	5.58	0.75	4.49	0.65	3.88
\$50-70k	0.25	1.14	0.92	6.19	0.81	4.38	0.99	4.75
\$70-100k	-0.10	-0.35	0.94	5.87	0.82	3.99	1.02	4.62
\$100-200k	-3.09	-1.82	-0.86	-1.06	0.71	0.62	-2.21	-2.03
Over \$200k	-3.67	-2.02	-0.70	-0.72	-0.89	-0.59	-0.36	-0.33
Net worth								
\$35-60k	1.13	1.97	1.05	2.51	1.11	1.90	1.01	1.88
\$60-100k	1.77	2.97	1.52	3.63	1.63	2.85	1.42	2.59
\$100-200k	2.16	3.96	1.79	4.61	1.77	3.37	1.78	3.52
Over \$200k	1.29	2.10	1.23	3.04	1.09	1.98	1.35	2.53
Occupation								
Finance professional	2.29	2.88	1.65	2.32	0.22	0.26	3.05	2.95
Self-employed	0.54	1.47	0.61	2.07	0.13	0.36	1.05	2.43
Government	0.97	2.95	0.85	3.84	1.05	3.43	0.72	2.48
Advisor FEs	No		Yes		Yes		Yes	
Age groups	Yes (Fig. 1)		Yes		Yes		Yes	
Year FEs	Yes		Yes		Yes		Yes	
Province FEs	Yes		Yes		Yes		Yes	
# of observations	758,058		758,058		327,235		427,242	
# of investors	174,609		174,609		92,314		111,520	
# of advisors	5,083		5,083		2,829		2,546	
Adjusted R^2	12.2%		30.2%		28.3%		29.6%	
w/o advisor FEs	.		12.2%		7.3%		13.5%	

Panel B: Dependent variable = Home bias

Independent variable	All advisors		All advisors		Low-dispersion advisors		High-dispersion advisors	
	\hat{b}	t	\hat{b}	t	\hat{b}	t	\hat{b}	t
Constant	64.86	18.38	59.57	22.60	57.05	11.04	59.56	18.89
Risk tolerance								
Low	0.34	0.19	-0.72	-0.43	2.98	0.65	-0.81	-0.47
Low to Moderate	-2.01	-1.12	-1.09	-0.70	0.42	0.11	-1.13	-0.70
Moderate	-0.55	-0.34	-0.80	-0.54	0.01	0.00	-0.68	-0.44
Moderate to High	-4.82	-2.81	-4.72	-3.17	-3.67	-1.02	-4.69	-3.03
High	-17.67	-9.30	-15.44	-9.68	-13.69	-3.78	-15.87	-8.97
Fin. knowledge								
Moderate	1.11	1.96	-0.78	-3.65	-0.74	-2.44	-0.81	-2.99
High	0.97	1.11	-1.70	-3.76	-2.06	-3.21	-1.46	-2.51
Time horizon								
Short	0.11	0.09	0.57	0.64	2.21	1.36	0.10	0.10
Moderate	0.61	0.53	1.42	1.67	2.82	1.79	1.06	1.10
Long	-0.03	-0.02	1.84	2.12	3.55	2.25	1.19	1.20
Female	0.65	2.60	0.34	2.12	0.14	0.59	0.51	2.44
French speaking	2.40	1.40	1.52	2.03	1.68	1.34	1.47	1.65
Salary								
\$30-50k	-0.32	-1.21	-0.26	-1.36	-0.16	-0.59	-0.33	-1.31
\$50-70k	-1.32	-3.92	-1.29	-5.70	-1.08	-3.21	-1.44	-5.03
\$70-100k	-2.86	-6.16	-1.97	-7.72	-1.82	-5.19	-2.09	-6.17
\$100-200k	-2.70	-1.19	-2.22	-1.71	-2.22	-1.12	-2.07	-1.24
Over \$200k	0.64	0.29	-1.86	-1.47	-0.67	-0.39	-2.67	-1.64
Net worth								
\$35-60k	0.88	1.07	0.88	1.38	0.71	0.73	0.87	1.11
\$60-100k	0.32	0.40	-0.15	-0.25	-0.58	-0.64	0.05	0.06
\$100-200k	-0.01	-0.02	-0.01	-0.01	-0.24	-0.27	0.07	0.10
Over \$200k	-0.06	-0.08	-0.13	-0.22	-0.35	-0.39	-0.09	-0.13
Occupation								
Finance professional	-1.33	-0.94	-0.84	-0.71	-0.54	-0.37	-1.23	-0.76
Self-employed	-0.98	-1.69	-0.42	-0.94	-0.17	-0.30	-0.79	-1.25
Government	1.44	2.72	0.78	2.43	0.60	1.32	0.86	2.05
Advisor FEs	No		Yes		Yes		Yes	
Age groups	Yes		Yes		Yes		Yes	
Year FEs	Yes		Yes		Yes		Yes	
Province FEs	Yes		Yes		Yes		Yes	
# of observations	739,687		739,687		321,707		414,531	
# of investors	171,145		171,145		90,993		108,664	
# of advisors	5,055		5,055		2,826		2,542	
Adjusted R^2	4.1%		27.9%		29.3%		27.1%	
w/o advisor FEs	.		4.1%		4.9%		4.1%	

