Can Market Regimes Really be Timed with Historical Volatility?

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Abstract

Recent research findings suggest long-term investor utility benefits through scaling expected returns by recent realized volatility. We test for utility gains to volatility timing using a utility regime-based methodology to classify investor-specific market investment regimes based solely on recent realized volatility levels. Under this framework we find limited informational content in using recent realized volatility to forecast utility regimes for the market index. To reconcile our findings we replicate work by Moreira and Muir (2017) and find that their reported Sharpe ratio gains through volatility-managing the US market factor do not appear to be statistically significant. We find that their scheme under-performs buy and hold in terms of Sharpe ratio over 30 of the 70 twenty year sub-periods in our sample (58 out of 70 for an un-leveraged investor). Furthermore, the historical out-performance of volatility management for the market index is highly sensitive to the timing of re-balancing within a month, suggesting that the strategy may not be robust to the precise timing of key market events relative to volatility changes. Strategy adopters should be aware that this timing is not guaranteed to line up favorably over future investment periods.

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1 Introduction

Recent research findings suggest that there are long-run utility gains from volatility managed portfolios that reduce investment weighting in risky assets in high volatility periods and increase weighting in low volatility periods (see Moreira and Muir, 2017). The approach is also referred to as volatility targeting (see Harvey, Hoyle, Korgaonkar, Rattray, Sargaison, and Van Hemert, 2018). The negative relationship between volatility and returns, often referred to as a leverage effect, runs counter to the positive risk premium one would expect under classical financial theory for bearing increased risk. The result is consistent, however, with findings in the regime classification literature that associate high volatility and negative returns with bear market regimes (see e.g. Maheu and McCurdy, 2000; Pagan and Sossounov, 2003; Candelon, Piplack, and Straetmans, 2008; Gordon and St-Amour, 2000).

In this article we apply an alternative methodology to volatility targeting to test the informational content of recent realized volatility. Specifically, we apply a volatility threshold to market-time investment between two different regime-specific optimal portfolios - a high volatility (bear) portfolio and a low volatility (bull) portfolio. The volatility threshold is determined through a utility-maximizing optimization and this is updated in a rolling window in-sample period, avoiding look-ahead bias. For a candidate volatility threshold the in-sample returns are partitioned into bull (below the threshold) and bear (volatility above the threshold) sub-periods. For each sub-period a utility-maximizing optimization is run to select optimal bull and bear portfolio weights. The selected threshold is then used to partition the historical investment period between holding the optimal bull and bear portfolios. The realized utility from the market-timed strategy returns historically is calculated and the volatility threshold with the maximum realized utility is selected for use out-of-sample to select between the optimal bull and bear portfolio in the following out-of-sample holding period (using the contemporaneous volatility estimate on the re-balancing date and the optimal volatility threshold to date).

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The time series of optimal thresholds over time partition expected returns into bull and bear regime returns allowing ex-post statistical testing of the efficacy of historical volatility in classifying future return regimes. This provides a large sample of classification decisions from the volatility rule allowing enhanced statistical analysis over existing research that looks at long run utility gains or risk factor regression 'α' intercepts (which may be driven by a small number of extreme events in the sample).

Our main contribution is to test whether recent realised volatility can be used to predict/market time investor utility regimes. This is the basic premise of any volatility management strategy. The second contribution of our paper is replication of Moreira and Muir (2017) and the application of a battery of robustness tests designed to help to inform practitioners as to whether the favorable utility gains reported are likely to be repeated for adopters of their strategy in the future.

2 Volatility Management

Moreira and Muir (2017) take the perspective of a mean-variance investor however they do not run portfolio optimisations to estimate optimal portfolio weights, rather they make an approximation that the ideal weights of a mean-variance investor can be estimated as:

$$w_t^* \propto \frac{E_t[f_{t+1}]}{\hat{\sigma}^t} \tag{1}$$

They use the previous month's realized variance as the volatility estimate. Their investment weight in an index is selected as:

$$w_t = \frac{c}{\hat{\sigma_t}^2} \tag{2}$$

where c is selected such that the scaled returns $(w_t * r_t)$ have the same conditional variance as the original r_t. The w_t are selected in sample (with look-ahead bias) and this is justified in

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the paper as having no effect on the strategy Sharpe. Their main results pointing to utility gains are presented in a portfolio that allows leverage.

In this article we will also consider un-leveraged (long-only) Sharpe ratio optimizing investors. While Moreira and Muir (2017) don't provide an exact mechanism through which volatility timing works they do state that: *results are uniquely driven by the time-series relationship between risk and return.* In Moreira and Muir (2019) they present a partial equilibrium model with potentially independent dynamically time varying expected returns and volatility. Under the model volatility timing is not optimal when there is a relationship between high volatility and the degree of mean-reversion in returns (the 'buying at a discount' narrative). They test for this empirically and find no difference in the auto-correlation in returns in high and low volatility regimes, therefore suggesting there are utility gains to a long term investor to volatility timing under their model. In Section 3 we analyze regimes in the risk-return relationship to test volatility management under an independent regime-based framework, with limited assumptions, to capture volatility-dependent investment regimes.

2.1 Market Factor Replication Analysis

Moreira and Muir (2017) use a very simple volatility estimator, estimated as the variance of the daily returns.[1] In this section we replicate their results using the same data set from Kenneth French's website covering the period 1926 to 2015. Figure 1 below plots the log of the resulting wealth through compounding the (simple) returns of the scheme. The plot can be seen to visually match Figure 3 of their paper closely. The volatility managed scheme has an annualized Sharpe ratio of 0.5225 versus a Sharpe ratio of 0.4155 for the buy and hold scheme over the same period. We obtain a risk factor regression α of 6.42% (annualized) when regressing the volatility managed returns against the original market factor.

In the following sections we apply robustness tests to the original scheme returns to investigate the performance dependence on leverage; the sensitivity to the timing of portfolio

[1]Without including a Bessel correction to de-bias the estimate for the estimation of the sample mean or the assumption of zero daily returns often used with financial data.

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Label #	Period	Description of volatility management gains/losses
1.	1927-1929	Gain: Low volatility, high leverage in a bull market.
2.	1930-1932	Gain: High volatility, low leverage in a bear market.
3.	1933-1934	Loss: High volatility, low leverage, mis-times market recovery.
4.	1965-1966	Gain: Low volatility, high leverage in a bull market.
5. & 6.	1974-1976	Gain: High volatility, low leverage in a bear market followed by a Loss as low leverage misses the recovery.
7., 8. & 9.	1994-2000	Gain: High leverage at the start of the dot com bubble followed by Losses and high volatility reduces leverage too soon before the end of the bubble, before finally gaining from the low leverage in the market crash.
10.	2003-2004	Loss: Low leverage misses the start of the market recovery.
11.	2008	Gain: High volatility, low leverage in a bear market.
12.	2009	Loss: High volatility, low leverage, mis-times market recovery.

Table 1. Description of the periods of gains/losses for the volatility managed strategy over the buy and hold strategy, labelled 1-12 in Figure 3

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re-balancing and we perform a sub-period analysis assuming an investment lifetime of 20 years rather than the 90 year period reported in the original results.

2.1.1 Dependence on Leverage. In this section we introduce a no-leverage constraint. The results for the scheme are illustrated in Figure 2. The Sharpe ratio of the volatility scaled scheme is marginally higher than the buy and hold before transaction costs are considered (0.427 Vs 0.416), however much of the gain occurs at the start of the period in the late 1920's. Furthermore, the scheme performs badly when considered over a range of 20 year investment sub-periods, under-performing in terms of Sharpe ratio in 58 out of 70 sub-periods (See Section 2.1.3).

2.1.2 Timing Sensitivity and Re-balancing date. In this section we change the portfolio re-balancing date from the end of the month to N days before the end of the month for N in the range [1,15]. The difference between the buy and hold and volatility scaled Sharpe ratios is plotted in Figure 4 for each re-balancing date. The out-performance of the strategy

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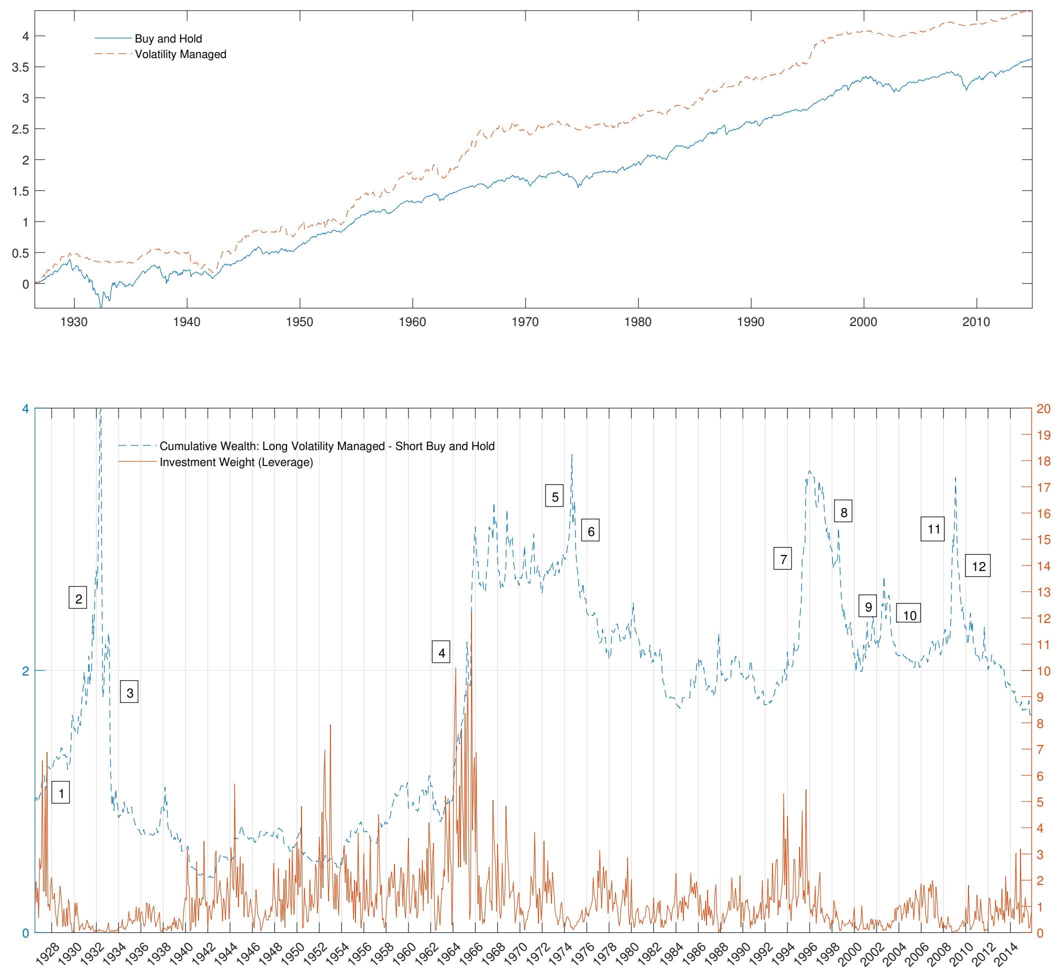

Figure 1.
The log (base 10) of cumulative wealth investing in the buy and hold market and the volatility managed strategy of Moreira and Muir (2017). The results closely match those in Figure 3 in their paper. The second plot shows where the additional performance of the voaltility managed strategy comes from, the figure plots the cumulative wealth from going long the vol managed portfolio and short the buy and hold returning the spread between the two. The numbered periods of gains/losses are described in Table 1.

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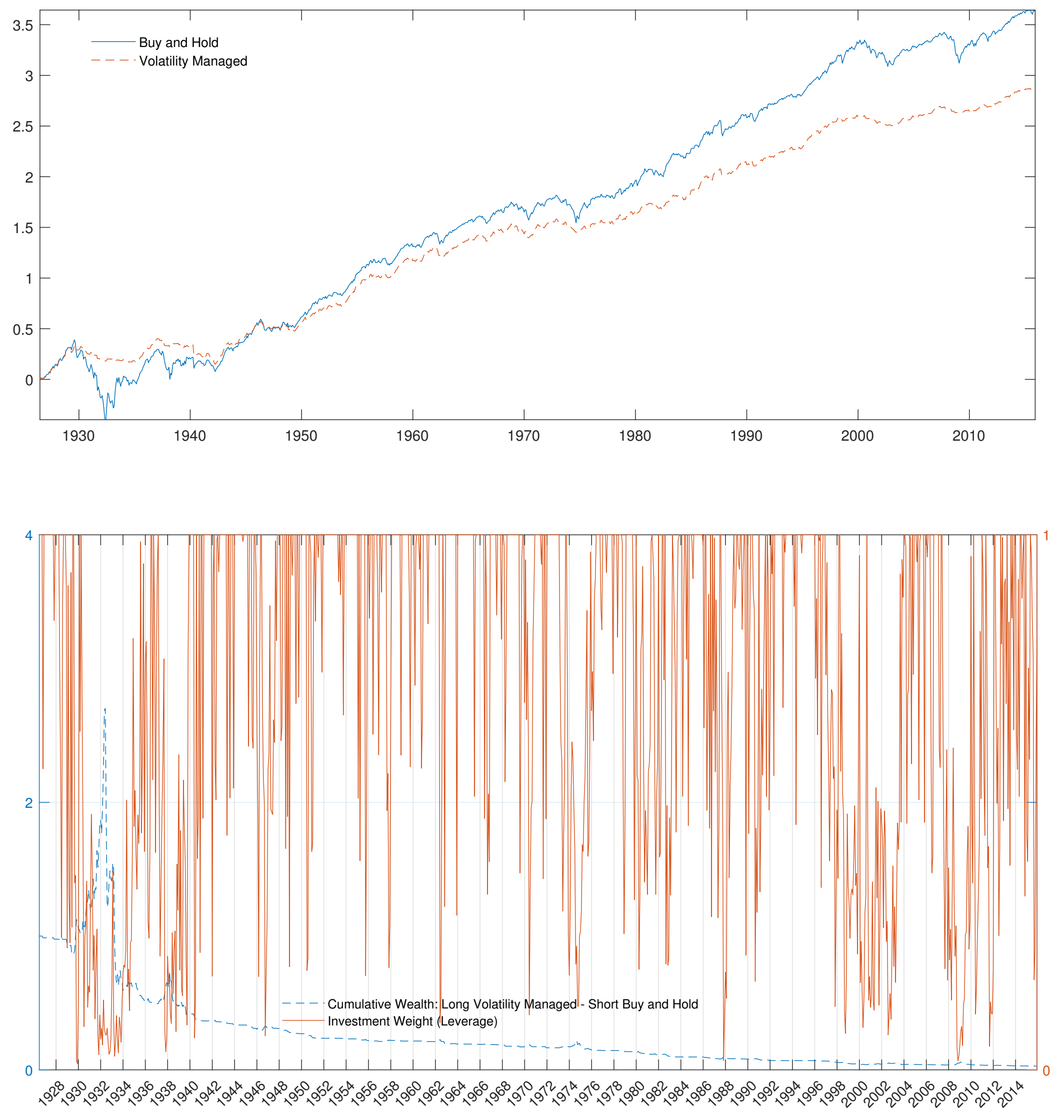