Source: Stephen Foerster, Juhani T. Linnainmaa, Brian T. Melzer, Alessandro Previtero. Retail Financial Advice: Does One Size Fit All? Chicago Booth Paper No. 14-38. *Journal of Finance*, forthcoming.

Table 3

Regressions of Advisor Fixed Effects on Advisor Attributes

This table reports estimates from regressions of advisor fixed effects on advisor attributes: age, gender, language, risk tolerance, the average number of clients and the risky share and home bias in the advisor's own portfolio. The fixed-effect estimates are from the second regression in Table 2.

Panel A: Dependent variable = Risky-share fixed effect

Independent variable	Regression					
	(1)		(2)		(3)	
	\hat{b}	t	\hat{b}	t	\hat{b}	t
Age, 25–29	6.98	1.52	7.23	1.55	6.69	1.59
30–34	3.76	0.84	4.70	1.04	6.09	1.51
35–39	5.63	1.28	6.07	1.37	6.72	1.70
40–44	7.63	1.75	7.28	1.66	8.02	2.05
45–49	7.74	1.78	8.00	1.82	9.31	2.37
50–54	8.58	1.98	8.72	1.99	10.09	2.58
55–59	8.08	1.84	8.26	1.87	9.39	2.39
60–64	11.30	2.57	11.71	2.65	12.83	3.25
65–69	11.33	2.51	11.98	2.61	13.34	3.24
70–74	18.93	4.11	18.38	3.95	19.17	4.49
75–79	6.14	0.58	13.52	2.25	15.18	2.86
Female	0.79	1.20	1.04	1.58	1.24	2.01
French speaking	–3.71	–2.33	–4.26	–2.91	–4.52	–2.99
log(# of clients)	–0.37	–1.90	–0.37	–1.80	–0.40	–2.05
Risk tolerance						
Moderate			3.32	2.03	–1.37	–0.84
Moderate to High			1.80	1.10	–3.28	–2.03
High			2.90	1.79	–3.38	–2.09
Advisor's risky share					25.17	15.51
Advisor province FEs	Yes		Yes		Yes	
# of observations	2,956		2,631		2,631	
Adjusted R^2	5.1%		5.6%		17.4%	

Panel B: Dependent variable = Home-bias fixed effect

Independent variable	Regression					
	(1)		(2)		(3)	
	\hat{b}	t	\hat{b}	t	\hat{b}	t
Age, 25–29	–4.43	–0.56	–2.40	–0.31	–1.77	–0.27
30–34	–17.96	–2.37	–16.95	–2.28	–15.02	–2.43
35–39	–11.54	–1.54	–10.57	–1.44	–7.33	–1.21
40–44	–11.90	–1.60	–11.96	–1.65	–7.95	–1.32
45–49	–13.81	–1.86	–14.34	–1.98	–9.98	–1.66
50–54	–14.06	–1.90	–14.18	–1.96	–9.16	–1.53
55–59	–7.97	–1.07	–7.59	–1.05	–4.88	–0.81
60–64	–8.15	–1.09	–6.69	–0.92	–4.50	–0.75
65–69	–9.40	–1.23	–9.82	–1.32	–7.77	–1.26
70–74	–9.03	–1.11	–6.79	–0.86	–4.69	–0.70
75–79	–2.35	–0.28	–2.24	–0.27	–3.11	–0.45
Female	2.27	2.16	2.54	2.33	1.10	1.12
French speaking	–0.48	–0.20	–0.19	–0.07	–1.17	–0.54
log(# of clients)	0.38	1.19	0.20	0.60	0.39	1.28
Risk tolerance						
Moderate			6.77	2.49	5.25	2.05
Moderate to High			8.41	3.13	7.90	3.11
High			6.06	2.28	9.12	3.62
Advisor's home bias					33.83	22.67
Advisor province FEs	Yes		Yes		Yes	
# of observations	2,947		2,626		2,599	
Adjusted R^2	2.7%		4.0%		22.5%	

Source: Stephen Foerster, Juhani T. Linnainmaa, Brian T. Melzer, Alessandro Previtero. Retail Financial Advice: Does One Size Fit All? Chicago Booth Paper No. 14-38. *Journal of Finance*, forthcoming.