Figure 2.
The log (base 10) of cumulative wealth investing in the buy and hold market and the volatility managed strategy of Moreira and Muir (2017) without leverage. The Sharpe ratio of the volatility scaled scheme is marginally higher than the buy and hold before transaction costs are considered (0.427 Vs 0.416), however much of the gains occur at the start of the period in the late 1920's.

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Date	$\hat{\sigma}$	w_t	Mkt return	Date	$\hat{\sigma}$	w_t	Mkt return
28-May-1965	5.94	3.09	-5.16	13-May-1965	2.94	11.47	-6.33
31-Aug-1965	3.05	12.21	3.17	16-Aug-1965	6.75	2.27	3.28
30-Sep-1955	25.45	0.19	-2.50	15-Sep-1955	5.15	3.90	-7.41
28-Feb-1969	8.78	1.48	3.10	12-Feb-1969	5.30	3.69	-5.92
31-Aug-1953	8.89	1.44	0.36	14-Aug-1953	6.18	2.71	-7.13

Table 2. The 5 worst volatility management mis-times when re-balancing 11 business days prior to the end of the month rather than at month end. The first entries in the table show the re-balancing date the volatility over the previous 22 days, $\hat{\sigma}$ the resulting volatility managed investment weight w_t and the market return to the following re-balancing date.

over the buy and hold can be seen to be highly sensitive to the day of the month that the re-balancing is carried out on with the volatility managed scheme under-performing buy and hold in terms of Share ratio if an investor re-balances between 9 and 12 business days before the end of each month over the same sample period. This sensitivity to re-balance date suggests the possibility that the original timing may have out-performed through chance.

2.1.3 Investor Horizon. In reality few investors are likely to have an investment horizon of 90 years. In this section we consider how an investor would have fared applying volatility management over a more realistic time frame of 20 years. We evaluate seventy sub-periods starting at each year in the sample from 1926 through to 1996. Detailed results are given in Table 3 & 4. The Sharpe ratio for the volatility managed scheme never exceeds the 10 percentile level obtained for the buy and hold Sharpe ratio using boot strapping of the factor returns. The regression intercept of the volatility managed returns against the original factor (α) is statistically significant for a minority of sub-periods that include the 1920's and the 2008 crisis. For many sub-periods, the volatility managed Sharpe ratio is actually worse than the buy and hold (highlighted rows in both tables).

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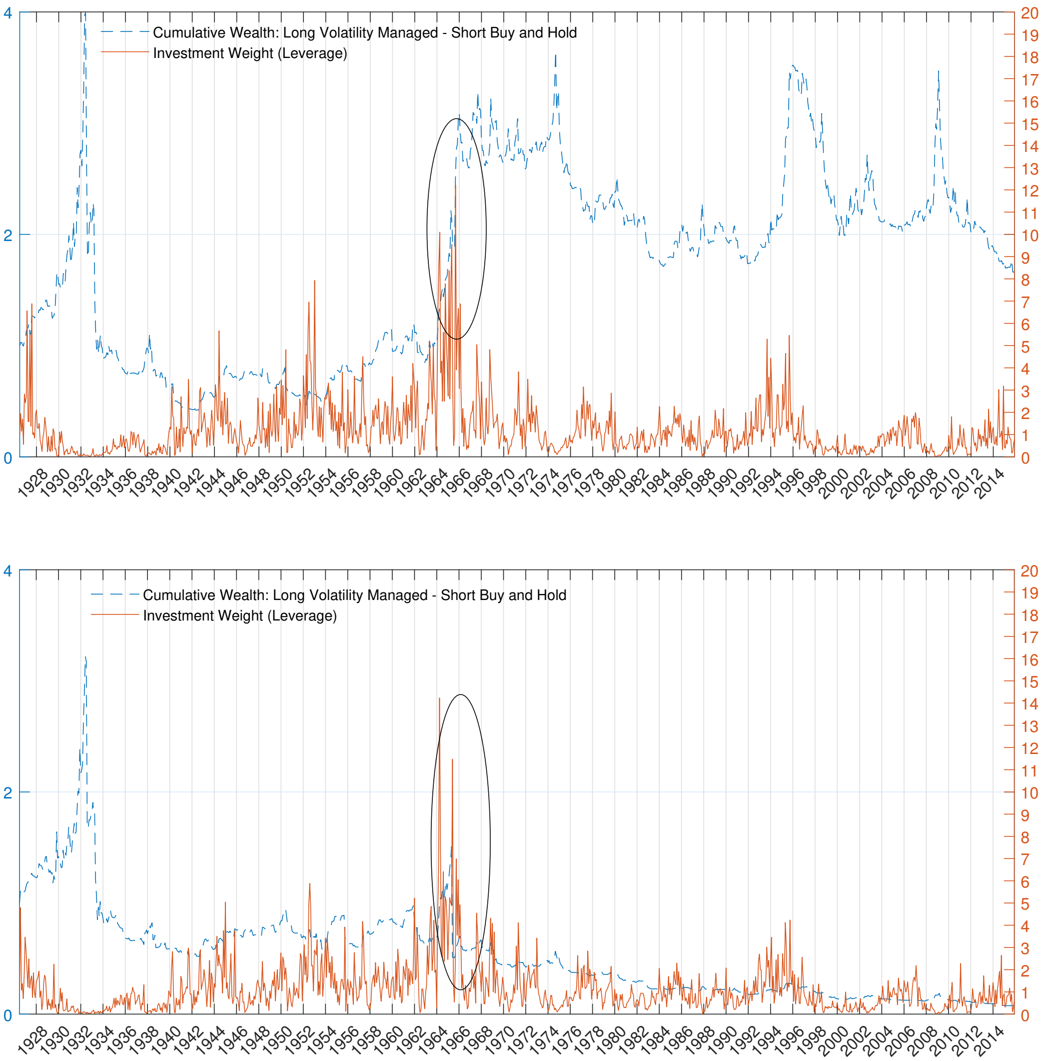

Figure 3.

A comparison of the log cumulative wealth obtained by investing in a strategy long the volatility managed portfolio and short buy & hold. The first plot illustrates performance for end of the month re-balancing, the second for a re-balancing date 11 days before the end of the month. The largest difference is circled in both plots, the top five return differentials between the two re-balancing date are described in Table 2. The large difference in performance highlights the sensitivity of volatility managed performance to timing and in particular timing a small number of key market events.

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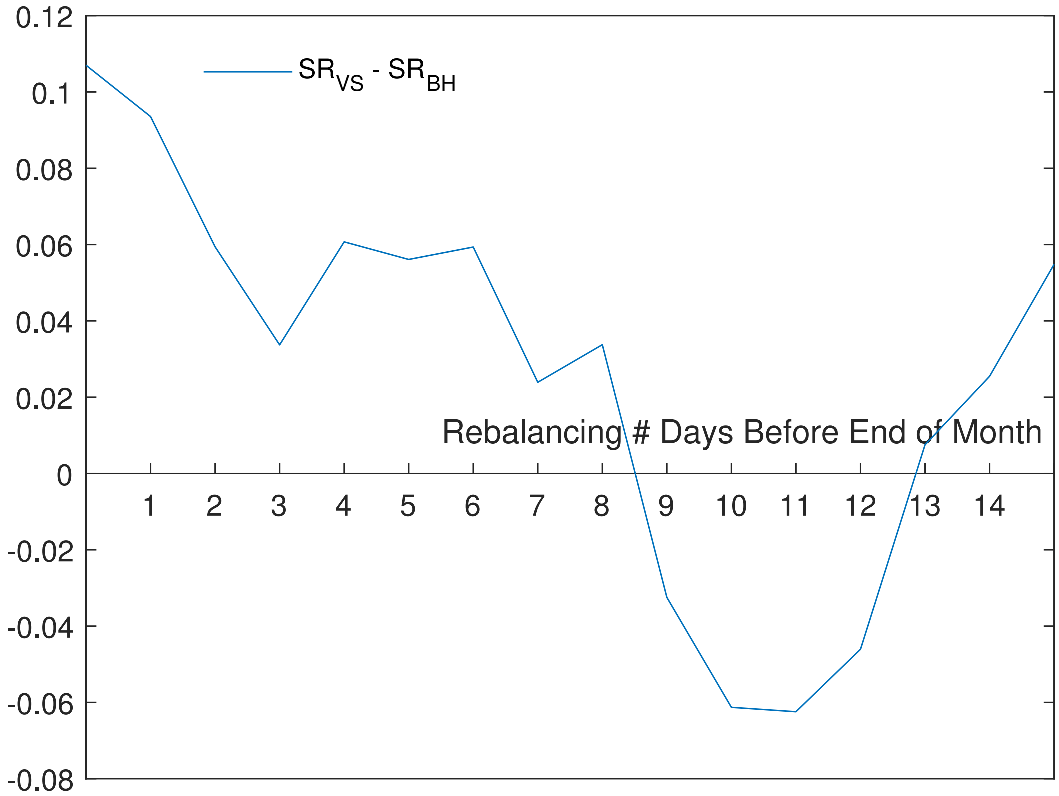

Figure 4. Plot of the out-performance of the volatility scaled sharpe ratio over the buy and hold sharpe ratio for the market factor for the sample from 1926-2015. The strategy replicates that of Moreira and Muir (2017), however the re-balancing day is changed from the last business date of the month to N days before the last business day of the month for N in the range [0,15]. Re-balancing between 9 and 12 days before the end of the month leads to under-performance of the volatility scaled strategy returns and performance is shown to be sensitive to the exact timing of the re-balancing suggesting that the strategy is not very robust.

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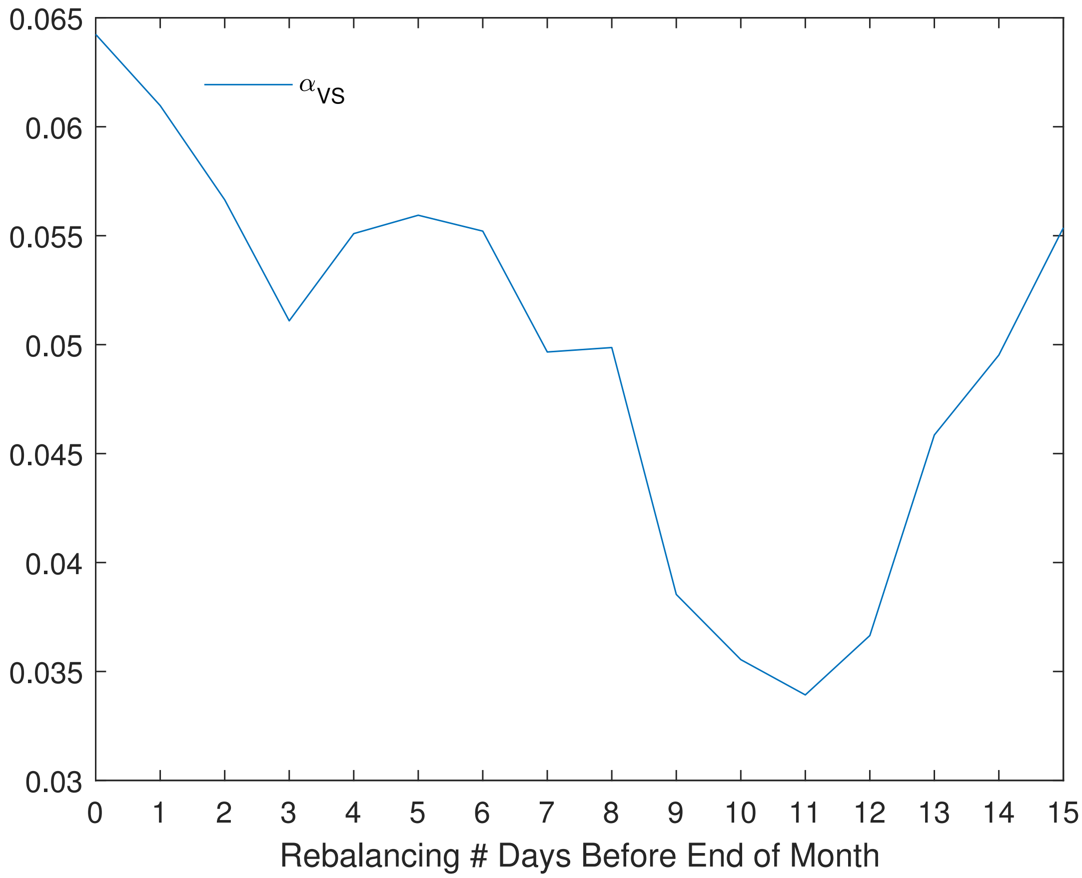

Figure 5. Plot of the α intercept of the volatility scaled market returns regressed against the buy and hold market returns for the sample from 1926-2015. The strategy replicates that of Moreira and Muir (2017), however the re-balancing day is changed from the last business date of the month to N days before the last business day of the month for N in the range [0,15]. Performance is shown to be sensitive to the exact timing of the re-balancing suggesting that the strategy is not very robust.