Table 4**Who Benefits From Trades?**

This table reports estimates of how often advisors and clients benefit from trade. The data consist of 6.9 million transactions in which the client moves from one mutual fund to another. An advisor benefits from a trade if he earns a sales commission, charges a front-end load, or increases the trailing commission; a client benefits from the trade if it lowers the management expense ratio or the investment is moved to another type of a fund; and the trade is costly to the client if he pays a deferred sales charge or a front-end load, or if the management expense ratio increases. We use the 54-category classification of Funddata to define fund types. We estimate the proportions of different combinations for each advisor, and then average across advisors. Standard errors, clustered by advisor, are reported in square brackets.

Advisor benefits	Client benefits	Costly to client	
		No	Yes
No (69.6%)	No	32.1% [0.7%]	2.9% [0.2%]
	Yes	38.9% [0.8%]	26.1% [0.6%]
Yes (30.4%)	No	16.2% [1.6%]	5.4% [0.8%]
	Yes	56.7% [2.3%]	21.7% [1.6%]

Source: Juhani T. Linnainmaa, Brian T. Melzer and Alessandro Previtero. Costly Financial Advice: Conflicts of Interest or Misguided Beliefs? December 2015.

Table 5

Measures of Trading Behavior: Clients versus Advisors

This table reports estimates of how clients and advisors trade. The measures are defined as follows: (i) Turnover is the market value of purchases and sales divided by the beginning of month market value of holdings, annualized by multiplying by 12; (ii) Active management is the portfolio share of index funds and target-date funds; (iii) Return chasing is the average percentile rank of prior one-year returns for funds bought; (iv) Disposition effect is the proportion of gains realized (PGR) minus the proportion of losses realized (PLR), computed using data on those months when the investor sells something; (v) Home bias is the fraction of Canadian equity mutual fund purchases out of all equity fund purchases; and (vi) growth tilt is the fraction of growth funds bought minus the fraction of value funds bought. We compute these measures for each client and then aggregate the data to advisor level. The last row reports the average percentile fee rank of the funds that clients and advisors purchase. We compute within-fund type percentile ranks using the 54-category classification of Funddata. The sample includes those advisors who also appear as clients in the dealer data, and who advise clients for at least two years.

Behavior	Clients		Advisors		Difference, <i>t</i> -value	<i>N</i>
	Mean	SE	Mean	SE		
Turnover						
Retirement accounts	29.1%	0.4%	33.4%	0.6%	7.23	3,276
Open accounts	28.1%	0.5%	45.4%	1.1%	15.68	2,035
Active management	99.1%	0.1%	99.1%	0.1%	0.26	3,254
Return chasing	58.7%	0.1%	60.7%	0.2%	11.98	3,170
Disposition effect						
Retirement accounts	-2.1%	0.3%	-1.0%	0.5%	2.13	2,118
Open accounts	-6.0%	0.8%	-7.6%	1.4%	-1.11	673
Home bias						
Retirement accounts	46.1%	0.3%	41.5%	0.4%	-11.95	3,205
Open accounts	40.7%	0.7%	36.3%	0.9%	-6.04	1,497
Growth tilt	9.1%	0.2%	6.9%	0.3%	-7.25	3,252
Percentile fee						
within fund type	38.3%	0.2%	39.6%	0.4%	3.28	1,854

Source: Juhani T. Linnainmaa, Brian T. Melzer and Alessandro Previtero. Costly Financial Advice: Conflicts of Interest or Misguided Beliefs? December 2015.

Table 6

Explaining Cross-Sectional Variation in Client Behavior with Client Attributes, Advisor Fixed Effects, Investor Fixed Effects, and Advisor Behavior

Panel A evaluates the importance of client attributes, advisor fixed effects, investor fixed effects, year fixed effects, and province fixed effects in explaining cross-sectional variation in client behavior. The measures of behavior are described in text and summarized in Table 4. Each measure is computed at the client-year level. Turnover, disposition effect, and home bias regressions are estimated separately for retirement accounts and open accounts; the other regressions pool trades and holdings across all accounts. The estimates in the full sample columns use data on all clients and advisors. The estimates in the two-way fixed effects columns limit the sample to clients who are forced to switch advisors when their old advisor dies, retires, or leaves the industry. In these regressions, the unit of observation is an advisor-client pair. Panel B reports estimates from regressions of advisor fixed effects on advisor attributes: age, gender, language, risk tolerance, the average number of clients and the advisor's behavior in his own portfolio. In the return-chasing regression, for example, this regressor is the return chasing estimate computed from the advisor's personal trades. The fixed-effect estimates are from the second regression in Panel A.