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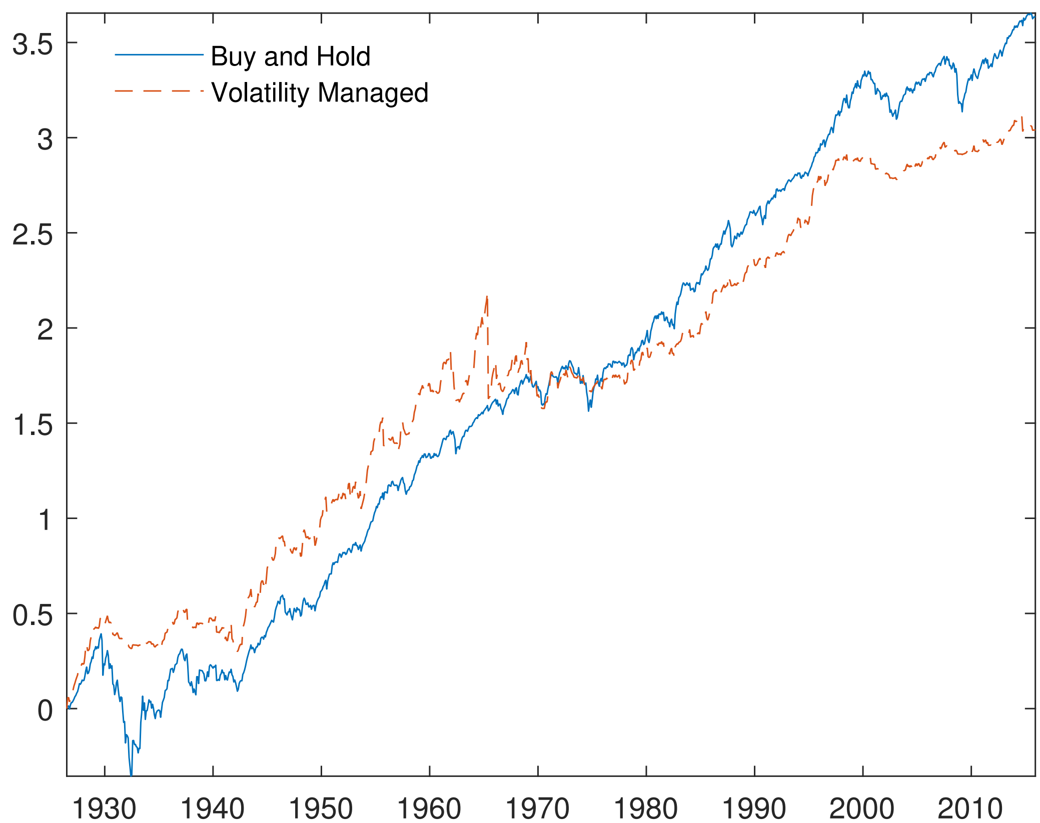

Figure 6. Log of the cumulative wealth from the volatility scaled and buy and hold schemes when the portfolio is re-balanced 11 days before the end of the month, the worst re-balancing timing found in the sample.

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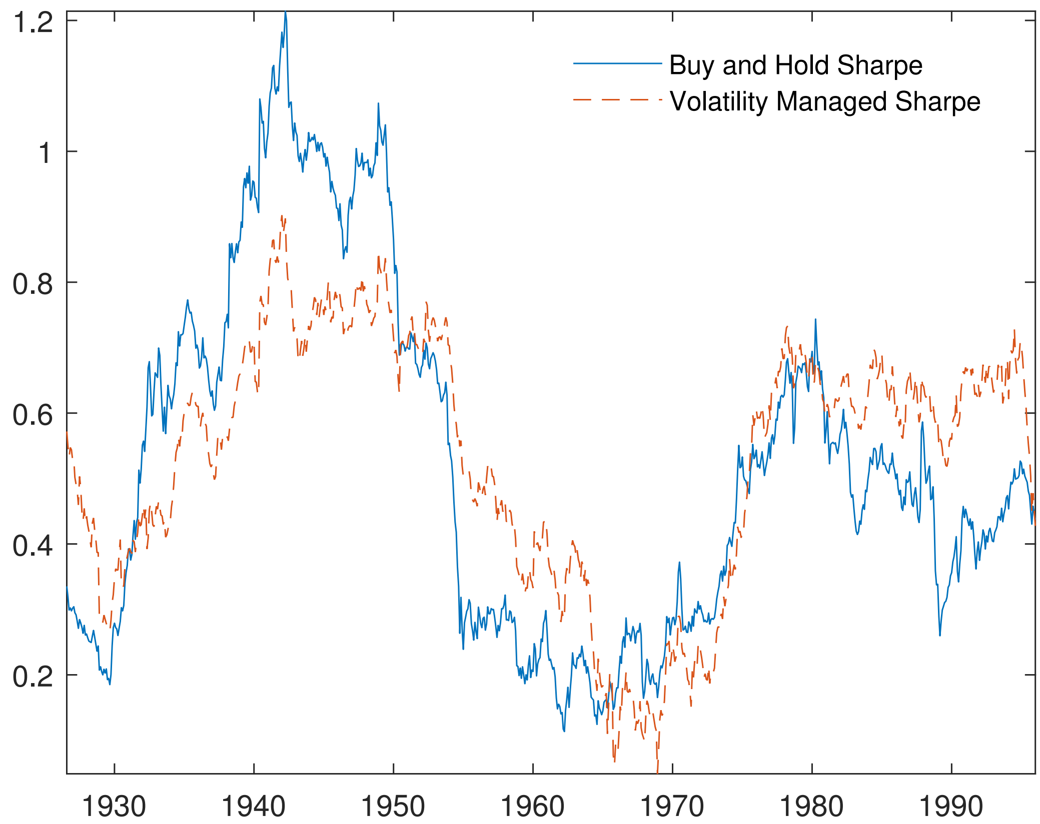

Figure 7. The realized Sharpe ratio of an investor with a 20 year investment horizon starting on the listed dates. The volatility scaled returns have a better Sharpe ratio for 56% of the start dates.