Panel A: Client attributes, advisor fixed effects, and investor fixed effects

Behavior	Model		Two-way FEs	
	Client attributes	Client attributes + advisor FEs	Investor FEs	Investor FEs + advisor FEs
Turnover				
Retirement accounts	0.9%	3.4%	4.8%	16.5%
Open accounts	0.9%	3.7%	23.4%	30.0%
Active management	1.1%	9.2%	38.8%	50.5%
Return chasing	2.3%	8.3%	15.3%	33.9%
Disposition effect				
Retirement accounts	1.0%	3.5%	36.0%	40.1%
Open accounts	0.7%	4.2%	28.8%	25.8%
Home bias				
Retirement accounts	3.7%	16.8%	53.6%	60.3%
Open accounts	10.7%	29.6%	61.7%	69.8%
Growth bias	0.5%	8.1%	54.8%	61.2%
Percentile fee within fund type	2.9%	16.1%	54.9%	63.6%

Panel B: Regressions of advisor fixed effects on advisor attributes

Independent variable	Turnover		Active management	Return chasing	Disposition effect	
	Retirement	Open			Retirement	Open
	Coefficients					
Log(Age)	-18.23	-14.06	-0.59	1.62	-0.42	-1.07
Female	1.09	-2.16	0.07	0.48	-1.06	2.48
French speaking	-0.78	-0.13	0.65	-0.64	1.82	2.49
log(# of clients)	-0.55	-0.07	2.54	-1.82	2.23	0.81
Risk tolerance						
Moderate	-4.63	5.22	-0.28	-0.68	-4.13	0.66
Moderate to High	-1.46	2.53	-0.07	0.28	-5.18	-2.03
High	-2.50	6.48	-0.04	0.54	-4.72	-0.72
Advisor's behavior	0.12	0.07	0.32	0.21	0.07	0.06
	t-values					
Log(Age)	-5.73	-4.87	-1.65	2.63	-0.29	-0.24
Female	0.64	-1.26	0.36	1.31	-1.21	0.94
French speaking	-0.55	-0.07	2.54	-1.82	2.23	0.81
log(# of clients)	-2.80	-3.14	-0.04	-0.29	0.58	1.63
Risk tolerance						
Moderate	-0.76	1.35	-0.57	-0.73	-1.83	0.11
Moderate to High	-0.25	0.79	-0.13	0.30	-2.29	-0.41
High	-0.43	2.05	-0.08	0.62	-2.20	-0.15
Advisor's behavior	5.31	2.94	4.24	15.99	5.88	2.41
<i>N</i>	3,163	1,964	3,095	3,101	1,813	612
Adjusted <i>R</i> ²	6.3%	5.7%	24.0%	14.6%	3.0%	1.2%

(cont'd)

Independent variable	Home bias		Growth bias	Percentile fee
	Retirement	Open		
Coefficients				
Log(Age)	5.60	-0.88	0.57	2.81
Female	0.16	1.75	-0.07	-0.10
French speaking	-1.10	0.53	0.03	1.34
log(# of clients)	-1.39	0.34	0.05	1.78
Risk tolerance				
Moderate	7.23	8.80	0.56	-0.09
Moderate to High	8.44	9.26	0.28	-1.63
High	8.87	13.25	0.84	-0.01
Advisor's behavior	0.29	0.34	0.19	0.14
t-values				
Log(Age)	4.04	-0.33	0.54	2.21
Female	0.20	1.20	-0.12	-0.16
French speaking	-1.39	0.34	0.05	1.78
log(# of clients)	-0.73	-2.20	-0.47	-0.69
Risk tolerance				
Moderate	3.56	1.71	0.42	-0.06
Moderate to High	4.19	1.89	0.20	-1.12
High	4.61	2.80	0.66	-0.01
Advisor's behavior	20.48	18.70	11.04	8.49
<i>N</i>	3,182	1,451	3,224	1,727
Adjusted R^2	17.0%	24.2%	7.8%	6.4%

Source: Juhani T. Linnainmaa, Brian T. Melzer and Alessandro Previtero. Costly Financial Advice: Conflicts of Interest or Misguided Beliefs? December 2015.

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