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Period	SR_{VS}	SR_{BH}	SR_{BH} Bootstrap 10 prctile	TC Sharpe Equivalence	α_{VS}	α pval
1926-1946	0.572	0.335	0.618	70.5	8.1^{**}	(0.026)
1927-1947	0.486	0.279	0.562	73.9	6.7^{*}	(0.055)
1928-1948	0.421	0.258	0.538	60.2	5.6	(0.107)
1929-1949	0.268	0.195	0.477	25.4	3.4	(0.336)
1930-1950	0.334	0.298	0.580	11.5	3.8	(0.317)
1931-1951	0.405	0.433	0.714	-8.5	3.9	(0.316)
1932-1952	0.463	0.640	0.912	-51.5	2.8	(0.497)
1933-1953	0.436	0.609	0.920	-44.0	1.1	(0.777)
1934-1954	0.564	0.724	1.059	-39.9	1.8	(0.633)
1935-1955	0.632	0.741	1.071	-26.1	2.9	(0.447)
1936-1956	0.580	0.683	1.023	-23.7	2.6	(0.499)
1937-1957	0.559	0.650	0.979	-20.3	2.7	(0.492)
1938-1958	0.611	0.830	1.165	-48.7	0.8	(0.810)
1939-1959	0.710	0.951	1.307	-52.1	-1.3	(0.704)
1940-1960	0.766	1.043	1.359	-56.5	-1.9	(0.565)
1941-1961	0.830	1.088	1.408	-52.7	-1.4	(0.684)
1942-1962	0.791	1.074	1.389	-59.1	-1.9	(0.584)
1943-1963	0.699	0.991	1.310	-59.6	-2.2	(0.528)
1944-1964	0.769	1.014	1.338	-46.4	-1.1	(0.755)
1945-1965	0.747	0.954	1.273	-36.0	-0.4	(0.922)
1946-1966	0.727	0.854	1.167	-21.8	1.2	(0.760)
1947-1967	0.792	0.978	1.295	-32.0	-0.2	(0.968)
1948-1968	0.745	0.978	1.293	-38.9	-1.4	(0.723)
1949-1969	0.777	0.938	1.249	-27.8	-0.2	(0.965)
1950-1970	0.708	0.708	1.013	-0.3	3.1	(0.437)
1951-1971	0.704	0.676	0.985	4.6	3.3	(0.399)
1952-1972	0.709	0.668	0.967	6.8	3.6	(0.347)
1953-1973	0.719	0.627	0.919	16.1	4.6	(0.226)
1954-1974	0.563	0.366	0.657	35.3	5.7	(0.124)
1955-1975	0.473	0.278	0.566	35.8	6.3	(0.113)
1956-1976	0.475	0.268	0.559	39.3	6.5	(0.099)
1957-1977	0.444	0.267	0.556	33.4	5.7	(0.138)
1958-1978	0.422	0.282	0.569	26.7	4.9	(0.208)
1959-1979	0.332	0.192	0.484	27.1	4.2	(0.274)
1960-1980	0.411	0.255	0.547	30.0	5.3	(0.166)
1961-1981	0.345	0.197	0.484	28.7	4.9	(0.200)
1962-1982	0.349	0.150	0.438	38.2	5.6	(0.117)
1963-1983	0.367	0.232	0.517	26.7	4.9	(0.167)
1964-1984	0.176	0.124	0.416	11.0	2.4	(0.472)
1965-1985	0.199	0.183	0.474	3.9	2.0	(0.511)
1966-1986	0.177	0.242	0.532	-19.1	0.6	(0.813)
1967-1987	0.131	0.265	0.557	-39.0	-0.4	(0.852)
1968-1988	0.144	0.201	0.492	-18.0	1.2	(0.606)
1969-1989	0.247	0.289	0.588	-12.9	1.9	(0.366)
1970-1990	0.277	0.347	0.645	-20.8	1.5	(0.477)
1971-1991	0.221	0.301	0.605	-24.2	1.4	(0.477)
1972-1992	0.213	0.294	0.591	-25.6	1.4	(0.472)
1973-1993	0.286	0.343	0.652	-17.7	1.9	(0.333)
1974-1994	0.385	0.468	0.778	-22.3	1.8	(0.404)
1975-1995	0.539	0.514	0.827	5.9	2.7	(0.219)
1976-1996	0.562	0.505	0.821	13.7	3.2	(0.178)
1977-1997	0.664	0.617	0.946	11.2	3.5	(0.139)
1978-1998	0.684	0.637	0.964	10.8	3.8^{*}	(0.099)
1979-1999	0.677	0.662	0.993	2.8	4.0^{*}	(0.095)
1980-2000	0.644	0.650	0.974	-2.7	3.7	(0.118)
1981-2001	0.606	0.553	0.880	13.5	4.5^{*}	(0.060)
1982-2002	0.660	0.550	0.881	29.6	5.5^{**}	(0.024)
1983-2003	0.589	0.451	0.767	36.4	5.7^{**}	(0.019)
1984-2004	0.685	0.541	0.863	38.9	6.1^{**}	(0.013)
1985-2005	0.637	0.522	0.855	30.0	5.5^{**}	(0.021)
1986-2006	0.583	0.472	0.801	27.6	5.1^{**}	(0.024)
1987-2007	0.608	0.441	0.768	40.4	5.8^{***}	(0.010)
1988-2008	0.604	0.469	0.771	32.8	5.3^{**}	(0.018)
1989-2009	0.520	0.312	0.614	49.0	6.0^{***}	(0.008)
1990-2010	0.594	0.369	0.675	52.4	6.4^{***}	(0.005)
1991-2011	0.650	0.413	0.720	54.4	$6.6^{p}***$	(0.003)
1992-2012	0.653	0.403	0.717	59.7	6.6^{***}	(0.003)
1993-2013	0.672	0.455	0.767	53.4	6.4^{***}	(0.004)
1994-2014	0.681	0.499	0.816	48.3	5.9^{***}	(0.005)
1995-2015	0.534	0.474	0.787	13.6	3.7^{**}	(0.034)

Table 3. Sub-period analysis for an investor with an investment horizon of 20 years, comparing the volatility scaling strategy with buy and hold using the realized Sharpe ratio. The 10 percentile level of the buy and hold Sharpe ratio obtained using boot strapping is included for comparison. The α value of a regression of the volatility scaled returns with the un-scaled returns is also included alongside the associated p value in brackets. Periods where the volatility-scaled Sharpe ratio is lower than the buy and hold Sharpe ratio are highlighted in gray and correspond to 30 out of the 70 sub-periods.

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Period	SR_{VS}	SR_{BH}	SR_{BH} Bootstrap 10 prctile	TC Sharpe Equivalence	α_{VS}	α pval
1926-1946	0.505	0.335	0.620	94.1	4.0^{**}	(0.044)
1927-1947	0.404	0.279	0.557	66.9	3.1	(0.111)
1928-1948	0.336	0.258	0.537	42.4	2.4	(0.217)
1929-1949	0.208	0.195	0.475	6.8	1.3	(0.509)
1930-1950	0.330	0.298	0.578	17.5	1.8	(0.350)
1931-1951	0.453	0.433	0.715	10.8	2.2	(0.258)
1932-1952	0.555	0.640	0.909	-45.4	1.6	(0.402)
1933-1953	0.559	0.609	0.923	-26.9	1.0	(0.521)
1934-1954	0.679	0.724	1.058	-24.6	1.1	(0.468)
1935-1955	0.742	0.741	1.081	1.1	1.6	(0.269)
1936-1956	0.673	0.683	1.013	-4.9	1.4	(0.332)
1937-1957	0.608	0.650	0.976	-25.0	1.0	(0.486)
1938-1958	0.681	0.830	1.167	-89.4	-0.8	(0.486)
1939-1959	0.812	0.951	1.310	-86.3	-0.7	(0.482)
1940-1960	0.942	1.043	1.363	-61.0	-0.4	(0.575)
1941-1961	1.002	1.088	1.403	-56.7	-0.2	(0.778)
1942-1962	1.017	1.074	1.400	-42.3	0.3	(0.743)
1943-1963	0.912	0.991	1.304	-54.5	0.1	(0.888)
1944-1964	0.946	1.014	1.331	-45.9	0.3	(0.726)
1945-1965	0.882	0.954	1.268	-49.7	0.2	(0.774)
1946-1966	0.791	0.854	1.172	-44.7	0.2	(0.764)
1947-1967	0.867	0.978	1.292	-79.1	-0.3	(0.695)
1948-1968	0.863	0.978	1.292	-83.1	-0.3	(0.733)
1949-1969	0.849	0.938	1.249	-79.8	-0.0	(0.953)
1950-1970	0.646	0.708	1.015	-54.2	0.3	(0.738)
1951-1971	0.637	0.676	0.982	-37.0	0.5	(0.546)
1952-1972	0.615	0.668	0.972	-44.6	0.4	(0.618)
1953-1973	0.579	0.627	0.933	-41.2	0.5	(0.556)
1954-1974	0.355	0.366	0.672	-10.6	0.9	(0.284)
1955-1975	0.229	0.278	0.571	-38.9	1.1	(0.314)
1956-1976	0.202	0.268	0.551	-52.0	1.0	(0.363)
1957-1977	0.203	0.267	0.559	-50.9	1.1	(0.331)
1958-1978	0.199	0.282	0.568	-68.0	0.9	(0.429)
1959-1979	0.069	0.192	0.488	-100.6	0.3	(0.765)
1960-1980	0.120	0.255	0.545	-107.1	0.4	(0.704)
1961-1981	0.027	0.197	0.481	-130.1	0.3	(0.814)
1962-1982	-0.044	0.150	0.440	-140.2	0.1	(0.901)
1963-1983	0.028	0.232	0.527	-145.4	0.1	(0.922)
1964-1984	-0.112	0.124	0.413	-165.8	-0.3	(0.765)
1965-1985	-0.026	0.183	0.476	-148.4	0.0	(0.994)
1966-1986	0.033	0.242	0.522	-149.9	0.0	(0.988)
1967-1987	0.033	0.265	0.546	-154.1	-0.3	(0.817)
1968-1988	-0.002	0.201	0.498	-131.9	0.3	(0.823)
1969-1989	0.113	0.289	0.584	-112.0	0.8	(0.499)
1970-1990	0.159	0.347	0.645	-118.3	0.7	(0.549)
1971-1991	0.088	0.301	0.597	-124.9	0.5	(0.692)
1972-1992	0.079	0.294	0.592	-124.1	0.5	(0.708)
1973-1993	0.137	0.343	0.647	-118.7	0.6	(0.615)
1974-1994	0.242	0.468	0.778	-130.6	0.5	(0.701)
1975-1995	0.322	0.514	0.830	-113.7	0.1	(0.907)
1976-1996	0.325	0.505	0.827	-108.1	0.2	(0.836)
1977-1997	0.425	0.617	0.954	-111.2	0.2	(0.841)
1978-1998	0.428	0.637	0.960	-114.5	0.2	(0.859)
1979-1999	0.433	0.662	0.995	-122.6	0.4	(0.701)
1980-2000	0.412	0.650	0.983	-124.3	0.4	(0.705)
1981-2001	0.361	0.553	0.876	-97.2	1.0	(0.419)
1982-2002	0.431	0.550	0.873	-59.2	1.7	(0.157)
1983-2003	0.337	0.451	0.770	-56.8	1.8	(0.137)
1984-2004	0.457	0.541	0.872	-39.4	2.1	(0.084)
1985-2005	0.416	0.522	0.853	-48.4	1.8	(0.128)
1986-2006	0.352	0.472	0.798	-53.1	1.6	(0.157)
1987-2007	0.348	0.441	0.766	-41.7	1.9^{*}	(0.075)
1988-2008	0.356	0.469	0.782	-48.1	1.7	(0.108)
1989-2009	0.260	0.312	0.621	-21.6	2.4^{**}	(0.042)
1990-2010	0.329	0.369	0.682	-15.5	2.4^{**}	(0.030)
1991-2011	0.413	0.413	0.727	0.9	2.7^{**}	(0.014)
1992-2012	0.429	0.403	0.713	10.4	2.8^{**}	(0.011)
1993-2013	0.490	0.455	0.759	12.5	2.8^{***}	(0.009)
1994-2014	0.552	0.499	0.821	17.8	3.0^{***}	(0.007)
1995-2015	0.483	0.474	0.789	2.0	2.4^{**}	(0.023)

Table 4. Sub-period analysis for an *un-leveraged* investor with an investment horizon of 20 years, comparing the volatility scaling strategy with buy and hold using the realized Sharpe ratio. The 10 percentile level of the buy and hold Sharpe ratio obtained using boot strapping is included for comparison. The α value of a regression of the volatility scaled returns with the un-scaled returns is also included alongside the associated p value in brackets. Periods where the volatility-scaled Sharpe ratio is lower than the buy and hold Sharpe ratio are highlighted in gray and correspond to 58 out of the 70 sub-periods.

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3 Utility regime timing using realized volatility

In this section we use a custom regime switching framework to investigate whether market timing with realized volatility results in utility gains. We apply a back-testing framework to construct optimal portfolio decisions point-in-time based on recent realized volatility. We test whether switching between high and low volatility utility regime specific optimal portfolios leads to utility gains where both the optimal level of volatility to switch at and the constitution of the two regime specific portfolios are selected in an optimization over a backward looking window. Our rolling back-test window with out of sample evaluation of performance mimics the information set of a real-world investor point-in-time.

3.1 Methodology

The full data set runs from January 1927 to December 2016. The in-sample (henceforth IS) is initially defined as the period up to November 1969 (half of the sample), and is updated in a rolling window to create a time series of out-of-sample returns for each month from December 1969 to the end of the sample.

For each month in the out-of-sample period (henceforth OS), the IS period up to that month is used to select utility optimal portfolios and the volatility threshold used to classify bull and bear regimes:

I/ The range of the time series of IS monthly volatilities is split into 41 percentile values in the percentile range [50,90]. Each percentile value is treated as a candidate volatility threshold and the following steps are performed for each:

- Each month in the historical sample is classified as either a bull month ($volatility < threshold$) or a bear month ($volatility > threshold$).
- For the set of bull returns the mean variance efficient frontier is estimated using quadratic programming with full investment and no leverage constraints. The

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portfolio weights with the best sharpe ratio are saved as the bull portfolio weights.

- The same procedure is carried out for the set of bear months resulting in the sharpe-optimal bear portfolio weights.
- A complete set of IS returns is generated by switching between the bull and bear portfolios depending on the level of volatility relative to the volatility threshold.
- The Sharpe ratio for the regime switched returns is stored as the Sharpe ratio for that candidate threshold.

II/ The candidate volatility threshold with the best Sharpe ratio in sample is selected for use in the following out-of-sample (OS) month. Depending on the level of volatility on the re-balancing date either the corresponding bull or bear portfolio weights from in sample (and for that threshold) will be applied to earn an OS return for the strategy.

3.2 Results

Figure 8 illustrates the optimal volatility threshold over time for a mean variance investor switching between bull and bear portfolios dependent on the level of recent realized volatility. The threshold is picked in a backward-looking sample avoiding look-ahead bias. Figure 8 also illustrates bear regimes defined by the volatility threshold, it is clear from the plot that there are periods of positive return associated with high recent volatility levels such as the dot com bubble leading up to 2000. The volatility threshold is surprisingly low and the optimal bear portfolio weights selected are a 100% switch to treasuries when it is breached. The optimal Bull portfolios have an average weighting of 44 % in the index and 56 % in treasuries. There wasn't a large differential in return in the bull and bear utility regimes (11.7% Vs 10.9% per annum) suggesting that the leverage relationship between recent realized volatility and returns is not strong when considered over the whole sample.

Table 5 compares the performance in terms of Sharpe ratio of the regime switched strategy; buy & hold of the index; a standard mean variance optimisation updated in a rolling window and the Moreira and Muir (2017) strategy. Neither of the volatility managed port-

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Strategy	μ (Annualized %)	σ (Annualized %)	Sharpe (Annualized)
Buy & Hold	11.291	15.399	0.436
Mean Variance	9.168	9.823	0.46
Regime Timed	6.148	3.634	0.433
Moreira & Muir	7.339	9.984	0.277

Table 5. Out of sample results for the second half of the data sample (November 1969-December 2016).

folios out-perform the buy & hold strategy. The empirical evidence is showing that there are no utility gains to volatility management over the period.

4 Conclusion

We present a utility-optimal regime-switching framework for investment when an informative signal contains information to classify whether an upcoming period is a bull or bear investment regime from the perspective of an investor with a defined utility function. The classification threshold in the information signal and the corresponding bull and bear portfolio constitution are simultaneously optimized to select the in-sample settings with maximum utility gains for investors of a specified utility function. We apply the methodology in a rolling out-of-sample window to test whether information about the level of recent realized volatility leads to utility gains when market timing.

Our results are not promising for volatility management of the market factor, both our regime switching strategy and the strategy of Moreira and Muir (2017) under-perform the buy & hold strategy over our out-of-sample period (1969-2016). Furthermore, when we replicate Moreira and Muir (2017) over the full sample, we find that the utility gains for the market are not statistically significant and that they are highly sensitive on the rebalancing timing of the portfolio and on the availability of leverage. Our findings suggest that practitioners looking to adopt volatility management should be aware that past results may have been dependent on the fortuitous relative timing of key market events that may

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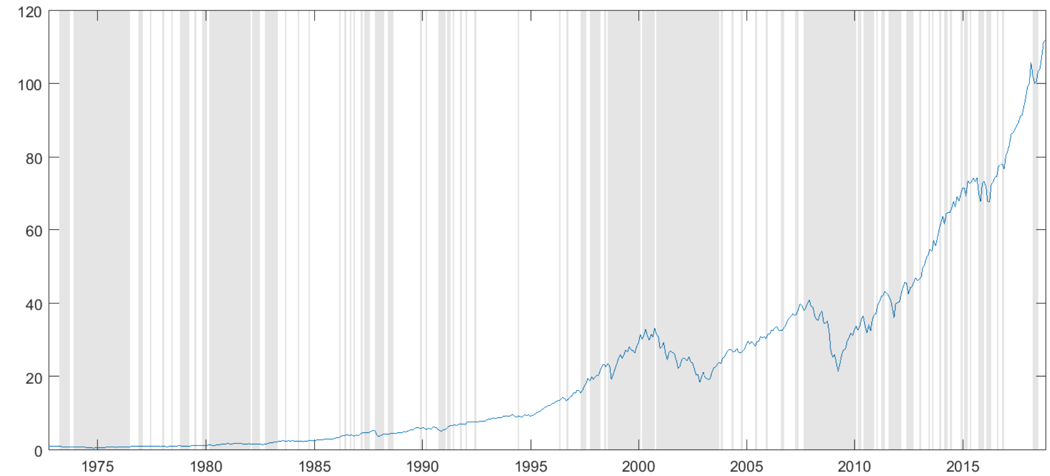

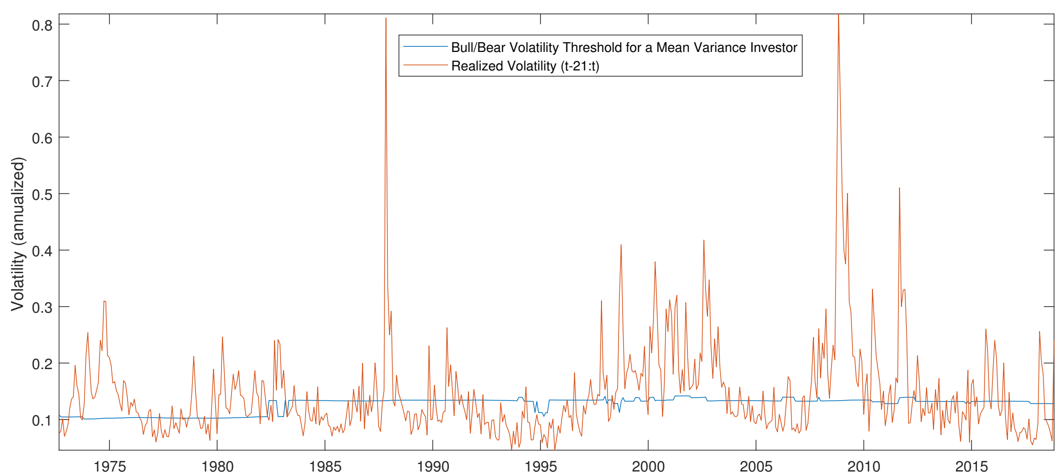

Figure 8.

The first plot illustrates the market index with the periods marked as utility bear regimes shaded. the second plot illustrates the corresponding historical volatility level used to classify the regime and the optimal volatility threshold obtained for a mean variance investor switching between a bull and a bear portfolio dependent ont he level of recent realized volatility.

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not be replicated in the future.

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